Scientific Research Publishing

Open Journal of Statistics, 2016, 6, 873-881
http://www.scirp.org/journal/ojs
ISSN Online: 2161-7198
ISSN Print: 2161-718X

Estimation of Reliability for Stress-Strength Cascade Model

Rohit R. Mutkekar[1], Surekha B. Munoli[2]

[1]Goa Institute of Management, Sanquelim, India
[2]Department of Statistics, Karnatak University, Dharwad, India
Email: rohitrm@gim.ac.in, sbmunoli@yahoo.co.in

How to cite this paper: Mutkekar, R.R. and Munoli, S.B. (2016) Estimation of Reliability for Stress-Strength Cascade Model. *Open Journal of Statistics*, **6**, 873-881.
http://dx.doi.org/10.4236/ojs.2016.65072

Received: March 25, 2016
Accepted: October 18, 2016
Published: October 21, 2016

Abstract

The study endeavors to provide statistical inference for a (1 + 1) cascade system for exponential distribution under joint effect of stress-strength attenuation factors. Estimators of reliability function are obtained using Maximum Likelihood Estimator (MLE) and Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the parameters. Asymptotic distribution of the parameters is also obtained. Comparison between estimators is made using data obtained through simulation experiment.

Keywords

Cascade System, Strength Attenuation Factor, Reliability Modelling, Life Testing Experiment, Estimators of Reliability Function

1. Introduction

As the complexity of a system increases, its reliability decreases unless compensatory measures are taken. System reliability can be increased by increasing the reliability of its associated components, but sometimes this cannot be achieved beyond certain limits. An alternative way to increase the reliability in such situation is to have redundant configuration of components in the system.

Cascade system is one such special type of standby system. Cascade redundancy is a hierarchical standby redundancy, where an array of components (finite in number) are arranged in the order of activation. Here, the first component is active and the remaining components are at standby. The brunt of attack, in the first instance is borne by the active component. If it survives the attack, the system also survives with no loss and is ready to face the next attack. However, if the active component fails then the next component in the array has to face and withstand the "cushioned" attack on it. The

stress acting on the subsequent active component will be "k" times the stress of the previous failed components, where "k" denotes stress attenuation factor.

Research works on reliability modelling and assessment related to cascade model as studied in the literature are quite exhaustive, Pandit and Sriwastav (1975) have featured relevance of geometric distribution in the study of behavior of a cascade system [1]. Raghavachar, Rao and Ramacharyulu (1983) presented a closed form solution of stress attenuated reliability function for n-cascade system when both stress and strength follow identical distributions [2]. Maheshwari, Rekha, Rao and Raghavachar (1993) studied stress attenuated reliability for n-cascade system whose stress and strength follow normal and exponential distributions respectively [3]. Rekha and Shyam Sunder (1997) have also highlighted a similar cascade system where stress and strength follow gamma and exponential distributions respectively. They showed that for higher parametric values and lower attenuation factors a high degree of reliability could be attained [4]. Rekha and ChechuRaju (1999) endeavored to present a closed form solution of stress attenuated reliability function for n-cascade system with exponential stress and standby strengths following Rayleigh and exponential distributions [5]. Shyam Sunder (2012) has studied stress attenuation for cascade system when both stress and strength follow Rayleigh distribution [6]. In most of the works mentioned in the literature on cascade model, study is carried out by considering the influence of stress attenuation factor only. This observation has motivated the present study of attempting to design reliability model for a cascade system under joint effect of stress as well as strength attenuation factors. Further, reliability assessment (estimation of reliability function) is carried using the standard methods [7]-[10].

2. Estimation of Reliability for a (1 + 1) Cascade Model

2.1. Stress-Strength Cascade Model

Let $X_1, X_2, \cdots, X_n$ denote the strengths of n-components in the order of activation and let $Y_1, Y_2, \cdots, Y_n$ be the corresponding stresses acting on them. In a n-cascade system after every failure the stress gets modified by a factor "k" (stress attenuation factor) such that $Y_2 = kY_1, Y_3 = kY_2 = k^2Y_1, \cdots, Y_i = k^{i-1}Y_1$ here, $k > 1$ and we assume that the strength gets modified by a factor "m" (strength attenuation factor) such that $X_2 = mX_1, X_3 = mX_2 = m^2X_1, \cdots, X_i = m^{i-1}X_1$ here, $0 < m < 1$.

