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Social Balance in the Triangle-Growing Networks

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ABSTRACT

In this article, we construct a triangle-growing network with tunable clusters and study the social balance dynamics in this network. The built network, which could reflect more features of real communities, has more triangle relations than the ordinary random-growing network. Then we apply the local triad social dynamics to the built network. The effects of the different cluster coefficients and the initial states to the final stationary states are discussed. Some new features of the sparse networks are found as well.

Keywords: Networks; Social Balance; Dynamics

1. Introduction

Since balance theory was first proposed by Heider [1], lots of researchers have been interested in this field and many important contributions were made [2-5]. Antal *et al.* [6] studied the balance dynamics on the social networks based on the notion of the balance theory. In their network, each node is connected to all the others, representing each person knows all the others in the society. Each edge in the network has two values, +1 and −1. If the edge s_{ij} is +1, it means the persons *i* and *j* are friendly with each other. If the edge s_{ij} is −1, it means the two persons are hostile towards each other. At every step, they choose a triangle randomly from the network. If the product of the three edges is +1, the triangle is stable. Otherwise, if the product is −1, the triangle is unstable. For the stable triangle, it satisfies 1) the friend of my friend being my friend; 2) the enemy of my friend being my enemy; 3) the friend of my enemy being my enemy; and 4) the enemy of my enemy being my friend. The unstable triangles always try to become stable, but the final state of the network depends on the edge flipping probability p, which is set manually. If $p > 1/2$, the network will reach the state of "paradise", with all relations being friendly. Several studies around the balance dynamics have been carried out, including the studies of the university class of triad dynamics [7] and the satisfiability problem of computer science [8], etc.

The researches on the dynamics of balance [6-8], which were based on the complete graph or the regular lattice, have revealed some interesting and important phenomena in the certain networks. Antal *et al.* [6] studied the local triad dynamics on the complete network and found the dynamical phase transition in this system. Apart from this dynamics, they also considered the constrained triad dynamics in which the number of imbalanced triads cannot increase in an update event, and the final state can either be balanced or jammed. They argued that the constrained triad dynamics can be applied to international relations. Radicchi *et al.* [7,8] generalized the topology from all-to-all to a regular one of a two-dimensional triangular lattice and generalized the triad dynamics to k-cycle dynamics for arbitrary integer k. From a finite-size scaling analysis they determined the critical exponents for triad dynamics. Meanwhile, they pointed out, the diluted k-cycle dynamics can be mapped on a certain satisfiability problem in computer science, the so-called k-XOR-SAT problem.

Although the significant contributions have been made in the work mentioned above, the social balance dynamics in some real communities has seldom been considered. Real communities have the features as follows, firstly, not all the people know each other, secondly, the person will know more and more people as he/she is growing, finally, most of the real networks are not completely connected. The aim of us is to study the social balance behavior closed to the reality. In article [9], we studied the dynamics and generalized it to ternary relationships on the small-world networks. We found that the system shows the behavior of the self-organized criticality. If there is a small disturbance, it will result in an avalanche. However, these studies were preparatory, real networks are more complicated. So the further researches are needed to capture the effect of the topology of the network to the social dynamics.

In this article, we construct a random network with tunable clusters to reflect the features of the real communities mentioned above. Then we study the social balance dynamics in the network. In the first part of the article, the statistical properties of the network are studied. We focus on the difference between the network and E-R random network. The degree distribution and cluster coefficient are discussed, and we find the network constructed in the paper is suitable for the issue of the dynamics of the social balance. Based on the network, the dynamics is studied, and some interesting results are found.

2. The Triangle-Growing Networks

An important aspect which is always present in social dynamics is the topology of the network. When applying social balance dynamics on specific topologies several nontrivial effects may arise [10]. In our work, we will discuss the effect of the topology to the dynamics. Up to now, lots of network models, such as small-world network, BA network, and some extension of them, were proposed to explain the features of the topologies [11], but most of them, which have small fraction triangles in the networks, can not reflect the growing features of the social networks. So we need to construct a network which can show the growing features of social communities.

