

Numerical Calculation of Transient Flow of Polymer Foam in Porous Media

Wei Zhao, Haiqing Cui, Keliang Wang

Key Laboratory of Enhanced Oil and Gas Recovery of Ministry of Education, Northeast Petroleum University, Daqing, China

Email: zhaoweinepu@163.com

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Abstract

Based on the mathematical model of one dimension transient flow of the polymer foam in porous media, the numerical calculation method of the flow mentioned above by using the finite difference method is given. Through the experiments of one dimension transient flow of HPAM (Hydrolytic Polyacrylamide) foam in the artificial sandstone core, the HPAM foam generation and coalescence coefficient of the mathematical model mentioned above are determined. The profiles of the liquid phase saturation, the pressure drop and the number density of one dimension transient flow of HPAM foam with the dimensionless time in artificial sandstone core are numerically calculated and analyzed by using the numerical calculation method.

Keywords

Polymer Foam, Porous Media, Transient Flow, Mathematical Model, Numerical Calculation Method, Experimental Research

1. Introduction

The researches about the flow of aqueous foam in porous media are published more in literatures and reports. Kovseck [1] established the mathematical model of the transient flow of the aqueous foam in porous media based on the population balance equation and gave the relevant numerical calculation method; Bertin [2] [3] presented the mathematical model of transient flow of the aqueous foam in porous media based on the bubble correlation equation and compared the numerical calculated results of transient flow of the aqueous foam based on the bubble correlation equation in porous media with those based on the population balance equation. And the relevant research about the transient flow of polymer foam in porous media is limited in the experiment research in most cases now, due to its non-Newtonian property of liquid phase causing the flow of polymer foam more complicated than that of the aqueous foam in porous media. Sydansk [4] derived po-

lymer-enhanced foams through the laboratory experiments, which were shown to exhibit, as compared with their counterpart conventional foams, the substantially higher effective viscosities and the much improved stability; Safouane, *et al.* [5] studied the drainage of foams made from Newtonian and non-Newtonian solutions of different viscosities and showed that foams made of solutions containing long flexible Poly Ethylene Oxide molecules counter-intuitively drained faster than foams made of Newtonian solutions of the same viscosity.

By using the finite difference method, one dimension transient flow of the polymer foam in porous media will be numerically calculated. In this paper, the profiles of the liquid phase saturation, the pressure drop and the number density of one dimension transient flow of HPAM foam with the dimensionless time in artificial sandstone core will be analyzed.

2. Mathematical Model

2.1. Hypothesis Conditions

- 1) The porous media is incompressible;
- 2) Polymer foam liquid phase is considered as the incompressible power-law fluid. The concentration of surfactant is constant. Gas phase obeys the ideal gas law;
- 3) Extensive Darcy law is adaptable, foam only influence on the gas phase relative permeability and viscosity;
- 4) Bubbles of the flow of the polymer foam in porous media are in the same size;
- 5) The effects of convection, diffusion, adsorption and gravitation of the polymer foam liquid phase in porous media are neglected;
- 6) The foam relative permeability is characterized by the gas relative permeability model.

2.2. Mass Balance Equation

The liquid and gas phase mass balance equations of one dimension transient flow of the polymer foam in porous media are [1] [6] [7]

$$\frac{\partial(\phi\rho_p S_p)}{\partial t} = \frac{\partial}{\partial x} \left(\rho_p \frac{KK_{rp}}{\mu_p} \left(\frac{\partial P_p}{\partial x} \right) \right) + q_p \quad (1)$$

$$\frac{\partial(\phi\rho_g S_g)}{\partial t} = \frac{\partial}{\partial x} \left(\rho_g \frac{KK_{rg}}{\mu_g} \left(\frac{\partial P_g}{\partial x} \right) \right) + q_g \quad (2)$$