The reliability function R_n of the system with 'n' components is defined as,

$$R_n = R(1) + R(2) + \cdots + R(n)$$

where,

$$R(r) = P\left[X_1 < Y_1, mX_1 < kY_1, \cdots, m^{r-2}X_1 < k^{r-2}Y_1, m^{r-1}X_1 \geq k^{r-1}Y_1\right] \text{ for } r = 1, 2, \cdots, n$$

Cascade model with more number of standby components is not recommended as the strength goes on depleting with the order of standby which leads to dead investment. In view of this fact, we have considered estimation of reliability for a (1 + 1) cascade model.

2.2. Reliability Function for a (1 + 1) Cascade Model

To determine reliability function for the model under study, let us consider the strength of the two components (basic and standby) to be X_1 and X_2 respectively, where X_1, X_2 are independently and identically distributed (i.i.d) exponential random variables with parameter "λ". Let Y_1 and Y_2 be the stress acting on the two components respectively, where Y_1, Y_2 are i.i.d exponential random variables with parameter 'μ'. To obtain the expression for reliability function, consider,

$$R(1) = P[X_1 \geq Y_1] = \int_0^\infty P[X_1 \geq y_1] g_{Y_1}(y_1)\mathrm{d}y_1 = \frac{\mu}{(\lambda+\mu)} \tag{1}$$

$$\begin{aligned} R(2) &= P[X_1 < Y_1, X_2 \geq Y_2] = P[X_1 < Y_1, mX_1 \geq kY_1] \\ &= \int_0^\infty P[X_1 < y_1] P[mX_1 \geq ky_1] g_{Y_1}(y_1)\mathrm{d}y_1 \\ &= \frac{\mu\lambda}{\left(\mu+\lambda\frac{k}{m}\right)\left\{\mu+\lambda\left(\frac{k}{m}+1\right)\right\}} \end{aligned} \tag{2}$$

Using results of (1) and (2), we obtain reliability function for the proposed (1 + 1) cascade model as,

$$R_2 = R(1) + R(2) = \frac{\mu}{(\lambda+\mu)} + \frac{\mu\lambda}{\left(\mu+\lambda\frac{k}{m}\right)\left\{\mu+\lambda\left(\frac{k}{m}+1\right)\right\}} \tag{3}$$

2.3. Life Testing Experiment

To obtain the estimators of "R_2", suppose "n" systems whose reliability function is defined as in expression (3) are put on life testing experiment. Here, $X_{1i}, X_{2i}\ (i = 1, 2, \cdots, n)$ are observed and X_{1i}, X_{2i} are i.i.d exponential random variables with parameters "λ" and "$\frac{\lambda}{m}$" respectively. Also, the data of stress $Y_{1i}, Y_{2i}\ (i = 1, 2, \cdots, n)$ are obtained separately from simulation of conditions of the operating environment and Y_{1i}, Y_{2i} are i.i.d exponential random variables with parameter "μ" and "$\frac{\mu}{k}$" respectively. The joint probability density function of the random variables $X_{1i}, X_{2i}\ (i = 1, 2, \cdots, n)$ and $Y_{1i}, Y_{2i}\ (i = 1, 2, \cdots, n)$ is given by,

$$\begin{aligned} L &= \prod_{i=1}^{n}\left[\left(\lambda \mathrm{e}^{-\lambda x_{1i}}\right)\left(\frac{\lambda}{m}\mathrm{e}^{-\frac{\lambda}{m}x_{2i}}\right)\right]\prod_{i=1}^{n}\left[\left(\mu \mathrm{e}^{-\mu y_{1i}}\right)\left(\frac{\mu}{k}\mathrm{e}^{-\frac{\mu}{k}y_{2i}}\right)\right] \\ &= \frac{\lambda^{2n}}{m^n}\mathrm{e}^{-\lambda\left[\sum_{i=1}^{n}x_{1i}+\frac{1}{m}\sum_{i=1}^{n}x_{2i}\right]}\frac{\mu^{2n}}{k^n}\mathrm{e}^{-\mu\left[\sum_{i=1}^{n}y_{1i}+\frac{1}{k}\sum_{i=1}^{n}y_{2i}\right]} \\ &= \frac{\lambda^{2n}}{m^n}\mathrm{e}^{-\lambda\left[x_1'+\frac{x_2'}{m}\right]}\frac{\mu^{2n}}{k^n}\mathrm{e}^{-\mu\left[y_1'+\frac{y_2'}{k}\right]} \end{aligned} \tag{4}$$

where,

$$x_1' = \sum_{i=1}^{n} x_{1i},\ x_2' = \sum_{i=1}^{n} x_{2i},\ y_1' = \sum_{i=1}^{n} y_{1i},\ y_2' = \sum_{i=1}^{n} y_{2i}$$

