

Forced Convection Thermal Boundary Layer Transfer for Non-Isothermal Surfaces Using the Modified Merk Series

Ayodeji Falana*, Richard Olayiwola Fagbenle

Department of Mechanical Engineering, University of Ibadan, Ibadan, Nigeria
Email: *falana.ayo@ui.edu.ng

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Abstract

The Chao and Fagbenle's modification of Merk series has been employed for the analysis of forced convection laminar thermal boundary layer transfer for non-isothermal surfaces. In addition to the Prandtl number (Pr) and the pressure gradient (Δ), a third parameter (temperature parameter, γ) was introduced in the analysis. Solutions of the resulting universal functions for the thermal boundary layer have been obtained for Pr of 0.70, 1.0 and 10.0 and for a range of Δ . The results obtained for the similarity equations agreed with published results within very close limits for all the Δ 's investigated.

Keywords

Boundary Layer, Non-Isothermal, Similarity Solutions, Power-Law

1. Introduction

Heat transfer in Newtonian fluids from external surfaces of bodies of various geometries has been the subject of numerous investigations during the past decades, since the pioneer work of Prandtl [1] [2] [3]. The interest in this subject still continues. However, in most of the works it has been assumed that the flow is over isothermal surfaces. In practice, it is sometimes found that the non-isothermal surface case is important because the isothermal case does not occur naturally. The present work is done to provide a theoretical means of analyzing the momentum and energy transport in such flows. The Merk-Chao-Fagbenle series solution technique is employed in this work, so a brief history of the method is appropriate.

*Corresponding author.

A procedure which belongs to the category of “wedge” methods and which provides a rigorous refinement of the local similarity concept is that of Merk [4]. Following Gortler and Meksyn [5] [6], Merk derived the momentum and energy boundary layer equations in the transformed coordinates (ξ, η) . In common with Gortler method, his series solution is also expressed in terms of universal functions. The point of departure is that Gortler expanded the “wedge” variable or the pressure gradient Λ in series of ξ while Merk chose the inverse expansion and adopted Λ as one of the independent coordinates. Merk evaluated the first term in the series by the asymptotic integration method propounded by Meksyn. As in Gortler’s series, the first term corresponds to the local similarity solution. Merks method is an extension or refinement of Meksyn’s procedure. An essential feature of Merk’s scheme is that it makes possible rapid calculations of the significant boundary layer quantities with the aid of a limited number of universal functions which can be tabulated once and for all [7] [8].

An advance in the accuracy of boundary-layer series solutions was therefore made possible by Merk in 1959. He refined the “wedge method” proposed by Meksyn by choosing to treat the wedge parameter, Λ as an independent variable rather than the stream wise coordinate, ξ . Thus the Merk series were expanded about the local similarity solution rather than the forward stagnation point of the body, as had been the convention in the past [5] [6]. However, an error in the form of series presented by Merk was found by independent researchers [7] [8]. Chao and Fagbenle therefore put forth a corrected form of Merk’s series and use it to perform a universal, laminar boundary layer analysis for the forced flow of Newtonian fluids over isothermal bodies. Since then, the Merk-Chao-Fagbenle’s approach has been used with success for a family of boundary-layer solutions.

Some of the latest applications of the Merk-Chao-Fagbenle series solution technique have been universal boundary layer analyses of the mixed convection to Newtonian fluids by Cameron, M.R. *et al.* [9] and the pure-forced and pure natural convection to non-Newtonian power-law fluids also by Cameron, M.R. *et al.* [10]. Meissner, D.L. *et al.* [11] extended the Merk-Chao-Fagbenle method to mixed convection to power-law fluids. Tien-Chen, A.C. *et al.* [12] applied the method to natural convection to power-law fluids from two-dimensional or axisymmetrical bodies of arbitrary contour. Falana, A. [13] [14] also applied the method for the analysis of thermal boundary layer for non-isothermal cases of flat plate and horizontal cylinder in cross flow.

The purpose of the present investigation is to employ the Merk-Chao-Fagbenle procedure for the analysis of thermal boundary layer on non-isothermal surfaces by providing firstly the sequence of the differential equations governing the universal functions associated with the method and, secondly, to provide a tabulation of these and other related functions.

