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An Application of Non-Linear Cobb-Douglas Production Function to Selected Manufacturing Industries in Bangladesh

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ABSTRACT

Recently, businessmen as well as industrialists are very much concerned about the theory of firm in order to make correct decisions regarding what items, how much and how to produce them. All these decisions are directly related with the cost considerations and market situations where the firm is to be operated. In this regard, this paper should be helpful in suggesting the most suitable functional form of production process for the major manufacturing industries in Bangladesh. This paper considers Cobb-Douglas (C-D) production function with additive error and multiplicative error term. The main purpose of this paper is to select the appropriate Cobb-Douglas production model for measuring the production process of some selected manufacturing industries in Bangladesh. We use different model selection criteria to compare the Cobb-Douglas production function with additive error term to Cobb-Douglas production function with multiplicative error term. Finally, we estimate the parameters of the production function by using optimization subroutine.

Keywords: Cobb-Douglas Production Function; Newton-Raphson Method; Manufacturing Industry; Bangladesh

1. Introduction

A developing country like Bangladesh which is facing enormous problems so far as industrialization policy is concerned does not follow the policy of Marxian Economy, neither thus it strictly follow the policy of a Capitalist country. The economy of Bangladesh actually turned out to be a mixed economy since a long time. The industry sector was severely damaged during the war of liberation in 1971. After a series of adjustments and temporary changes in state policy, the government finally adopted a new industrial policy in 1982. Every industrialist tries to produce goods with maximum profit but with minimum cost. In order to do this, it has to be decided what to produce, how much to produce and how to produce. The industries need various inputs such as labor, raw material, machines etc. to produces goods. An industry's production cost depends on the quantities of inputs it buy and on the prices of each input. Thus an industry needs to select the optimal combination of inputs, that is, the combination that enables it to produce the desired level of output with minimum cost and hence with maximum profitability. So the main objective of this paper is to select the appropriate Cobb-Douglas production function. We use different model selection criteria to compare the Cobb-Douglas production function with additive error term to Cobb-Douglas production function with multiplicative error term. Finally, we estimate the parameters of the production function by using optimization subroutine.

2. Cobb-Douglas Production Function

The Cobb-Douglas production function is the widely used function in Econometrics. A famous case is the well-known Cobb-Douglas production function introduced by Charles W. Cobb and Paul H. Douglas, although anticipated by Knut Wicksell and, some have argued, J. H. Von Thünen [1]. They have estimated it after studying different industries in the world, for this it is used as a fairly universal law of production.

The Cobb-Douglas production function with multiplicative error term can be represented as,

$$p_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} u_t \qquad (2.1)$$

where, p_t is the output at time t; L_t is the Labor input; K_t is the Capital input; β_1 is a constant; u_t is the random error term. β_2 and β_3 are positive parameters.

The Cobb-Douglas production function with additive error term can be represented as,

$$p_t = \beta_1 L_t^{\beta_2} K_t^{\beta_3} + u_t \quad (2.2)$$

where, p_t is the output at time t; L_t is the Labor input; K_t is the Capital input; β_1 is a constant; u_t is the random error term. β_2 and β_3 are positive parameters.

3. Literature Review

Production functions for the industrial sector as a whole as well as for seven important industries in India are worked out based on cross-section data relating to individual firms for the two years 1951 and 1952 (V. N. Murti and V. K. Sastry) [2]. De-Min Wu [3], derives the exact distribution of the indirect least squares estimator of the coefficients of the Cobb-Douglas production function within the context of a stochastic production model of Marschak-Andrews type. The stochastic term in Cobb-Douglas type models is either specified to be additive or multiplicative (Stephen M. Goldfeld and Richard E. Quandt) [4]. They developed a model in which a Cobb-Douglas type function is coupled with simultaneous multiplicative and additive errors. This specification is a natural generalization of the "pure" models in which either additive or multiplicative stochastic terms are introduced. A Cobb-Douglas type function with both multiplicative and additive errors has been proposed by Goldfeld and Quandt [5]. They suggested a maximum likelyhood approach to the estimation of a Cobb-Douglas type model when the model includes both multiplicative and additive disturbance terms. As expected, an analytical expression for the solution to the maximization problem did not exist. Indeed, because of the complexity of the likelihood function, their maximization algorithm had to be used in conjunction with a numerical integration technique.

