Theoretical Economics Letters, 2011, 1, 111-113
doi:10.4236/tel.2011.13023 Published Online November 2011 (http://www.SciRP.org/journal/tel)

Are Sunspots Stabilizing?

Paul Shea
University of Kentucky, Lexington, USA
E-mail: *paul.shea@uky.edu*
Received August 18, 2011; *revised October* 4, 2011; *accepted October* 12, 2011

Abstract

The reduced form solutions of indeterminate rational expectations models often include extraneous expectational errors or "sunspots". Sunspots are usually modeled as independent of the model's fundamentals, and are often presumed to result in excess volatility. An alternate approach, however, is to assume that sunspots include both an overreaction or underreaction to fundamentals, as well as genuine extraneous noise. This paper uses a simple linear model to formally show how the relationship between sunspots and fundamentals affects aggregate volatility. Sunspots reduce volatility if 1) they include an undereaction to fundamentals, 2) the variance of genuine extraneous noise is sufficiently small, and 3) the root that causes indeterminacy is sufficiently far from one.

Keywords: Sunspots, Indeterminacy, Volatility,

1. Introduction

It is well known that linear rational expectations models may be indeterminate, implying that a continuum of equilibria paths exist. Often these equilibria depend on extraneous expectational errors, known as sunspots. In their seminal paper examining the general equilibrium effects of sunspots, Cass and Shell (1983) [1] define a sunspot as "represent[ing] extrinsic uncertainty, that is, random phenomena that do not affect tastes, endowments, or production possibilities." Although it is debatable whether an overreaction or underreaction to fundamentals fits with this definition, it is clear that a sunspot that includes such a reaction is consistent with a rational expectations equilibrium under indeterminacy.[1] The most common approach for modeling sunspots is to assume that they are independent of fundamentals, and to treat the variance of sunspots as a parameter to be calibrated.[2] This is equivalent to assuming that sunspots include an underreaction to fundamentals.

Benhabib and Farmer (1999) [4] write that sunspots are of interest because they both add an additional source of volatility, and because they allow for richer propagation dynamics. It follows from the former property that sunspots are typically destabilizing. In the Real Business Cycle (RBC) literature, sunspots are therefore often viewed as a mechanism for reducing the volatility of productivity shocks needed to match the volatility of output, and thus reducing the probability of productivity regressing. Most notably, Farmer and Guo (1994) [5] show that a RBC model that includes only sunspot shocks does arguably as well at matching the data as a standard RBC which includes only fundamentals.[3] Likewise, a major literature in monetary economics views ensuring a unique equilibrium, and thus avoiding excess volatility, as a primary goal of monetary policy.[4]

This paper uses a simple linear model to show that sunspots may either reduce or increase volatility relative to the model's minimum state variable (MSV) solution. I decompose sunspots into a linear combination that includes an overreaction or underreaction to fundamentals, as well as genuinely extraneous noise. I prove that sunspots reduce volatility if: 1) they include a small enough underreaction to fundamentals, 2) the ratio of the variance of genuine noise to the variance of fundamental shocks is sufficiently small, and 3) the root responsible for indeterminacy is sufficiently far from one.

[1]Throughout the paper, overreactions and underreactions are defined as relative to the response to fundamentals that occurs in the model's minimum state variable (MSV) solution. McCallum (1983) [2] proposes the minimum state variable solution as a selection criteria for models with multiple equilibria. The MSV solution, by definition the most parsimonious, does not depend on extraneous noise.
[2]See Farmer (1999) [3] for a textbook treatment of indeterminacy.
[3]Other RBC models that include sunspots in equilibrium include Benhabib and Farmer (1996) [6], Schmitt-Grohe (1997) [7], Schmitt-Grohe and Uribe (2000) [8], and Shea (2011) [9].
[4]See Woodford (2003) [10] for an overview of this literature.

2. Model

The precise conditions for sunspots to be stabilizing are a function of the specific model being analyzed. I thus focus on a simple, univariate linear model to make the paper's main point. The main result, however, easily extends to more complex linear models.

A variable, y_t, depends on its one-period ahead expectation as well as a fundamental shock, e_t :

$$y_t = \beta E_t\left[y_t + 1\right] + \mathrm{e}_t \tag{1}$$

$$j \xrightarrow{\lim} \infty E_t\left[y_t + j\right] = 0 \tag{2}$$

where e_t is *iid*, and mean-zero. If $\left|\beta\right| < 1$, then equilibrium is unique and follows:

$$y_t = \mathrm{e}_t \tag{3}$$

If $\left|\beta\right| < 1$, however, then equilibrium is indeterminate and may be represented as:

$$y_t = \beta^{-1}\left(y_{t-1} - e_{t-1}\right) + \xi_t \tag{4}$$

where $\xi_t = y_t - E_{t-1}\left[y_t\right]$ is agents' expectational error. Equations (1) and (2) hold as long as ξ_t is an *iid* mean-zero process. The literature defines ξ_t as a sunspot. The sunspot may be decomposed into two elements: $\xi_t = \mathcal{T}e_t + v_t$. $\mathcal{T}e_t$ is an overreaction or underreaction to fundamentals and v_t is the part of the sunspot that depends on truly extraneous noise. Iterating Equation (4) backwards yields the following representation of the solution:

$$y_t = \mathcal{T}e_t + \sum_{\mathrm{i}=1}^{\infty} \beta^{-\mathrm{i}}\left(\mathcal{T} - 1\right)e_{t-\mathrm{i}} + \sum_{\mathrm{i}=0}^{\infty} \beta^{-\mathrm{i}} v_{t-\mathrm{i}} \tag{5}$$