Our inspiration of solving this problem comes from the random graph proposed by Eröds and Rényi [11] and the random growing network discussed by Barabási and Albert [12]. E-R random graph is defined as N labeled nodes connected by n edges which are chosen randomly from $N(N-1)/2$ possible edges. The clustering coefficient of a random graph is

$$C_{\text{rand}} = p = \frac{<k>}{N},$$

where p is connection probability and $<k>$ is the average degree. Bollobás [13] derived degree distribution of the random networks and get the conclusion that for large N, the degree distribution follows the Poisson distribution. Moreover, many authors have studied the diameter of a random graph [14]. For E-R network, when $<k>$ is larger, the cluster becomes larger. However, the number of adjacent triangle clusters is not so giants.

The random-growing network is the limiting case of the scale-free network [12]. This model keeps the growing character of a network without preferential attachment. A new node connects with equal probability to the nodes which are already present in the system. The degree distribution decays exponentially. This network also has small fraction triangles. Our aim is to construct a network that has more triangles than the models built before. We build the graph by growth like random growing network. But to get more triangles, some new rules are introduced. The rules of the construction are as follows:

1) To start with, the network consists of m_0 vertices which are linked completely (This rule represents that all people know each other in a small community or a company).

2) Every step, one vertex v with 2 edges is added. One of the edges is attached to an existing vertex w randomly, and the other edge is attached to a neighbor of the vertex w: After N-m_0 steps, there are N vertices in the network (This rule shows that a new individual will know a person randomly and know another person by the acquaintance of the random selected person).

3) Then we let the existing vertices grow two links with p_A, the links will find the targets. The finding rule is the same as step 2. By repeating this step, we may get a network which approximates to the completely graph with very large cluster coefficient (This rule means that the old members in the company will know others randomly by the acquaintance).

The 2nd step is the key of the rules. Every added point will engender more than one triangle in the graph. In sociology the clustering coefficient can be defined as the fraction of transitive triples [15]. According to this mechanism, the network will have a large number of triads and the large cluster coefficient. **Figure 1** gives the cluster coefficient. After N steps of the second rule, compared to the random-growing network, the triangle growing one has larger coefficient with the same average degree $<k> \approx 3.9$. The reason of this phenomenon is that

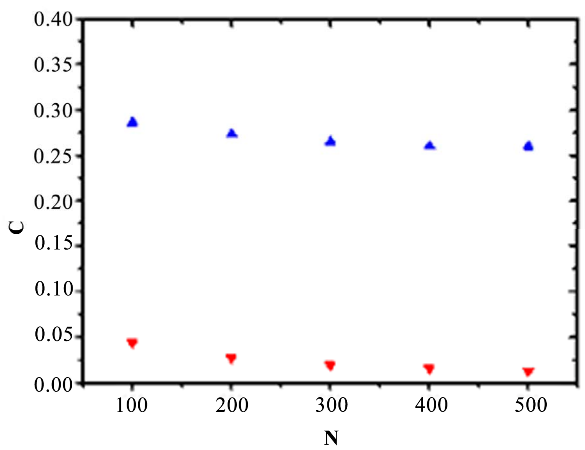

Figure 1. The cluster coefficient of random-growing graph and triangle-growing graph with different size of the network. The blue ▲ is triangle-growing case and the red ▼ is random-growing case.

the triangle-growing network is added triangles in every step, while random one is only added links in each step.

If the edges are added according to the step 3, the cluster coefficient and the triangles will vary with the increasing average degree. **Figure 2** gives the cluster coefficient of these two networks, with different average degree. Form the figure, we can get the conclusion that the triangle growing network's cluster coefficient is larger than the random growing one's, but with the degree's increase, the coefficient is closing to each other. For the triangle growing network, the edges are added according to the step 3, so the vertices are linked more and more, and the triangles increase. Meanwhile, for the random-growing network, the vertices are also linked more and more as the increasing degree. So the numbers of the triads of these two networks get closer, especially at large average degree. Because at this moment, these two networks both approximate to the completely connected graph.

