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A Simulation Study on the Performances of Classical Var and Sims-Zha Bayesian Var Models in the Presence of Autocorrelated Errors

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Abstract

It is well known that a high degree of positive dependency among the errors generally leads to 1) serious underestimation of standard errors for regression coefficients; 2) prediction intervals that are excessively wide. This paper set out to study the performances of classical VAR and Sims-Zha Bayesian VAR models in the presence of autocorrelated errors. Autocorrelation levels of (−0.99, −0.95, −0.9, −0.85, −0.8, 0.8, 0.85, 0.9, 0.95, 0.99) were considered for short term (T = 8, 16); medium term (T = 32, 64) and long term (T = 128, 256). The results from 10,000 simulation revealed that BVAR model with loose prior is suitable for negative autocorrelations and BVAR model with tight prior is suitable for positive autocorrelations in the short term. While for medium term, the BVAR model with loose prior is suitable for the autocorrelation levels considered except in few cases. Lastly, for long term, the classical VAR is suitable for all the autocorrelation levels considered except in some cases where the BVAR models are preferred. This work therefore concludes that the performance of the classical VAR and Sims-Zha Bayesian VAR varies in terms of the autocorrelation levels and the time series lengths.

Keywords

Simulation, Performances, Vector Autoregression (VAR), Classical VAR, Sims-Zha Prior, Bayesian VAR (BVAR), Autocorrelated Errors

1. Introduction

Autocorrelation plays significant role in both time series and cross sectional data [1]. More often autocorrelation

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renders the inferences and decision making about the estimated parameters invalid [2]. In addition, it is well known that a high degree of positive dependency among the errors generally leads to 1) serious underestimation of standard errors for regression coefficients; 2) prediction intervals that are excessively wide [3]. While Gujarati, [4] identified the several reasons that make autocorrelation to occur. They are Inertia, Specification Bias, Excluded variables, incorrect functional form, cobweb phenomenon, lags, data transformation/manipulation, and nonstationarity.

In the time series literature, the standard linear regression model, autocorrelation of the disturbances leads to inefficient but still unbiased estimates of the coefficient [5], while the Least squares estimation of parameters in the general linear model may be highly inefficient in the presence of autocorrelated errors [6]. In the work of Smith, Wong and Kohn, [7] revealed that when a regression model is fitted to time series data the errors are likely to be autocorrelated. In a recent work of Garba *et al*. [8], they observed that the autocorrelation problem usually afflict time series data, while in a similar study carried out by Adenomon & Oyejola [9], they concluded that classical VAR model tend to forecast where there is no autocorrelation while the Bayesian VAR models with harmonic decay forecast better for both negative and positive autocorrelation level.

This paper therefore studied the forecasting performances of the classical VAR and some versions of Sims-Zha Bayesian VAR with quadratic decay models for bivariate time series with AR(1) error terms using Monte-Carlo experiment.

2. Model Description

2.1. Vector Autoregression (VAR) Model

Given a set of *k* time series variables, $y_t = [y_{it}, \cdots, y_{Kt}]'$, VAR models of the form

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t \quad (1)$$

provide a fairly general framework for the Data General Process (DGP) of the series. More precisely this model is called a VAR process of order p or VAR(p) process. Here $u_t = [u_{1t}, \cdots, u_{Kt}]'$ is a zero mean independent white noise process with non singular time invariant covariance matrix $\sum_u$ and the A_i are $(k \times k)$ coefficient matrices. The process is easy to use for forecasting purpose though it is not easy to determine the exact relations between the variables represented by the VAR model in Equation (1) above [10]. Also, polynomial trends or seasonal dummies can be included in the model.

The process is stable if

$$\det\left(I_K - A_1 z - \cdots - A_p z^p\right) \neq 0 \text{ for } |z| \leq 1 \quad (2)$$

In that case it generates stationary time series with time invariant means and variance covariance structure. The basic assumptions and properties of a VAR processes is the stability condition. A VAR(*p*) processes is said to be stable or fulfils stability condition, if all its eigenvalues have modulus less than 1 [11].