The log-likelihood function of Equation (4) is obtained as,

$$\log L = 2n\log\lambda - n\log m - \lambda\left[x_1' + \frac{x_2'}{m}\right] + 2n\log\mu - n\log k - \mu\left[y_1' + \frac{y_2'}{k}\right] \tag{5}$$

2.4. Estimators of Reliability Function (MLE & UMVUE)

Differentiating the log-likelihood function given in Equation (5) partially with respect to λ, m and equating it to zero, we get,

$$\frac{\partial \log L}{\partial \lambda} = 0 \Rightarrow \frac{2n}{\lambda} - \left[x_1' + \frac{x_2'}{m}\right] = 0 \tag{6}$$

$$\frac{\partial \log L}{\partial m} = 0 \Rightarrow \frac{-n}{m} + \frac{\lambda x_2'}{m^2} = 0 \Rightarrow -n + \frac{\lambda x_2'}{m} = 0 \tag{7}$$

Solving Equations ((6) and (7)) simultaneously, we get the Maximum Likelihood Estimator (MLE) of λ and m as,

$$\hat{\lambda} = \frac{n}{x_1'} \tag{8}$$

$$\hat{m} = \frac{x_2'}{x_1'} \tag{9}$$

Similarly, differentiating the log-likelihood function given in Equation (5) with respect to μ, k and equating it to zero, we get,

$$\frac{\partial \log L}{\partial \mu} = 0 \Rightarrow \frac{2n}{\mu} - \left[y_1' + \frac{y_2'}{k}\right] = 0 \tag{10}$$

$$\frac{\partial \log L}{\partial k} = 0 \Rightarrow \frac{-n}{k} + \frac{\mu y_2'}{k^2} = 0 \Rightarrow -n + \frac{\mu y_2'}{k} = 0 \tag{11}$$

Solving Equations ((10) and (11)) simultaneously, we get the MLE of μ and k as,

$$\hat{\mu} = \frac{n}{y_1'} \tag{12}$$

$$\hat{k} = \frac{y_2'}{y_1'} \tag{13}$$

Using the invariance property of MLE, the MLE of reliability function '$\widehat{R_2}$' is obtained by substituting the MLEs of λ, m, μ, k in Equation (3) and is given by,

$$\widehat{R_2} = \frac{\hat{\mu}}{\left(\hat{\lambda} + \hat{\mu}\right)} + \frac{\hat{\mu}\hat{\lambda}}{\left(\hat{\mu} + \hat{\lambda}\dfrac{\hat{k}}{\hat{m}}\right)\left\{\hat{\mu} + \hat{\lambda}\left(\dfrac{\hat{k}}{\hat{m}} + 1\right)\right\}} \tag{14}$$

Here, $\widehat{R_2}$ denotes the estimator of reliability function obtained through MLE of the parameters. Further, estimator of the reliability function "R_2^*" attained through the Uniformly Minimum Variance Unbiased Estimator (UMVUE) of the parameters is obtained as follows.

We know that, $x_{1i} \sim \text{Exponential}(\lambda)$, implies $x_1' = \sum_{i=1}^{n} x_{1i} \sim \text{Gamma}(n, \lambda)$

$$\Rightarrow E\left[\frac{1}{x_1'}\right]=\int_0^{\infty}\frac{1}{x_1'}\frac{\lambda^n e^{-\lambda x_1'}\left(x_1'\right)^{n-1}}{\Gamma n}dx_1'=\frac{\lambda^n}{\Gamma n}\int_0^{\infty}e^{-\lambda x_1'}\left(x_1'\right)^{(n-1)-1}dx_1'$$

$$\Rightarrow E\left[\frac{(n-1)}{x_1'}\right]=\lambda$$

$$\Rightarrow \lambda^*=\left[\frac{(n-1)}{x_1'}\right] \text{ is the UMVUE of parameter } \lambda \tag{15}$$