Another important statistical feature of networks is degree distribution. The random-growing network and the triangle-growing one are very different. Especially, the triangle-growing network undergoes vary of the to pology. If there are only 2 edges of an added node, an arbitrary vertex w increases its degree with rate

$$\frac{\Delta k_w}{\Delta t} = \mathrm{II}_R\left(k_w\right) + \mathrm{II}_N\left(k_w\right) \quad (1)$$

where $\mathrm{II}_R(k_w)$ denotes the probability of the random link, and $\mathrm{II}_N(k_w)$ is proportional to the probability that a vertex in the neighborhood w is linked in the random link step before. So we can get

$$\mathrm{II}_R\left(k_w\right) = \frac{1}{m_0 + t - 1} \quad (2)$$

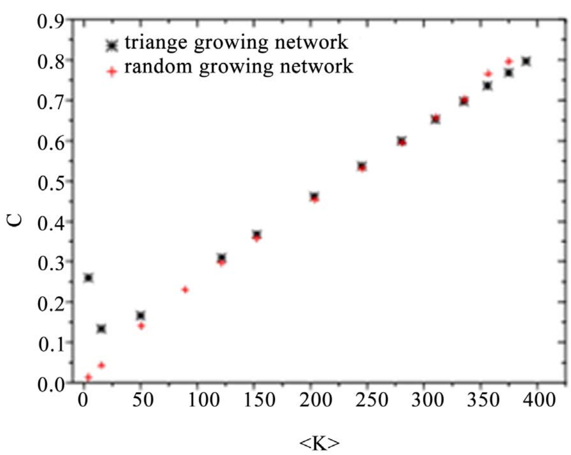

Figure 2. The cluster coefficient of random-growing graph and triangle-growing graph with different average degree. The black * is triangle growing case and the red + is random growing case.

$$\mathrm{II}_N\left(k_w\right) = \frac{\sum_{\mathrm{V}\in\Gamma_w} k_v\left(1/k_v\right)}{\sum k_v} \quad (3)$$

Where Γ_w is the neighborhood of w: We can get the conclusion that the degree distribution which is similar with the BA network. In **Figure 3** the degree distributions of triangle-growing and random-growing networks are displayed. From the figure, we can see that when $<k>$ is small, the triangle growing graph is heterogeneous. Moreover, if the edges are added following the step 3, the cluster coefficient and the fraction of the triangles get larger. And as the links are increasing, the degree distribution will change.

3. Social Balance in Triangle-Growing Networks

In this section, we will study the local triad dynamics (LTD), which has been introduced in article [6]. In each update step of LTD, we first choose a triad at random, if this triad is balanced ($s_{ij}s_{jk}s_{ki} > 0$), nothing happens. If the selected triad is imbalanced ($s_{ij}s_{jk}s_{ki} < 0$), one of the links of the triad will be changed form s to $-s$. The rules of the evolution are as follows:

1) If all the links are −1, we choose a link randomly and change it.

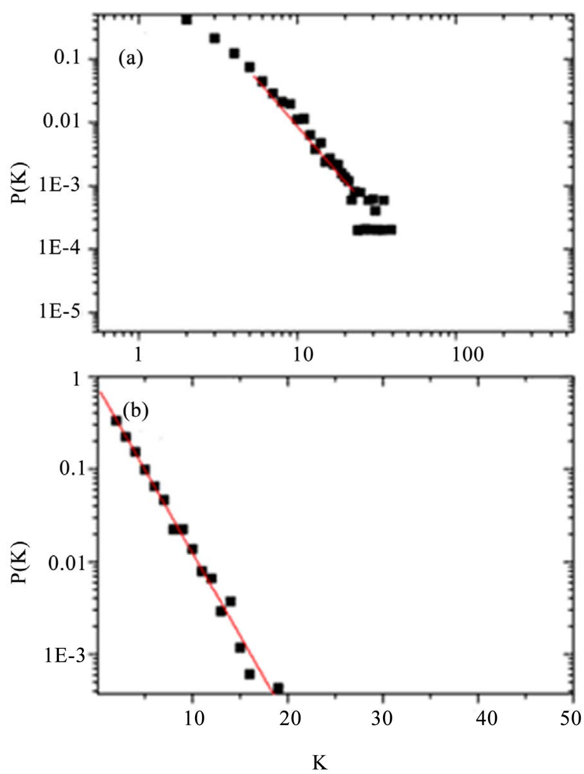

Figure 3. (a) Is the degree distribution of the triangle-growing networks and (b) is the distribution of the random-growing network.

