Open Journal of Modern Hydrology, 2016, 6, 1-7
Published Online January 2016 in SciRes. http://www.scirp.org/journal/ojmh
http://dx.doi.org/10.4236/ojmh.2016.61001

Impact of Global Warming on Intensity-Duration-Frequency (IDF) Relationship of Precipitation: A Case Study of Toronto, Canada

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Received 27 May 2015; accepted 9 January 2016; published 12 January 2016

Abstract

Annual maximum rainfall intensity for several duration and return periods has been analyzed according to the Gumbel distribution. The Intensity-Duration-Frequency (IDF) curves before and after 1980 have been computed and compared. For the city of Toronto, it is shown that the rainfall intensities after 1980 are lower than those from before this date. This is especially clear for those of short duration. Comparing our results with those of other authors, it appears that, for the moment, no general law on the impact of global warming on the curves intensity duration frequency cannot be made. It appears that the impact of global warming on rainfall varies with geographic location and that it is not possible to draw some general conclusions across the planet.

Keywords

Climate Change, Duration, Frequency, Intensity, Rainfall

1. Introduction

Regarding civil engineering, the knowledge and understanding of climate change is important because, if there are changes in the variables related to hydrological systems, it could imply changes in design criteria, as these are frequently based upon the assumption of the hydrological series stationary. Not doing so, could mean the under or over design of hydraulic infrastructures, thus creating performance deficiencies or over expensive solutions.

How to cite this paper: Carlier, E. and El Khattabi, J. (2016) Impact of Global Warming on Intensity-Duration-Frequency (IDF) Relationship of Precipitation: A Case Study of Toronto, Canada. *Open Journal of Modern Hydrology*, **6**, 1-7.
http://dx.doi.org/10.4236/ojmh.2016.61001

The IDF (Intensity Duration Frequency) relationship constitutes an objective tool to quantify precipitation uncertainty, especially in circumstances when a design rainfall event must be determined for a particular water resources project. To perform the analysis, long-term precipitation data from a recording rain gage must be available. The prediction of uncertain environmental variables is often a hydrologic problem of significance in water resources management and water resources design projects. The Gumbel distribution, named after one of the pioneer scientists in practical applications of the Extreme Value Theory (EVT), the German mathematician Emil Gumbel (1891-1966), has been extensively used in various fields including hydrology for modeling extreme events [1]-[3]. Gumbel applied EVT on real world problems in engineering and in meteorological phenomena such as annual flood flows [4].

Figure 1 shows that the temperature increases significantly since 1980. The objective of this present work is to study the impact of this increase on the intensity of the rainfall at Toronto for several return periods and durations.

2. Statistical Analysis of the Rainfall

The Gumbel distribution is very suitable for modeling extreme event [5] [6]. The cumulative distribution function (CDF) is given by Equation (1):

$$F_X(x) = P(X \le x) = \exp\left(-\exp\left(-\frac{x-a}{b}\right)\right) \quad (1)$$

where X is a random variable. In our case, X is the rainfall intensity or the rainfall depth for a given duration.

The Gumbel variable is defined by Equation (2)

$$u = \frac{x-a}{b} \quad (2)$$

The parameters a and b are defined by:

$$b = \frac{\sqrt{6}}{\pi}\sigma$$
$$a = \mu - b\gamma \quad (3)$$

where σ is the standard deviation and μ is the mean of the variable.

The empirical distribution of Hazen is used:

