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Mathematical Communication by 5th Grade Students' Gestures in Lesson Study and Open Approach Context

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The objective of this research was to explore the Mathematical Communication by 5th grade students' gestures in Lesson Study and Open Approach context. This study was conducted at Nongtoom-nong-ngu-lerm School, and Ban-Beung-neum-beung-krai-noon School, Muang District, Khon Kaen Province in Project for Professional development of Mathematics teachers through Lesson Study and Open Approach, using qualitative research: Ethnographic Study, in-depth interview, Video Analysis supported by protocol analysis, and Descriptive Analysis. The research findings found that there were 7 kinds of students' Mathematical Communication by students' Gestures including 1) rigorousness by students' beat gesture; 2) rigorousness by students' metaphoric gesture; 3) economy by students' deictic gesture; 4) economy by students' iconic gestures; 5) freedom by students' deictic gesture; 6) freedom by students' iconic gesture; and 7) freedom by students' deictic and iconic gestures in explaining students' Mathematical Ideas, and the most commonly used economically by deictic gestures, and students' self learning in Open Approach. Furthermore, the schools in Lesson Study and Open Approach context, the students had opportunity in learning based on their potentiality, being able to think, perform, and express. They preferred to express divergent think.

Keywords: Mathematical Communication; Gesture; Lesson Study; Open Approach

Introduction

The National Education Act in A.D. 1999, the Educational Reform Act for Further Development of the Thai People, had provided the national education guidelines to focus on. The Educational Management had to be focused on rationale that every student had learning ability, competency in self development, and the students were the most important person. As a result of the National Education Act, and the Basic Education Reform in A.D. 2001, determined the mathematical learning process as a part of curriculum needed to teach mathematics learning processes were problem solving, communication, representation, reasoning, and connection which were the things the teachers were not familiar as well as were very confused. In addition, the teacher shad to prepare for lesson in most of mathematics teachers in Thailand, who worked alone, and wanted to teach complete content as scheduled in the program. So, this group of teachers used lecturing, discussing, and demonstrating to their students. They also assigned a lot of homework assignments in order to practice thinking based on pattern specified by the teacher or techniques in the text to be skilful and memorized (Inprasitha, 2003) and the Basic Education Core Curriculum in A.D. 2008, the current reform movements in Thai school Mathematics still kept this mathematical learning process. But, the learning and teaching were not changed as well as be able to emphasize on the students' learning process. Most of classrooms still offer traditional teaching model including teaching sequences as: the teacher reviewed lesson, studying new knowledge in worksheet, or lecturing new knowledge, doing exercise, examining the answer, and concluding the lesson (Pasjuso, Thinwiengtong, & Kongthip, 2010). Those classrooms was a communication pattern started by the teacher's question, the students' answers, and the teacher's evaluation in students' responses (Wood, 1998). The above communication pattern was the teacher's attempt to transfer knowledge to students as one way communication (Wertsch & Toma, 1995 cited in Wood, 1998). Mathematics teaching by explaining or giving an example for students, did not support the students' meaningful learning (Wood, 1998). Mathematics teaching for supporting the students' meaningful learning needed to be based on including the classroom innovation using Lesson Study, and Open Approach as the classroom offering various opportunities for every student to learn Mathematics meaningfully according to one's own potentiality as well as student-centered classroom (Inprasitha, Loipha, & Silanoi, 2006).

Lesson Study was a teaching professional cycle which the teachers worked together. As a result, the teachers better understood the mathematics content and teaching as well as the students' thinking (Lewis, 2002). Most of Lesson Study included multi stages of practice, and work relating to research (Fernanadez & Yoshida, 2004). In Thailand, Assistant Professor Dr. Maitree Inprasitha brought the model and method from Japan by adjusting to Thai classroom culture into cycle of Lesson Study to emphasize the teaching profession in school by bringing major principles to adjust into only 3 phases as: 1) Collaboratively design research lesson (plan)—term consists of teachers, researchers, school-coordinator, and coach formulated

open-ended problem; 2) Collaboratively observing the research lessons (do) bringing lesson plan from 1) to implement teaching in classroom including the teacher as a group member to teach, all of members had objective in observing the learning management as: to observe the students' thinking method using in participating in activities of problem solving the open-ended problems,; and 3) Collaboratively Reflection or Post-discussion (see), every member from 1) discussed after teaching in order to improve the lesson plan together. The important part of problem situation development as open-ended problems, Assistant Professor Dr. Maitree Inprasitha integrated the Lesson Study with mathematics teaching model focusing on the Open Approach (Inprasitha, 2006).

