

# An Assessment of Ionospheric Error Mitigation Techniques for GNSS Estimation in the Low Equatorial African Region

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## ABSTRACT

Single frequency GNSS receivers are the most widely used tools for tracking, navigation and geo-referencing around the world. It is estimated that over 75% of all GNSS receivers used globally are single frequency receivers and users experience positioning error due to the ionosphere. To enable GNSS Single Frequency Precise Point Positioning (SFPPP), accurate *a-prior* information about the ionosphere is needed. The variation of the ionosphere is larger around the magnetic equator and therefore depends on latitude. It will be expected that SFPPP works better on latitude further from the magnetic equator. This present study aims to investigate the accuracy of some ionospheric error mitigation approaches used in single frequency precise point positioning (SFPPP) at several GNSS station in the new Nigerian GNSS Network (NIGNet) and two IGS sites in the low equatorial African region. This study covers two epochs of observation. The first consists of observation from three consecutive days (GPS week 1638; days 0, 1 and 2) that belongs to a period of low solar activities. The second epoch consists of observation from three consecutive days (GPS week 1647; days 2, 3 and 4) that belongs to a high solar activity and intense geomagnetic conditions. The estimated position for the GNSS stations from dual frequency measurement and their known ITRF solutions were used as a benchmark to assess the accuracy of SFPPP under four conditions *i.e.*, SFPPP without ionospheric correction, SFPPP using final GIM models from the Centre for Orbit Determination in Europe( CODE), SFPPP with Klobuchar model, and SFPPP with a computed (local) model at each station. All computation was done using Leica Geo-office software. The result of the study clearly demonstrates the significance of removing or correcting for the effect of the ionosphere, which can result in up to 7 m displacement. It was recommended that GIMs from different organization should be investigated and also efforts should be towards improvement in algorithms and clock error modeling.

## KEYWORDS

Global Ionosphere Map (GIM); Single Frequency Precise Point Positioning (SFPPP); GNSS; Dual Frequency Measurement; Klobuchar Model; Position Accuracy

## 1. Introduction

Most users of GPS data use the differential technique due to its higher accuracy. However, there are some limitations in relative GPS technique: two or more receivers are required to be available, and the true coordinates of the reference station should be known. Moreover, increasing the distance between the two receivers causes a decrement in the quality of positioning. A new technique

in GPS positioning known as precise point positioning (PPP) shows that a user with a single receiver can attain positioning accuracy at centimetre or decimetre level, as compared to differential technique. PPP is very cost-effective since there is no need for observations from local or regional reference stations [1].

Precise point positioning uses the globally available GPS precise orbit and clock data and can be applied to

static and kinematic mode of observation, providing centimetre accuracy in static and decimetre level accuracy in kinematic mode, as long as there is continuous GPS observation without any interruption and that all mechanisms are available to process the precise GPS observations in post-processing mode. Precise point positioning is a positioning technique determined from a single station or receiver, when using a long series of observation, it also gives the user an opportunity to acquire site positioning in a reference frame of the utilised GPS products and can also be used to investigate the stability of a site over time as well as the GPS products [2]. It also gives homogeneous positioning accuracy on a global scale.

PPP is capable of providing centimetre level point positioning for static applications and decimetre level for kinematic applications using a dual frequency, geodetic receiver [3]. As for single frequency observations, the accuracy of the estimated point positioning decreases [4], particularly in the height component. One main factor for this degradation in accuracy is the effect of un-modelled ionospheric error. The ionosphere is the single largest error source in point positioning after the application of precise GNSS orbit and clock products, and there are a number of mathematical models that have been proposed to mitigate its effects.

