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Propagation Characteristics of Airy-Gaussian Beams Passing through a Misaligned Optical System with Finite Aperture

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Abstract

Propagation characteristics of finite Airy-Gaussian beams through an apertured misaligned first-order *ABCD* optical system are studied. In this work, the generalized Huygens-Fresnel diffraction integral and the expansion of the hard aperture function into a finite sum of complex Gaussian functions are used. The propagation of Airy-Gaussian beam passing through: an unapertured misaligned optical system, an apertured aligned *ABCD* optical system and an unapertured aligned *ABCD* optical system are derived here as particular cases of the main finding. Some numerical simulations are performed in the paper.

Keywords

Airy-Gaussian Beams, Huygens-Fresnel Diffraction Integral, Aperture, Misalignment

1. Introduction

Airy beam is initially predicted theoretically, in quantum physics, as a solution of force-free Schrödinger equation by Berry and Balazs [1] in 1979. It is a non-spreading wave packet that remains invariant during propagation and contains infinite energy. Airy beam can exhibit a self-healing property after being obscured by an obstacle placed in its propagation path [2] and a self-accelerating feature even in the absence of any external potential [3]. Yet, Airy beam is propagating along parabolic trajectory, while preserving its amplitude structure in-

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definitely [4]. The original Airy beam which contains infinite energy is not realizable in practice. However, in 2007, Siviloglou *et al*. [3] and Siviloglou and Christodoulides [5] have started the first observation of Airy optical beam that presents a finite energy and demonstrates experimentally the unusual features of the new finite Airy beam. In the literature, several methods were used to produce the finite Airy beam, including cubic phase, 3/2 phase only pattern [6]-[9], and three-wave mixing processes in an asymmetric nonlinear photonic crystals [10] [11]. In the past few years, the propagation characteristics of Airy family have been examined widely in free space [12] [13], in fractional Fourier transform and quadratic index medium [13]-[16], in turbulence [17] [18], in a unixial crystals [19] and in other media [20]-[22]. Among of these, in [13], Bandres and Gutierrez-Vega have introduced for the first time, the so-called generalized Airy-Gaussian beam and treated its propagation properties through different complex paraxial optical systems characterized by *ABCD* matrices. This generalized Airy-Gaussian beam carries a finite energy and can be realized experimentally. The Airy beam devoted by Berry and Balazs [1] and the finite Airy invented and produced by Siviloglou *et al.* [3] [4] are regarded as special cases of the study of Bandres and Gutiérrez-Vega [13].

On the other hand, most practical optical systems are more or less slightly misaligned, due to displacement or angle misalignment. Then, it is necessary to take the misalignment of the optical system into consideration. Various laser beams passing through misaligned optical systems with or without aperture have been treated by researchers [23]-[30]. To the best of our knowledge, the research of Airy-Gaussian beam propagating through an apertured misaligned optical has not been reported elsewhere.

In this paper, by expanding a hard-edged aperture function into a finite sum of complex Gaussian functions and the generalized Huygens-Fresnel diffraction integral, an approximate formula for the propagation of Airy-Gaussian beam in any misaligned optical system with a hard-edged aperture is developed in the coming section. The propagation of Airy-Gaussian beam through: unapertured misaligned, unapertured and apertured aligned optical systems are deduced as particular cases in Section 3. Some numerical results are performed and discussed in Section 4. The work is finished by a simple conclusion in Section 5.

2. Theory

The field distribution $E(x_0, z=0)$ of finite Airy-Gaussian beam at plane source in the rectangular coordinate system is expressed as follows [13] [31]

$$E(x_0, z=0) = E_0 Ai(x_0/\omega_0)\exp(a_0 x_0/\omega_0)\cdot\exp(-x_0^2/\omega_0^2), \tag{1}$$

where $Ai(.)$ is the Airy function of the first kind, ω_0 is the waist width (is a characteristic parameter of finite Airy beam) at waist plane $z=0$ and a_0 is the modulation parameter (aperture coefficient).

Figure 1 illustrates a comparison between intensity distributions of finite Airy beam and finite Airy-Gaussian beam for different aperture coefficients a_0 $(0<a_0<1)$. Depicted plots show that ideal Airy beam (finite Airy beam with $a_0=0$) carry an infinite energy and its intensity profile presents infinity of oscillations, side-lobes and zeros in the negative part of the transverse x-coordinate and principle lobe shifted from the propagation axis z. Intensity oscillations vanish gradually with the increase of a_0 and totally disappear when a_0 approaches to 1. A modulation of finite Airy beam by a Gaussian transmittance avoid the oscillations and secondary lobes whatever value of a_0. Furthermore, it should be noted that the intensity maximum decreases with the increasing of a_0, in the both cases: finite Airy and finite Airy-Gaussian beams. However, the velocity of diminution of intensity amplitude of finite Airy beam modulated by Gaussian envelope is very small compared with that of no-modulated one. Also, an increasing in a_0 leads to a movement of principle lobe towards optical axis for $a_0=0.8$.