The Open Approach consisted of 4 phases in sequence as follows: 1) Posing open-ended problem; 2) Students' self learning; 3) Whole class discussion and comparison; and 4) Summarization through connecting students' mathematical ideas emerged in the classroom (Inprasitha, 2010) based on basis of management the students for obtaining: 1) the motivation for students' interest to enjoy mathematics, the best opportunity as well as environment should be provided for students to learn; 2) the type of open-ended problem, the unfamiliar problems were used. In addition, the problem should provide open process, open product, and guidelines for developing the open-ended problem which was called open-ended problem; 3) the evaluation in students' guidelines of answers should be various ways (Nohda, 2000). The students were courage to think, do, express themselves, and concluded knowledge by themselves (Inprasitha & Loipha, 2007). In the Open Approach, communication among students in classroom was developed with worth (Isoda, Shimizu, & Ohtani, 2007). Many researchers stated that the mathematical communication was necessary for mathematics learning (Sierpinska, 1998) gave an importance to characteristics of mathematical communication and interaction participated by students in learning (Emori, 2005; Sierpinska, 1998) Furthermore, it was depended on basis of communication (Emori, 2005). Mathematics communication was mathematical learning process as an important part of techniques for sharing one's ideas which could help students to learn meaningfully (National Council of Teacher of Mathematics, 2000). Consequently, the teachers should encourage their students to discuss as well as shared their ideas with each other (Cooke & Buchholz, 2005).

The researchers and teacher didn't give an importance to Mathematical Communication much since they paid their attention to the students' number of speaking rather in classroom without considering the quality of thinking, and expression technique (Emori, 2005). The students communicated their comprehension through speaking and gestures in expressing for sharing the meaning from the work task (Pire, 1998). The Gesture was a language leading to thinking (McNeill, 1992 cited in Arzarello & Edwards, 2005). It was a part of communication which the sender using for learning very well, and decreasing the mistaken (Lozano & Tversky, 2006) and extending the communication to be successful (Thurston, 1994). It was one' body movement considered as the extension part of human's attention (Kendon, 2000; Rasmussen, Stephan & Allen, 2004). It might include the writing of symbols, graph, formula, table, chart, picture drawing, calculating, etc. (Radford, 2005; Thurston, 1994). If there was a systematic observation, the students' gestures were not only to fill the gap of speaking, but also to present worthy information of thinking (Kendon, 1997; Scherr, 2008). It was like a bridge connecting the speaking, and associating the action, viewing, memory, language, and written description (Bjuland, Cestare, & Bergensen, 2007; Edwards, 2005). Most of people expressed while they were speaking. So, the gestures included one's thought and language (Nunez, 2004) as well as the stimulator for speaking expression (Wu & Coulson, 2007). When the students had obstacle in speaking for communicating their ideas with the others, the gesture was a part in expressing that approach of student (So, Kita, & Goldin Meadow, 2009). It could be seen that the gesture had strong point in development of human beings, perception, learning, and communication. But, it was surprising that there was very little number of research studies regarding to gestures in learning and teaching area (Roth, 2001). Since the past to present, the gestures were overlooked in communication (Bjuland et al., 2007; Edwards, 2005).

It would be viewed that the knowledge couldn't be directly transferred to the others. So, the lecture wasn't successful in learning and teaching. Since both of communication and gesture were important in learning meaningfully. Furthermore, the communication could complete the gap of speaking. But, it was surprising that there was very little number of research studies regarding to gestures in learning and teaching area. But, in classroom using process of Lesson Study and Open Approach at the Center for Research in Mathematics Education, Faculty of Education, Khon Kaen University, were used in classroom, providing opportunity for students to learn based on their own potentiality. Therefore, the researcher was interested in surveying the mathematics communication by students' gestures under context of Lesson Study and Open Approach context.

Research Question

How many kinds were there in Mathematical Communication by the 5^{th} grade students' gestures in Lesson Study and Open Approach Context?

Research Objective

The objective of this research was to explore the Mathematical Communication by 5^{th} grade students' gestures in Lesson Study and Open Approach context.