Also, $y_{1i}\sim \text{Exponential}(\mu)$, implies $y_1'=\sum_{i=1}^{n}y_{1i}\sim \text{Gamma}(n,\mu)$
On similar grounds we have,

$$\mu^*=\left[\frac{(n-1)}{y_1'}\right] \text{ is the UMVUE of parameter } \mu \tag{16}$$

Similarly, $x_{2i}\sim \text{Exponential}\left(\frac{\lambda}{m}\right)$, implies $x_2'=\sum_{i=1}^{n}x_{2i}\sim \text{Gamma}\left(n,\frac{\lambda}{m}\right)$

$$\Rightarrow E\left[x_2'\right]=\int_0^{\infty}x_2'\frac{\left(\frac{\lambda}{m}\right)^n e^{-\frac{\lambda}{m}x_2'}\left(x_2'\right)^{n-1}}{\Gamma n}dx_2'=\frac{\left(\frac{\lambda}{m}\right)^n}{\Gamma n}\int_0^{\infty}e^{-\frac{\lambda}{m}x_2'}\left(x_2'\right)^{(n+1)-1}dx_2'$$

$$\Rightarrow E\left[x_2'\right]=\frac{mn}{\lambda}$$

Substituting $\lambda=\lambda^*$ [result as mentioned in Equation (15)], we get,

$$\Rightarrow E\left[x_2'\right]=\frac{mnx_1'}{n-1}$$

$$\Rightarrow E\left[\frac{(n-1)x_2'}{nx_1'}\right]=m$$

$$\Rightarrow m^*=\left[\frac{(n-1)x_2'}{nx_1'}\right] \text{ is the UMVUE of parameter } m \tag{17}$$

Also, $y_{2i}\sim \text{Exponential}\left(\frac{\mu}{k}\right)$, implies $y_2'=\sum_{i=1}^{n}y_{2i}\sim \text{Gamma}\left(n,\frac{\mu}{k}\right)$

On similar grounds we have,

$$k^*=\left[\frac{(n-1)y_2'}{ny_1'}\right] \text{ is the UMVUE of parameter } k \tag{18}$$

Substituting the UMVUEs of λ,m,μ,k in Equation (3), we get estimator of the reliability function "R_2^*" obtained through the UMVUE of the parameters.

2.5. Asymptotic Distribution

To obtain the asymptotic distribution of $\hat{\lambda},\hat{m},\hat{\mu},\hat{k}$, let us denote the Fisher Information Matrix of λ,m,μ,k as $I(\lambda,m,\mu,k)$.

$$I(\lambda,m,\mu,k)=-\begin{bmatrix} E\left(\frac{\partial^2 L}{\partial\lambda^2}\right) & E\left(\frac{\partial^2 L}{\partial\lambda\partial m}\right) & E\left(\frac{\partial^2 L}{\partial\lambda\partial\mu}\right) & E\left(\frac{\partial^2 L}{\partial\lambda\partial k}\right) \\ E\left(\frac{\partial^2 L}{\partial m\partial\lambda}\right) & E\left(\frac{\partial^2 L}{\partial m^2}\right) & E\left(\frac{\partial^2 L}{\partial m\partial\mu}\right) & E\left(\frac{\partial^2 L}{\partial m\partial k}\right) \\ E\left(\frac{\partial^2 L}{\partial\mu\partial\lambda}\right) & E\left(\frac{\partial^2 L}{\partial\mu\partial m}\right) & E\left(\frac{\partial^2 L}{\partial\mu^2}\right) & E\left(\frac{\partial^2 L}{\partial\mu\partial k}\right) \\ E\left(\frac{\partial^2 L}{\partial k\partial\lambda}\right) & E\left(\frac{\partial^2 L}{\partial k\partial m}\right) & E\left(\frac{\partial^2 L}{\partial k\partial\mu}\right) & E\left(\frac{\partial^2 L}{\partial k^2}\right) \end{bmatrix}$$