Assuming a hard-edge rectangular aperture of radius a located at waist plane of $z=0$. The corresponding window is

$$H(x_0) = \begin{cases} 1 & \text{if } |x_0| \le a, \\ 0 & \text{else.} \end{cases} \tag{2}$$

According to the method proposed by Wen and Breazeale [32], the hard-edged function can be expanded into a finite sum of complex Gaussian functions [23] [32] as

$$H(x_0) = \sum_{m=1}^{M} A_m \exp\left(-\frac{B_m}{a^2}x_0^2\right), \tag{3}$$

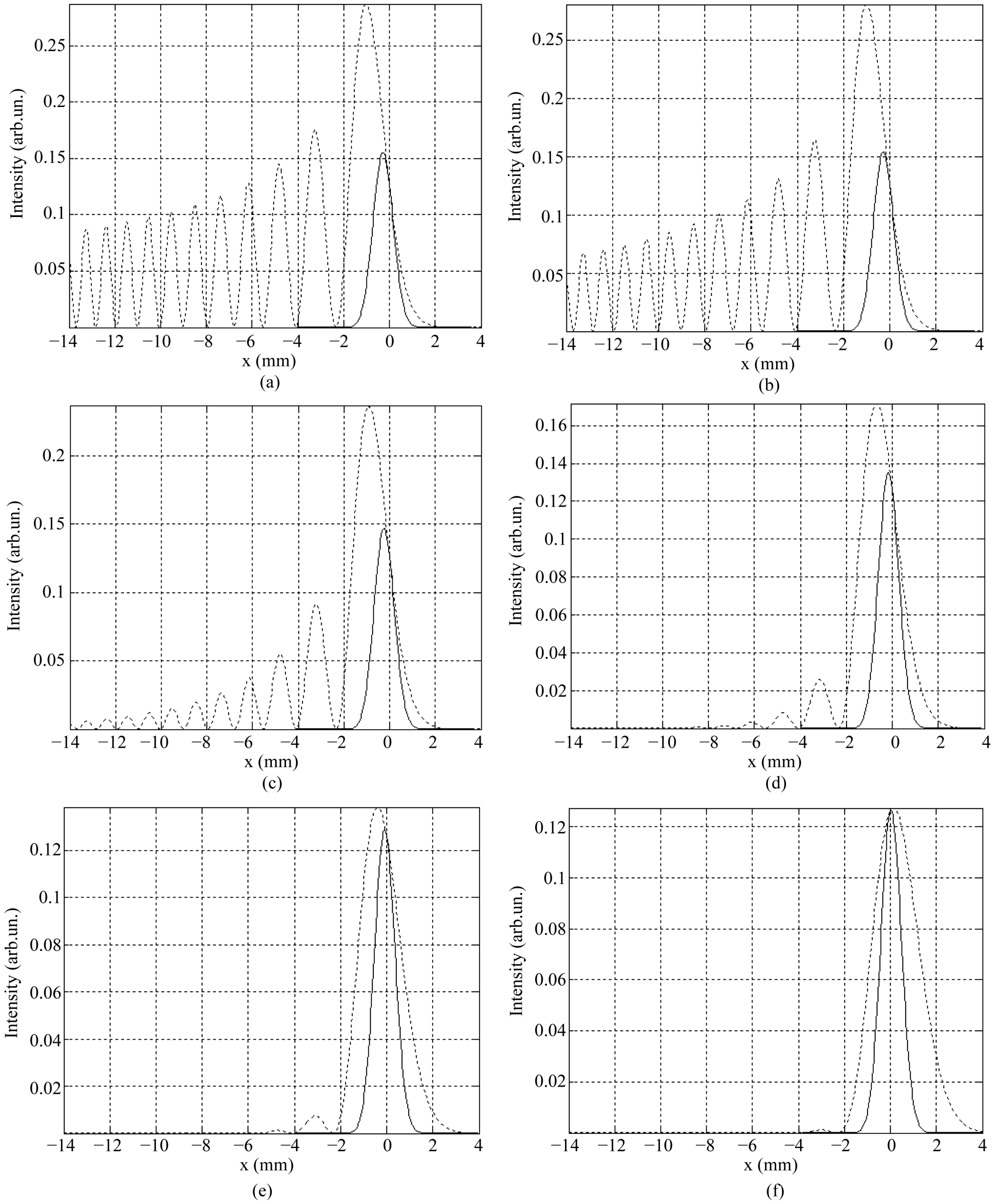

Figure 1. Intensity distributions of finite Airy beam (doted line) and finite Airy-Gaussian beam (solid line) at emitter plane $(z=0)$ versus transverse coordinate x for different aperture coefficients a_0: (a) $a_0=0$; (b) $a_0=0.01$; (c) $a_0=0.1$; (d) $a_0=0.3$; (e) $a_0=0.5$; (f) $a_0=0.8$ with $\omega_0=1\text{ mm}$.

where A_m and B_m are the expansion and Gaussian coefficients, respectively, which could be obtained by optimization-computation directly. M is the number of complex Gaussian terms.