Kelejian [6] generalize and simplify the work by Goldfeld and Quandt [5]. Specifically, an estimation technique is suggested which does not require the specification of the disturbance terms beyond their means and variances, which does not require the compounding of a maximization algorithm with a numerical integration technique, but yet leads to asymptotically efficient estimates of the parameters of the regression function. In addition, the procedure readily lends itself to interpretation. For instance, it will become evident that if the distribution of the multiplicative disturbance term is not known, the scale parameter of the model (unlike the other parameters) will not be identified. The origins of the Cobb-Douglas form date back to the seminal work of Cobb and Douglas [1], who used data for the US manufacturing sector for 1899-1922 (although, as Brown [7]), Sandelin [8], and Samuelson [9], indicate, Wicksell should have taken the credit for its "discovery", for he had been working with this form in the 19th century).

Cheema [10], Kemal [11], and Wizarat [12], showed the performance of large-scale manufacturing sector of Pakistan. Moreover tax holidays also encourage investment in the industrial sector. For example, Azhar and Sharif [13], and Bond [14], empirically proved the positive correlation between tax holidays and industrial output. But it is also worth to note that tax rate and output of manufacturing sector are inversely related. The authors used the data of West Virginia State of United States. However, Radhu [15], showed the positive correlation between indirect tax rates and prices of the commodities. Vijay K. Bhasin and Vijay K. Seth [16], estimate production functions for Indian manufacturing industries and to find whether plausible and meaningful estimates can be obtained for returns to scale, substitution, distribution, and efficiency parameters. Some studies are based on data collected through surveys specially designed for estimate the levels of technical efficiency (TE) (e.g., Little *et al*., [17], Page [18]). Many of the studies are concerned with estimating and explaining variations in TE only in Small-Scale Industrial Units by fitting either a deterministic or a stochastic production frontier (e.g., Bhavani [19], Goldar [20], Neogi and Ghosh [21], Nikaido [22]). A review of other studies in this area may be found in Goldar [23].

All these studies, however, use data relating to years prior to the economic reforms. For instance, Bhavani [19] uses data collected under the first Census of small scale industrial units in 1973 to estimate the TE of firms at the 4-digit level industries of metal product groups by fitting a deterministic translog production frontier with three inputs-capital, labor and materials- and observes a very level of average efficiency across the four groups. Similarly, on the basis of the data made available by the Second All India Census of Small Scale Industrial Units in 1987-1988, Nikaido [22] fits a single stochastic production frontier, considering firms under all the 2-digit industry groups and using intercept dummies to distinguish different industry groups. He finds little variation in TEs across industry groups and a high level of average TE in each industry groups. Neogi and Ghosh [21] examine the inetrtemporal movement of TE using panel industry-level summary data for the years 1974-1975 to 1987-1988 and observe TEs to be falling over time.

In recent years, there has been an increasing interest in the examination of productivity from different parts of the economy such as industry, agriculture, and services. Numerous studies have attempted to explain productivity in the economic sector, for example, productivity growth in Swedish manufacturing (Carlsson [24]), the impact of regional investment incentives on employment and pro-

ductivity in Canada (M. Daly, Gorman, Lenjosek, MacNevin, & Phiriyapreunt [25]), productivity and imperfect competition in Italian firms (Contini, Revelli, & Cuneo [26]), explaining total factor productivity differentials in urban manufacturing of US (Mullen & Williams, [27]). Total factor productivity growth in manufacturing has been examined by applied parametric and non-parametric approaches. In most of the studies have used non-parametric approach, wherein total factor productivity growth has decomposed into efficiency change and technological change. Efficiency change measures "catching-up" to the isoquant while technological change measures shifts in the isoquant. For example, see Weber and Domazlicky [28]; Nemoto and Goto [29]; (Maniadakis and Thanassoulis [30] and Radam [31].