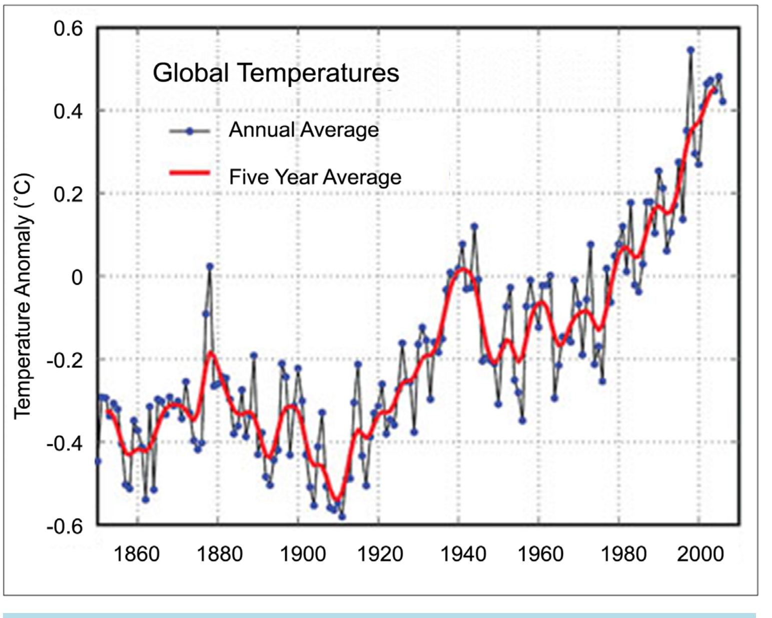

Figure 1. Global temperature anomaly (source: Institute Creation Research (ICR)).

$$F\left(x_i\right) = P\left(i \le x_i\right) = \frac{i - 0.5}{n} \tag{4}$$

where i is the rank of a given data and n is the total number of data.

The rainfall data of Toronto (Canada) has been used for computing the IDF curves before and after 1980.

The rainfall data are from 1940 to 2007. The rainfall station is located at latitude 43.67˚N and longitude 79.4˚W. Its elevation is 112 m. The duration of rainfall are 5 min, 10 min, 15 min, 30 min, 1 h, 2 h, 6 h, 12 h and 24 h. Equation (2) shows that if the Gumbel distribution is valid, it has to a linear relationship between the empirical intensity x and the Gumbel variable u.

Validity of the Gumbel Distribution

Starting from Equation (4), the Gumbel variable is computed by:

$$u = -\ln\left[-\ln\left(\frac{i - 0.5}{n}\right)\right] \tag{5}$$

where ln is the natural logarithm, i is the rank of a given data and n is the total number of data.

Figure 2 shows the experimental rainfall depth versus the Gumble variable. The fit of the data by the Gumbel distribution is suitable.

The IDF curves

The IDF curves are computed for five return periods T (2, 5, 10, 20 and 50 years). For these return periods, the probability associated with the not exceedance is computed by:

$$P\left(X < x\right) = 1 - \frac{1}{T} \tag{6}$$

The Gumbel variable are computed by:

$$u = -\ln\left[-\ln\left(\frac{i - 0.5}{n}\right)\right] \tag{7}$$

Table 1 gives the results of Equations (6) and (7).

For each rainfall duration, there are a specific standard deviation σ and a specific mean μ. Therefore, there are a specific parameters a and b defined by Equation (3).

Table 2 and **Table 3** give the different values of these parameters.

Equation (2) enable to compute the rainfall depth x for the different durations and return periods. Finally, the rainfall intensity is calculated by dividing the rainfall depth by the duration. **Table 4** and **Table 5** give the computed intensity before and after 1980 and **Figures 3-7** shows the IDF curves.

Examination of **Table 4** and **Table 5**, and **Figures 3-7**, it appears that the impact of global warming on the IDF curves is not very clear, however, their analysis enables to note that the rainfall intensities after 1980 are lower than those from before this date.

Table 1. Gumbel variables for several return periods.

Return periods (year)	2	5	10	20	50
Probability associated with the not exceedance	0.5	0.8	0.9	0.95	0.98
Gumbel variable	0.36651292	1.49993999	2.25036733	2.97019525	3.90193866

Table 2. Values of the gumbel variable parameters for several durations (5 min - 1 h).

Duration	5 min	10 min	15 min	30 min	1 h
Mean	9.76393443	13.4672131	16.5032787	20.9852459	25.4163934
Standard deviation	4.02181687	4.81233558	6.60978989	8.77967038	10.0157573
b	3.13738827	3.75406581	5.15624603	6.84895304	7.81321486
a	7.95303392	11.3003663	13.5270935	17.0320302	20.9066058

Table 3. Values of the gumbel variable parameters for several durations (2 h - 24 h).

Duration	2 h	6 h	12 h	24 h
Mean	29.6983607	36.59	43.3305085	48.3409836
Standard deviation	10.6580563	12.7690662	13.7554786	14.7781976
b	8.31426738	9.96104988	10.7305426	11.5283578
a	24.8993655	30.840482	37.1368393	41.6868155

Table 4. Rainfall intensity before 1980 for several durations and return periods.