Now, let us consider a misaligned optical system *ABCD* as schematized in **Figure 2**. The transformation of a light laser beam by such optical system with an aperture is expressed by the generalized Huygens-Fresnel dif-

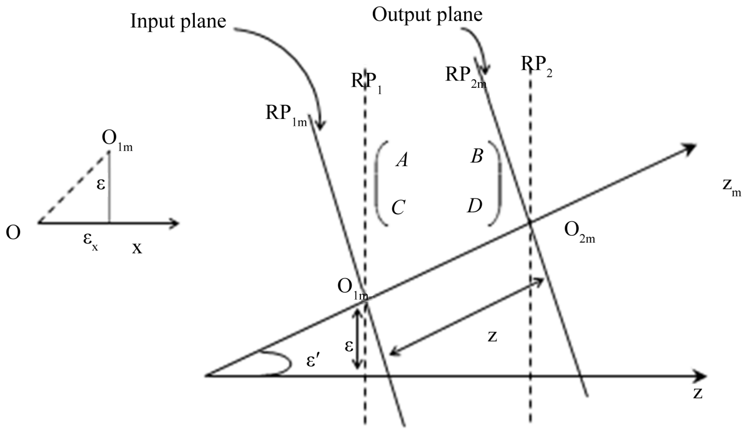

Figure 2. Schematic representation of a misaligned paraxial *ABCD* optical system.

fraction integral formulae for a misaligned optical system of the form [23] [33]-[35]

$$E(x,z)=\sqrt{\frac{i}{\lambda B}}\exp(-ikz)\cdot\int_{-\infty}^{+\infty}E(x_0,0)H(x_0)\exp\left[-\frac{ik}{2B}\left(Ax_0^2-2xx_0+Dx^2+Ex_0+Gx\right)\right]\mathrm{d}x_0, \quad (4)$$

where $k=\frac{2\pi}{\lambda}$ is the wave number and λ being the wavelength.

The coefficients *A*, *B* and *D* are elements of transfer matrix corresponding to the *ABCD* optical system after the aperture. $H(x_0)$ is the finite hard aperture function. The parameters *E* and *G* are elements characterizing the system misalignment and take the following expressions

$$E=2\left(\alpha_T\varepsilon_x+\beta_T\varepsilon_x'\right), \quad (5a)$$

$$G=2\left(B\gamma_T-D\alpha_T\right)\varepsilon_x+2\left(B\delta_T-D\beta_T\right)\varepsilon_x', \quad (5b)$$

where ε_x is the displacement and ε_x' is the tilting angle of the element. α_T, β_T, γ_T and δ_T represent the misaligned matrix elements determined by

$$\alpha_T=1-A, \quad (6a)$$

$$\beta_T=l-B, \quad (6b)$$

$$\gamma_T=-C, \quad (6c)$$

and

$$\delta_T=1-D. \quad (6d)$$

Substituting Equations (1) and (3) into Equation (4), the exiting beam in the observation plane of the apertured misaligned optical system is obtained as

$$E(x,z)=C_0(x,z)\sum_{m=1}^{M}A_m\int_{-\infty}^{+\infty}Ai\left(\frac{x_0}{\omega_0}\right)\cdot\exp\left[-\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)x_0^2+\left(\frac{a_0}{\omega_0}+\frac{ik}{B}\left(x-\frac{E}{2}\right)\right)x_0\right]\mathrm{d}x_0, \quad (7)$$

where

$$C_0(x,z)=\sqrt{\frac{i}{\lambda B}}\exp(-ikz)\cdot\exp\left[-\frac{ik}{2B}\left(Dx^2+Gx\right)\right]E_0. \quad (8)$$

In order to determine the above integral (7), the Airy function can be rewritten into representation integral as [36]

$$Ai(x)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\exp\left(i\left(\frac{u^3}{3}+xu\right)\right)\mathrm{d}u. \quad (9)$$

Inserting this equation into Equation (7) the field distribution of the outgoing beam of a finite Airy-Gaussian beam passing from an apertured misaligned optical system becomes

$$E(x,z) = C_0(x,z)\sum_{m=1}^{M} A_m \int_{-\infty}^{+\infty}\left[\exp\left(\frac{iu^3}{3}\right)\left(\int_{-\infty}^{+\infty}\exp\left[-\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)x_0^2+\left(\frac{a_0}{\omega_0}+\frac{iu}{\omega_0}+\frac{ik}{B}\left(x-\frac{E}{2}\right)\right)x_0\right]\mathrm{d}x_0\right)\right]\mathrm{d}u. \quad (10)$$

By means the well known integrals [36] [37]

$$\int_{-\infty}^{+\infty}\exp\left(-p^2y^2 \pm qy\right)\mathrm{d}y = \frac{\sqrt{\pi}}{p}\exp\left(\frac{q^2}{4p^2}\right) \qquad \left[\operatorname{Re} p^2 > 0\right], \quad (11)$$

and

$$\int_{-\infty}^{+\infty}\exp\left[i\left(\frac{t^3}{3}+\alpha t^2+\beta t\right)\right]\mathrm{d}t = 2\pi\exp\left[i\alpha\left(\frac{2\alpha^2}{3}-\beta\right)\right]\cdot Ai\left(\beta-\alpha^2\right), \quad (12)$$

the exiting electric field of a finite Airy-Gaussian beam propagating through an apertured misaligned optical system is obtained as

$$E(x,z) = 2\pi C_0(x,z)\sum_{m=1}^{M} A_m \frac{\sqrt{\pi}}{\sqrt{\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}}}\exp\left(\frac{1}{96\omega_0^6\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^3}\right)\exp\left(\frac{a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)}{8\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^2}\right)$$
$$\times\exp\left(\frac{\left(a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)\right)^2}{4\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)}\right)\times Ai\left(\frac{a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)}{2\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)}+\frac{1}{16\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^2}\right). \quad (13)$$

This last equation is the main result of the current work. It is the general analytical expression of the outgoing electric field of a finite Airy-Gaussian beam propagating through an apertured misaligned optical system at the receiver plane. From this result, it can easily be seen that the out-put beam at the observation plane of the misaligned optical system becomes decentred. The principle spot center is deviated away from the origin of the emitted plane by $E/2$ in transverse x-direction coordinate.