The studies by Golder *et al*. [32], Lall and Rodrigo [33], and Mukherjee and Ray [34], however, relate to the post-reform era. Using panel data for 63 firms in the engineering industry from 1990-1991 to 1999-2000 drawn from the Prowess database (version 2001) of the Centre for Monitoring Indian Economy, Goldar *et al*. [32] fit a translog stochastic production frontier to estimate firm-level TE scores in each year. They find the mean TE of foreign firms to be higher than that of domestically owned firms but do not find any statistically significant variation in mean TE across public and private sector firms among the latter group. They can attempt to explain variation in TEs in terms of economic variables, including export and import intensity and the degree of vertical nitration. Lall and Rodrigo [33] examine TE variation across four industrial sectors in India during 1994 and consider TE in relation to scale, location extent of infrastructure investment and other determinants.

Md. Zakir Hossain, M. Ishaq Bhatti, Md. Zulficar Ali, [35], reviews some models recently used in the literature and selects the most suitable one for measuring the production process of 21 major manufacturing industries in Bangladesh. In particular, they estimates and tests the coefficients of the production inputs for each of the selected manufacturing industries using Bangladesh Bureau of Statistics annual data over the period 1982-1983 through 1991-1992. Cheng-Ping Lin [36] analyzes the cost function of construction firms with due consideration of their available resources by using Cobb-Douglas Production and Cost Functions. Moosup Jung, *et al.* [37] made a study on Total Factor Productivity of Korean Firms and Catching up with the Japanese Firms. They measured and compared the TFP of both Korean and Japanese listed firms of 1984 to 2004. They found that the average TFP of Korean firms grew about 44.1% between 1984 and 2005, with 2.1% annual growth rates. Industry was observed to be outstanding.

Danish A. Hashim [38] made research on "Cost and Productivity in Indian Textiles" for Indian Council for Research on International Economic Relations. His observations and findings are: there is an inverse relationship between the unit cost and productivity: Industry and States, which witnessed higher productivity (growth) experienced lower unit cost (growth) and vice-versa. Better capacity utilization, reductions in Nominal Rate of Protection and increased availability of electricity are found to be favorably affecting the productivity in all the three industries. M. Z. Hossain and K. S. Al-Amri, [39] find that for most of the selected industries the C-D function fits the data very well in terms of labor and capital elasticity, return to scale measurements, standard errors, economy of the industries, high value of R^2 and reasonably good Durbin-Watson statistics. The estimated results suggest that the manufacturing industries of Oman generally seem to indicate the case of increasing return to scale. Of the nine industries, seven exhibit increasing return to scale and only the rest two show decreasing return to scale. They also find that no industry with constant return to scale.

4. Estimation Procedure

Equation (2.1) is nearly always treated as a linear relationship by making a logarithmic transformation, which yields:

$$\log p_t = \log \beta_1 + \beta_2 \log L_t + \beta_3 \log K_t + \log u_t \qquad (3.1)$$

where, $\log u$ is treated as an additive random error with a zero mean. In this form the function is a single equation which is linear in the unknown parameters: $\log \beta_1$, β_2 and β_3.

In the case of Equation (2.2), the minimization of, $\sum u^2$ is no longer a simple linear estimation problem. To estimate the production function we need to know different types of non-linear estimation. In non-linear model it is not possible to give a closed form expression for the estimates as a function of the sample values, *i.e*., the likelihood function or sum of squares cannot be transformed so that the normal equations are linear. The idea of using estimates that minimize the sum squared errors is a data-analytic idea, not a statistical idea; it does not depend on the statistical properties of the observations. Newton-Raphson method is one of the method which are used to estimate the parameters in non-linear system.