Intensity mm/h	2 years	5 years	10 years	20 years	50 years
5 min	113.4117503	161.4166204	193.2000214	223.6874228	263.1502366
10 min	77.58690533	105.544399	124.0546925	141.8102105	164.7929072
15 min	62.53175359	87.23827087	103.5961351	119.2869912	139.5971949
30 min	40.15905929	55.73317723	66.04459846	75.93556127	88.73839806
1 h	24.92161479	33.28195029	38.81721985	44.1267833	50.99946856
2 h	14.73000025	19.2794554	22.29159057	25.18090295	28.92082144
6 h	6.084464197	7.9127997	9.123316892	10.28447405	11.78747288
12 h	3.611399512	4.608573879	5.268790077	5.902085304	6.721821017
24 h	2.012225697	2.541975305	2.89271564	3.229154191	3.664639384

Table 5. Rainfall intensity after 1980 for several durations and return periods.

Intensity mm/h	2 years	5 years	10 years	20 years	50 years
5 min	103.1542708	135.6783017	157.2120401	177.867719	204.6043765
10 min	73.74453206	95.29262978	109.5593454	123.2443216	140.9581195
15 min	60.28632165	81.87858867	96.17454824	109.8875759	127.6376836
30 min	37.38801884	53.05351799	63.42544152	73.37443959	86.25239708
1 h	22.0043123	31.57485927	37.91139413	43.98955049	51.85710044
2 h	12.8216983	17.70357911	20.935809	24.03624151	28.04943337
6 h	5.218241187	7.131638583	8.39847415	9.61365324	11.18657793
12 h	3.132853403	4.139250597	4.805573109	5.44472566	6.272043077
24 h	1.76292092	2.314501059	2.679695105	3.029998002	3.483429171

This is especially clear for those of short duration. Their intensity decreased, particularly for the return period of 5, 10, 20 and 50 years.

3. Conclusions

Examination of **Table 4** and **Table 5**, and **Figures 3-7**, it appears that the rainfall intensities after 1980 are lower than those from before this date. This is especially clear for those of short duration.

Fallot and Hertig [7] carried out Gumbel analysis of rainfall depth at 429 locations in Switzerland. They computed rainfall depth for a return period of 500 years and concluded that rainfall depth obtained for the period 1961 to 2010 are overall higher than 15% than estimated from the rainfall series from 1901 to 1970 for all stations in Switzerland. Vaz [8] studied annual maximum daily rainfall series from 23 rain gages in Portugal.

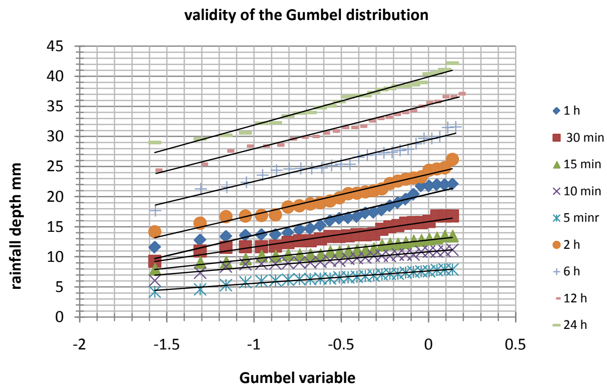

Figure 2. Validity of the Gumbel distribution.

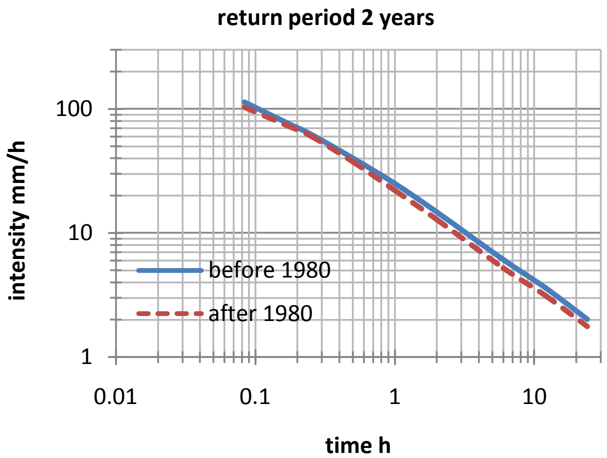

Figure 3. IDF curves before and after 1980. T = 2 years.