The main objective of this paper is to assess the positioning accuracy that can be achieved from Single Frequency Precise Point Positioning (SFPPP) using available ionospheric correction models. This was demonstrated through a series of comparisons between estimated positions that were performed using a different approach to the correction for the effect of the ionospheric corrections. The following options were considered: 1) no correction 2) Klobuchar model 3) Computed model 4) Global Ionospheric Map 5) PPP Dual frequency solution and 6) Differential Dual Frequency solutions. The study is based on data from the Nigerian GNSS Network (NIGNET) and two IGS sites all located in the low equatorial African region. The Known ITRF, PPP Dual frequency and Differential Dual Frequency solutions were used as a benchmark in assessing the accuracy of positioning estimation at the stations under investigation.

## 2. Materials and Method

### Data Used

This study covers two epochs of observation, the first consists of observation from three consecutive days (GPS week 1638; days 0, 1 and 2) that belongs to a period of low solar activities. The second epoch consists of observation from three consecutive days (GPS week 1647; days 2, 3 and 4) that belongs to a high solar activity and intense geomagnetic conditions as reported by

[www.spaceweather.com](http://www.spaceweather.com). They were chosen in order to have an estimate of possible worst conditions for SFPPP in the low equatorial African region. GNSS receivers from eight different sites were used. Six of which are GNSS sites in the Nigerian GNSS network (BKFP, CGGT, FUTY, OSGF, RUST, and UNEC). The other two sites NKLK and BJCO are IGS sites in Africa. The data for the NIGNET sites were downloaded from [www.nignet.net](http://www.nignet.net) which is the official website of the Office of the Surveyor General of Nigeria (OSGOF). Also, the data from IGS sites were obtained from the official website of the Scripps Orbit and Permanent Array Centre (SOPAC). **Figure 1** shows the spatial location of the eight GNSS sites under investigation.

## 3. Method

There are a number of different mitigation methods for single frequency GNSS users to correct for the ionospheric error. In this study three different approaches were used, namely: the Klobuchar model, Computed (local) model, and Global Ionospheric Maps.

The Klobuchar model is something of a compromise between computational complexity and accuracy. Klobuchar model uses eight ionospheric coefficients which are broadcasted as part of the navigation message. During normal operation, the parameters of the model are updated at least once every six days [5]. This algorithm can be used in real-time and it was designed to provide a correction for approximately 50 percent Root Mean Square (RMS) of the ionospheric range delay [6]. Since mid July 2000, the Centre for Orbit Determination in Europe (CODE) has been providing post-fit Klobuchar ionospheric coefficients that best fit the GIMs data estimated by CODE. Currently, the post-fit Klobuchar ionospheric coefficients have a latency of several days. Thus, for the purpose of this investigation, the Klobuchar model with the broadcast ionospheric coefficients was used instead. It is worth noting that the CODE has also been estimating predicted Klobuchar-style coefficients. However, the improvement was found to be not as significant as the post-fit coefficients [7,8].

Global ionosphere maps (GIM) are generated on a daily basis at CODE using data from about 200 GPS/GLONASS sites of the IGS and other institutions. The vertical total electron content (VTEC) is modeled in a solar-geomagnetic reference frame using a spherical harmonics expansion up to degree and order 15. Piece-wise linear functions are used for representation in the time domain. The time spacing of their vertices is 2 hours, conforming with the epochs of the VTEC maps. Instrumental biases, so-called differential P1-P2 code biases (DCB), for all GPS satellites and ground stations are estimated as constant values for each day, simultaneously



Figure 1. The location of the eight GNSS stations used in this study.

with the 13 times 256, or 3328 parameters used to represent the global VTEC distribution. The DCB datum is defined by a zero-mean condition imposed on the satellite bias estimates. P1-C1 bias corrections are taken into account if needed. GIMs represent a new tool for monitoring global patterns of ionospheric weather, a key component of the space weather, which is driven by changes in solar ultra-violet radiation, the interplanetary particle stream known as the solar wind, and the underlying composition, wind patterns and electrodynamics of the thermosphere (the upper atmosphere at altitudes between 100 and 1000 km). GIMs are being used for global ionospheric delay calibrations, for scientific investigations of the upper atmosphere. For the purpose of this study daily GIM were obtained from University of Bern in Switzerland through an FTP account ([FTP.UNIBE.CH](ftp://ftp.unibe.ch) or 130.92.4.48). Leica Geo office software only supports files from University of Bern, which are in Bernese format.