$$=\begin{bmatrix} I_{11} & I_{12} & I_{13} & I_{14} \\ I_{21} & I_{22} & I_{23} & I_{24} \\ I_{31} & I_{32} & I_{33} & I_{34} \\ I_{41} & I_{42} & I_{43} & I_{44} \end{bmatrix} \text{(say)}$$

where,

$$I_{11}=-\left[E\left(\frac{\partial^2 L}{\partial\lambda^2}\right)\right]=\frac{2n}{\lambda^2}$$

$$I_{22}=-\left[E\left(\frac{\partial^2 L}{\partial m^2}\right)\right]=\frac{n}{m^2}$$

$$I_{33}=-\left[E\left(\frac{\partial^2 L}{\partial\mu^2}\right)\right]=\frac{2n}{\mu^2}$$

$$I_{44}=-\left[E\left(\frac{\partial^2 L}{\partial k^2}\right)\right]=\frac{n}{k^2}$$

$$-E\left(\frac{\partial^2 L}{\partial\lambda\partial m}\right)=-E\left(\frac{\partial^2 L}{\partial m\partial\lambda}\right)\Rightarrow I_{12}=I_{21}=\frac{-n}{m\lambda}$$

$$-E\left(\frac{\partial^2 L}{\partial\mu\partial k}\right)=-E\left(\frac{\partial^2 L}{\partial k\partial\mu}\right)\Rightarrow I_{34}=I_{43}=\frac{-n}{k\mu}$$

$$-E\left(\frac{\partial^2 L}{\partial\lambda\partial\mu}\right)=-E\left(\frac{\partial^2 L}{\partial\mu\partial\lambda}\right)=0\Rightarrow I_{13}=I_{31}=0$$

$$-E\left(\frac{\partial^2 L}{\partial\lambda\partial k}\right)=-E\left(\frac{\partial^2 L}{\partial k\partial\lambda}\right)=0\Rightarrow I_{14}=I_{41}=0$$

$$-E\left(\frac{\partial^2 L}{\partial m\partial\mu}\right)=-E\left(\frac{\partial^2 L}{\partial\mu\partial m}\right)=0\Rightarrow I_{23}=I_{32}=0$$

$$-E\left(\frac{\partial^2 L}{\partial m\partial k}\right)=-E\left(\frac{\partial^2 L}{\partial k\partial m}\right)=0\Rightarrow I_{24}=I_{42}=0$$

Thus, we have the Fisher Information Matrix as,

$$I(\lambda, m, \mu, k) = \begin{bmatrix} \frac{2n}{\lambda^2} & \frac{-n}{m\lambda} & 0 & 0 \\ \frac{-n}{m\lambda} & \frac{n}{m^2} & 0 & 0 \\ 0 & 0 & \frac{2n}{\mu^2} & \frac{-n}{k\mu} \\ 0 & 0 & \frac{-n}{k\mu} & \frac{n}{k^2} \end{bmatrix}$$

From the asymptotic properties of MLE under regularity conditions and multivariate central limit theorem we have,

$$\left[(\hat{\lambda}-\lambda),(\hat{m}-m),(\hat{\mu}-\mu),(\hat{k}-k)\right] \to Normal\left(0, I^{-1}\right)$$

I^{-1} is inverse of Fisher Information Matrix "$I(\lambda, m, \mu, k)$" where,

$$I^{-1} = \begin{bmatrix} \frac{\lambda^2}{n} & \frac{m\lambda}{n} & 0 & 0 \\ \frac{m\lambda}{n} & \frac{2m^2}{n} & 0 & 0 \\ 0 & 0 & \frac{\mu^2}{n} & \frac{k\mu}{n} \\ 0 & 0 & \frac{k\mu}{n} & \frac{2k^2}{n} \end{bmatrix}$$

3. Simulation Experiment

For the i^{th} system, the random variables x_{1i}, x_{2i} (with respect to strength) and random variables y_{i1}, y_{i2} (with respect to stress) are generated independently as follows:

Step 1: Initialize $i_1 = 1, n = n_0, \lambda = \lambda_0, m = m_0$ for the 1st and 2nd component of the system. Uniform random numbers $U_1[i_1]$ is generated from $U(0,1)$. Further, exponential random variable $x_{1i} = \left(\frac{-1}{\lambda_0}\right)\log\left(1-U_1[i_1]\right)$ is obtained for the 1st component of the i^{th} system. Another uniform random numbers $U_2[i_1]$ is generated from $U(0,1)$. Further, exponential random variable $x_{2i} = \left(\frac{-m_0}{\lambda_0}\right)\log\left(1-U_2[i_1]\right)$ is obtained for the 2nd component of the i^{th} system.