With the availability of such tabulation, the determination of the local wall shear and the surface heat transfer rates over the non-isothermal simple geometrical surfaces become a simple matter, once the outer stream velocity distribution is known. The development of the boundary layer and details of the velocity and temperature fields can be obtained with equal ease.

2. Governing Equations and Mathematical Analysis

Here, consideration is given to the conservation equations for steady, laminar, non-dissipative, constant property boundary layer flow over two-dimensional or rotationally symmetrical (or axisymmetric) bodies of non-uniform surface temperature, $T_w(x)$, situated in an infinite ambient fluid of undisturbed temperature, T_∞ . The coordinate x denotes the distance along the body surface from the forward stagnation point, and the coordinate y denotes the normal distance from the surface. Accordingly, the velocity components u and v are in x - and y -directions respectively. For axisymmetric flows, $r(x)$ represents the distance from the axis of symmetry to the body surface, and for two-dimensional flows, $r = L$, the reference length. Following Chao and Fagbenle, the boundary layer equations are reproduced below:

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) u = u \frac{du}{dx} + v \frac{\partial^2 u}{\partial y^2} \quad (1)$$

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial y} = 0 \quad (2)$$

$$\left(u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} \right) T = \alpha \frac{\partial^2 T}{\partial y^2} \quad (3)$$

The boundary conditions considered by Merk-Chao-Fagbenle are,

$$\text{for } y = 0; \quad u = v = 0 \text{ and } T = T_w \tag{4}$$

$$\text{for } y \rightarrow \infty; \quad u \Rightarrow u_\infty \text{ and } T \Rightarrow T_\infty \tag{5}$$

where T_∞ is a constant.

The continuity equation is identically satisfied by introducing a stream function $\psi(x, y)$ defined by

$$u = \frac{1}{r} \frac{\partial \psi}{\partial y}, \quad v = -\frac{1}{r} \frac{\partial \psi}{\partial x} \tag{6}$$

In (6) and other equations which follow, one needs only to set $r = L$ for two-dimensional flows.

Following Chao and Fagbenle, the (x, y) coordinate system is transformed into a new dimensionless coordinate system by adopting the dimensionless variables.

$$\xi = \int_0^x \frac{u(x)r^2}{U_\infty L^2} \cdot \frac{dx}{L}; \quad \eta = \left[\frac{\text{Re}}{2\xi} \right]^{\frac{1}{2}} \frac{u(x)r}{U_\infty L} \cdot \frac{y}{L} \tag{7}$$

A dimensionless stream function f is introduced, such that

$$\psi(x, y) = U_\infty L \left[\frac{2\xi}{\text{Re}} \right]^{\frac{1}{2}} f(\xi, \eta) \tag{8}$$

From (6), (7) and (8), we obtain

$$u = \frac{\partial f}{\partial \eta} \tag{9a}$$

$$v = \left(\frac{r}{L} \right) \frac{u}{(2\xi \text{Re})^{\frac{1}{2}}} \left[f + 2\xi \frac{\partial f}{\partial \xi} + \left(\frac{2\xi}{r} \frac{dr}{d\xi} + \wedge - 1 \right) \eta \frac{\partial f}{\partial \eta} \right] \tag{9b}$$

where

$$\wedge = \frac{2\xi}{u} \frac{du}{d\xi} \tag{10}$$

is the wedge variable.

It was named by Görtler as the principal function.

Using (9) in (1), the momentum equation with associated boundary conditions may be reduced to the following system:

$$f''' + ff'' + \wedge [1 - (f')^2] = 2\xi \frac{\partial(f', f)}{\partial(\xi, \eta)} \tag{11}$$

and the boundary conditions are

$$\left. \begin{aligned} f = f' = 0 & \quad \text{for } \eta = 0 \text{ and} \\ f' = 1 & \quad \text{for } \eta \rightarrow \infty \end{aligned} \right\} \tag{12}$$

The primes denote differentiation with respect to η while $\frac{\partial(f', f)}{\partial(\xi, \eta)}$ is the Jacobian.