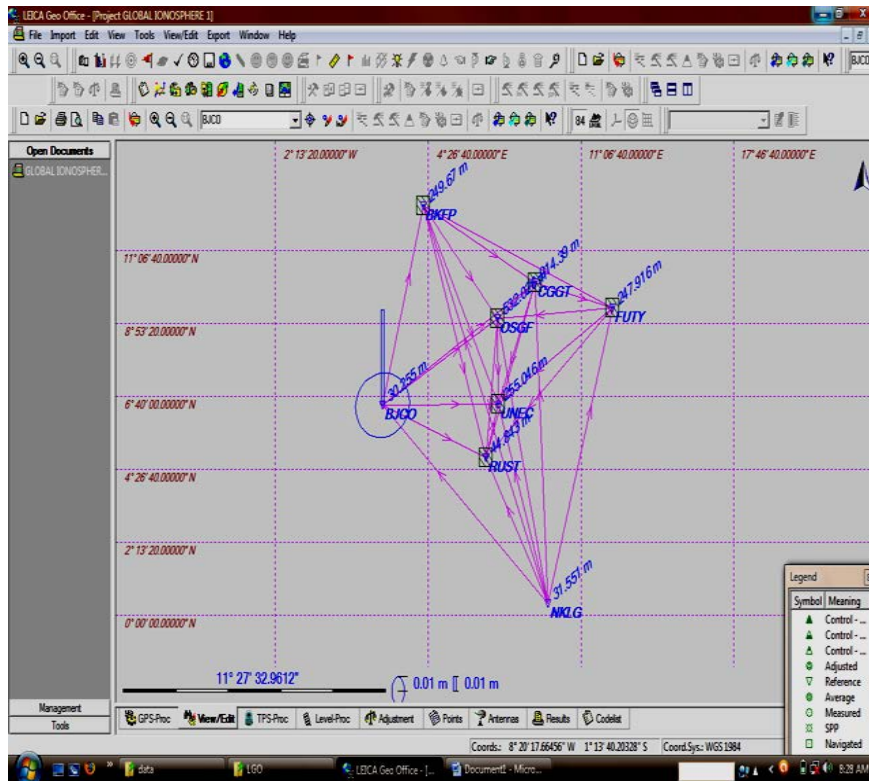
The computed (local) model uses static or rapid static dual frequency data collected at the reference station to compute an ionospheric model. This model is advantageous, as the model computed is in accordance with conditions prevalent at the time and position of observation. This model is well documented in Leica [9].

In order to assess the effectiveness of each of the approach highlighted above, certain parameters were adjusted in the software used (Leica Geo office version 7.0). Six options of coordinate estimation was considered for the GNSS stations under investigation *i.e.*, option of no ionospheric correction applied to processed coordinates, use of Klobuchar model for correcting ionospheric error, Computed model for correcting ionospheric error, Global Ionospheric Map for correcting ionospheric error, PPP Dual frequency solution without for correcting ionospheric error and finally Differential Dual Frequency solutions for correcting ionospheric error. The coordinate estimate from each of the approaches was compare to the ITRF 2005 solutions at each station. **Table 1** summarized all the parameter considering in the processing of coordinate of each GNSS station under investigation.