Step 2: The whole procedure in Step 1 is repeated for $n = n_0$ number of systems and the statistics $x_1' = \sum_{i=1}^{n} x_{1i}, x_2' = \sum_{i=1}^{n} x_{2i}$ are obtained.

Step 3: Initialize $i_2 = 1, n = n_0, \mu = \mu_0, k = k_0$ for the 1st and 2nd component of the system. Uniform random numbers $V_1[i_2]$ are generated from $U(0,1)$. Further, exponential random variable $y_{1i} = \left(\frac{-1}{\mu_0}\right)\log\left(1-V_1[i_2]\right)$ is obtained for the 1st component of the i^{th} system. Another uniform random numbers $V_2[i_2]$ are generated from

$U(0,1)$. Further, exponential random variable $y_{2i} = \left(\frac{-k_0}{\mu_0}\right)\log\left(1-V_2[i_2]\right)$ is obtained for the 2nd component of the i^{th} system.

Step 4: The whole procedure in Step 3 is repeated for $n = n_0$ number of systems and the statistics $y_1' = \sum_{i=1}^{n} y_{1i}, y_2' = \sum_{i=1}^{n} y_{2i}$ are obtained.

Step 5: With the help of the statistics x_1', x_2', y_1' and y_2' the MLE of parameters λ, m, μ, k of the model are obtained. Using these MLEs in the expression of reliability function, the MLE of reliability function is obtained.

Step 6: With the help of the statistics x_1', x_2', y_1' and y_2' the UMVUE of parameters λ, m, μ, k are obtained. Using these UMVUEs in the expression of reliability function, estimator of the reliability function based on UMVUE of the parameters is obtained.

Table 1 and Table 2 give the results of the above simulation experiment for different values of λ, m, μ, k and n.

4. Conclusion

From the above results (as shown in Table 1 and Table 2), we observe that reliability of

Table 1. $\lambda = 0.5, \mu = 1.0, n = 5$.

m	k	R	x_1'	x_2'	y_1'	y_2'	$\hat{R}$ MLE	$\hat{R}$ UMVUE	MSE MLE	MSE UMVUE
0.25	1.00	0.810	13.412501	6.562061	6.706251	3.124121	0.855	0.818	2.03×10^{-3}	6.40×10^{-5}
0.25	1.25	0.792	10.290507	2.620377	5.145254	3.800942	0.831	0.799	1.52×10^{-3}	4.90×10^{-5}
0.25	1.50	0.778	09.181533	2.998768	5.590766	3.368335	0.816	0.784	1.44×10^{-3}	3.60×10^{-5}
0.50	1.00	0.867	09.778324	3.125278	4.889162	1.915278	0.909	0.860	1.76×10^{-3}	4.90×10^{-5}
0.50	1.25	0.848	15.892459	6.225975	6.912290	1.256218	0.885	0.853	1.37×10^{-3}	2.50×10^{-5}
0.50	1.50	0.833	12.241618	9.066720	6.120809	3.797080	0.868	0.837	1.23×10^{-3}	1.60×10^{-5}
0.75	1.00	0.897	14.691534	6.720832	5.345767	2.380555	0.931	0.892	1.16×10^{-3}	2.50×10^{-5}
0.75	1.25	0.881	13.931526	7.937316	5.965763	1.581096	0.913	0.877	1.03×10^{-3}	1.60×10^{-5}
0.75	1.50	0.867	15.716419	6.075062	6.428210	1.075062	0.898	0.870	9.61×10^{-4}	9.00×10^{-6}

Table 2. $\lambda = 0.5, \mu = 1.0, n = 10$.