Two special cases merit discussion. First, if $\mathcal{T} = 0$, then sunspots are entirely genuine extraneous noise that is unrelated to fundamentals. This is the most common approach to modeling sunspots.[5] Notably, setting $\sigma_v^2 = 0$ does not eliminate the effects of indeterminacy under this approach. Instead, this parameterization results in self-fulfilling perfect foresight where contemporaneous stochastic shocks have no effect in equilibrium. Second, $\mathcal{T} = 1$ and $\sigma_v^2 = 0$ results in the MSV solution, identical to the unique equilibrium under determinacy. For the remainder of the paper, I define $\mathcal{T} > 1$ as an overreaction (relative to the MSV solution) to fundamentals, and $\mathcal{T} < 1$ as an underreaction.

Setting $\mathcal{T} = 0$ has a simple appeal. Under indeterminacy, any martingale difference sequence may represent agents' self-fulfilling beliefs and is thus consistent with rational expectations. The fundamental shock is thus just one of an infinite number of candidates and is accorded no special consideration under this approach. σ_v^2 is treated as a parameter to be calibrated, possibly by choosing the value that best fits the data as in Farmer and Guo (1994) [5].

There are, however, two compelling justifications for assuming that $\mathcal{T} \neq 0$. First, in modern macroeconomics, forward-looking equations such as Equation (1) usually result from agents solving an optimization problem. Agents are typically presumed to understand that, for any $E_t\left[y_{t+1}\right]$, they must respond to a one unit increase in e_t by also increasing y_t by one unit in order to be optimizing. The fundamental shock is the unique stochastic process that appears in the structural model and it is therefore unlikely that its contemporaneous value does not appear in the solution. For it not to appear, it must be the case that the direct effect of e_t in Equation (4) is perfectly offset by its effect on agents' self-fulfilling beliefs. There typically is no apparent reason for this knife edge case to occur.

To demonstrate the second justification for $\mathcal{T} \neq 0$, consider the following example. y_t is output in a neoclassical model where money does not ordinarily matter. e_t is a fundamental shock to productivity. Although, in principle, ξ_t may be either a popular macroeconomic variable such as a monetary aggregate, or the population of penguins in Antarctica, it is far more likely that agents will coordinate their beliefs on the former than the latter. Suppose that agents, persuaded by decades of monetary economics, set ξ_t as a monetary aggregate. The monetary aggregate may itself be a function of productivity. $\mathcal{T}$ thus represents the response of the monetary authority to productivity while v_t is the part of the monetary aggregate that is independent of the fundamental.

Lubik and Schorfheide (2004) [11] provide an example of setting $\mathcal{T} \neq 0$. In that paper, the authors endogenize $\mathcal{T}$ and σ_v^2 so that the solution under indeterminacy and the MSV solution are identical at the boundary between the determinate and indeterminate regions. Away from the boundary, however, $\mathcal{T}$ and σ_v^2 are treated as parameters to estimate. This ensures that small changes to the model's parameters which switch the solution from indeterminate to determinate, or vice-versa, do not have large effects on the model's equilibrium behavior. It also aids the authors with their main purpose of empirically estimating a New Keynesian model where part of the parameter space yields indeterminacy.

3. Results and Analysis

It is straightforward to calculate the volatility of y_t under determinacy and indeterminacy:

$$Var\left(y_t\right)^{\det} = \sigma_v^2 \tag{6}$$

[5] This is also the approach taken by Farmer's (1999) [3] textbook on indeterminacy.

$$Var\left(y_t\right)^{in\det} = \left[\mathcal{T}^2 + \frac{\left(\mathcal{T}-1\right)^2}{1-\beta^{-2}}\right]\sigma_e^2 + \frac{1}{1-\beta^{-2}}\sigma_v^2 \quad (7)$$

Comparing Equations (6) and (7) shows that indeterminacy may stabilize y_t. Result 1 provides necessary and sufficient conditions for indeterminacy to be stabilizing:

Result 1: Indeterminacy will reduce the variance of y_t, relative to the MSV solution, if:

$$\mathcal{T} \in \left(\begin{array}{l} \beta^{-2} - \sqrt{\beta^{-4} - \beta^{-2} + 1 - \left(\frac{\sigma_v}{\sigma_v}\right)^2}, \\ \beta^{-2} + \sqrt{\beta^{-4} - \beta^{-2} + 1 - \left(\frac{\sigma_v}{\sigma_v}\right)^2} \end{array}\right)$$

Result 1 shows that three conditions are necessary for indeterminacy to reduce volatility. First, the sunspot must include an underreaction to fundamentals. The underreaction, however, must not be too strong. $\left|\mathcal{T}\right| < 1$ is thus a necessary condition for stabilizing sunspots. Second, the relative variance of genuine noise $\left(\frac{\sigma_v}{\sigma_v}\right)^2$ must be sufficiently small. Finally, the root under indeterminacy, β^{-1} from Equation (4), must be sufficiently far from one.