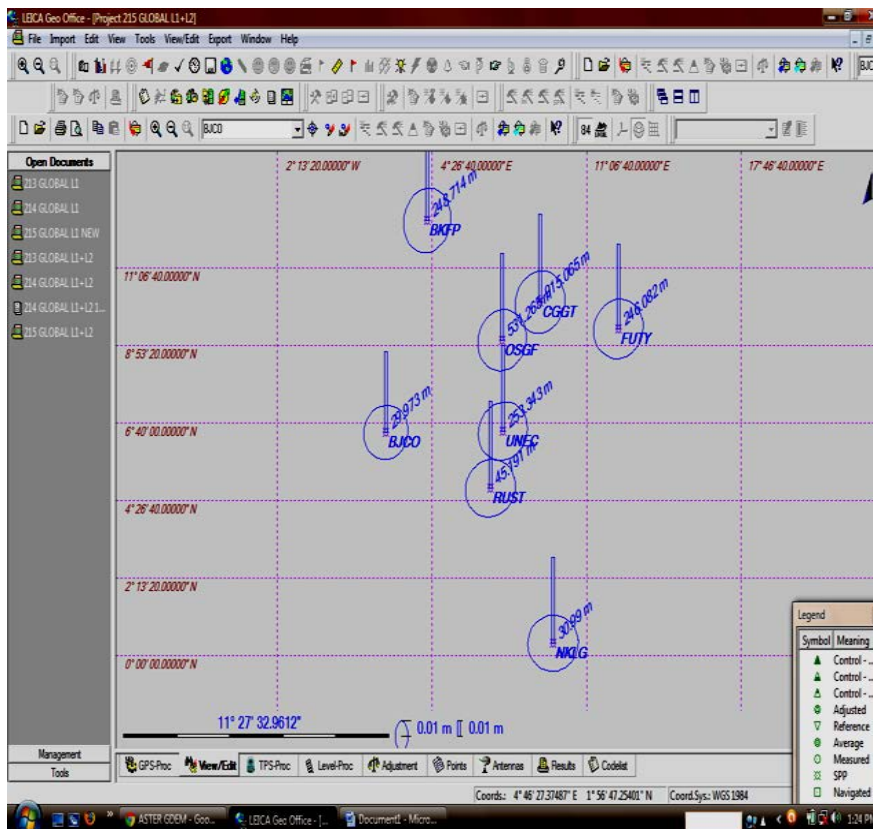
Also, **Figures 2(a)** and **(b)** show snapshots of the processing window in Leica geo office for relative network solution and SFPPP solution.

#### 4. Results and Discussions

**Table 2** gives the ITRF 2005 solution of the GNSS stations under investigation. The coordinate estimate from each of the approaches under investigation for the two different epochs was subjected to quality control (toler-



(a)



(b)

Figure 2. (a) Snapshot screen for relative network solution using dual frequency; (b) Snapshot screen for SFPPP.



**Table 1. Summary of processing parameter used in the study.**

Parameters	Description/Value
Mask (Cut off) Angle	Default (usually 10 or 15 degree)
Orbit type	Precise
Tropospheric model	Hopfield Method (default)
Coordinate seeding strategy	By time
Frequency used positioning method	L1 alone for PPP, L1 + L2 combined for relative (differential) positioning
Adjustment type	Inner constrained
Coordinate system	ITRF 2005 solution
RINEX Data Used	30 seconds sampling rate
Ionospheric correction	Post fit Klobuchar model with coefficients broadcasted from CODE, Computed (local) model, and Global Ionospheric Map from CODE

**Table 2. Known ITRF coordinates of GNSS stations under investigation.**

GNSS Station	X(m)	Y(m)	Z(m)
<b>BJCO</b>	6333076.505	270973.252	704551.808
<b>BKFP</b>	6211960.354	459365.467	1368115.024
<b>CGGT</b>	6201032.286	995277.236	1113815.499
<b>FUTY</b>	6145058.507	1362078.867	1029389.894
<b>NKLG</b>	6287385.799	1071574.422	39132.804
<b>OSGF</b>	6246471.262	820848.732	994267.908
<b>RUST</b>	6308859.059	772229.918	530354.436
<b>UNEC</b>	6284298.315	827900.505	708988.565

ance) check for gross error detection (outliers) which may have resulted from undetected cycle slip error or snooping. The 2RMS test was adopted for the quality control check (*i.e.*, average difference  $\pm$  1.96 standard deviation of the difference), after all the station coordinates were subjected to the test, the mean (average) values for the days under investigation for the two different epochs are computed in **Tables 3-7**.