m	k	R	x_1'	x_2'	y_1'	y_2'	$\hat{R}$ MLE	$\hat{R}$ UMVUE	MSE MLE	MSE UMVUE
0.25	1.00	0.810	18.036022	04.895907	09.018011	08.965815	0.833	0.816	5.29×10^{-4}	3.60×10^{-5}
0.25	1.25	0.792	18.058302	04.234054	09.029151	09.985135	0.810	0.796	3.24×10^{-4}	1.60×10^{-5}
0.25	1.50	0.778	15.606312	07.447487	09.303156	15.842461	0.795	0.781	2.89×10^{-4}	9.00×10^{-6}
0.50	1.00	0.867	20.403873	06.592115	10.201937	05.012115	0.886	0.862	3.61×10^{-4}	2.50×10^{-5}
0.50	1.25	0.848	26.671318	12.450741	13.335659	05.463426	0.865	0.844	2.89×10^{-4}	1.60×10^{-5}
0.50	1.50	0.833	22.037254	08.622803	11.018627	07.934204	0.849	0.835	2.56×10^{-4}	4.00×10^{-6}
0.75	1.00	0.897	23.010950	10.241815	11.505475	02.827877	0.915	0.893	3.24×10^{-4}	1.60×10^{-5}
0.75	1.25	0.881	16.283512	08.869339	10.941756	02.691160	0.898	0.878	2.89×10^{-4}	9.00×10^{-6}
0.75	1.50	0.867	21.313683	11.199248	10.656841	07.099248	0.883	0.865	2.56×10^{-4}	4.00×10^{-6}

the system improves for larger values of strength attenuation factor (m) and for lower values of stress attenuation factor (k). Here, we also observed the estimates of reliability improves for larger value of the sample size "n". This indicates that reliability of a system can be enhanced by strengthening the inbuilt mechanism of the system, which ultimately withstands the effects of the external environment in which it operates.

Further, on comparing the efficiencies of MLE of reliability function with reliability estimator obtained using UMVUEs of the parameters, we observed reliability estimator obtained from the UMVUEs of the perform better than the MLE of reliability function in terms of Mean Square Error (MSE) for the given data set. This emphasizes the need to strengthen the processes such that they are least affected by effects of the variation factors which intern boost the reliability of the operating system.

References

[1] Pandit, S.N.N. and Sriwastav, G.L. (1975) Studies in Cascade Reliability—1.*IEEE Transactions on Reliability*, **R-24**, 53–57. http://dx.doi.org/10.1109/TR.1975.5215330

[2] Raghavachar, A.C.N., Kesava Rao, B. and Ramacharyulu Pattabhi, N.C. (1983) Survival Function under Stress Attenuation in Cascade Reliability. *Opsearch*, **20**, 190-207.

[3] Maheshwari, U., Rekha, T.S., Anjan Rao, E. and Raghava Char, A.C.N. (1993) Reliability of a Cascade System with Normal Stress and Exponential Strength. *Microelectronics Reliability*, **33**, 929-936. http://dx.doi.org/10.1016/0026-2714(93)90289-B

[4] Rekha, A. and Shyam Sundar, T. (1997) Reliability of a Cascade System with Exponential Strength and Gamma Stress. *Microelectronics Reliability*, **37**, 683-685. http://dx.doi.org/10.1016/S0026-2714(97)87650-0

[5] Rekha, A. and Chenchu Raju, V.C. (1999) Cascade System Reliability with Rayleigh Distribution. *Botswana Journal of Technology*, **8**, 14-19.

[6] Shyam Sundar, T. (2012) Case Study of Cascade Reliability with Rayleigh Distribution. *International Journal of Computer Technology & Electronics Engineering*, **2**, 78-87.

[7] Hanagal, D.D. (1996) Estimation of System Reliability in a Two Component Stress-Strength Models. *Economic Quality Control*, **11**, 145-154.

[8] Kunchur, S.H. and Munoli, S.B. (1994) Estimation of Reliability for a Multi Component Survival Stress-Strength Model Based on Exponential Distribution. *Communications in Statistics— Theory & Methods*, **23**, 3273-3283. http://dx.doi.org/10.1080/03610929408831446

[9] Kundu, D. and Gupta, R.D. (2005) Estimation of P(Y<X) for Generalized Exponential Distribution. *Metrika*, **61**, 291-308. http://dx.doi.org/10.1007/s001840400345

[10] Munoli, S.B. and Mutkekar, R.R. (2011) Estimation of Reliability for a Two Component Survival Stress-Strength Model. *International Journal of Quality, Statistics & Reliability*, **2011**, Article ID: 721962. http://dx.doi.org/10.1155/2011/721962