4.1. Newton-Raphson Method

Newton-Raphson is one of the popular Gradient methods of estimation. In Newton-Raphson method we find the values of β_j that maximize a twice differentiable concave function, the objective function $g(\beta)$. In this me-

thod we approximate $g(\beta)$ at β^t by Taylor series expansion up to the quadratic terms

$$g(\beta) \approx g(\beta^t) + \boldsymbol{G}(\beta^t)(\beta - \beta^t) + \frac{1}{2}(\beta - \beta^t)' \boldsymbol{H}(\beta^t)(\beta - \beta^t)$$

where, $\boldsymbol{G}(\beta^t) = \left[\frac{\partial g}{\partial \beta_i}\right]_{\beta^t}$ is the gradient vector and $\boldsymbol{H}(\beta^t) = \left[\frac{\partial^2 g}{\partial \beta_i \partial \beta_k}\right]_{\beta^t}$ is the Hessian matrix. This Hessian matrix is positive definite, the maximum of the approximation $g(\beta)$ occurs when its derivative is zero.

$$\begin{aligned} &\boldsymbol{G}(\beta^t) + \boldsymbol{H}(\beta^t)(\beta - \beta^t) = 0 \\ &\beta = \beta^t - \left[\boldsymbol{H}(\beta^t)\right]^{-1} \boldsymbol{G}(\beta^t) \end{aligned} \tag{3.11}$$

This gives us a way to compute β^{t+1}, the next value in iterations is,

$$\beta^{t+1} = \beta^t - \left[\boldsymbol{H}(\beta^t)\right]^{-1} \boldsymbol{G}(\beta^t)$$

The iteration procedures continue until convergence is achieved. Near the maximum the rate of convergence is quadratic as define by

$$\left|\beta_i^{t+1} - \hat{\beta}_i\right| \le c\left|\beta_i^t - \hat{\beta}_i\right|^2$$

for some $c \ge 0$ when β_i^t is near $\hat{\beta}_i$ for all i. Thus we get estimates $\hat{\beta}_i$ by Newton-Raphson methods.

For the model (2.2), to estimate the parameters we minimize the following error sum squares

$$S(\beta) = \sum_{t=1}^{n} \left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)^2$$

In case of nonlinear estimation we use the score vector and Hessian matrix. The elements of score vector are given below:

$$\frac{\partial S(\beta)}{\partial \beta_1} = -2 * \sum_{t=1}^{n}\left[\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(L_t^{\beta_2} K_t^{\beta_3}\right)\right]$$

$$\frac{\partial S(\beta)}{\partial \beta_2} = -2 * \sum_{t=1}^{n}\left[\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(L_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right]$$

$$\frac{\partial S(\beta)}{\partial \beta_3} = -2 * \sum_{t=1}^{n}\left[\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(K_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right]$$

Also the elements of Hessian matrix are given below:

$$\frac{\partial^2 S(\beta)}{\partial \beta_1^2} = 2 * \sum_{t=1}^{n}\left(L_t^{\beta_2} K_t^{\beta_3}\right)^2$$

$$\frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2} = 2 * \sum_{t=1}^{n}\left[\left(\left(\ln(L_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(L_t^{\beta_2} K_t^{\beta_3}\right)\right) - \left(\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(L_t)\right) * \left(L_t^{\beta_2} K_t^{\beta_3}\right)\right)\right]$$

$$\frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3} = 2 * \sum_{t=1}^{n}\left[\left(\left(\ln(K_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(L_t^{\beta_2} K_t^{\beta_3}\right)\right) - \left(\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(K_t)\right) * \left(L_t^{\beta_2} K_t^{\beta_3}\right)\right)\right]$$

$$\frac{\partial^2 S(\beta)}{\partial \beta_2^2} = 2 * \sum_{t=1}^{n}\left[\left(\left(\ln(L_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right)^2 - \left(\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(L_t)\right)^2 * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right)\right]$$

$$\frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3} = 2 * \sum_{t=1}^{n}\left[\left(\left(\ln(K_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(L_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right) - \left(\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(K_t)\right) * \left(\ln(L_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right)\right]$$

$$\frac{\partial^2 S(\beta)}{\partial \beta_3^2} = 2 * \sum_{t=1}^{n}\left[\left(\left(\ln(K_t)\right) * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right)^2 - \left(\left(p_t - \beta_1 L_t^{\beta_2} K_t^{\beta_3}\right) * \left(\ln(K_t)\right)^2 * \left(\beta_1 L_t^{\beta_2} K_t^{\beta_3}\right)\right)\right]$$

