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The Modigliani-Miller Theorem for Equity Participation

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ABSTRACT

The paper shows that the use of an equity participation loan has no effect on the value of the firm, and that taxation of the borrowing firm and lender reduces firm value. The paper includes the assumption that firms borrow at an interest rate that is greater than the rate at which they can lend, so the value of the firm declines with the amount borrowed. Also, it is assumed that the firm may go bankrupt, which introduces the need for financial intermediation, as discussed by McDonald [1]. A state-preference model is employed.

Keywords: Equity Participation; Valuation; Modigliani-Miller Theorem

1. Introduction

The purpose of this paper is to investigate the effect of equity participation and taxation on the value of firms (and other financial investments). The paper also includes the assumption that the rate of interest at which a firm can borrow exceeds the interest rate at which it can lend. According to the classic Modigliani-Miller theorem [2], borrowing by untaxed entities has no effect on market value. However, this conclusion depends upon the assumption that the firm's borrowing rate equals its lending rate. Alteration of this assumption produces a different conclusion; if the borrowing rate is greater than the lending rate, then the value of the firm will decline with the amount borrowed. However, the value of the untaxed firm is unchanged by equity participation even if the borrowing rate and lending rate are unequal. Conversion of some debt to equity increases the value of the firm, and the value of the firm is reduced by taxation. A state-preference model is employed to establish these propositions.

It is now widely recognized that the supply of financial intermediation services is an important element to include in financial models. Joseph Stiglitz [3] reexamined the Modigliani-Miller propositions, and found that two assumptions of their model are important for their proof; individuals and firms borrow at the same interest rate, and there is no bankruptcy. He states [3, p. 784] that, "It should be clear that these assumptions are not independent." However, he did not pursue the possibility that bankruptcy introduces the need for financial intermediaries to provide the service of qualifying and monitoring borrowers and their investments. A model that reviews literature and incorporates these elements was provided by McDonald [1].

The topic of equity participation does not have an extensive literature. Theoretical studies consist of Schnabel [4], Ebrahim [5], Alvayay, Harter, and Smith [6], and Ebrahim, Shackketon, and Wojakowski [7]. Schnabel [4] used an agency-theoretic model to demonstrate that an equity-participation feature can increase the value of a firm and control the under-investment problem. Ebrahim [5] showed that equity participation can improve social welfare through the sharing of ownership and risk.

Alvayay, Harter, and Smith [6] used option-pricing theory to develop a model of the extent of lender equity participation. Their model of lender behavior is based on the assumption that a portion of standard debt is converted to equity for the lender, and that the value of lender's asset remains constant. This procedure combines two factors—the decision to undertake equity participation and the decision to reduce the amount of standard debt. These actions are treated separately in this paper. The Alvayay-Harter-Smith model is not used to explore the effects of equity participation on the value of the borrower's asset and its expected rate of return. Nor is the model used to investigate the effect of equity participation on the reservation value of the property involved. Ebrahim, Shackleton, and Wojakowski [7] also employ option-pricing theory to model the value of participation mortgage loans. Their model of lender behavior also is based on the assumptions that equity participation is combined with a reduction in the size of the standard loan and that the lender's asset has a constant value with or without equity participation. They find that a higher level of participation is associated with a greater reduction in the interest rate on the loan, but that the reduction in the interest rate is moderated by a higher loan-to-value ratio. A major focus of their study is the effect of time to maturity on the level of participation; the effect is strongly negative. The question of the effect of time to maturity is not addressed in this paper.

2. State-Preference Presentation of the Base Case

A demonstration of the basic model can be formulated using the state-preference approach as adopted by Stiglitz [3], Sargent [8], and McDonald [1] to illustrate the Modigliani-Miller Proposition I. This section is a more detailed discussion of a model presented briefly in McDonald [1]. Assume that there is only one date in the "future", and that there are N possible future states of the world. The index of future states of the world is $\theta = 1, \ 2, \cdots N$. An individual has a concave utility function U, and utility depends upon the future state of the world and the amount of money M in his/her possession at that time:

$$U = U\left[M\left(\theta\right)\right] \quad (1)$$

The individual has a set of subjective probabilities over the states of the world $\pi(1)$, $i(2)$, $\cdots$,$\pi(n)$ that sum to 1.0. Individual are assumed to maximize expected utility V:

$$V = \sum\nolimits_{\theta} \pi\left(\theta\right) U\left[M\left(\theta\right)\right] \quad (2)$$

The individual is assumed to have an endowment of M_0 at the present that is invested to provide for future consumption.