Therefore, to estimate the VAR model, one can write a VAR(*p*) with a concise matrix notation as

$$Y = BZ + U$$

$$\text{where } Y = [y_1, \cdots, y_T], \quad Z_{t-1} = \begin{bmatrix} y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix}, \quad Z = [Z_o, \cdots, Z_{T-1}] \quad (3)$$

Then the Multivariate Least Squares (MLS) for *B* yields

$$\hat{B} = (ZZ')^{-1} Z'Y \quad (4)$$

2.2. Bayesian Vector Autoregression with Sims-Zha Prior

In recent times, the BVAR model of Sims and Zha [12] has gained popularity both in economic time series and political analysis. The Sims-Zha BVAR allows for a more general specification and can produce a tractable multivariate normal posterior distribution. Again, the Sims-Zha BVAR estimates the parameters for the full system in a multivariate regression [13].

Given the reduced form model

$$y_t = c + y_{t-1}B_1 + \cdots + y_{t-p}B_p + u_t$$

where $c = dA_0^{-1}$, $B_l = -A_l A_0^{-1}$, $l = 1, 2, \cdots, p$, $u_t = \varepsilon_t A_0^{-1}$ and $\Sigma = A_0^{-1\prime} A_0^{-1}$

The matrix representation of the reduced form is given as

$$\underset{T\times m}{Y} = \underset{T\times(mp+1)}{X} \underset{(mp+1)\times m}{\beta} + \underset{T\times m}{U}, \quad U \sim MVN(0, \Sigma)$$

We can then construct a reduced form Bayesian SUR with the Sims-Zha prior as follows. The prior means for the reduced form coefficients are that $B_1 = I$ and $B_2, \cdots, B_p = 0$. We assume that the prior has a conditional structure that is multivariate Normal-inverse Wishart distribution for the parameters in the model. To estimate the coefficients for the system of the reduced form model with the following estimators

$$\hat{\beta} = \left(\Psi^{-1} + X'X\right)^{-1}\left(\Psi^{-1}\bar{\beta} + X'Y\right)$$

$$\hat{\Sigma} = T^{-1}\left(Y'Y - \hat{\beta}'\left(X'X + \Psi^{-1}\right)\hat{\beta} + \bar{\beta}'\Psi^{-1}\bar{\beta} + \bar{S}\right)$$

where the Normal-inverse Wishart prior for the coefficients is

$$\beta/\Sigma \sim N\left(\bar{\beta}, \Psi\right) \text{ and } \Sigma \sim IW\left(\bar{S}, v\right)$$

This representation translates the prior proposed by Sims and Zha form from the structural model to the reduced form ([13] [14] and [12] [15]).

The summary of the Sims-Zha prior is given in **Table 1**.

3. Simulation Procedure

A bivariate time series data that have autocorrelated error of order 1 were simulated using the VAR (2) process of the form:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_t = \begin{bmatrix} 5.0 \\ 10.0 \end{bmatrix} + \begin{bmatrix} 0.5 & 0.2 \\ -0.2 & -0.5 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-1} + \begin{bmatrix} -0.3 & -0.7 \\ -0.1 & 0.3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}_{t-2} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}_t$$

Such that $u_1 = u_2 = \delta\varepsilon_{t-1} + \varepsilon_t$ where $\varepsilon_t \sim N(0,1)$. the choice here is similar to the work and illustration of Cowpertwait, [16]. This work considered ten autocorrelated levels as δ = (−0.99, −0.95, −0.9, −0.85, −0.8, 0.8, 0.85, 0.9, 0.95, 0.99) for short term (T = 8, 16); medium term (T = 32, 64) and long term (T = 128, 256). Sample of generated data are presented in **Table 2**.

Table 1. Hyperparameters of sims-zha reference prior.

Parameter	Range	Interpretation
λ_0	[0, 1]	Overall scale of the error covariance matrix
λ_1	>0	Standard deviation around A_1 (persistence)
λ_2	=1	Weight of own lag versus other lags
λ_3	>0	Lag decay
λ_4	≥0	Scale of standard deviation of intercept
λ_5	≥0	Scale of standard deviation of exogenous variable coefficients
μ_5	≥0	Sum of coefficients/Cointegration (long-term trends)
μ_6	≥0	Initial observations/dummy observation (impacts of initial conditions)
v	>0	Prior degrees of freedom

Source: Brandt and Freeman, [13].

Table 2. Sample of generated data for T = 8.