The quantity \wedge is a function of ξ , *i.e.* x can be calculated explicitly if $U(x)$ is given.

The above momentum Equation (11) is unchanged for the uniform surface temperature problem.

In order to transform Equation (3), we write

$$T_w(x) - T_\infty = (T_0 - T_\infty) e^{\frac{ax}{L}} \tag{13}$$

The boundary conditions are:

For $y = 0; u = v = 0$ and

$$y = 0 \Rightarrow T_w(x) = (T_0 - T_\infty)e^{\frac{ax}{L}} \tag{14}$$

For

$$y \rightarrow \infty; U(x) = U_\infty \text{ and } T = T_\infty \tag{15}$$

The dimensionless temperature is defined as:

$$\frac{T(x, y) - T_\infty}{T_w(x) - T_\infty} = \theta(\xi, \eta) \tag{16}$$

Substituting (13) in (16), gives

$$T(x, y) = T_\infty + (T_0 - T)e^{ax/L}\theta(\xi, \eta) \tag{17}$$

Making use of (9) and (17) the transformation of (3) is now easily obtained as

$$\theta'' + \text{Pr } f\theta' - \text{Pr } \gamma f'\theta = 2\xi \text{Pr } \frac{\partial(\theta, f)}{\partial(\xi, \eta)} \tag{18}$$

while the boundary conditions (4) and (5) become:

$$\theta(\xi, 0) = 0; \theta(\xi, \eta) \rightarrow \infty \text{ as } \eta \rightarrow \infty \tag{19}$$

The temperature parameter, γ is given by

$$\gamma = \frac{2\xi \frac{d}{d\xi}(T_w(x) - T_\infty)}{e^{ax/L}}$$

$$\gamma = \frac{2\xi \frac{d}{d\xi}(T_w(x) - T_\infty)}{(T_w(x) - T_\infty)} \tag{20}$$

It can now be seen that with the variable wall temperature proposed in (13), Equation (18) contains an additional term:

$$-\text{Pr } \gamma f'\theta$$

With $T_w(x) = \text{constant}$, Equation (18) reduces to the same equation considered by Chao and Fagbenle.

Herewith, the transformations are complete and we have now to solve (11) with boundary conditions (12) and (18) with boundary conditions (19).

3. Solution Methodology

The appropriate series solution for (11) and (12) is that used by Chao and Fagbenle. It is given by

$$f(\xi, \eta) = f_0(\wedge, \eta) + 2\xi \frac{d\wedge}{d\xi} f_1(\wedge, \eta) + 4\xi^2 \frac{d^2\wedge}{d\xi^2} f_2(\wedge, \eta)$$

$$+ \left(2\xi \frac{d\wedge}{d\xi}\right)^2 f_3(\wedge, \eta) + \left(2\xi \frac{d\wedge}{d\xi}\right)^2 \left(4\xi^2 \frac{d^2\wedge}{d\xi^2}\right) f_{1,2}(\wedge, \eta) \tag{21a}$$

The appropriate series solution for (18) and (19) according to Chao and Fagbenle is stated as follows:

$$\theta(\xi, \eta) = \theta_0(\wedge, \eta) + 2\xi \frac{d\wedge}{d\xi} \theta_1(\wedge, \eta) + 4\xi^2 \frac{d^2\wedge}{d\xi^2} \theta_2(\wedge, \eta)$$

$$+ \left(2\xi \frac{d\wedge}{d\xi}\right)^2 \theta_3(\wedge, \eta) + \left(2\xi \frac{d\wedge}{d\xi}\right) \left(4\xi^2 \frac{d^2\wedge}{d\xi^2}\right) \theta_{1,2}(\wedge, \eta) + \dots \tag{21b}$$