Hence the Score vector is

$$\boldsymbol{G}(\beta) = \left[\frac{\partial S(\beta)}{\partial \beta_1}, \frac{\partial S(\beta)}{\partial \beta_2}, \frac{\partial S(\beta)}{\partial \beta_3}\right]'$$

and Hessian matrix is

$$\boldsymbol{H}(\beta) = \begin{bmatrix} \frac{\partial^2 S(\beta)}{\partial \beta_1^2} & \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2} & \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_2} & \frac{\partial^2 S(\beta)}{\partial \beta_2^2} & \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3} \\ \frac{\partial^2 S(\beta)}{\partial \beta_1 \partial \beta_3} & \frac{\partial^2 S(\beta)}{\partial \beta_2 \partial \beta_3} & \frac{\partial^2 S(\beta)}{\partial \beta_3^2} \end{bmatrix}$$

4.2. Model Selection Criteria

To find the appropriate production function we use model selection criterion. The model that minimizes the criterion is the best model. The recent years several criteria for choosing among models have proposed. These entire take the form of residual sum of squares (ESS) multiplied by a penalty factor that depend on the complexity of the model. Some of these criteria are discuss below.

4.2.1. Finite Prediction Error (FPE)

Akaike (1970) developed Finite Prediction Error procedure, which is known as FPE. The statistic of this procedure can be represented as,

$$\text{FPE} = \left(\frac{ESS}{T}\right)\frac{T+K}{T-K}$$

where, T is the number of observations and K is the number of estimated parameter (Ramanathan [40]).

4.2.2. Akaike Information Criteria (AIC)

Akaike (1974) also developed another procedure which is known as Akaike Information Criteria. The form of this statistic is given below,

$$\text{AIC} = \left(\frac{ESS}{T}\right)\text{e}^{\left(\frac{2K}{T}\right)}$$

The value of AIC deceases when some variables are dropped (Ramanathan [40]).

4.2.3. Hunnan and Quinn (HQ) Criterion

Hunnan and Quinn (1979) developed a procedure which is known as HQ criteria. The statistic of this procedure can be represented as

$$\text{HQ} = \left(\frac{ESS}{T}\right)(\ln T)^{(2k/T)}$$

The value of HQ will decrease provided there are at least 16 observations (Ramanathan [40]).

4.2.4. SCHWARZ Criterion

Craven and Wahba (1978) developed a procedure which is known as SCHWARZ (BIC) criteria. The form of this procedure is represented as

$$\text{SCHWARZ} = \left(\frac{ESS}{T}\right)T^{K/T}$$

The value of SCHWARZ will also decrease provided there are at least 8 observations (Ramanathan [40]).

4.2.5. SHIBATA Criterion

Craven and Wahba (1981) developed a procedure which is known as SHIBATA criteria. The form of this procedure is represented as

$$\text{SHIBATA} = \left(\frac{ESS}{T}\right)\frac{T+2K}{T}.$$

When some variables dropped SHIBATA will increase (Ramanathan [40]).

4.2.6. Generalized Cross Validation (GCV)

Generalized Cross Validation (GCV) is another procedure which is developed by Craven and Wahba (1979). The form of the statistic is given below

$$\text{GCV} = \left(\frac{ESS}{T}\right)\left[1-\left(\frac{K}{T}\right)\right]^{-2}$$

If one or more variables are dropped then GCV will decrease (Ramanathan [40]).

4.2.7. Rice Criterion

The model selection criteria Rice developed by Craven and Wahba (1984). The form of this criterion can be represented as

$$\text{RICE} = \left(\frac{ESS}{T}\right)\left[1-\left(\frac{2K}{T}\right)\right]^{-1}.$$

(Ramanathan [40]).