Time series data for T = 8		Autocorrelated errors $\delta = 0.95$	
y_1	y_2		
5.00505541	10.917722	0.005055408	−1.001469533
8.68460254	4.482366	−0.271733404	−1.034463062
0.82311917	9.929715	−2.226441998	−1.381950068
0.62000249	5.215348	−0.892427694	−0.108602706
−3.07110720	9.393832	0.917721826	0.942238044
0.12451823	7.026048	1.133007326	−0.131419986
−0.07930944	10.020091	−0.771095435	−0.393861135
1.90017153	7.003259	0.432758733	−0.097920078

Model Specification

The time series were generated data using a VAR model with lag 2. The choice here is to obtain a bivariate time series with the true lag length. While the VAR and BVAR models of lag length of 2 was used for modeling and forecasting purpose.

For the BVAR model with Sims-Zha prior, we consider the following range of values for the hyperparameters given below and the Normal-Inverse Wishart prior was employed.

We consider two tight priors and two loose priors as follows:

The Tight priors are as follows

$$\text{BVAR1} = \left(\lambda_0 = 0.6,\ \lambda_1 = 0.1,\ \lambda_3 = 2,\ \lambda_4 = 0.1,\ \lambda_5 = 0.07,\ \mu_5 = \mu_6 = 5\right)$$

$$\text{BVAR2} = \left(\lambda_0 = 0.8,\ \lambda_1 = 0.1,\ \lambda_3 = 2,\ \lambda_4 = 0.1,\ \lambda_5 = 0.07,\ \mu_5 = \mu_6 = 5\right)$$

The Loose priors are as follows

$$\text{BVAR3} = \left(\lambda_0 = 0.6,\ \lambda_1 = 0.15,\ \lambda_3 = 2,\ \lambda_4 = 0.15,\ \lambda_5 = 0.07,\ \mu_5 = \mu_6 = 2\right)$$

$$\text{BVAR4} = \left(\lambda_0 = 0.8,\ \lambda_1 = 0.15,\ \lambda_3 = 2,\ \lambda_4 = 0.15,\ \lambda_5 = 0.07,\ \mu_5 = \mu_6 = 2\right)$$

where $n\mu$ is prior degrees of freedom given as $m + 1$ where m is the number of variables in the multiple time series data. In work $n\mu$ is 3 (that is two (2) time series variables plus 1(one)).

Our choice of Normal-Inverse Wishart prior for the BVAR models follow the work of Kadiyala & Karlsson, [17] that Normal Wishart prior tends to performed better when compared to other priors. In addition Sims and Zha, [12] proposed Normal-Inverse Wishart prior because of its suitability for large systems while Breheny, [18] reported that the most advantage of wishart distribution is that it guaranteed to produce positive definite draws. Our choice of the overall tightness $\lambda_0 = 0.6$ and 0.8 is in line with work of Brandt, Colaresi and Freeman [19]. In this work we assumed that the bivariate time series follows a quadratic decay. The Quadratic Decay (QD) model has many attractive theoretical properties that is why it is been applied to many fields of endeavour ([20]-[22]).

The following are the criteria for Forecast assessments used:

1) Mean Absolute Error (*MAE*) has a formular $MAE_j = \frac{\sum_{i=1}^{n} |e_i|}{n}$. This criterion measures deviation from the series in absolute terms, and measures how much the forecast is biased. This measure is one of the most common ones used for analyzing the quality of different forecasts.

2) The Root Mean Square Error (*RMSE*) is given as $RMSE_j = \sqrt{\frac{\sum_{i}^{n} \left(y_i - y^f\right)^2}{n}}$ where y_i is the time series data and y^f is the forecast value of y [23].

For the two measures above, the smaller the value, the better the fit of the model [24].

In this simulation study, $RMSE = \frac{\sum_{j}^{N} RMSE_j}{N}$ and $MAE = \frac{\sum_{j}^{N} MAE_j}{N}$ where N = 10,000. Therefore, the model with the minimum *RMSE* and *MAE* result as the preferred model.

4. Results and Discussion

The entire simulation and analysis was carried out in R environment. The values of the RMSE and MAE for short, medium and long terms are presented in **Tables A1-A3** respectively in **Appendix A**. While the ranks for short, medium and long terms are presented in **Tables B1-B3** respectively in **Appendix B**. In general the values of the RMSE and MAE increased as a result of increase in the autocorrelated levels. In addition the values of the RMSE and MAE decreased as a result of increase in the time series lengths.