Consider a competitive economy in which there are n markets for contingent (Arrow-Debreu) securities, where each one promises to pay one dollar if the corresponding state of the world θ occurs. The price of a security, $p\left(\theta\right)$, is the price of the claim on one dollar should state θ occur. The units of $p\left(\theta\right)$ are dollars in the current period per dollar in state θ in the future. The price of a certain dollar in the future is $\sum\nolimits_{\theta} p\left(\theta\right)$, which is the reciprocal of one plus the risk-free interest rate. Perfect markets for contingent securities in all states of the world mean that it is possible to insure against any risk.

The individual faces the Arrow-Debreu budget constraint that states:

$$M_0 = \sum\nolimits_{\theta} p\left(\theta\right) M\left(\theta\right) \quad (3)$$

Since it is assumed that a complete set of markets for contingent securities exists, and if there is general agreement about the probabilities for the future states of the world, the prices for the contingent securities are actuarially fair; *i.e.*,

$$p\left(\theta_i\right) / p\left(\theta_j\right) = \pi\left(\theta_i\right) / \pi\left(\theta_j\right) \quad (4)$$

where $p\left(\theta_i\right)$ and $p\left(\theta_j\right)$ are the prices of the contingent securities for states of the world i and j, and $\pi\left(\theta_i\right)$ and $\pi\left(\theta_j\right)$ are the probabilities of states of the world i and j. Maximization of utility subject to the budget constraint produces the condition for the marginal rate of substitution between money in any two states of the world is

$$\begin{aligned} MRS &= \frac{\left[\dfrac{\partial V}{\partial M\left(\theta_i\right)}\right]}{\left[\dfrac{\partial V}{\partial M\left(\theta_j\right)}\right]} \\ &= \pi\left(\theta_i\right) U'\left(M\left(\theta_i\right)\right) / \pi\left(\theta_j\right) U'\left(M\left(\theta_j\right)\right) \\ &= p\left(\theta_i\right) / p\left(\theta_j\right) \end{aligned} \quad (5)$$

Since the market for contingent securities is actuarially fair, this first-order condition reduces to

$$U'\left(M\left(\theta_i\right)\right) = U'\left(M\left(\theta_j\right)\right) \quad (6)$$

Assuming that the utility function is the same regardless of the future state of the world, Equation (6) means that $M\left(\theta_i\right) = M\left(\theta_j\right)$. In short, the individual acts to insure that the level of money in the future is the same amount regardless of the future state of the world. The individual invests M_0 in Arrow-Debreu securities based on equity in firms and bonds to generate this amount of money in the future, and earns the risk-free rate of return. It is important for understanding the model to recall that individuals invest in a complete set of Arrow-Debreu securities based on both stocks and bonds. Firms and issuers of bonds are intermediaries that provide the investment vehicles upon which Arrow-Debreu securities are based.

Now consider firms that produce output that individuals purchase in the future. We assume an absence of taxes. A firm produces a return net of current labor and materials costs that depends upon the state of the world; $X\left(\theta\right)$. The firm issues bonds in the amount of B dollars, and promises now to pay $\mathrm{B}\left(1+r-c\right)$ to its bond holders (the individual investors) at the future date, provided that the firm is not bankrupt at that time; *i.e.*, $X\left(\theta\right) \geq B\left(1+r-c\right)$. The rate at which firms and individuals borrow is r, and the rate that firms pay its lenders is $r-c<r$. The firms goes bankrupt if

$$X\left(\theta\right) < B\left(1+r\right),$$

so the realized returns to bonds depend upon the state of the world as follows.

$$\begin{aligned} 1+r\left(\theta\right)-c &= 1+r-c \ \text{ if } X\left(\theta\right) \geq B\left(1+r\right) \ \text{ or} \\ &= \left[X\left(\theta\right)/B\right]-c \ \textit{if} \ X\left(\theta\right) < B\left(1+r\right) \end{aligned} \quad (7)$$

The model includes possible bankruptcy so that there is a need for financial intermediation. The amount cB is the cost of providing the financial intermediation services in which it was determined that the firm was in fact eligible to borrow amount B. It is assumed that this cost must be paid in full unless $X(\theta) < B(1+c)$; in this case the lender suffers a loss. The case in which the future value of the firm is less than the outstanding balance of the loan occurs if $X(\theta) < B$ (in real estate known as being under water).