Upon substituting (21a) into (11) and (12) and collecting terms, free of $d\Lambda/d\xi$ and then terms common to $\xi \frac{d\Lambda}{d\xi}$, $\xi^2 \frac{d^2\Lambda}{d\xi^2}$ etc, leads to the following hierarchy of differential equations:

$$\left. \begin{aligned} f_0''' + f_0 f_0'' + \Lambda [1 - (f_0')^2] &= 0 \\ \text{with} \\ f_0 = f_0' = 0 \text{ for } \eta = 0, \text{ and } f_0' = 1 \text{ for } \eta \rightarrow \infty \end{aligned} \right\} \quad (22)$$

$$\left. \begin{aligned} f_1''' + f_0 f_0'' - 2(1 + \Lambda) f_0' f_1' + 3 f_0'' f_1 &= \frac{\partial(f_0', f_0)}{\partial(\Lambda, \eta)} \\ \text{with} \\ f_1 = f_1' = 0 \text{ for } \eta = 0, \text{ and } f_1' = 1 \text{ for } \eta \rightarrow \infty \end{aligned} \right\} \quad (23)$$

$$\left. \begin{aligned} f_2''' + f_0 f_2'' - 2(2 + \Lambda) f_0' f_2' + 5 f_0'' f_2 &= \frac{\partial(f_1', f_0)}{\partial(\Lambda, \eta)} \\ \text{with} \\ f_1 = f_1' = 0 \text{ for } \eta = 0, \text{ and } f_2' = 0 \text{ for } \eta \rightarrow \infty \end{aligned} \right\} \quad (24)$$

An inspection of the above set of equations shows that they can be integrated as if they were ordinary differential equations since, for any given stream wise location, Λ is fixed. Furthermore, all f_i 's ($i = 0, 1, 2, \dots$) are universal in the sense that they depend on a single parameter. As such, they can be tabulated once and for all.

For energy Equations (18) and (19), series (21b) leads to hierarchy of differential equations which depends on 3 parameters Pr , Λ and γ . It is rather too bulky to tabulate the resulting universal functions.

We seek a better series solution for (18) and (19). A better series solution which leads to only a 2-parameter universal functions dependence is:

$$\begin{aligned} \theta(\xi, \eta) = & \theta_0(\Lambda, \eta) + 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta_1(\Lambda, \eta) + \gamma \theta_2(\Lambda, \eta) + \gamma 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta_3(\Lambda, \eta) \\ & + 4\xi^2 \left(\frac{d^2\Lambda}{d\xi^2} \right) \theta_4(\Lambda, \eta) + \dots \end{aligned} \quad (26)$$

Substituting (26) in (18) and (19) leads to the following hierarchy of differential equations:

$$Pr^{-1} \theta_0'' + f_0 \theta_0' = 0 \quad (27)$$

$$Pr^{-1} \theta_1'' + f_0 \theta_1' - 2 f_0' \theta_1' = -3 f_1 \theta_0' + \frac{\partial(\theta_0, f_0)}{\partial(\Lambda, \eta)} \quad (28)$$

$$Pr^{-1} \theta_2'' + f_0 \theta_2' - 2 f_0' \theta_2' = f_0' \theta_0' \quad (29)$$

$$Pr^{-1} \theta_3'' + f_0 \theta_3' - 2 f_0' \theta_3' = f_0' (\theta_1 + 2\theta_2) + f_1 \theta_0' + 2 f_1' \theta_2 - 3 f_1 \theta_2' + f_0' \frac{\partial \theta_2}{\partial \Lambda} - \theta_2' \frac{\partial f_0}{\partial \Lambda} \quad (30)$$

$$Pr^{-1} \theta_4'' + f_0 \theta_4' - 4 f_0' \theta_4' = f_0' \theta_1' - f_1 \theta_0' - 5 f_2 \theta_0' \quad (31)$$

With boundary conditions:

$$\text{for } i = 0; \theta_i = 0 \text{ at } \eta = 0 \text{ and } \theta_i = 0 \text{ as } \eta \rightarrow \infty \text{ and for } i > 0; \theta_i = 0 \text{ at } \eta = 0 \text{ and } \theta_i = 0 \text{ as } \eta \rightarrow \infty \quad (32)$$

Thus all the temperature functions in the sequence dependent on 2 parameters, Pr and Λ .

The dependence of the temperature functions on the temperature parameter γ is indirect through the series solution expansion. This parameter γ would only need to be evaluated at each point of the flow and does not come into the tabulation of the universal functions.