where

$$K_{rp} = k_{rp}^o \left(\frac{S_p - S_{pr}}{1 - S_{pr}} \right)^f \quad (3)$$

$$K_{rg} = k_{rg}^o \left(X_f \left(1 - \frac{S_p - S_{pr}}{1 - S_{pr}} \right) \right)^g \quad (4)$$

$$\rho_g = \rho_{ga} \frac{P_g}{P_{ga}} \quad (5)$$

$$\mu_p = \left(K_c \left[\frac{3n+1}{n+1} \frac{1}{\sqrt{8c'K\phi}} \left(-KK_{rp} \left(\frac{\partial P_p}{\partial x} \right) \right) \right]^{n-1} \right)^{\frac{1}{n}} \quad (6)$$

$$\mu_g = \mu_{g0} + \frac{\alpha n_f}{V_f^c} = \mu_f \quad (7)$$

where ϕ is the porosity; ρ_p the liquid phase mass density, kg/m^3 ; S_p the saturation of liquid phase; K the permeability of porous media, m^2 ; K_{rp} the liquid phase relative permeability; μ_p the apparent viscosity of liquid phase, $\text{Pa}\cdot\text{s}$; P_p the pressure of liquid phase, Pa; q_p the source-sink term of liquid phase, kg/m^3 ; ρ_g the gas mass density, kg/m^3 ; S_g the saturation of gas phase; K_{rg} the gas phase relative permeability; μ_g the gas viscosity, $\text{Pa}\cdot\text{s}$; P_g the pressure of gas phase, Pa; q_g the source-sink term of gas phase, kg/m^3 ; S_f the gas phase saturation of flowing foam; n_f the number density of flowing foam, m^{-3} ; S_t the gas phase saturation of trapped foam; n_t the number density of trapped foam, m^{-3} ; u_f the velocity of the flowing foam in porous media, m/s ; v_f the local interstitial velocity of flowing foam, m/s ; k_{rp}^o the liquid relative permeability where $S_p = S_{pr}$; S_{pr} the connate liquid phase saturation; f the index; k_{rg}^o the gas relative permeability where $S_p = S_{pr}$; X_f the fraction of the gas phase flow rate; g the index; ρ_{ga} the air density under standard atmosphere, kg/m^3 ; P_{ga} the pressure under standard atmosphere, Pa; n flow behavior index of liquid phase; K_c the consistency coefficient of liquid phase, $\text{Pa}\cdot\text{s}^n$; c' the pore tortuosity; μ_{g0} the initial gas phase viscosity, $\text{Pa}\cdot\text{s}$; α the constant of proportionality dependent primarily on the surfactant system, $\text{Pa}\cdot\text{s}^{2/3}\cdot\text{m}^{10/3}$; n_f the number density of flowing foam, m^{-3} ; v_f the local interstitial velocity of flowing foam, m/s ; c the index; μ_f the foam effective viscosity, $\text{Pa}\cdot\text{s}$.

2.3. Population Balance Equation

The population balance equation of one dimension transient flow of the polymer foam in porous media is [1]

$$\frac{\partial(\phi(S_f n_f + S_t n_t))}{\partial t} = -\frac{\partial(u_f n_f)}{\partial x} + \phi S_g \left(k_1 v_f^{1/3} v_p - k_{-1}^o \frac{(1-S_p)}{(S_p - S_{pc})} v_f n_f \right) + H_b \quad (8)$$

$$S_f = X_f S_g \quad (9)$$

$$S_t = X_t S_g \quad (10)$$

$$u_f = -\frac{KK_{rf}}{\mu_f} \frac{\partial P_g}{\partial x} \quad (11)$$

$$X_f + X_t = 1 \quad (12)$$

$$X_t = X_{t,\max} \left(\frac{\beta n_t}{1 + \beta n_t} \right) \quad (13)$$

where S_f is the gas phase saturation of flowing foam; S_t the gas phase saturation of trapped foam; n_t the number density of trapped foam, m^{-3} ; u_f the velocity of the flowing foam in porous media, m/s ; k_1 the foam generation coefficient, $\text{m}^{1/3}\cdot\text{s}^{-13/3}$; v_p the local interstitial liquid velocity, m/s ; k_{-1}^o the foam coalescence coefficient, m^{-1} ; S_{pc} the liquid phase saturation relevant to the limiting capillary pressure; H_b the source-sink term for foam bubbles in units of number per unit volume per unit time, m^{-3} ; X_t the fraction of the gas phase trapped rate; K_{rf} the relative permeability of the foam that is flowing; $X_{t,\max}$ the maximum fraction of the gas phase that trapped, β trapped parameter.