Three conditions are each sufficient to ensure that sunspots are destabilizing. If the response to fundamentals is too strong ($\left|\mathcal{T}\right| > 1$), if Equation (4) is too close to a random walk ($\left|\beta\right| \approx 1$), or if the variance of genuine noise (σ_v^2) is too high, then y_t will be more volatile than under the MSV solution.

Conventional wisdom suggests that sunspots add volatility. Result 1 shows, however, that under the common approach to modeling sunspots ($\mathcal{T} = 0$), this may or may not be the case. Another special case is to assume that sunspots include the same response to fundamentals as the MSV solution ($\mathcal{T} = 1$). In this case, for any $\sigma_v^2 > 0$, sunspots only add noise to the system and are necessarily destabilizing. **Figure 1** illustrates the region where sunspots stabilize y_t for three values of β.

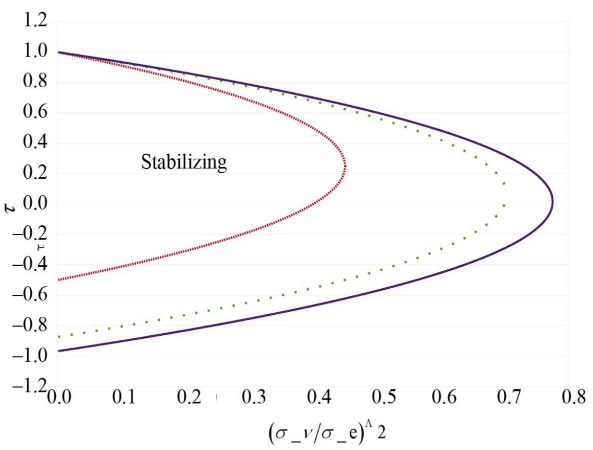

Figure 1. Region Where Indeterminacy Stabilizes y_t. (Red (Dashed) is β=2, Green (Dots) is β=4, and Blue (Solid) is β=8.)

4. Conclusions

The model of this paper is very simple. The result, however, is straightforward and easily extends to more complex linear models. If sunspots weaken the response to fundamentals, if indeterminacy does not result in near random walk behavior, and if sunspots do not add too much genuine noise into the model, then sunspots reduce rather than augment volatility.

5. References

[1] D. Cass and K. Shell, "Do Sunspots Matter?" *Journal of Political Economy*, Vol. 29, 1983, pp. 209-226.

[2] B. McCallum, "On Non-Uniqueness of Rational Expectations Models: An Attempt at Perspective," *Journal of Monetary Economics*, Vol. 11, No. 2, 1983, pp. 139-168. doi:10.1016/0304-3932(83)90028-4

[3] R. Farmer. "The Macroeconomics of Self-Fulfilling Prophecies," MIT Press, Cambridge, 1999.

[4] J. Benhabib and R. Farmer, "Indeterminacy and Sunspots in Macroeconomics," In: J. Taylor and M. Woodford, Eds., *The Handbook of Macroeconomics*, North Holland, Amsterdam, 1999.

[5] R. Farmer and J. Guo, "Real Business Cycles and the Animal Spirits Hypothesis," *Journal of Economic Theory*, Vol. 63, No. 1, 1994, pp. 42-72. doi:10.1006/jeth.1994.1032

[6] J. Benhabib and R. Farmer, "Indeterminacy and Sector-Specific Externalities," *Journal of Monetary Economics*, Vol. 37, No. 3, 1996, pp. 421-433. doi:10.1016/0304-3932(96)01257-3

[7] S. Schmitt-Grohe, "Endogenous Business Cycles and the Dynamics of Output, Hours, and Consumption," A*merican Economic Review*, Vol. 90, No. 5, 2000, pp. 1136-1159. doi:10.1257/aer.90.5.1136

[8] S. Schmitt-Grohe and M. Uribe, "Balanced Budget Rules, Distortionary Taxes, and Aggregate Instability," *Journal of Political Economy*, Vol. 105, No. 5, 1997, pp. 976-1000. doi:10.1086/262101

[9] P. Shea, "Short-Sighted Managers and Aggregate Volatility," *Mimeo*, University of Kentucky, Lexington, 2011.

[10] M. Woodford, "Interest and Prices," Princeton University Press, Princeton, 2003.

[11] T. Lubik and F. Schorfheide, "Testing for Indeterminacy: An Application to U.S. Monetary Policy," *American Economic Review*, Vol. 94, No. 1, 2004, pp. 190-217. doi:10.1257/000282804322970760