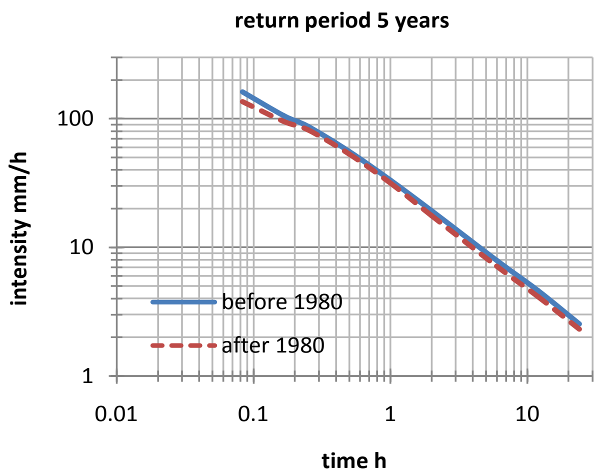

Figure 4. IDF curves before and after 1980. T = 5 years.

The research carried out showed that the samples of intensive rainfalls do not exhibit trends, as to affirm or contradict the effects often attributed to the climate change phenomenon (*i.e.* heavier rainfalls with smaller duration). The study found out that all kinds of behaviors can occur: some samples denote the trends often considered as resulting from the climate change, while exhibit the exact opposite, not allowing the identification of any of the consequences attributed to such phenomenon.

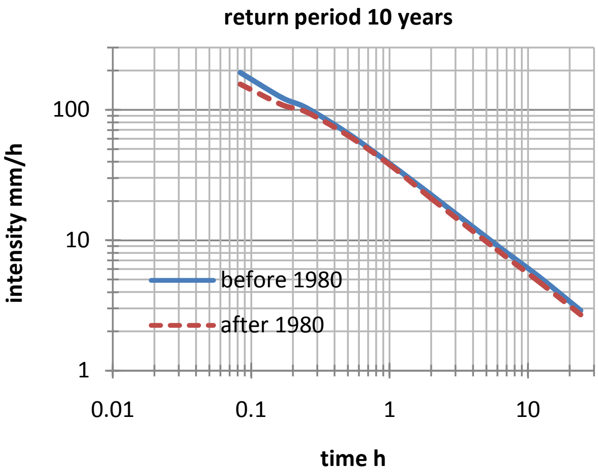

Figure 5. IDF curves before and after 1980. T = 10 years.

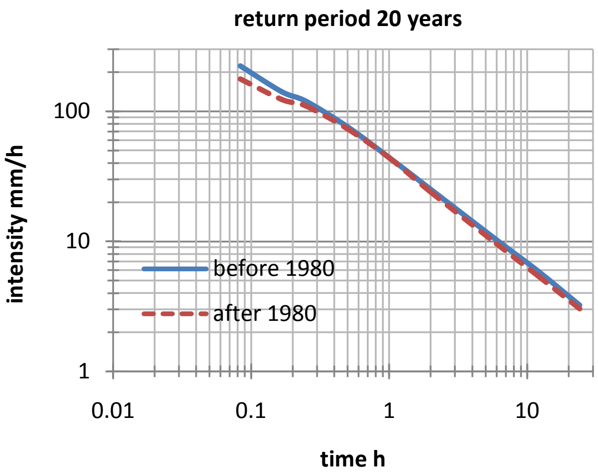

Figure 6. IDF curves before and after 1980. T = 20 years.

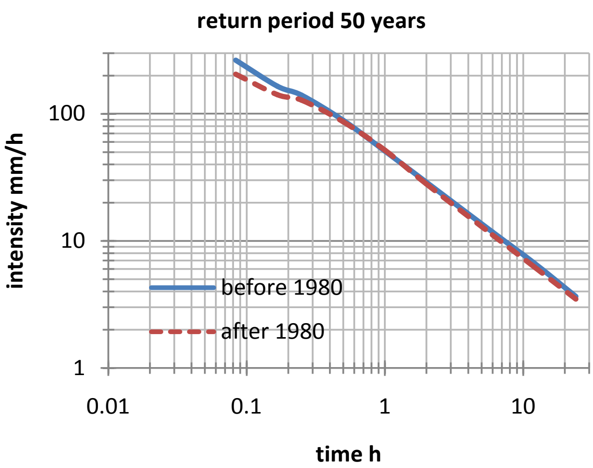

Figure 7. IDF curves before and after 1980. T = 50 years.

Comparing our results with those of other authors, it appears that, for the moment, no general law on the impact of global warming on the intensity duration frequency relationships can be made. It appears that the impact of global warming on rainfall varies with geographic location and that it is not possible to draw some general conclusions across the planet

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