Methodology

The target group of this study included 27 fifth grade students, Nong-tum-nong-ngu-lerm School and 33 fifth grade students, Beung-neum-beung-krai-noon School which were the schools participating in Project for Professional development of Mathematics teachers through Lesson Study and Open Approach, under supervision of the Center for Research in Mathematics Education, Faculty of Education, Khon Kaen University, and Mathematics Communication by students' gestures as follows:

Phase 1: Collaboratively design research lessons (plan), term consisted of teachers, researchers, school coordinator, and under supervision by coach, starting from determination of activities in mathematics problems by using Open-ended Problem from Mathematics Textbooks using in the project in aligned with designing and establishing the teaching media and material, and discussing the teaching sequence through Open Approach by considering students' gestures as well as mathematics Communication which would occur in teaching sequence by

Open Approach which the lesson plans were written together every week.

Phase 2: Collaboratively observing the research lessons (do), during this session, the details of phase 1 in writing the lesson management plans, were used in classroom. A teacher in team was a representative of teaching. The rest of members were classroom observers or witnesses in teaching sequence using the 4 phases of Open Approach including: 1) Posing open-ended problem; 2) Students' self learning through problem solving while the teacher take notes students' idea for later discussion; 3) Whole class discussion and comparison; and 4) Summarization through connecting students' mathematical ideas emerged in the classroom aiming to observe mathematical approach of students communicating by gestures expressing during the phase of self learning, and group discussion. The teacher's teaching ability wasn't considered. The audio tape and video tape were recorded during sequence of teaching.

Phase 3: Collaboratively Reflection or Post-discussion (see), the collaboration in discussion was performed after teaching practice in order to consider the findings after observing the learning management for improving the lesson planning, and teaching in next year. This session, was performed every week. The reflections were ranked in order by allowing the teacher reflect one's own teaching for the first person. Then, the classroom observation team discussed the existing approach in teaching sequence through the Open Approach, gestures, and Mathematics Communication occurring in classroom.

Since 2007, Center for Research in Mathematics Education had started implementing the lesson study and open approach in Nongtoom-nong-ngu-lerm School, and Ban-Beung-neum-beung-krai-noon School were funded by the Office of The Basic Education Commission in Research Program on "Model for Fostering Students' Mathematical Thinking by Implementing Lesson Study and Open Approach". I and team introduced our information to teachers and teacher introduced team to students in the classroom. The team could implement following processes of lesson study and open approach, observe behavior of students both inside and outside classroom as participation observation, record video and audio tap in the classroom and reflect students' thinking in school meeting every week. In 2009, these schools were funded by the Office of The Basic Education Commission in Project for Professional development of Mathematics teachers through Lesson Study and Open Approach. I used participation observation and informal interview to find target group in this research. I record my data following my framework at Nongtoom-nong-ngu-lerm School in 2009 and at Ban-Beung-neum-beung-krai-noon School in 2010 by recording video and audio taps. Video provided a more comprehensive understanding of the students' learning. I reviewed the video and audio taps to select data and posing problem which brought opening them and asked students in interviewing students. Data analysis used video analysis supported by protocol analysis, and used analytical description for students' behavior in mathematical communication by gesture to analysis data.

Instruments Using in the Study

The implementation of this study, the instruments for data collection as well as data analysis was used as follows:

Instruments using for data collection included the lesson plans developed by Open Approach, Filed Note form, Video Tape Recorder, Audio Tape Recorder, Notebook Computer, and students' work pieces.

Instruments using for data analysis included the analysis of video tape supported protocol, findings of filed note, in-depth interview, target group's demographic data, students' performances.

Data Analysis

For Qualitative Data Analysis, Video Analysis supported by Protocol was an opening of Video Tape based on teaching steps of Open Approach in order to see movement as well as speaking by teacher and students while the students were solving the open-ended problem. Then, they were deciphered into Protocol. The word "Protocol" referred to deciphering the behavior obtaining from audio tape and video tape into narration including pictures and word describing the occurred gestures in classroom according to the teaching incidence by Open Approach by using the word "Item" which referred to one's behaviors including each one's spoken language and gestures, writing, and body movement. The word "Episode" referred to the behavior groups expressed by the students during their Mathematical communication, and used in analytical describing the students and teacher's behaviors expressing in classroom by analyzing the students' gestures under context of Lesson Study and Open Approach respectively through the steps of Open Approach and being based on data analysis unit as basic cycle of dyad feedback, by adapting from Emori (2005) in order to consider the students' gesture that it was a message sending or receiving by students while they were solving mathematical problem. So, the sent or received message might be either spoken words or gestures using in investigating the students' common understanding whether they had.