3. Particular Cases

3.1. Unapertured Misaligned Optical System

This special case can be obtained when $a \to \infty$, under this condition Equation (13) reduces to

$$E(x,z) = 2\pi C_0(x,z)\frac{\sqrt{\pi}}{\sqrt{\frac{1}{\omega_0^2}+\frac{ikA}{2B}}}\exp\left(\frac{1}{96\omega_0^6\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^3}\right)\exp\left(\frac{a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)}{8\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^2}\right)$$
$$\times\exp\left(\frac{\left(a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)\right)^2}{4\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)}\right)\times Ai\left(\frac{a_0+\frac{ik\omega_0}{B}\left(x-\frac{E}{2}\right)}{2\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)}+\frac{1}{16\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^2}\right). \quad (14)$$

This is the formula of an Airy-Gaussian beam passing through an unapertured misaligned optical system.

3.2. Apertured Aligned Optical System

When $\varepsilon_x = \varepsilon'_x = 0$, one find that the misalignment parameters are null, $E = G = 0$, the optical system arrives aligned and Equation (13) reduces to

$$E(x,z) = 2\pi\sqrt{\frac{i}{\lambda B}}\exp(-ikz)\cdot\exp\left[-\frac{ik}{2B}Dx^2\right]E_0\sum_{m=1}^{M}A_m\frac{\sqrt{\pi}}{\sqrt{\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}}}\exp\left(\frac{1}{96\omega_0^6\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^3}\right)$$
$$\times\exp\left(\frac{a_0+\frac{ik\omega_0}{B}x}{8\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^2}\right)\exp\left(\frac{\left(a_0+\frac{ik\omega_0}{B}x\right)^2}{4\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)}\right) \tag{15}$$
$$\times Ai\left[\frac{a_0+\frac{ik\omega_0}{B}x}{2\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)}+\frac{1}{16\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{B_m}{a^2}+\frac{ikA}{2B}\right)^2}\right].$$

This is the analytical formula of outgoing electric field of the Airy-Gaussian beam passing through an aligned paraxial *ABCD* optical system with a finite hard aperture.

3.3. Unapertured Aligned Optical System

This situation could be obtained if $a \to \infty$ and $\varepsilon_x = \varepsilon'_x = 0$. Under these conditions, Equation (13) becomes

$$E(x,z) = 2\pi\sqrt{\frac{i}{\lambda B}}\exp(-ikz)\cdot\exp\left[-\frac{ik}{2B}Dx^2\right]E_0\frac{\sqrt{\pi}}{\sqrt{\frac{1}{\omega_0^2}+\frac{ikA}{2B}}}\exp\left(\frac{1}{96\omega_0^6\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^3}\right)$$
$$\times\exp\left(\frac{a_0+\frac{ik\omega_0}{B}x}{8\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^2}\right)\exp\left(\frac{\left(a_0+\frac{ik\omega_0}{B}x\right)^2}{4\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)}\right)Ai\left(\frac{a_0+\frac{ik\omega_0}{B}x}{2\omega_0^2\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)}+\frac{1}{16\omega_0^4\left(\frac{1}{\omega_0^2}+\frac{ikA}{2B}\right)^2}\right). \tag{16}$$

This closed-form expression characterizes the propagation of Airy-Gaussian beam through an unapertured aligned paraxial *ABCD* optical system.

4. Numerical Simulations and Discussions

According to the obtained analytical expression established in Equation (13), the properties of an Airy-Gaussian beam through an apertured misaligned optical system are investigated numerically in this section. Let us consider an Airy-Gaussian beam propagating through an apertured misaligned circular thin lens placed at waist plane, $z = 0$, followed by a free space. The matrix corresponding to this optical system has the form

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} -\frac{z-f}{f} & z-\frac{z-f}{f}s \\ -\frac{1}{f} & 1-\frac{s}{f} \end{pmatrix}, \tag{17}$$

where s is the axial distance between the plane waist and the thin lens. In our situation, we take $s = 0$, f is the thin lens focus length, and z is the distance from the input plane to the observation plane (is the propagation distance). The parameters used in the simulations are: the wavelength $\lambda = 632.8\ \text{nm}$, the waist size of the incident beam $\omega_0 = 1\ \text{mm}$, the angle misalignment of the lens with respect to the optical propagation axis chosen as $\varepsilon'_x = 0$. The misalignment parameters α_T, β_T, γ_T and δ_T take the following expressions

$$\alpha_T = \frac{z}{f}, \quad \beta_T = 0, \quad \gamma_T = \frac{1}{f} \quad \text{and} \quad \delta_T = 0, \tag{18}$$

and the corresponding parameters E and G are

$$E = 2\frac{z\varepsilon_x}{f} \quad \text{and} \quad G = 0. \tag{19}$$

In order to validate the theoretical finding, in the following we will discuss the effect of some factors including elements system displacement ε_x, propagation distance z and thin lens focal length f on deviation of the out-put beam at the observation plane.