A statistical test was conducted to investigate the statistical agreement between the coordinate estimates from the different methods of ionospheric correction and the known ITRF solution of each station. The Bland-Altman plot [10,11] was used. The Bland-Altman plot, or difference plot, is a graphical method to compare two measurements techniques. In this graphical method, the differences (or alternatively the ratios) between the two techniques are plotted against the averages of the two techniques. Alternatively, according to Krouwer [12], the differences can be plotted against one of the two methods, if this method is a reference or “standard” method. As in this case, the known ITRF solutions of the station under investigation were considered as the reference method. Thirty (30) difference plots were generated for the two campaigns of observation; samples of the difference plot are presented in **Figures 3-5**. All computation was done using Analyse-it software, version 2.12.

The bias was also determined for each of the plots. The bias is the average difference between variables and

should ideally be zero. Thus, near zero, values signify the agreement between the measurement technique and the standard method. **Tables 8** and **9** show the biases for the different ionospheric mitigation approaches when compared with the reference (ITRF solutions).

From **Tables 8** and **9**, the biases in the (X, Y, Z) components for each approach can be considered as points in the Euclidean three space that deviate from the reference (Known ITRF solution) which has an assumed value of (0, 0, 0) for the three components. Thus the displacements from the reference or standard can be considered as the Euclidean distance between each of the approaches and the reference. **Figure 6** shows a plot of the displacements from the reference value, and the displacements were computed as the Euclidean distance between each approach and reference method. From **Figure 6**, it is evident that the effect of the ionosphere on position estimation can be severe if not corrected for, it can cause displacement of up to 7 m or more during intense ionospheric activities. The different mitigation approaches have all reduced the effect of the ionosphere in both epochs. The GIM from CODE proves to be a very efficient approach since it performs better than the DGPS approach in the two epochs of observation. Ideally, the DGPS is supposed to provide the best result, and the poor performance of the method can be attributed to the sparse of the network which has resulted in large inter-station distance. Generally, the results presented in **Figure 6**

**Table 3. Mean coordinates of GNSS stations estimated without ionospheric correction.**

		<b>BJCO</b>	<b>BKFP</b>	<b>CGGT</b>	<b>FUTY</b>	<b>NKLG</b>	<b>OSGF</b>	<b>RUST</b>	<b>UNEC</b>
<b>X</b>	Epoch 1	6333082.750	6211966.452	6201038.522	6145064.050	6287390.514	6246477.325	6308865.021	6284304.210
	Epoch 2	6333084.443	6211967.376	6201039.428	6145064.752	6287391.838	6246478.249	6308866.054	6284305.154
<b>Y</b>	Epoch 1	270973.472	459365.644	995277.774	1362079.895	1071575.070	820849.250	772230.403	827901.031
	Epoch 2	270973.527	459365.935	995277.932	1362080.185	1071575.280	820849.567	772230.786	827901.406
<b>Z</b>	Epoch 1	704552.940	1368116.088	1113816.397	1029390.881	39132.521	994268.916	530355.086	708989.373
	Epoch 2	704553.183	1368116.212	1113816.492	1029391.034	39132.380	994269.085	530355.276	708989.533

**Table 4. Mean coordinates of GNSS stations estimated with the computed ionospheric correction.**

		<b>BJCO</b>	<b>BKFP</b>	<b>CGGT</b>	<b>FUTY</b>	<b>NKLG</b>	<b>OSGF</b>	<b>RUST</b>	<b>UNEC</b>
<b>X</b>	Epoch 1	6333077.876	6211960.943	6201034.077	6145059.141	6287387.141	6246477.287	6308859.963	6284299.295
	Epoch 2	6333077.993	6211960.610	6201033.922	6145058.724	6287387.373	6246472.003	6308859.839	6284299.003
<b>Y</b>	Epoch 1	270973.278	459365.276	995277.060	1362078.769	1071574.421	8208848.604	772229.701	827900.341
	Epoch 2	270973.271	459365.269	995276.927	1362078.648	1071574.421	820848.575	772229.642	827900.311
<b>Z</b>	Epoch 1	704552.138	1368114.865	1113815.583	1029389.774	39132.875	994267.871	530354.408	708988.596
	Epoch 2	704552.122	1368114.738	1113815.497	1029389.666	39132.850	994267.807	530354.374	708988.544