4.2.8. SGMASQ Criterion

The form of this criterion can be represented as

$$\text{SGMASQ} = \left(\frac{ESS}{T}\right)\left[1-\left(\frac{K}{T}\right)\right]^{-1}.$$

If SGMASQ decreases (that is $\bar{R}^2$ increases) when one or more variable dropped, then GCV and RICE will also decreases (Ramanathan [40]).

5. Selected Manufacturing Industries of Bangladesh for This Study

In recent publications of "Statistical Yearbook of Bangladesh [41]" and "Report on Bangladesh Census of Manufacturing Industries (CMI) [42]" published by BBS, we get the published secondary data for the major manufacturing industries of Bangladesh over the period 1978-2002. We have chosen the following manufacturing industries for the ongoing analysis.

1) Manufacturing of Textile;
2) Manufacturing of Leather & Leather Products;
3) Manufacturing of Leather Footwear;
4) Manufacturing of Wood & Cork Products;
5) Manufacturing of Furniture & Fixtures (Wooden);
6) Manufacturing of Paper & Paper Products;
7) Manufacturing of Printing & Publications;
8) Manufacturing of Drugs & Pharmaceuticals;
9) Manufacturing of Industrial Chemical;
10) Manufacturing of Plastic Products;
11) Manufacturing of Glass & Glass Products;
12) Manufacturing of Iron & Steel Basic Industries;
13) Manufacturing of Fabricated Metal Products;
14) Manufacturing of Transport Equipment;
15) Manufacturing of Beverage;
16) Manufacturing of Tobacco.

6. Results and Discussion

In case of Cobb-Douglas production function with multi-

plicative error terms *i.e.*, for intrinsically linear model and additive errors *i.e.*, for intrinsically nonlinear model, we get the following estimates by using different model selection criteria discussed in Section 4.2.

From **Table 1**, we observe that, the Cobb-Douglas production function with additive error (2.2) performs better for the selected manufacturing industries based on the data under study period. Thus the strictly nonlinear models (which are nonlinear with additive error terms) seem to be better than intrinsically linear model (which are nonlinear with multiplicative error terms).

Now we estimate the parameters of the Cobb-Douglas production function with additive errors by using optimization subroutine. The estimates are given in **Table 2**.

There are economies of scale in the manufacturing of Drugs & pharmaceuticals, Furniture & fixtures (wooden), Iron & steel basic, Leather footwear, Fabricated metal products, Plastic products, Printing & publications, Tobacco since $\gamma < 1$ for these industries. There are diseconomies of scale in the Beverage, Chemical, Glass & glass products, Leather & leather products, Paper & paper products, Textile, Wood & crock products industries, Transport equipment since $\gamma > 1$ for these industries.

Table 1. Values of different model selection criteria of two models under study.

Name of the industry	FPE		AIC	
	Model (2.1)	Model (2.2)	Model (2.1)	Model (2.2)
Beverage	1360488	213713	1358701	213432
Chemical	1614804	957644	1612683	956386
Drugs	58414018	3029017	58337287	3025038
Furniture	4256943	193762	4251351	193507
Glass	16250	9608	16228	9595
Iron	20604099	18929094	20577034	18904229
Leather footwear	9973064	1967261	9959964	1964676
Leather products	1589512	1371610	1587424	1369808
Fabricated metal	1192242	885584	1190676	884420
Paper	1980514	1613608	1977913	1611488
Plastic	121323	75752	121164	75653
Printing	1065074	409231	1063675	408694
Textile	87418640	69914464	87303810	69822627
Tobacco	24044986	11143840	24013401	11129201
Transport	22949982	19106587	22919836	19081489
Wood	69177	30634	69086	30593