The characteristics of mathematical communication: Rigorousness, Economy, or Freedom which they participated in communication by deictic, iconic, beat, or metaphoric gestures.

The characteristics of mathematical communication including Rigorousness which referred to one's opinion expression, speaking and talking, and discussing for sharing one's mathematical ideas expressing step by step in mathematical problem solving, and being able to send and receive message congruently with one's idea. Economy referred to one's opinion expression for sharing the mathematical ideas concisely to the others in mathematical problem solving, and being able to send and receive the concise message as well as make the communication participants have common understanding. Freedom referred to one's opinion expression, speaking and talking, and discussing for sharing various or new mathematical ideas in mathematical problem solving.

Students' Gestures referred to the students' observable body movement including: Deictic gesture as their body movement showing the mathematical approach in determining the existing or visible objects. Iconic gesture referred to one's body movement expressing the mathematical ideas in drawing picture referring to the lesson content. Beat gesture referred to one's repeated or rhythm body movement to emphasize that idea. Metaphoric gesture referred to one's body movement based on mathematical ideas regarding to the abstract content.

We used basic cycle of dyad feedback which was unit of analysis messages as gesture and verbal language of sender and receiver communicated mathematical ideas following mathematical communication framework.

Example Findings

Mathematical communication by students' gestures found all steps of Open approach as teaching approach. Example of research findings found following: Freedom Mathematical communication by students' deictic and iconic gestures in Classroom Discussion and Comparison step and Economically Mathematical communication by students' iconic gesture in Summarization through connecting students' mathematical ideas emerged in the classroom step of Open approach.

Classroom Discussion and Comparison Step.

In this stage, the students expressed their opinion by various communication techniques in Mathematics Communication through gestures as follows:

Item 459-469, is video analysis supported by Protocol in which the teachers asking the students' performances. The students in group responded by many techniques.

Item Name Messages

459 Teacher: Then, what is going on? It is like this. What's going on?

460 M: (Point at the picture.)

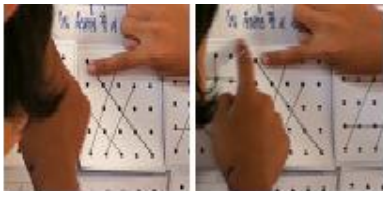
461 Teacher: It comes close. It comes close, quick!

462 M: (Express gesture of parallel in vertical line and horizontal line.

463 Students: (Laugh)

464 J: (Point at her performance.) These two lines could be vertical pattern. This line could be horizontal one. This line would be a diagonal pattern. It the cross with each other, there would be a rectangular.

465 M: These two lines could be a vertical pattern. These two lines could be a horizontal pattern. These two lines could be a diagonal pattern. These two lines could be a horizontal pattern.



466 J: (Pick her performance)

467 M: These two lines make a diagonal pattern.

468 J: (Bring microphone to her friend's mouth, her friend escaped it.) Well, this line is an inclined pattern. It's the longest. Well, this one is a short diagonal pattern. These two lines make a short diagonal pattern. They cross each other as a rectangular.

469 Teacher: How many patterns do we have?

According to the Episode in Item 459-468 as the above, for Item 459, the teacher asks what characteristic this group made. The teacher sends a message to students to answer the questions that: 1) Item 460 M, points at the picture. But, in Item 461, the teacher stimulates "It comes close. It comes close, quick." The students respond; 2) Item 462 M, use her thumb and index make the claw of a crab. Then, she draws it in vertical line, and horizontal line; 3) Item 464 J, points to the group's performance and said that: "These two lines would make vertical pattern. This line would make a horizontal line. This line would make a diagonal pattern. When, they are crossed, it would be a rectangular pattern"; 4) Item 465 M, point at the line as group's work; and 5) item 468 J points at the line which is group's performance, in Item 460 Item 462 Item 465 and Item 468 by specific gesture, for Item 464 includes a physical gesture in responding the teacher's question that the made rectangular as 2 parallel lines in diagonal pattern, horizontal pattern, or vertical line including various patterns. Then, the teacher passes this issue into other ones. According to the above Episode, it could be viewed that the student uses gesture as pointing as well as picture in Mathematical Communication in various approaches. It could be concluded that it was Freedom Mathematical Communication by students' deictic and iconic gestures.