2.4. Assistant Equation

2.4.1. Relationship between Pressure

$$P_c = P_g - P_p \quad (14)$$

$$P_c = \sigma \sqrt{\frac{\phi}{K}} J(S_p) \quad (15)$$

where [3] P_c is the capillary pressure, Pa; σ the equilibrium surface tension, N/m; $J(S_p)$ the Leverett J function.

2.4.2. Relationship between Saturation

$$S_p + S_g = 1 \quad (16)$$

2.4.3. Relationship between the Number Density of Flowing Foam and the Number Density of Trapped Foam

$$n_t = \frac{K_f A_f n_f}{1 + K_f n_f} \quad (17)$$

where K_f , A_f are the bubble trapped parameters [1].

2.5. Definite Condition

2.5.1. Initial Condition

$$P_p(x, 0) = P_{pi} \quad (18)$$

$$P_g(x, 0) = 0 \quad (19)$$

$$S_p(x, 0) = 1 \quad (20)$$

$$S_g(x, 0) = 0 \quad (21)$$

$$n_f(x, 0) = 0 \quad (22)$$

$$n_t(x, 0) = 0 \quad (23)$$

where P_{pi} is the initial liquid phase pressure, Pa.

2.5.2. Boundary Condition

$$-\frac{K \cdot K_{rp}}{\mu_p} \frac{\partial P_p}{\partial x} \Big|_{t=t, x=0} = u_{pi} \quad (24)$$

$$-\frac{K \cdot K_{rg}}{\mu_g} \frac{\partial P_g}{\partial x} \Big|_{t=t, x=0} = u_{gi} \quad (25)$$

$$P_p(L, t) = P_0 \quad (26)$$

$$P_g(L, t) = P_0 \quad (27)$$

$$n_f(0, t) = n_f(L, t) = 0 \quad (28)$$

$$n_t(0, t) = n_t(L, t) = 0 \quad (29)$$

where u_{pi} is the injection velocity of liquid phase, m/s; u_{gi} the injection velocity of gas phase, m/s; L the length of porous media, m, P_0 the backpressure, Pa.

The mass balance Equations (1) and (2), the bubble population balance Equation (8), the assistant equations (14), (16), (17), the definite condition (18)-(29) mentioned above consist of the mathematical model of one dimension transient flow of the polymer foam in porous media.

It is generally considered that the polymer foam generation coefficient k_1 and the coalescence coefficient k_{c1}^o of the transient flow of the polymer foam in porous media are influenced by the physical properties of the polymer foam liquid phase, the polymer foam gas phase and the porous media, which needed to be determined by experiments.

3. Numerical Calculation Method

The mathematical model mentioned above is numerically calculated by using the finite difference method.

The mass balance Equations (1) and (2) of the mathematical model mentioned above are solved by using IMPES (Implicit Pressure Explicit Saturation) method. And the bubble population balance Equation (8) is solved by using TVD (Total Variation Diminish) three order methods [6] [8] [9].

4. Experiment Research

Through the laboratory experiments, with measuring the time-average pressure drop $\overline{\Delta P}^*$ of one dimension transient flow of the HPAM foam in the artificial sandstone core, the generation coefficient k_1^* and the coalescence coefficient k_{-1}^{o*} of the mathematical model mentioned above in the artificial sandstone core are determined.