The value of the firm's bonds is equal to the sum of the values of the contingent securities on which the bond consists implicitly. States of the world in which the firm does not go bankrupt are indexed as $\theta(a)$, and states of the world in which the firm goes bankrupt are indexed as $\theta(b)$. The value of the firm's bonds to the lenders is:

$$B_L = [1+r-c]B\sum_{\theta(a)} p(\theta) + B\left\{\sum_{\theta(b)}\left[\frac{X(\theta)}{B}\right]-c\right\}p(\theta). \tag{8}$$

The price vector $p(\theta)$ is set by the market so that the lender earns the risk-free rate of return. The value of the firm's equity is:

$$E = \sum_{\theta(a)}\left[X(\theta)-(1+r)B\right]p(\theta). \tag{9}$$

Therefore, the value of the firm V is:

$$V = E + B_L = \sum_{\theta}\left[X(\theta)-cB\right]p(\theta) \text{ so } \partial V/\partial B < 0 \text{ and } \partial V/\partial c < 0. \tag{10}$$

The value of the firm decreases with both the amount borrowed and the cost of financial intermediation. If the borrowing and lending rates are equal, then $c = 0$ and the value of the firm does not depend upon borrowing. This is, of course, Modigliani-Miller Proposition I.

3. Equity Participation

Equity participation involves the lender accepting a reduction in the interest rate on the loan for a share of the return to equity. Assume that the share of the return to equity for the lender is s and the new (lower) interest rate on the loan is r^*. The value of that equity share E_L is

$$E_L = s\sum_{\theta(a)} X(\theta)p(\theta), \tag{11}$$

and the value of the firm's equity is now

$$E^* = \sum_{\theta(a)}\left[(1-s)X(\theta)-(1+r^*)B\right]p(\theta). \tag{12}$$

The value of the firm's bond to the lender is

$$B_L^* = \left[1+r^*-c\right]B\sum_{\theta(a)} p(\theta) + B\left\{\sum_{\theta(b)}\left[X(\theta)/B\right]-c\right\}p(\theta). \tag{13}$$

Therefore the value of the firm is now

$$V = E^* + E_L + B_L^* = \sum_{\theta}\left[X(\theta)-cB\right]p(\theta). \tag{14}$$

The value of the firm is unchanged, and the terms of the equity participation do not matter (*i.e.*, the choices of s and r^*). Indeed, the interest rate on the loan r^* could be greater than r and still the value of the firm is unchanged.

The conversion of a portion of debt to equity is a transaction with two parts. As shown in Equation (10) above, a reduction in debt increases the value of the firm if the cost of financial intermediation is a function of the amount of the debt. The equity participation portion of the transaction has no effect on firm value. So conversion of debt to equity increases firm value in this model.

However, the lender will place a minimum condition on the terms of equity participation such that the same level of money is provided as with no equity participation. That condition is:

$$B_L = E_L + B_L^* \tag{15}$$

From Equations (8), (11), and (13), the lender's condition reduces to

$$s\sum_{\theta(a)} X(\theta)p(\theta) = (r-r^*)B\sum_{\theta(a)} p(\theta) > 0 \tag{16}$$

In short, the value of the lender's equity must equal the reduction in the change in the value of the bond arising from the change in the interest rate. This demonstrates that the lender will charge a lower rate of interest in exchange for equity participation. Equation (16) implies an explicit trade-off between and lender's share of equity s and the reduction in the interest rate ($r - r^*$).