2) If there is only one −1 link, we change the link of −1 to 1 with the probability p, or change one of the other two links to −1 with the probability $1 - p$.

Antal *et al.* [6] have studied the dynamics in fully connected graph, and get the conclusion that a finite network falls into a socially-balanced state. They also gave the results that the density of unfriendly links gradually decreases and the network undergoes a dynamical phase transition to an absorbing, paradise state for $p \geq 1/2$. However, many real social networks are not fully connected. So, we construct a triangle growing network which is not completely connected in the previous section. Our work in this section is to study the dynamics in the network.

The rules of the evolution are the same as in article [6]. The completely connected network will fall into the paradise state for $p \geq 1/2$ which has no enemy relations. But for the new topology, does the non-completely connected graph have the same feature? For answering this question, we calculate the density of friendly links ρ_+, which is defined as the ratio of friendly links to all the links. First, we let the network have the same quantity of the friendly and enemy links. That is the initial state ρ_{+0} = 0.5. Then we let the network evolve as the dynamics above. After a sufficiently long time, the system will get to the stationary state. Our interest is to study the influence of the topology to the evolution of the dynamics. Via increasing the adding probability p_A, the networks can be changed from sparse heterogeneous network with small degree to the homogeneous nearly fully connected ones.

Figure 4 shows the stationary state of the friendly re lations ρ_+ as a function of p. From the figure, we can see that the system never reach the Utopia state without enemy relations in any p when the adding probability p_A is small. However, with the increase of p_A, the terminal stationary state will have more and more friendly links. Especially, when p_A is greater than a certain value, the system shows the same feature that the network undergoes a dynamical phase transition to an absorbing, paradise state for $p \geq 1/2$. This value of p_A may be 0.5 from our calculations. The results indicate that for a certain scale network, the system is easier to get friendly state when the average degree is larger. This is because larger degree implies a link may belong to quite a number of triads. An enemy relation may not lead to imbalance in a triad, but other triads, which share this enemy relation, may be imbalanced because of the relation. So this results in a evolution in the whole network and the system is stable at the end. The best stationary state likes more friendly links.

Another feature we concern is whether the initial condition affects the result of the evolution. We simulate the dynamics in different networks with adding probability p_A=0.08 and p_A=0.5. The different initial conditions are considered and the results are drawn in **Figures 5** and **6**. It is found that when p_A is small, the final states de pend on the initial states. However, when p_A is large, the final states are the same in different initial states. For p_A=0.08, the relations between the individuals of the network are sparse. The network has lots of independent clusters.