**Table 5. Mean coordinates of GNSS stations estimated with GIM from CODE.**

		<b>BJCO</b>	<b>BKFP</b>	<b>CGGT</b>	<b>FUTY</b>	<b>NKLG</b>	<b>OSGF</b>	<b>RUST</b>	<b>UNEC</b>
<b>X</b>	Epoch 1	6333076.224	6211959.875	6201033.082	6145057.865	6287385.483	6246470.851	6308858.966	6284297.996
	Epoch 2	6333075.807	6211959.271	6201032.598	6145057.080	6287385.179	6246470.207	6308858.560	6284297.404
<b>Y</b>	Epoch 1	270973.247	459365.212	995276.909	1362078.614	1071574.124	820848.481	772229.654	827900.254
	Epoch 2	270973.230	459365.188	995276.722	1362078.452	1071574.027	820848.389	772229.570	827900.180
<b>Z</b>	Epoch 1	704551.940	1368114.918	1113815.721	1029389.770	39132.740	994267.817	530354.337	708988.464
	Epoch 2	704551.812	1368114.883	1113815.686	1029389.703	39132.671	994267.763	530354.264	708988.353

**Table 6. Mean coordinates of GNSS stations estimated with the Klobuchar ionospheric correction model.**

		<b>BJCO</b>	<b>BKFP</b>	<b>CGGT</b>	<b>FUTY</b>	<b>NKLG</b>	<b>OSGF</b>	<b>RUST</b>	<b>UNEC</b>
<b>X</b>	Epoch 1	6333077.993	6211960.610	6201033.922	6145058.724	6287387.373	6246472.003	6308859.839	6284299.033
	Epoch 2	6333077.940	6211960.740	6201033.179	6145059.323	6287388.486	6246472.402	6308859.256	6284298.844
<b>Y</b>	Epoch 1	270973.271	459365.269	995276.927	1362078.648	1071574.421	820848.575	772229.642	827900.317
	Epoch 2	270972.988	459364.964	995276.596	1362078.533	1071574.921	820848.042	772228.852	827900.732
<b>Z</b>	Epoch 1	704552.122	1368114.738	1113815.497	1029389.666	39132.850	994267.807	530354.374	708988.544
	Epoch 2	704552.721	1368113.929	1113815.287	1029389.492	39131.867	994267.793	530354.921	708987.893

**Table 7. Mean coordinates of GNSS stations estimated from dual frequency DGPS.**

		<b>BJCO</b>	<b>BKFP</b>	<b>CGGT</b>	<b>FUTY</b>	<b>NKLG</b>	<b>OSGF</b>	<b>RUST</b>	<b>UNEC</b>
<b>X</b>	Epoch 1	6333077.094	6211960.474	6201032.227	6145058.601	6287386.365	6246470.982	6308860.335	6284297.793
	Epoch 2	6333077.443	6211960.376	6201032.428	6145058.752	6287386.838	6246470.249	6308860.054	6284298.154
<b>Y</b>	Epoch 1	270972.731	459364.947	995276.792	1362078.446	1071575.123	820847.723	772230.048	827900.083
	Epoch 2	270973.527	459365.935	995277.932	1362076.185	1071575.280	820848.567	772230.786	827901.406
<b>Z</b>	Epoch 1	704552.179	1368114.898	1113815.431	1029389.763	39132.458	994267.998	530354.545	708988.720
	Epoch 2	704553.183	1368113.212	1113816.492	1029390.034	39132.380	994269.085	530355.276	708989.533

indicate a significant progress in the PPP mode of observation as it stands out to be very advantageous for user of networks or in a situation where no reference station is available.