The preferred model for short, medium and long terms are presented in **Tables 3(a)-(c)** respectively.

Table 3(a) revealed that the BVAR model with loose prior (BVAR4) is preferred for negative autocorrelation levels except in few cases, while BVAR model with tight prior (BVAR1) is preferred for positive autocorrelation levels in the short term

In **Table 3(b)**, the BVAR model with loose prior (BVAR4) is preferred for autocorrelation level of −0.99, −0.99 and from 0.9 to 0.99. The classical VAR (VAR(2)) is preferred for autocorrelation levels of −0.8 to 0.85 for T = 64. While in other autocorrelation levels the preferred models varies among BVAR models with tight prior, classical VAR and BVAR model with loose prior respectively.

Table 3. (a) The preferred models for short term (T = 8, 16); (b) The preferred models for medium short (T = 32, 64); (c) The preferred models for long short (T = 128, 256).

(a)

AUTOCORRELATION LEVELS (δ)	T = 8		T = 16	
	RMSE	MAE	RMSE	MAE
−0.99	BVAR4	BVAR4	BVAR4	BVAR4
−0.95	BVAR4	BVAR4	BVAR4	BVAR4
−0.9	BVAR4	BVAR4	BVAR4	BVAR4
−0.85	BVAR4	BVAR4	BVAR4	BVAR4
−0.8	BVAR4	BVAR2	BVAR2	BVAR4
0.8	BVAR1	BVAR1	BVAR1	BVAR1
0.85	BVAR1	BVAR1	BVAR1	BVAR1
0.9	BVAR1	BVAR1	BVAR1	BVAR1
0.95	BVAR1	BVAR1	BVAR1	BVAR1
0.99	BVAR1	BVAR1	BVAR1	BVAR1

(b)

AUTOCORRELATION LEVELS (δ)	T = 32		T = 64	
	RMSE	MAE	RMSE	MAE
−0.99	BVAR4	BVAR4	BVAR4	BVAR4
−0.95	BVAR4	BVAR4	BVAR4	BVAR4
−0.9	BVAR2	BAVR2	BVAR4	BVAR4
−0.85	BVAR1	BVAR2	VAR(2)	BVAR4
−0.8	VAR(2)	BVAR1	VAR(2)	VAR(2)
0.8	BVAR4	VAR(2)	VAR(2)	VAR(2)
0.85	BVAR4	BVAR4	VAR(2)	VAR(2)
0.9	BVAR4	BVAR4	BVAR4	BVAR4
0.95	BVAR4	BVAR4	BVAR4	BVAR4
0.99	BVAR4	BVAR4	BVAR3	BVAR3

(c)

AUTOCORRELATION LEVELS (δ)	T = 128		T = 256	
	RMSE	MAE	RMSE	MAE
−0.99	BVAR3	BVAR3	BVAR2	BVAR2
−0.95	BVAR4	BVAR2	BVAR2	BVAR2
−0.9	BVAR4	BVAR4	VAR(2)	VAR(2)
−0.85	VAR(2)	VAR(2)	VAR(2)	VAR(2)
−0.8	VAR(2)	VAR(2)	VAR(2)	VAR(2)
0.8	VAR(2)	VAR(2)	VAR(2)	VAR(2)
0.85	VAR(2)	VAR(2)	VAR(2)	VAR(2)
0.9	VAR(2)	VAR(2)	VAR(2)	VAR(2)
0.95	BVAR4	BVAR4	BVAR4	BVAR4
0.99	BVAR4	BVAR4	BVAR4	BVAR2

In **Table 3(c)**, the classical VAR (VAR(2)) model is preferred for autocorrelation levels of −0.85 to 0.9, the BVAR model with loose prior (BVAR4) is preferred for autocorrelation levels of 0.95 and 0.99. while in other autocorrelation levels the preferred models varies among BVAR models with loose prior, BVAR models with tight prior and the classical VAR model respectively.

5. Conclusions and Recommendation

In conclusion, the performances of the forecasting models depend on the autocorrelation levels and the time series length.

It is therefore recommended that the autocorrelation levels and the time series length should be considered in using an appropriate model for forecasting.