4.1. Experiment Meter

The main experiment meters include: DV-II + Pro viscometer, produced by American Brookfield Company, the measurement error is $\pm 1\%$; TX-500C Spining interfacial tension meter, produced by American Bowing Company, the measurement error is 0.001 mm.

4.2. Experiment Material

HPAM: foam liquid phase is the aqueous solution of 0.045 wt% HPAM, 0.200 wt% FL-605 and 0.830 wt% NaCl, gas phase is air. The core is the homemade artificial sandstone core of 0.045 m \times 0.045 m \times 0.30 m.

4.3. Experiment Programme

The experiment programme shows in **Figure 1**.

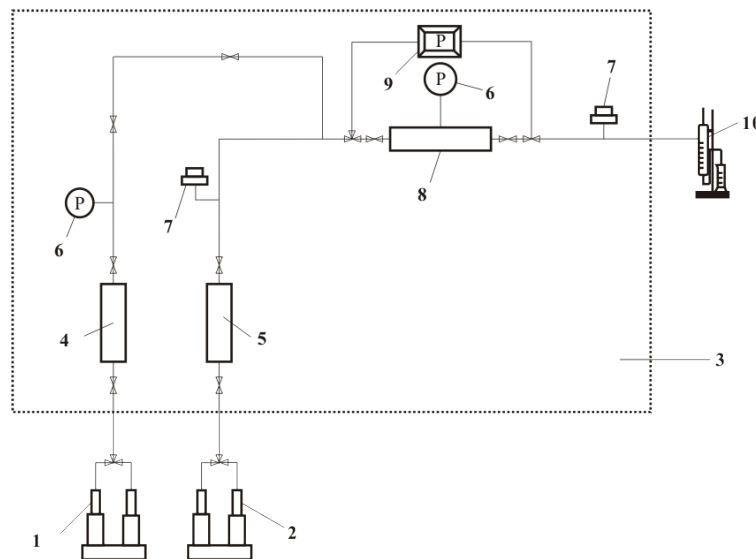


Figure 1. The experiment programme.

“1” D-250L liquid pump; “2” D-250L gas pump; “3” PMQ-2 foam performance measurement apparatus; “4” the liquid can; “5” the gas can; “6” digital manometer; “7” back-pressure valve; “8” the homemade artificial sandstone; “9” digital manometer; “10” cylinder.

Shut the back-pressure valve “7” of the outlet of the artificial sandstone core “8” of PMQ-2 foam performance measurement apparatus “3” and set the backpressure: Start D-250L liquid pump “1” and the liquid can “4” and set the liquid injection velocity u_p^* ; start D-250L gas pump “2” and the gas can “5” and set the air injection velocity u_g^* ; measure the time-average pressure drop $\overline{\Delta P}^*$ of the homemade artificial sandstone core “8” continuously by using the digital manometer “9” and “6” of “3”.

4.4. Experimental Result

4.4.1. Experimental Parameters

The experimental parameters of one dimension transient flow of the HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 and 0.083 wt% NaCl in the artificial sandstone core show as **Table 1** [1].

Table 1. Experimental parameters.

$\rho_p/\text{kg}\cdot\text{m}^{-3}$	1000.24	$\sigma\times 10^{-1}/(\text{N}/\text{m})$	0.28
n	0.7872	$\rho_{\text{ga}}\times 10^{-1}/\text{kg}\cdot\text{m}^{-3}$	11.03
$K_c\times 10^{-2}/(\text{Pa}\cdot\text{s}^n)$	0.5126	$\mu_{\text{go}}\times 10^{-4}/\text{Pa}\cdot\text{s}$	0.18
L/m	0.30	$\mu_{\text{po}}\times 10^{-1}/\text{Pa}\cdot\text{s}$	0.01
K/D	0.51	$\beta\times 10^{-9}/\text{m}^3$	1.00
ϕ	0.25	$X_{t,\text{max}}$	0.90
c'	2.30	$\alpha\times 10^{-14}/\text{Pa}\cdot\text{s}^{2/3}\cdot\text{m}^{10/3}$	6.00
k_{tp}^o	0.72	c	0.33
k_{tg}^o	1.00	P_0/MPa	4.80
f	2.99	P_{pi}/MPa	4.80
g	3.02	P_{ga}/MPa	0.10
S_{pr}	0.33	$u_p^*/(\text{m}/\text{d})$	0.046
S_{pi}	1.00	$u_g^*/(\text{m}/\text{d})$	0.430
S_{pc}	0.26	$\overline{\Delta P^2}/\text{kPa}$	1129.42