Figure 3 displays the normalized intensity of finite Airy-Gaussian beam through an apertured misaligned

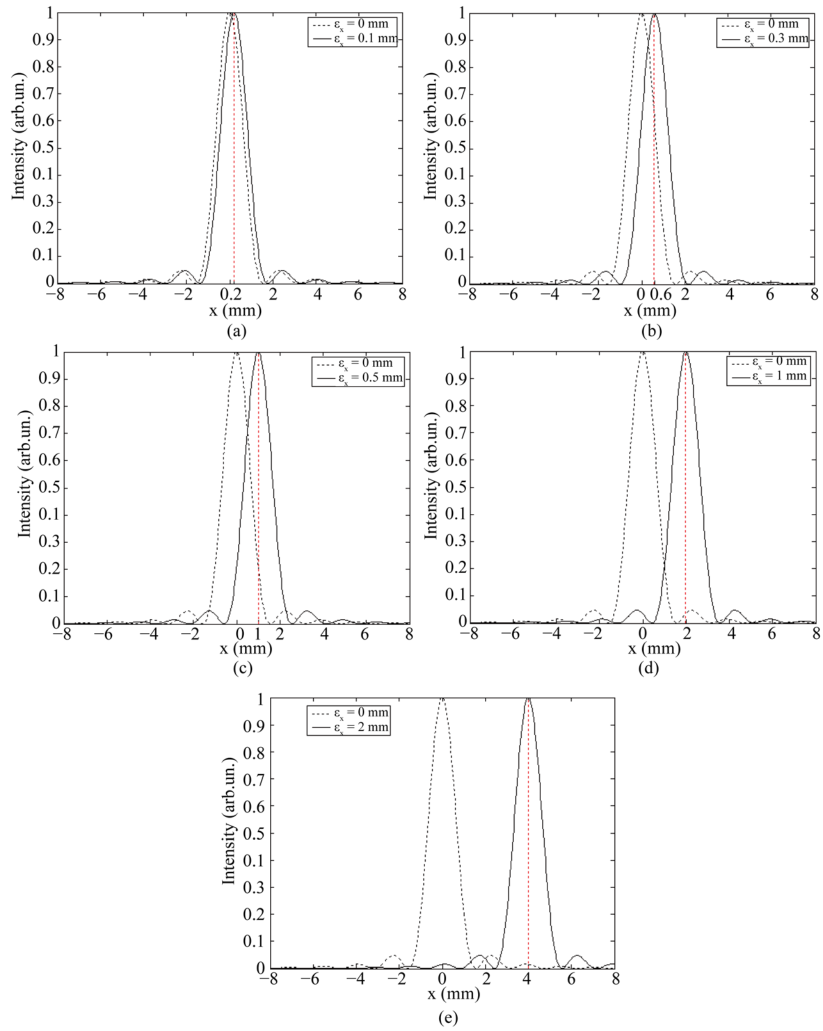

Figure 3. Intensity distribution at the output plane of Airy-Gaussian beams passing through an aligned (solid line) and misaligned (doted line) thin lenses for different thin lens displacements ε_x: (a) $\varepsilon_x = 0.1$; (b) $\varepsilon_x = 0.3$; (c) $\varepsilon_x = 0.5\ \text{mm}$; (d) $\varepsilon_x = 0.1\ \text{mm}$ and (e) $\varepsilon_x = 2\ \text{mm}$, with $\omega_0 = 1\ \text{mm}$, $\lambda = 0.6328\ \mu\text{m}$, $a_0 = 0.8\ \text{mm}$, a = 0.1 mm, f = 250 mm and $z = 500\ \text{mm}$. Red vertical doted line indicates the spot deviation quantity.

optical system versus the transverse coordinate x for different elements displacement ε_x. The other parameters are fixed at $a = 0.1$ mm, $f = 250$ mm and $z = 500$ mm. From the curves of this figure, it appears that the center of this exiting beam is shifted effectively. Elements optical displacements $\varepsilon_x = 0.1$, 0.3, 0.5, 1 and 2 mm lead to deviation of exiting beam by $\Delta x = 0.2$, 0.6, 1.2 and 4 mm, respectively. Theses deviations correspond, in each time, to $E/2 \ \left(= z\varepsilon_x / f\right)$.

Figure 4 is the same as **Figure 3**, but in this time for fixed $f = 150$ mm, $a = 0.1$ mm and $\varepsilon_x = 0.1$ mm and for different propagation distances $z = 125$, 250, 500 and 750 mm. Their corresponding outgoing beam displacements are $E/2 = 0.5$, 1, 2 and 4 mm, respectively.

Figure 5 is similar to **Figure 3** and **Figure 4**, but in this time for fixed z, ε_x and a and for various propagation distances f. The centre of the output beam is shifted inversely in proportion to the thin lens focal length f = 125, 250, 500 and 750 mm lead to exiting beam shift $\Delta x = 4$, 2, 1 and 0.5 mm.

Generally, a displacement of element optical system affects a shift of the exiting beam. The deviation degree increases with an increase in elements optical system displacement ε_x or with a fixed $\varepsilon_x \neq 0$ accompanied with an augmentation in propagation distance z or a diminution in thin lens focal length. The deviation quantity is proportional to optical system elements displacement, to propagation distance and inversely proportional to thin lens focal length.