Observational data, in addition to navigation and clock information from eight GNSS stations in the low equatorial region of West Africa, were processed using SFPPP technique. Different options for correcting for the effect

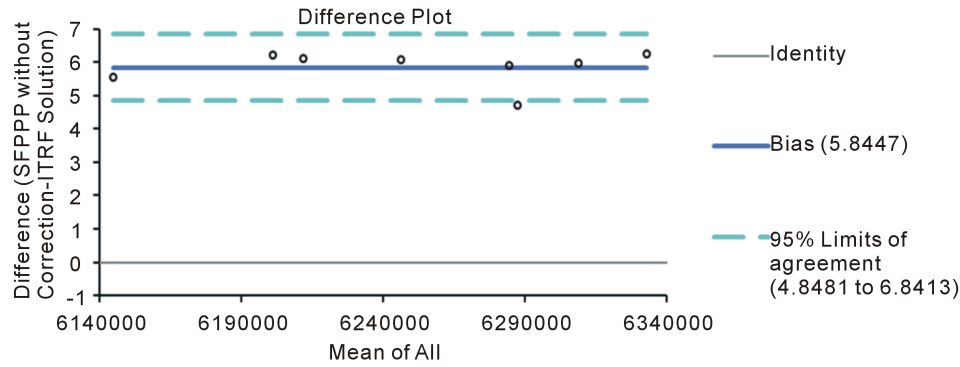


Figure 3. A sample of the Bland Altman plot (difference plot) depicting the bias when ionospheric correction was not applied at all the stations for epoch 1 (X-component).

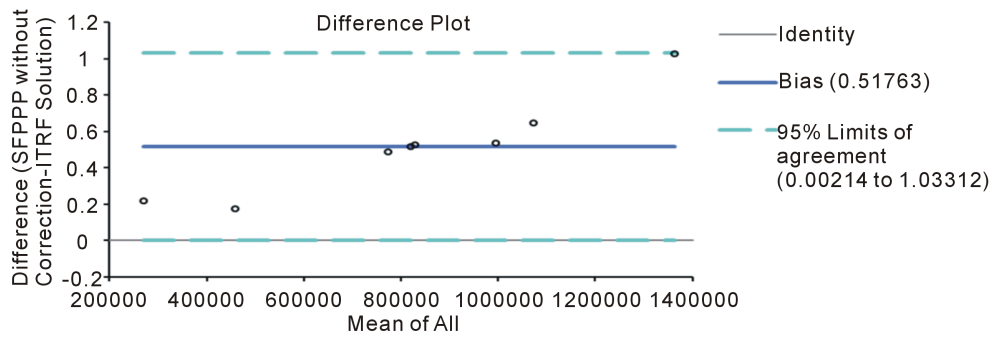


Figure 4. A sample of the Bland Altman plot (difference plot) depicting the bias when ionospheric correction was not applied at all the stations for epoch 1 (Y-component).

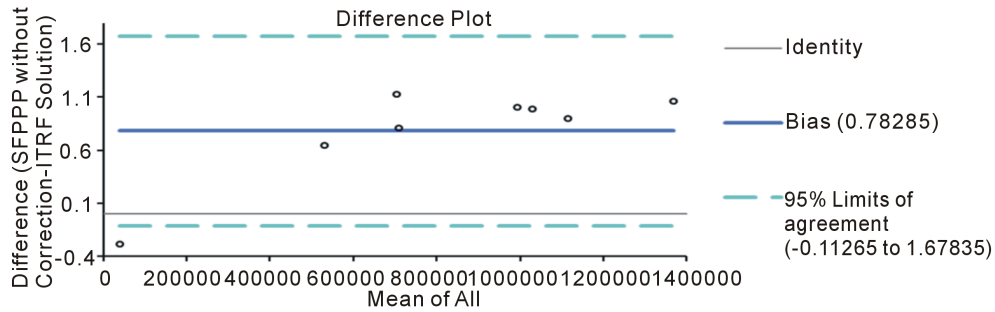


Figure 5. A sample of the Bland Altman plot (difference plot) depicting the bias when ionospheric correction was not applied at all the stations for epoch 1 (Z-component).