4.4.2. Determination of the Generation and Coalescence Coefficient

By using the mathematical model, the relevant numerical calculated method, **Table 1** mentioned above and B (Bisection) iterated algorithm, the generation coefficient k_1^* and the coalescence coefficient k_{-1}^{o*} of one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core can be determined as $k_1^* = 10.59 \times 10^{13} \text{ m}^{1/3} \cdot \text{s}^{-13/3}$, $k_{-1}^{o*} = 60.83 \text{ m}^{-1}$ [1] [6].

5. Numerical Calculation Results

By using the mathematical model mentioned above, the relevant numerical calculated method, **Table 1** and the generation coefficient k_1^* and the coalescence coefficient k_{-1}^{o*} , the profiles of the liquid phase saturation S_p , the pressure drop ΔP_p , the number density n_f of one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core with the dimensionless time PV are numerical calculated and analyzed.

5.1. Profiles of Liquid Phase Saturation

The profiles of the liquid phase saturation S_p of the one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core with the dimensionless time PV show as **Figure 2**.

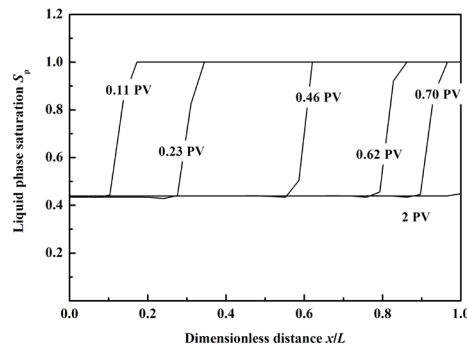


Figure 2. Profiles of liquid phase saturation S_p of HPAM foam with the dimensionless time PV.

In **Figure 2**, the liquid phase saturation S_p of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl at the flow front appears to be steep and sharp, the liquid phase saturation S_p of the upstream of the flow front is about 0.42, and 1.00 of the downstream. The gas phase breaks through at about dimensionless time 0.70 PV. The liquid phase saturation S_p at steady state is about 0.42. HPAM foam flows in a piston-like fashion in the artificial sandstone core. After HPAM foam flow front passing a particular location of the core, the liquid phase saturation S_p changes very little.

5.2. Profiles of Pressure Drop

The profiles of the pressure drop ΔP_p of the one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core with the dimensionless time PV show as **Figure 3**.

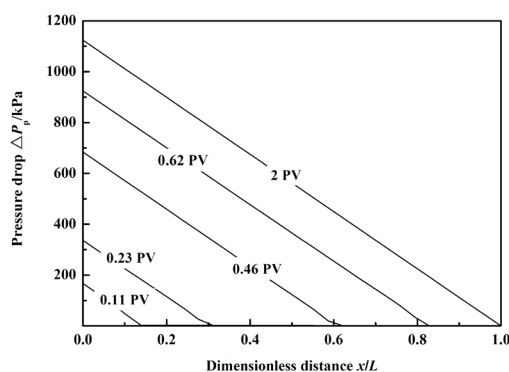


Figure 3. Profiles of pressure drop ΔP_p of HPAM foam with the dimensionless time PV.