Practically, the misalignment of the optical system can be a tool or a technique for the determination of a thin lens focal length. Knowing the elements displacement ε_x and propagation distance z and the coordinates of

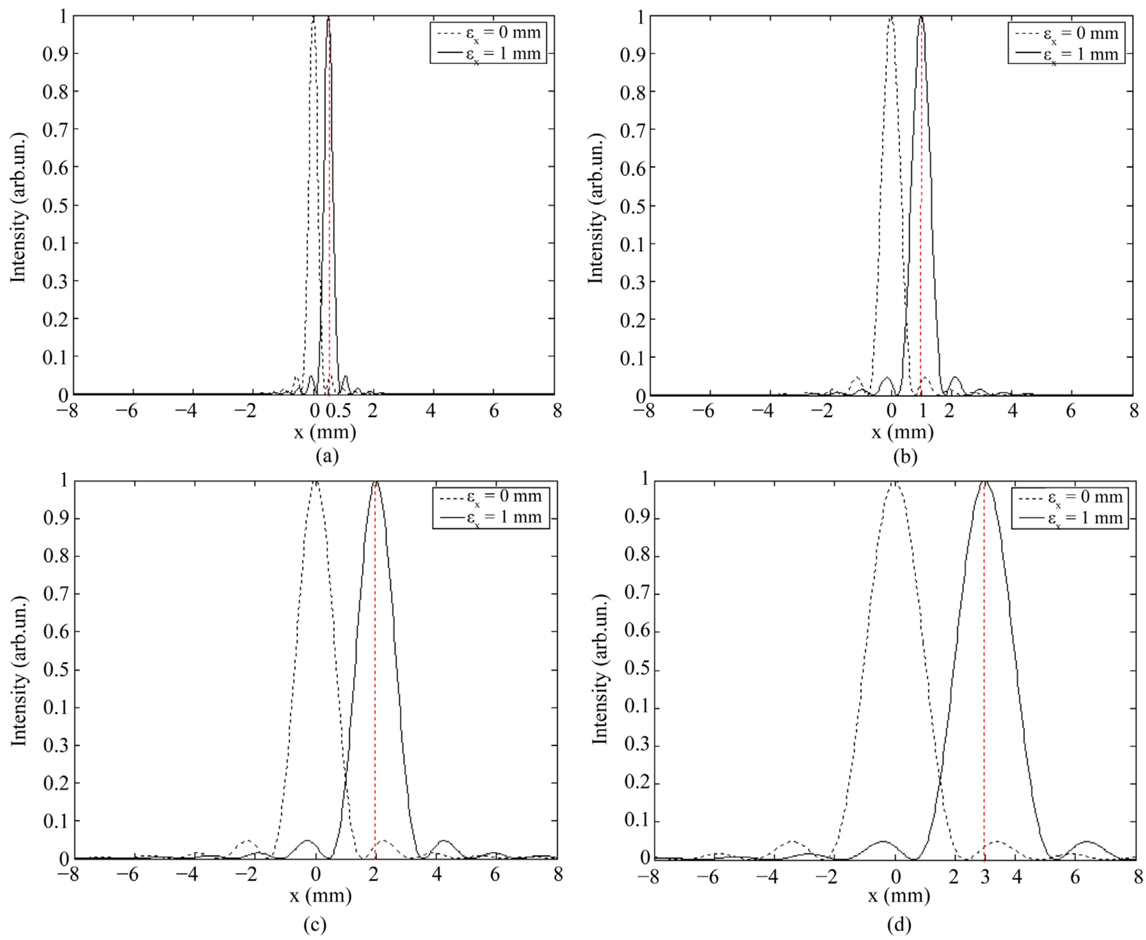

Figure 4. Intensity distribution at the output plane of Airy-Gaussian beams passing through an aligned (solid line) and misaligned (doted line) thin lenses for different propagation distances z: (a) $z = 125$ mm; (b) $z = 250$ mm; (c) $z = 500$ mm and (d) z = 750 mm, with $\omega_0 = 1$ mm, $\lambda = 0.6328$ µm, $a_0 = 0.8$, $a = 0.1$ mm, f = 250 mm and $\varepsilon_x = 1$ mm. Red vertical doted line indicates the spot deviation quantity.