Continued

Name of the industry	HQ		SCHWARZ	
	Model (2.1)	Model (2.2)	Model (2.1)	Model (2.2)
Beverage	1412832	221936	1574260	247294
Chemical	1676933	994489	1868537	1108118
Drugs	60661469	3145557	67592579	3504964
Furniture	4420727	201217	4925834	224208
Glass	16875	9977	18803	11117
Iron	21396832	19657382	23841609	21903411
Leather footwear	10356773	2042950	11540126	2276375
Leather products	1650668	1424382	1839271	1587130
Fabricated metal	1238113	919656	1379578	1024735
Paper	2056714	1675691	2291712	1867153
Plastic	125991	78667	140386	87655
Printing	1106052	424976	1232428	473534
Textile	90782030	72604389	101154681	80900084
Tobacco	24970105	11572593	27823160	12894865
Transport	23832972	19841704	26556100	22108794
Wood	71838	31812	80047	35447

Name of the industry	SHIBATA		GCV	
	Model (2.1)	Model (2.2)	Model (2.1)	Model (2.2)
Beverage	1322696	207777	1382083	217105
Chemical	1569948	931043	1640436	972844
Drugs	56791406	2944877	59341225	3077096
Furniture	4138695	188380	4324514	196838
Glass	15798	9341	16508	9760
Iron	20031763	18403286	20931148	19229556
Leather footwear	9696034	1912614	10131367	1998487
Leather products	1545359	1333510	1614743	1393382
Fabricated metal	1159124	860984	1211166	899641
Paper	1925500	1568786	2011951	1639221
Plastic	117953	73648	123249	76955
Printing	1035488	397864	1081980	415727
Textile	84990345	67972396	88806238	71024218
Tobacco	23377069	10834288	24426652	11320726
Transport	22312483	18575849	23314268	19409866
Wood	67255	29783	70275	31120

Continued

Name of the industry	RICE		SGMASQ	
	Model (2.1)	Model (2.2)	Model (2.1)	Model (2.2)
Beverage	1410876	221629	1209322	189967
Chemical	1674611	993112	1435381	851239
Drugs	60577500	3141203	51923572	2692459
Furniture	4414608	200938	3783949	172233
Glass	16851	9964	14444	8540
Iron	21367214	19630171	18314755	16825861
Leather footwear	10342437	2040122	8864946	1748676
Leather products	1648383	1422411	1412900	1219209
Fabricated metal	1236399	918383	1059771	787186
Paper	2053867	1673371	1760457	1434318
Plastic	125817	78558	107843	67335
Printing	1104521	424388	946732	363761
Textile	90656368	72503889	77705458	62146190
Tobacco	24935541	11556574	21373321	9905635
Transport	23799982	19814239	20399984	16983633
Wood	71739	31768	61491	27230

7. Hypothesis Testing

To investigate the model that is the model is well fitted or not, we have consider the following the null hypothesis, $H_0 : \theta = 0$, *i.e.*, the model is not fitted well, against the alternative hypothesis, $H_0 : \theta \neq 0$, *i.e.*, the model is fitted well, where θ is the vector of parameters, *i.e.*, $\theta = (\beta_1 \;\; \beta_2 \;\; \beta_3)'$ for the model (2.2).

Under the null hypothesis, the test statistic is,

$$F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)}$$

where, k is the number of parameter and n is the number of observations.

We reject H_0, if $F = \frac{R^2/(k-1)}{(1-R^2)/(n-k)} > F_{0.05,(k-1),(n-k)}$,

which implies that model is fitted well.

The analytical results of the hypothesis testing are presented in **Table 3**.

From **Table 3**, we observe that R^2 is highly significant for all the manufacturing industries, we can say that the intrinsically nonlinear model (2.2) is fitted well according to the null hypothesis $H_0 : \theta = 0$.

Table 2. The estimates of Cobb-Douglas production function with additive error term (intrinsically nonlinear) of the industries under study.