In **Figure 3**, the pressure drop ΔP_p of the flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl decreases away from the core entrance with the dimensionless distance x/L . The pressure drop ΔP_p of the downstream of the flow front is close to 0.00 kPa. The pressure drop ΔP_p doesn't increase up to 1130.21 kPa along with dimensionless time PV elapsing until about 2 PV. The steady pressure drop ΔP_p is 1130.21 kPa.

5.3. Profiles of Number Density of Flowing HPAM Foam

The profiles of the number density n_f of the one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core with the dimensionless time PV show as **Figure 4**.

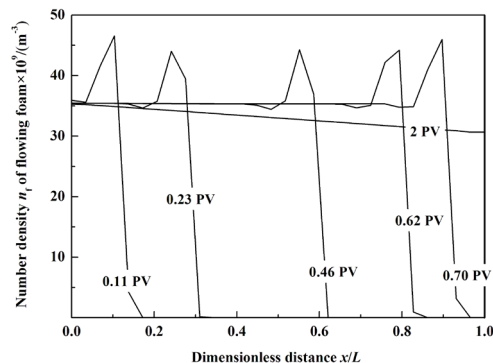


Figure 4. Profiles of the number density n_f of HPAM foam with the dimensionless time PV.

In **Figure 4**, there is the flow front of the flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core. The number density n_f of flowing HPAM foam reaches the peak value at the flow front. The steady state profile of the number density n_f of flowing HPAM foam appears at 2 PV, after which the number density n_f of flowing HPAM foam decreases from the inlet to the outlet.

6. Experiment Verification

By using the mathematical model, the relevant numerical calculated method, **Table 1** and the generation coefficient k_1^* , the coalescence coefficient k_{-1}^{o*} mentioned above, the curves of the calculated pressure drop ΔP and the experiment pressure drop ΔP^* of one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 and 0.830 wt% NaCl with dimensionless time PV in the artificial sandstone core are compared as **Figure 5**.

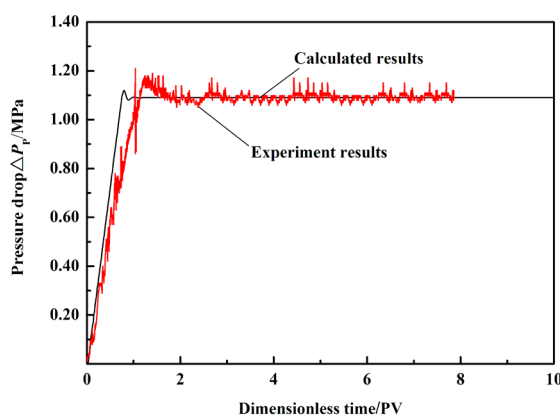


Figure 5. Comparison between the curves of the numerical calculated pressure drop ΔP and the experiment pressure drop ΔP^* of HPAM foam.

In **Figure 5**, the curves of the calculated pressure drop ΔP curve and the experiment pressure drop ΔP^* of one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 and 0.830 wt% NaCl in artificial sandstone core are basically coincided, which show that the mathematical model of one dimension transient flow of polymer foam in porous media and the relevant numerical calculation method in the paper are correct [10].

7. Conclusions

- 1) Based on the mathematical model of one dimension transient flow of the polymer foam in porous media, the numerical calculation method of the flow mentioned above by using the finite difference method is given;
- 2) Through the experiments, the generation coefficient and the coalescence coefficient of the transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 surfactant and 0.830 wt% NaCl in the artificial sandstone core are determined;
- 3) The profiles of the liquid phase saturation, the pressure drop and the number density of one dimension transient flow of HPAM foam of 0.045 wt% HPAM, 0.200 wt% FL-605 and 0.830 wt% NaCl with the dimensionless time in the artificial sandstone core are numerically calculated and analyzed.
- 4) Through the experiments, the mathematical model of one dimension transient flow of polymer foam in porous media and the relevant numerical calculation method are verified.
- 5) Based on the research results mentioned above, it needs the future study to continue working and researching for the polymer concentration influencing on the transient flow of the polymer foam in porous media.

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