Summarization through connecting students' mathematical ideas emerged in the classroom step.

In this stage, the student expresses her various opinions as well as techniques in Mathematical Communication by gestures while they were concluding the lessons as follows:

Item 393-398, is video analysis supported by Protocol in which the teacher asking student to explain why six channels were obtained the same. The student explains picture (d).

Item Name Messages

393 Teacher: Well, are there six channels? One, two, three, four, five, and six. There are six channels. How about this one? Picture b. For this one, some students count them as six channels. Some students cut this part, and put it this side. We still count the same. For this one, c, bring it here (the left hand). This one is informed by a friend to cut at this point, and put it here. (The right hand) or we can cut it here (the right hand). Put it here. How many channels we would get? Six channels again. For this one. For (the left hand), this one, we can cut it here, can't we? Isn't it? How can we cut it? Let this group show how to cut it, it will be better. Chicken Group, please cut it. Where would you draw? Where do you draw it? Write it down.

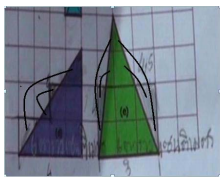
394 Student:

395 Teacher: The channels are full the same.

396 Student: Count the channels drawn by her friend.

397 Student: Walk to see her Group's work.

398Teacher: This picture, point to the student's work.



We get this picture, don't we? Then, we can put it here. Is it a rectangular? Well, let's see our friend's. At first, our friend cut it like this. How many channels are there? There are six channels.

According to the Episode in Item 393-398 as the above, in Item 393, the teacher asks the students to explain why do they get six channels the same. The student explain picture (d) as Item 394, (cross the picture. Then paint it). The gesture is iconic. As the Item 398, the teacher shows the Group's performance for their friends to consider again. According to the above Episode, it could be seen that the student uses gesture in concise messages in Mathematical Communication. So, the participants have common understanding. It could be concluded that it is Economically Mathematical Communication by student's iconic gestures.

Conclusion and Discussions

The research findings found that there were 7 kinds of students' Mathematical Communication by students' Gestures including 1) rigorousness by students' beat gesture-Mathematical Communication in which the students wanted to emphasize the statements or pictures as their own ideas, or communicate with their friends or teacher; 2) rigorousness by students' metaphoric gestures—gesture referring to content or concepts of the lesson; 3) economy by students' deictic gesture—this kind of gesture could be easily performed economically with common understanding; 4) economy by students' iconic gesture—this kind of gesture was to draw a picture to communicate economically with each other for common understanding; 5) freedom by students' deictic gesture—the students used various kinds of pointing, and had freedom to express their mathematical ideas; 6) freedom by students' iconic gesture—the students could use many kinds in drawing picture as well as have freedom to express their mathematical ideas; and 7) freedom by students' deictic and iconic gestures—the students used both of pointing, and drawing picture to communicate their ideas as well as had freedom in express their mathematical ideas, and the most commonly used the characteristic of economically mathematical communication by deictic gestures, and Students' self learning through problem solving while the teacher take notes students' idea for later discussion in Open Approach. The teachers used gestures for observing students' learning in the classroom and students had happiness in the classroom. Furthermore, the schools in Lesson Study and Open Approach context, the students had opportunity in learning based on their potentiality, being able to think, perform, and express. They preferred to express divergent think.

Recommendations

The teachers and Educational Staffs could use the behavioral or gestures observation technique in learning of students during they expressed Mathematical Communication with each other meaningfully which was an evaluation from the students' working process. In addition, their gestures could be able to be used in reasoning, explaining, and analyzing their performance, concluding their approaches, and evaluating their emotion or feeling during learning. Specifically, under Lesson Study, and Open Classroom situations, the students had freedom in expressing their ideas in various ways including both of verbal, and gestures meaningfully with themselves. We implement this result through pre-service teachers to bring this knowledge working group with teachers in schools and teacher learned these knowledge and I will write article to publish this result. The studies of students' Mathematical Communication by gestures in other class levels should be studied further. And each academic year had trained pre-service teachers about new knowledge to bring this knowledge working group with teachers in schools.

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