Industry name	Intercept	p-value	Capital elasticity (β_2)	p-value	Labor elasticity (β_3)	p-value	Return to scale ($\beta_2+\beta_3$)	$\gamma = \frac{1}{\beta_2+\beta_3}$
Beverage	5.848951	0.0650	0.683362	0.0001	0.230199	0.0458	0.913561	1.094618
Chemical	6.552999	0.0720	0.567255	0.0001	0.239483	0.0113	0.806738	1.23956
Drugs	1.418816	0.0107	0.578490	0.0350	0.583740	0.0321	1.16223	0.860415
Furniture	0.136145	0.3751	1.583382	0.0001	0.323816	0.0001	1.907198	0.524329
Glass	10.858785	0.0048	0.446118	0.0001	0.267905	0.0792	0.714023	1.400515
Iron	5.432328	0.5702	0.317029	0.0641	0.825566	0.0265	1.142595	0.875201
Leather footwear	9.975966	0.0001	0.168618	0.1296	0.851867	0.0001	1.020485	0.979926
Leather products	149.5248	0.0008	0.273520	0.0293	0.396121	0.0185	0.669641	1.493337
Fabricated metal	1.560328	0.1357	0.282128	0.0753	0.979802	0.0001	1.26193	0.792437
Paper	36.90303	0.4661	0.154744	0.5083	0.593256	0.0002	0.748	1.336898
Plastic	10.04537	0.0001	0.081875	0.5527	0.962046	0.0001	1.043921	0.957927
Printing	0.761334	0.0090	1.062223	0.0001	0.215724	0.0007	1.277947	0.782505
Textile	33.44288	0.2979	0.503446	0.0001	0.237309	0.0874	0.740755	1.349974
Tobacco	5.828218	0.1361	0.867991	0.0001	0.257396	0.0001	1.125387	0.888583
Transport	35.21922	0.4605	0.037898	0.7996	0.873132	0.0124	0.91103	1.097659
Wood	45.73787	0.0789	0.054236	0.6731	0.566334	0.0001	0.62057	1.611422

Table 3. The values of test statistic of intrinsically nonlinear model for selected manufacturing industries.

Name of Industry	R^2	F
Beverage	0.9709	350.3247
Chemical	0.9733	382.7584
Drugs	0.9956	2375.864
Furniture	0.9813	550.9973
Glass	0.9545	220.2692
Iron	0.752	31.83871
Leather footwear	0.9885	902.5435
Leather products	0.9642	282.7961
Fabricated metal	0.954	217.7609
Paper	0.7956	40.86986
Plastic	0.9155	113.7604
Printing	0.991	1156.167
Textile	0.958	239.5
Tobacco	0.9711	352.8218
Transport	0.7434	30.41972
Wood	0.931	141.6739

Table 4. Estimated intrinsically nonlinear Cobb-Douglas production functions for the manufacturing industries under study.

Name of industry	Estimated intrinsically nonlinear Cobb-Douglas production function
Beverage	$P = 5.848951K^{0.683362}L^{0.230199}$
Chemical	$P = 6.552999K^{0.567255}L^{0.239483}$
Drugs	$P = 1.418816K^{0.578490}L^{0.583740}$
Furniture	$P = 0.136145K^{1.583382}L^{0.323816}$
Glass	$P = 10.858785K^{0.446118}L^{0.267905}$
Iron	$P = 5.432328K^{0.317029}L^{0.825566}$
Leather footwear	$P = 9.975966K^{0.168618}L^{0.851867}$
Leather products	$P = 149.5248K^{0.27352}L^{0.396121}$
Fabricated metal	$P = 1.560328K^{0.282128}L^{0.979802}$
Paper	$P = 36.90303K^{0.154744}L^{0.593256}$
Plastic	$P = 10.04527K^{0.081875}L^{0.962046}$
Printing	$P = 0.761334K^{1.062223}L^{0.215724}$
Textile	$P = 33.44288K^{0.503446}L^{0.237309}$
Tobacco	$P = 5.828218K^{0.867991}L^{0.257396}$
Transport	$P = 35.21922K^{0.037898}L^{0.873132}$
Wood	$P = 45.73787K^{0.054236}L^{0.566334}$

In order to forecast the production of manufacturing industries, we use the production function (**Table 4**).

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