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Appendix A

Table A1. RMSE and MAE of the Models for short term (T = 8, 16)

AUTOCORRELATION LEVELS (δ)	Models	T = 8		T = 16	
		RMSE	MAE	RMSE	MAE
−0.99	VAR(2)	N/A	N/A	15.61070	12.46684
	BVAR1	8.268939	6.159536	10.247713	8.557597
	BVAR2	7.988318	6.030353	9.777068	8.162389
	BVAR3	6.828580	5.532507	9.411352	7.511767
	BVAR4	6.696952	5.424807	9.101631	7.228307
−0.95	VAR(2)	N/A	N/A	10.694678	8.404044
	BVAR1	6.868568	5.117551	7.565327	6.217715
	BVAR2	6.767191	5.096843	7.302214	6.018805
	BVAR3	5.970514	4.810123	7.124959	5.611988
	BVAR4	5.841910	4.712191	6.999561	5.492313
−0.9	VAR(2)	N/A	N/A	8.227285	6.347241
	BVAR1	5.761336	4.313672	5.953596	4.825443
	BVAR2	5.686742	4.294434	5.797857	4.714364
	BVAR3	5.284131	4.217560	5.730219	4.458697
	BVAR4	5.155838	4.132914	5.671135	4.401618
−0.85	VAR(2)	N/A	N/A	7.245699	5.314589
	BVAR1	5.053004	3.795841	5.050312	4.060008
	BVAR2	5.003829	3.777888	4.962080	4.000081
	BVAR3	4.778971	3.796238	4.941124	3.827773
	BVAR4	4.705957	3.752238	4.912545	3.795789
−0.8	VAR(2)	N/A	N/A	5.570151	4.202644
	BVAR1	4.552127	3.429420	4.472638	3.566182
	BVAR2	4.532001	3.428232	4.41249	3.52623
	BVAR3	4.454912	3.518556	4.445870	3.433351
	BVAR4	4.425705	3.510317	4.435451	3.418151
0.8	VAR(2)	N/A	N/A	4.604481	3.387338
	BVAR1	3.170552	2.328981	3.115189	2.402498
	BVAR2	3.206466	2.366413	3.134324	2.425714
	BVAR3	3.441772	2.677405	3.236948	2.572512
	BVAR4	3.462605	2.700449	3.195356	2.536710
0.85	VAR(2)	N/A	N/A	5.298748	3.917063
	BVAR1	3.205167	2.351426	3.262307	2.539760
	BVAR2	3.224667	2.376088	3.264149	2.542133
	BVAR3	3.475158	2.707142	3.384553	2.711048
	BVAR4	3.472026	2.703957	3.340094	2.668608
0.9	VAR(2)	N/A	N/A	8.085072	5.479093
	BVAR1	3.21618	2.36365	3.399855	2.660100
	BVAR2	3.241386	2.395118	3.426421	2.695679
	BVAR3	3.504280	2.731006	3.549209	2.858683
	BVAR4	3.512631	2.740081	3.497261	2.813078
0.95	VAR(2)	N/A	N/A	8.57251	5.99339
	BVAR1	3.249457	2.389992	3.571661	2.810530
	BVAR2	3.253510	2.403027	3.617310	2.865264
	BVAR3	3.525785	2.745731	3.747034	3.044939
	BVAR4	3.534562	2.763094	3.718684	3.013547
0.99	VAR(2)	N/A	N/A	11.691166	7.641045
	BVAR1	3.252371	2.385865	3.709390	2.915292
	BVAR2	3.273087	2.411261	3.799228	3.014138
	BVAR3	3.521370	2.729458	3.897329	3.155196
	BVAR4	3.543619	2.759944	3.848584	3.111145

Table A2. RMSE and MAE of the models for medium term (T = 32, 64).