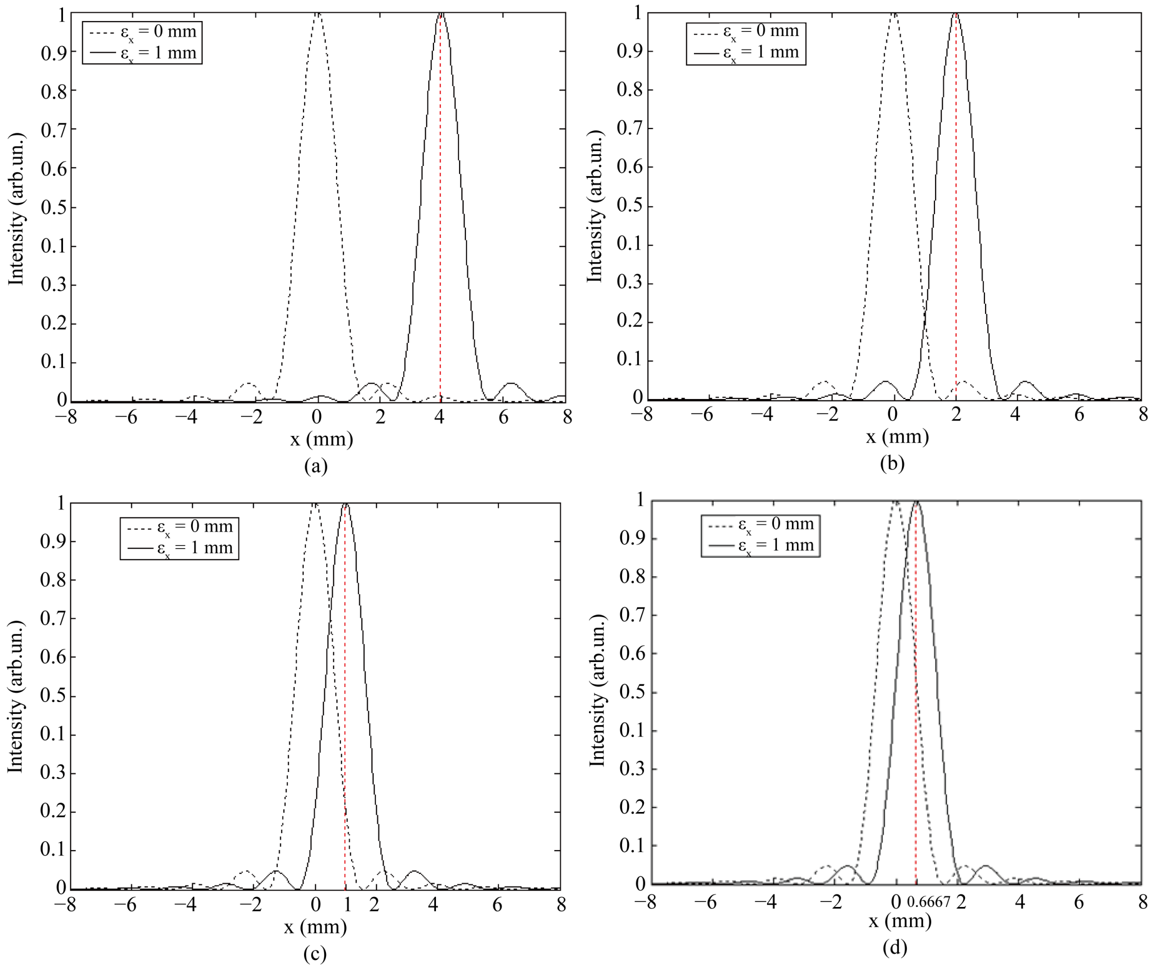

Figure 5. Intensity distribution at the output plane of Airy-Gaussian beams passing through an aligned (solid line) and misaligned (doted line) thin lenses for different thin lens focal length *f*: (a) $f = 125$ mm; (b) $f = 250$ mm; (c) $f = 500$ mm and (d) $f = 750$ mm, with $\omega_0 = 1\ \text{mm}$, $\lambda = 0.6328\ \mu\text{m}$, $a_0 = 0.8$, $a = 0.1\ \text{mm}$ and $z = 500\ \text{mm}$. Red doted line indicates the spot deviation quantity.

the center of exiting spot $(E/2, z)$ and with help the relationship $E = 2\dfrac{z\varepsilon_x}{f}$, one can easily deduce f.

To consolidate our theoretical and numerical finding concerning the deviation of the exiting beam from a misaligned optical system, we display in **Figure 6** the cross three-dimensional intensity distribution of the outgoing finite Airy-Gaussian beams intensity along the meridian plane (x, z). From the plots of this figure, we can find that the deviation degree of the beam in x-direction depends on the displacement quantity. For an indicated point located at (x, z) coordinates, the deviation degree of the spot is proportional to optical system displacement ε_x.

5. Conclusion

Based on the generalized Huygens-Fresnel diffraction integral and by expanding of the hard edged aperture function into a finite sum of complex Gaussian functions, we have come up with an approximate analytical expression for determining and analyzing the propagation properties of finite Airy-Gaussian beam through an apertured misaligned optical system. This study generalizes the cases of propagation of Airy-Gaussian beam through unapertured misaligned optical system, apertured aligned optical system and unapertured aligned optical system, which are regarded as special cases of our main investigation. The numerical simulations developed