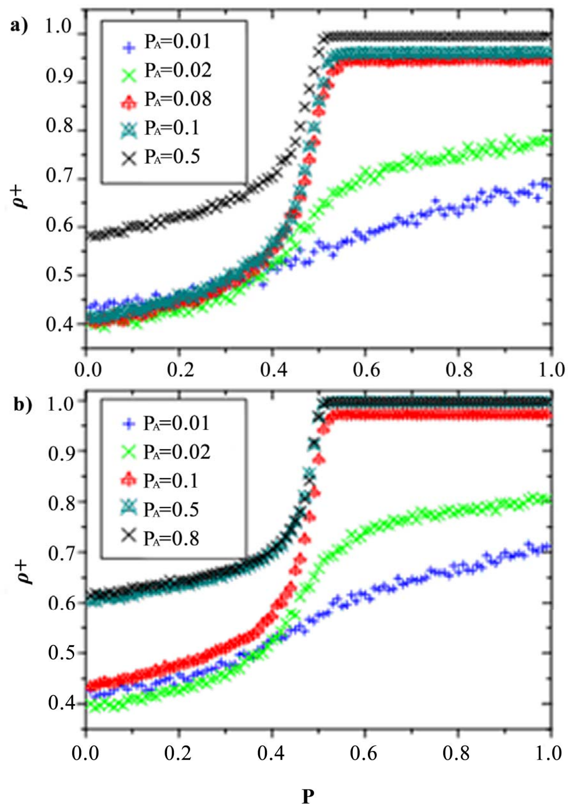

Figure 4. The stationary state of the friendly relations ρ+ as a function of p with 500 and 300 nodes.

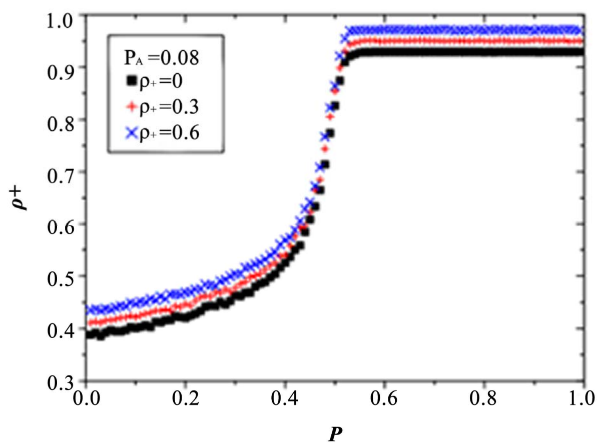

Figure 5. The stationary state of the friendly relations ρ_+ with different initial state ρ_{+0} in the sparse network.

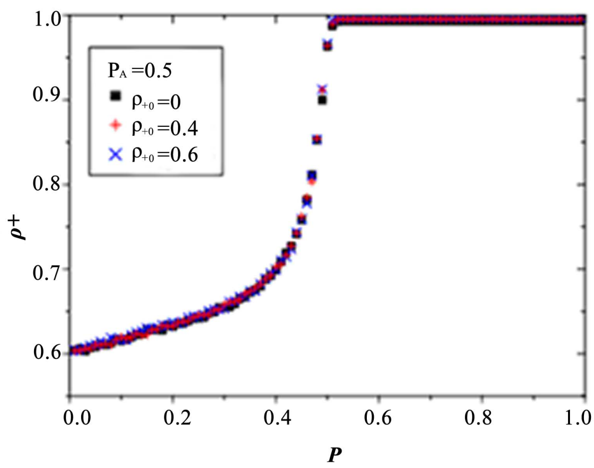

Figure 6. The stationary state of the friendly relations ρ_+ with different initial state ρ_{+0} in the dense network.

The evolution of a cluster affects the others difficultly. So the different initial conditions lead to the different final states. For $p_A = 0.5$, most of the clusters are adjacent to each other. The change of one link will affect lots of cluster of the network. So no matter the initial state is, the final state will be the same for the certain p_A.

4. Conclusion

The dynamics of the social balance is discussed in this article. The dynamics depends on the relations of three persons, so we construct a triangle-growing network, which has large cluster coefficient and lots of triangles. Another interesting feature of the network is the network has the power-law degree distribution although the growth is random. With the increase of the links, the network can get to the completely linked graph in which the dynamics has been researched in article [6]. We apply the social balance dynamics to the network constructed in our article. And some new phenomena are found. First of all, if the network is sparse, the Utopia states are never present in any evolution conditions. However, if we add more links to the sparse network, and make the network dense. The Utopia state appears when the cluster coefficient is large enough. This behavior is similar to the completely-linked graph, and the network also undergoes the dynamical phase transition. Another feature is that the final state of the sparse network depends on the initial state. This means that the balance of the network is not stable, and it is sensitive to the disturb form outside. So the group of the sparse relations will be easily affected. While the networks with condense relation are more stable. The disturbance from outside could affect the net limitedly.

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