AUTOCORRELATION LEVELS (δ)	Models	T = 32		T = 64	
		RMSE	MAE	RMSE	MAE
	VAR(2)	18.51937	15.32571	22.47805	18.50490
	BVAR1	13.44155	11.20306	16.85493	13.96209
−0.99	BVAR2	13.39895	11.13304	16.45158	13.60912
	BVAR3	12.029086	9.707249	14.19239	11.53329
	BVAR4	11.170522	8.978522	13.9217	11.4063
	VAR(2)	11.59479	9.37044	11.235892	9.146275
	BVAR1	8.204827	6.620134	9.048921	7.125616
−0.95	BVAR2	8.207244	6.605890	8.931793	7.063148
	BVAR3	8.372768	6.617752	9.034040	7.141074
	BVAR4	8.048093	6.376259	8.753774	6.974093
	VAR(2)	8.378650	6.574879	6.999085	5.643608
	BVAR1	6.167595	4.886693	6.645625	5.109532
−0.9	BVAR2	6.157462	4.850431	6.518469	5.025112
	BVAR3	6.380995	4.925847	6.673274	5.135679
	BVAR4	6.270225	4.876456	6.362430	4.949103
	VAR(2)	5.716131	4.613260	5.245832	4.195248
	BVAR1	5.177526	4.067993	5.598164	4.270608
-0.85	BVAR2	5.192059	4.051523	5.465794	4.176788
	BVAR3	5.483297	4.192094	5.641280	4.294666
	BVAR4	5.323288	4.089317	5.268442	4.041161
	VAR(2)	4.573355	3.688970	4.111588	3.267457
	BVAR1	4.579310	3.582714	5.007763	3.813774
−0.8	BVAR2	4.612613	3.584074	4.848271	3.692432
	BVAR3	4.928052	3.755175	5.043621	3.826875
	BVAR4	4.768427	3.644363	4.672554	3.565822
	VAR(2)	2.911991	2.226130	2.364223	1.830622
	BVAR1	3.061780	2.446032	2.960311	2.404376
0.8	BVAR2	3.029923	2.419689	2.900026	2.352158
	BVAR3	2.968358	2.389509	2.923218	2.387972
	BVAR4	2.896568	2.324630	2.789921	2.270791
	VAR(2)	3.664264	2.842395	2.819089	2.209700
	BVAR1	3.321767	2.670342	3.253465	2.647904
0.85	BVAR2	3.317626	2.671295	3.186185	2.588476
	BVAR3	3.174913	2.571598	3.127688	2.558588
	BVAR4	3.113732	2.516620	3.020891	2.463131
	VAR(2)	4.941591	3.819592	3.709760	2.977056
	BVAR1	3.729089	3.031137	3.710443	3.030718
0.9	BVAR2	3.725952	3.034129	3.656403	2.984484
	BVAR3	3.524312	2.877726	3.507946	2.876968
	BVAR4	3.471619	2.827012	3.405919	2.781972
	VAR(2)	8.949051	6.335658	6.951095	5.454921
	BVAR1	4.384650	3.619267	4.793900	3.963677
0.95	BVAR2	4.340558	3.572655	4.687418	3.859172
	BVAR3	4.099345	3.386899	4.433082	3.662572
	BVAR4	4.094079	3.381934	4.378921	3.606226
	VAR(2)	9.886464	7.534475	11.759527	9.301805
	BVAR1	4.964834	4.104314	6.687535	5.596650
0.99	BVAR2	5.000935	4.135764	6.650984	5.553134
	BVAR3	4.812514	3.998640	6.250987	5.223581
	BVAR4	4.79703	3.98559	6.314432	5.270220

Table A3. RMSE and MAE of the models for Long term (T = 128, 256).