4. Taxation

This section adds the corporate income tax to the basic model in Section 2. Both the lender and the borrowing firm are subject to tax rate t. The value of the firm's equity is:

$$E = \sum_{\theta(a)} p(\theta) \times\left\{X(\theta)(1-t)+tE_0-\left[1+r(1-t)\right]B\right\}. \tag{17}$$

Here E_0 is the original equity investment. Equation (17) states that the income and any capital gain for equity are taxed at rate t, and that interest on the loan is a deductible expense. The value of the bond for the lender is:

$$B_L = \left[1+(r-c)(1-t)\right]B\sum_{\theta(a)} p(\theta) + \left\{\sum_{\theta(b)}\left[X(\theta)\right]-cB-t\left[X(\theta)-B(1+c)\right]\right\}p(\theta) \tag{18}$$

Equation (18) shows that the lender pays taxes on interest earned after expenses if the firm is not bankrupt, pays taxes if $X(\theta) > B(1+c)$, and receives a tax deduction on the loss if $X(\theta) < B(1+c)$.

The value of the firm is $V = E + B_L$, or:

$$V = \sum_{\theta(a)} p(\theta)\left[X(\theta)(1-t) + tE_0 - c(1-t)B\right] + \sum_{\theta(b)} \left\{\left[X(\theta) - cB\right](1-t) + tB\right\} p(\theta) \quad (19)$$

$$= \sum_{\theta} \left\{\left[X(\theta) - cB\right](1-t)\right\} p(\theta) + \sum_{\theta(a)} tE_0 p(\theta) + \sum_{\theta(b)} tBp(\theta) \quad (20)$$

Compare Equation (20) to Equation (10), the value of the firm in the absence of taxation. Taxation reduces the value of the firm. If the firm is not bankrupt $X(\theta) < B(1+r)$, so the return to the firm after expenses exceeds the equity investment E_0 and $t\left[X(\theta) - cB\right]$ exceeds tE_0. If the firm is bankrupt $X(\theta) < B(1+r)$ and it exists no longer. The value of a loan made to a bankrupt firm is a more complex question. The question in this case is whether $tB - t\left[X(\theta) - cB\right]$ is less than or greater than zero; if this expression is negative the value of the firm clearly is smaller with taxation. However, if $tB - t\left[X(\theta) - cB\right]$ is positive it is possible that the value of the firm is greater with taxation than in the absence of taxation. This condition can be rewritten as

$$B(1+c) - X(\theta) > 0 \text{ or } < 0. \quad (21)$$

From Equation (7) there are four cases:

$$\begin{aligned} &B(1+r) > X(\theta) > B(1+c), \text{ so } B(1+c) < X(\theta), \\ &B(1+c) > X(\theta) > B, \text{ so } B(1+c) > X(\theta), \\ &B > X(\theta) > cB, \text{ so } B(1+c) > X(\theta), \text{ and} \\ &cB > X(\theta), \text{ so } B(1+c) > X(\theta). \end{aligned} \quad (22)$$

In the first of these cases taxation reduces the value of the firm, but in the last three cases taxation might actually increase the value of the firm. It seems unlikely that these cases can outweigh the other cases so that taxation produces an increase in the value of the firm. However, if there is a high probability that the borrowing firm will go bankrupt (so the equity has no value) and also be unable to pay back the loan plus the cost of financial intermediation $\left[\mathrm{B}(1+c) > X(\theta)\right]$, then the imposition of taxation with deductions for losses will increase the value of the loan to the lender. Do lenders that are subject to taxation grant more "bad" loans than lenders that are not subject to taxation? Empirical studies are needed. Equation (20) reduces to Modigliani-Miller Proposition I if both c and t are zero.

5. Conclusion

Equity participation is an arrangement in which a lender agrees to reduce the interest rate on a loan to a borrowing firm in exchange for a share of the return to equity. The primary new result in the paper is that use of an equity participation loan, instead of a conventional loan, has no effect on the value of the firm. The paper derives the condition that a lender would impose on an equity participation deal; the value of the equity participation must offset the decline in the interest on the loan. The paper also shows that the value of the firm is reduced by taxation at the entity level, although the possibility of bankruptcy makes the analysis somewhat complex. The paper has included the result from McDonald [1] that, if the borrowing rate exceeds the lending rate (as in the case of financial intermediation services), then the value of a firm declines with financial leverage. The value of the firm is reduced by the cost of the financial intermediation services. If the borrowing rate and the lending rates are equal, then the value of the firm is independent of financial leverage, as in Modigliani-Miller Proposition I. This proposition holds in the presence of the possibility of firm bankruptcy.

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