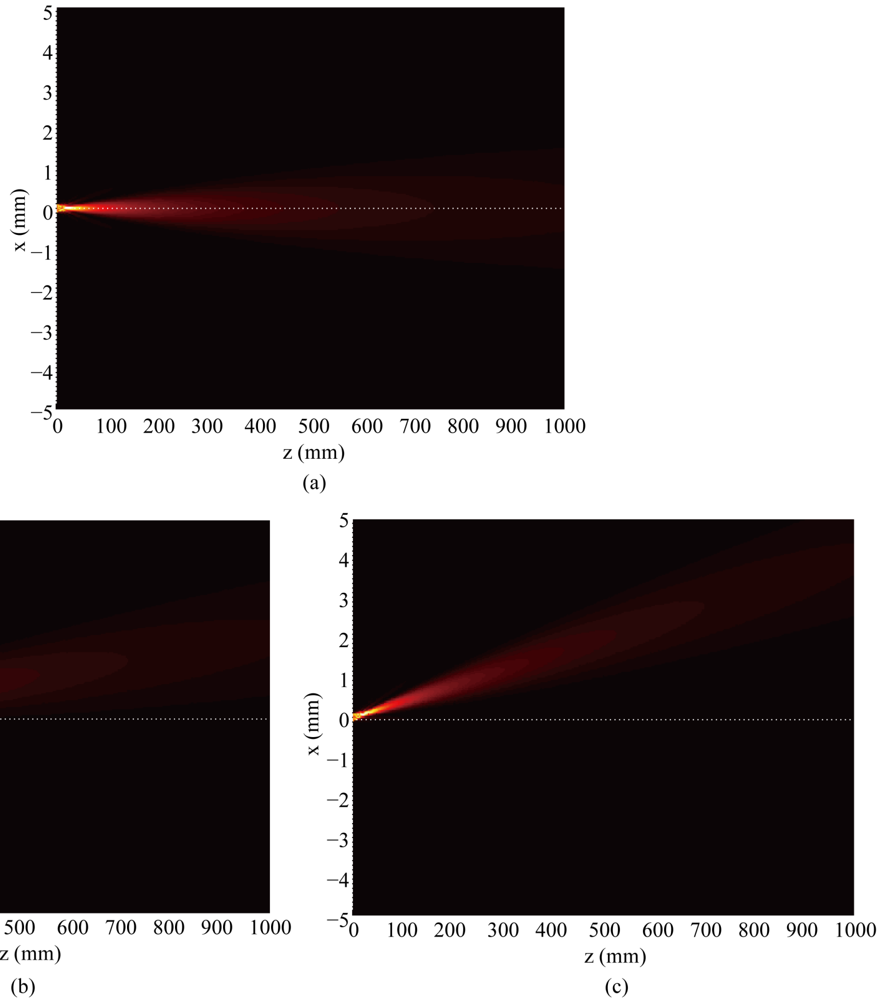

Figure 6. Intensity distribution at the output plane of finite Airy-Gaussian beams passing through a misaligned thin lens for different element displacement ε_x in the meridian plane (z, x), with $\omega_0 = 1$ mm, $\lambda = 0.6328$ µm, a_0 = 0.8, a = 0.1 mm and f = 250 mm. (a) ε_x = 0 mm; (b) ε_x = 0.5 mm and (c) ε_x = 1 mm.

in the paper show that the exiting beam keeps similar properties of its incident beam but it shifts from the propagation axis.

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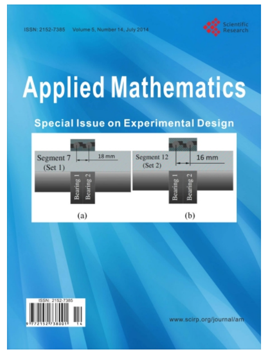
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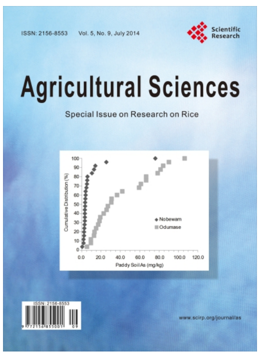
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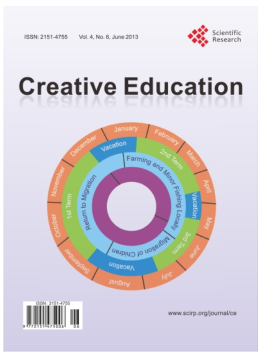
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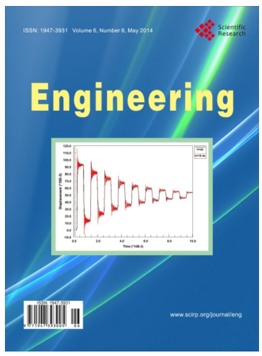
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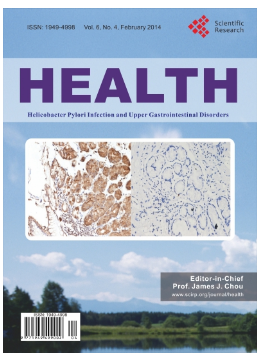
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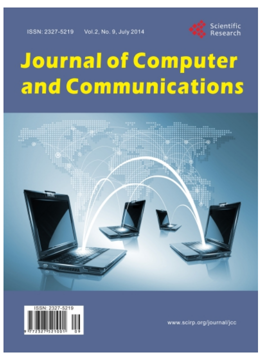
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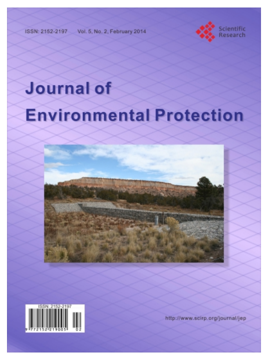
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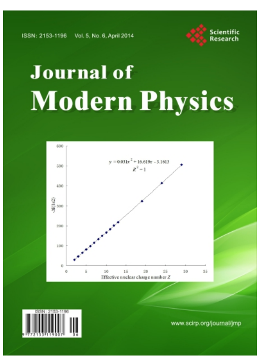
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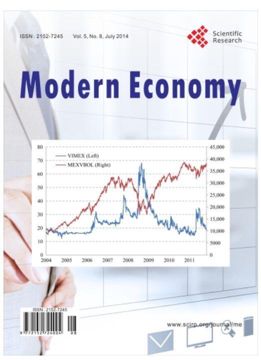
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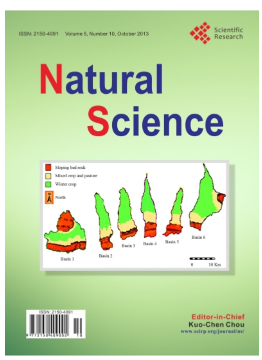
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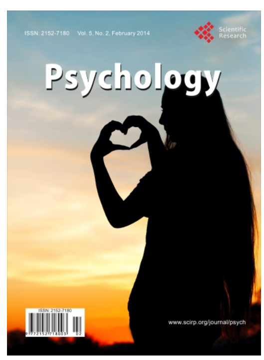
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