AUTOCORRELATION LEVELS (δ)	Models	T = 128		T = 256	
		RMSE	MAE	RMSE	MAE
	VAR(2)	25.29136	20.57803	24.89353	20.27533
	BVAR1	18.97229	15.49887	20.52371	16.50237
−0.99	BVAR2	18.42898	15.04846	20.34156	16.36560
	BVAR3	18.01198	14.71195	22.94825	18.66869
	BVAR4	19.21266	15.78985	25.37395	20.58512
	VAR(2)	10.608058	8.434172	9.521107	7.454405
	BVAR1	9.445834	7.330077	9.376191	7.226121
−0.95	BVAR2	9.157944	7.155402	9.161844	7.097149
	BVAR3	9.280705	7.288772	9.322954	7.246612
	BVAR4	9.130834	7.228411	9.377873	7.328429
	VAR(2)	6.416899	5.071596	6.055986	4.697850
	BVAR1	6.837341	5.195836	6.550906	4.979713
−0.9	BVAR2	6.498645	4.975510	6.257913	4.790203
	BVAR3	6.491610	4.994156	6.266058	4.815668
	BVAR4	6.226952	4.834336	6.128210	4.732092
	VAR(2)	4.721675	3.713432	4.645043	3.597282
	BVAR1	5.734427	4.332739	5.377073	4.066994
−0.85	BVAR2	5.363857	4.072418	5.007710	3.815148
	BVAR3	5.312746	4.049472	4.929922	3.772702
	BVAR4	4.982521	3.835193	4.755215	3.661254
	VAR(2)	3.858129	3.020307	3.80270	2.94512
	BVAR1	5.142641	3.881026	4.727391	3.570719
−0.8	BVAR2	4.739819	3.589527	4.298829	3.266398
	BVAR3	4.655187	3.535919	4.19855	3.20381
	BVAR4	4.269902	3.268957	3.957584	3.042249
	VAR(2)	2.218726	1.720205	2.176999	1.695073
	BVAR1	3.124369	2.580126	3.226327	2.693960
0.8	BVAR2	2.950443	2.428861	2.828942	2.337125
	BVAR3	2.972579	2.455268	2.722261	2.238176
	BVAR4	2.68660	2.19352	2.427032	1.950018
	VAR(2)	2.519725	1.970158	2.482135	1.938329
	BVAR1	3.339102	2.754286	3.427334	2.852653
0.85	BVAR2	3.172200	2.605337	3.054614	2.509326
	BVAR3	3.170801	2.611133	2.959596	2.419124
	BVAR4	2.913734	2.369758	2.691089	2.154252
	VAR(2)	3.147020	2.498148	3.016211	2.370721
	BVAR1	3.733826	3.072094	3.797471	3.135458
0.9	BVAR2	3.577652	2.924736	3.472985	2.827289
	BVAR3	3.552882	2.914901	3.379560	2.738709
	BVAR4	3.324255	2.694440	3.154451	2.516101
	VAR(2)	5.099671	4.142258	4.452966	3.562149
	BVAR1	4.768778	3.913935	4.735043	3.860720
0.95	BVAR2	4.651729	3.797358	4.479936	3.608397
	BVAR3	4.518151	3.696097	4.413060	3.545852
	BVAR4	4.360350	3.533585	4.253486	3.388638
	VAR(2)	13.44323	11.03769	12.76596	10.67239
	BVAR1	8.303355	6.915693	9.126759	7.511420
0.99	BVAR2	8.035857	6.681232	9.071670	7.436501
	BVAR3	7.899572	6.569464	9.094940	7.440822
	BVAR4	7.789944	6.479641	9.031439	7.446020

Appendix B

Table B1. Ranks of RMSE and MAE of the Models for short term (T = 8, 16).

AUTOCORRELATION LEVELS (δ)	Models	T = 8		T = 16	
		RMSE	MAE	RMSE	MAE
−0.99	VAR(2)	N/A	N/A	5	5
	BVAR1	4	4	4	4
	BVAR2	3	3	3	3
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
−0.95	VAR(2)	N/A	N/A	5	5
	BVAR1	4	4	4	4
	BVAR2	3	3	3	3
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
−0.9	VAR(2)	N/A	N/A	5	5
	BVAR1	4	4	4	4
	BVAR2	3	3	3	3
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
−0.85	VAR(2)	N/A	N/A	5	5
	BVAR1	4	3	4	4
	BVAR2	3	2	3	3
	BVAR3	2	4	2	2
	BVAR4	1	1	1	1
−0.8	VAR(2)	N/A	N/A	5	5
	BVAR1	4	2	4	4
	BVAR2	3	1	1	3
	BVAR3	2	4	3	2
	BVAR4	1	3	2	1
0.8	VAR(2)	N/A	N/A	5	5
	BVAR1	1	1	1	1
	BVAR2	2	2	2	2
	BVAR3	3	3	4	4
	BVAR4	4	4	3	3
0.85	VAR(2)	N/A	N/A	5	5
	BVAR1	1	1	1	1
	BVAR2	2	2	2	2
	BVAR3	4	4	4	4
	BVAR4	3	3	3	3
0.9	VAR(2)	N/A	N/A	5	5
	BVAR1	1	1	1	1
	BVAR2	2	2	2	2
	BVAR3	3	3	4	4
	BVAR4	4	4	3	3
0.95	VAR(2)	N/A	N/A	5	5
	BVAR1	1	1	1	1
	BVAR2	2	2	2	2
	BVAR3	3	3	4	4
	BVAR4	4	4	3	3
0.99	VAR(2)	N/A	N/A	5	5
	BVAR1	1	1	1	1
	BVAR2	2	2	2	2
	BVAR3	3	3	4	4
	BVAR4	4	4	3	3

Table B2. Rank of RMSE and MAE of the models for medium term (T = 32, 64).