Table 8. Bias estimate for each approach (Epoch 1).

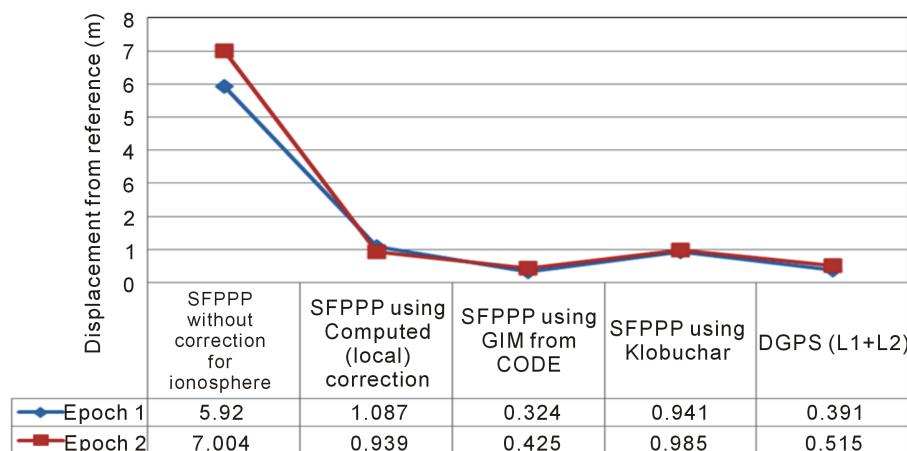
Methods	Bias in X component (m)	Bias in Y component (m)	Bais in Z component (m)
SFPPP without correction for ionosphere	5.845	0.518	-0.783
SFPPP using Computed (local) correction	1.080	-0.119	0.021
SFPPP using GIM from CODE	-0.218	-0.238	-0.033
SFPPP using Klobuchar	0.926	-0.166	-0.043
DGPS (L1 + L2)	0.223	-0.313	0.07

of the ionospheric were investigated to assess the positioning accuracy that can be achieved from SFPPP. The result of the study clearly demonstrates the significance of removing or correcting for the effect of the ionosphere.

As it has been observed that some residual errors still remain in the estimated user position even after using dual frequency receivers in a relative approach. User of single frequency receiver for PPP only needs to understand

**Table 9. Bias estimate for each approach (Epoch 2).**

Methods	Bias in X component (m)	Bias in Y component (m)	Bais in Z component (m)
SFPPP without correction for ionosphere	6.901	0.777	-0.907
SFPPP using Computed (local) correction	0.923	-0.167	-0.043
SFPPP using GIM from CODE	-0.248	-0.330	-0.101
SFPPP using Klobuchar	-0.886	-0.346	-0.255
DGPS (L1 + L2)	0.276	0.152	0.407



**Figure 6. Displacement of stations from the reference based on ionospheric correction approach.**

the behavior of the ionosphere at the time of their observation to attain acceptable limit of accuracy. It is believed that with expected improvements in satellite clock error bias estimation in the future, the accuracy of SFPPP will be greatly improved. As the results of this study have demonstrated, it will be of great importance if the accuracy of the different GIM produced by different organizations is compared and more effort should be made on the improvement of PPP algorithm to include effective modeling of clock errors and atmospheric effects. SFPPP can possibly replace the current widely used technique of relative DGPS in many applications, in view of the disadvantages of sparse networks and consequent long baseline errors associated with the DGPS technique. Finally, efforts are required to develop a RIM ionospheric model for the low equatorial African region and corrections from such are made available to users for Near Real Time (NRT) applications.

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