AUTOCORRELATION LEVELS (δ)	Models	T = 32		T = 64	
		RMSE	MAE	RMSE	MAE
	VAR(2)	5	5	5	5
	BVAR1	4	4	4	4
−0.99	BVAR2	3	3	3	3
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
	VAR(2)	5	5	5	5
	BVAR1	2	4	4	3
−0.95	BVAR2	3	2	2	2
	BVAR3	4	3	3	4
	BVAR4	1	1	1	1
	VAR(2)	5	5	5	5
	BVAR1	2	3	3	3
−0.9	BVAR2	1	1	2	2
	BVAR3	4	4	4	4
	BVAR4	3	2	1	1
	VAR(2)	5	5	1	3
	BVAR1	1	2	4	4
−0.85	BVAR2	2	1	3	2
	BVAR3	4	4	5	5
	BVAR4	3	3	2	1
	VAR(2)	1	4	1	1
	BVAR1	2	1	4	4
−0.8	BVAR2	3	2	3	3
	BVAR3	5	5	5	5
	BVAR4	4	3	2	2
	VAR(2)	2	1	1	1
	BVAR1	5	5	5	5
0.8	BVAR2	4	4	3	3
	BVAR3	3	3	4	4
	BVAR4	1	2	2	2
	VAR(2)	5	5	1	1
	BVAR1	4	3	5	5
0.85	BVAR2	3	4	4	4
	BVAR3	2	2	3	3
	BVAR4	1	1	2	2
	VAR(2)	5	5	4	3
	BVAR1	4	3	5	5
0.9	BVAR2	3	4	3	4
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
	VAR(2)	5	5	5	5
	BVAR1	4	4	4	4
0.95	BVAR2	3	3	3	3
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
	VAR(2)	5	5	5	5
	BVAR1	3	3	4	4
0.99	BVAR2	4	4	3	3
	BVAR3	2	2	1	1
	BVAR4	1	1	2	2

Table B3. Ranks of RMSE and MAE of the Models for Long term (T = 128, 256)

AUTOCORRELATION LEVELS (δ)	Models	T = 128		T = 256	
		RMSE	MAE	RMSE	MAE
	VAR(2)	5	5	4	4
	BVAR1	3	3	2	2
−0.99	BVAR2	2	2	1	1
	BVAR3	1	1	3	3
	BVAR4	4	4	5	5
	VAR(2)	5	5	5	5
	BVAR1	4	4	3	2
−0.95	BVAR2	2	1	1	1
	BVAR3	3	3	2	3
	BVAR4	1	2	4	4
	VAR(2)	2	4	1	1
	BVAR1	5	5	5	5
−0.9	BVAR2	4	2	3	3
	BVAR3	3	3	4	4
	BVAR4	1	1	2	2
	VAR(2)	1	1	1	1
	BVAR1	5	5	5	5
−0.85	BVAR2	4	4	4	4
	BVAR3	3	3	3	3
	BVAR4	2	2	2	2
	VAR(2)	1	1	1	1
	BVAR1	5	5	5	5
−0.8	BVAR2	4	4	4	4
	BVAR3	3	3	3	3
	BVAR4	2	2	2	2
	VAR(2)	1	1	1	1
	BVAR1	5	5	5	5
0.8	BVAR2	3	3	4	4
	BVAR3	4	4	3	3
	BVAR4	2	2	2	2
	VAR(2)	1	1	1	1
	BVAR1	5	5	5	5
0.85	BVAR2	4	3	4	4
	BVAR3	3	4	3	3
	BVAR4	2	2	2	2
	VAR(2)	1	1	1	1
	BVAR1	5	5	5	5
0.9	BVAR2	4	4	4	4
	BVAR3	3	3	3	3
	BVAR4	2	2	2	2
	VAR(2)	5	5	3	3
	BVAR1	4	4	5	5
0.95	BVAR2	3	3	4	4
	BVAR3	2	2	2	2
	BVAR4	1	1	1	1
	VAR(2)	5	5	5	5
	BVAR1	4	4	4	4
0.99	BVAR2	3	3	2	1
	BVAR3	2	2	3	2
	BVAR4	1	1	1	3