With the form of the modified Mark-Chao-Fagbenle series proposed in Equations (21), (26), the higher-order solutions depend on the lower-order solutions, but not vice-versa. An inspection of the resulting equations also shows that there are no terms containing the derivatives of the parameter Λ and therefore, the geometry of the surface is totally absorbed in these parameters. By assigning a fixed value of Λ for any given streamwise location, the equations can be integrated as if they were ordinary differential equations and the results tabulated once and for all.

When the solutions for the various f 's and θ 's are available, the parameters of physical interest for the present problem which can readily be obtained are the local skin friction coefficient C_f , displacement and momentum thickness, local Nusselt number, etc.

The local Nusselt number may be defined by

$$Nu = -L \left(\frac{\partial T}{\partial y} \right)_{y=0} / (T_w - T_\infty) = L \left(\frac{\partial \theta}{\partial y} \right)_{y=0}$$

and one finds, after transformation

$$Nu Re^{-1/2} = \frac{r U(x)}{L U_\infty} \frac{1}{(2\xi)^{1/2}} \left[\theta'_0(\eta, 0) + 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta'_1(\eta, 0) + \gamma \theta'_2(\eta, 0) + \gamma 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta'_3(\eta, 0) + 4\xi^2 \frac{d^2\Lambda}{d\xi^2} \theta'_1(\eta, 0) \right] \tag{33}$$

Chao and Fagbenle [7] [8] have already given extensive applications to the calculation of the local friction coefficient, displacement and momentum thickness. Tabulated values of the above mentioned functions are available with the authors. However, we shall confine ourselves here to the calculation of the local heat transfer quantities.

The f_i and the θ_i functions (1,2,3,...) are universal in character. The f_i functions depend on the single parameter Λ while the θ_i functions depend, additionally, on Pr. They are tabulated once and for all. Equations (22) to (31) with their respective boundary conditions therefore use the above technique. The governing ordinary differential equations for the f_i and the θ_i functions are integrated sequentially using the fourth order Runge-Kutta method.

We first convert the equations stated above into the following simultaneous linear equations of first order as follows.

$$Y_1 = f; Y_1' = f_1'; Y_5 = y_4^1 = \theta'; Y_3 = f'' = y_2^1; Y_3 = \theta; Y_5 = y_4^1 = \theta' \tag{34}$$

Then the Newtons method is applied to transform the problem into initial value one, where the initial conditions are:

$$Y_1(0) = 0, Y_2(0) = 0, Y_4(0) = 1.0 \tag{35}$$

$$Y_3(0) = a_0, Y_5(0) = b_0 \tag{36}$$

Hence, a_0 and b_0 are a priori unknown and has to be determined as part of the numerical solution. Once the problem is reduced to initial value problem, it is solved using the famous fourth-order Runge-Kutta method.

A computer program was written for the calculation of f_0, f_0', f_0'' for 20 values of Λ in the range $-0.15 \leq \Lambda \leq 1.0$. According to Chao and Fagbenle the selection of the lower limit is based on the anticipation that the Merk-Meksyn procedure is, in general, not applicable to regions of retarded flow near the separation point at which $\Lambda = -0.199$.

The computation began with $f_0(\Lambda, 0) = f_0'(\Lambda, 0) = 0$. Since $f_0''(\Lambda, 0)$ was unknown, Newton's iterative scheme was used starting from a guessed value of $f_0''(\Lambda, 0)$ say a_0 . Any reasonable initial guess would eventually lead to the desired result.

Very high accuracy is required of the basic function f_0 and its derivatives since θ_0, f_1 and hence, other functions of the higher order in the hierarchy of the differential equations are sensitive to its variation. The fact is that the differential equation for f_1 contains derivatives of f_0 and f_0' with respect to Λ . After a number of trials, the integration step size used is: $\Delta\eta = 0.1$ for $\eta \leq 15.0$.

The iteration was to continue until the following conditions was met:

$$|1 - f_0'(\Lambda, \eta)| < 10^{-6} \tag{37}$$

where η designates or represents some maximum large value of η .

The results obtained (for the similarity equations) as a result agreed with the published data of Chao and Fagbenle within very close limits for all Λ 's investigated. This is demonstrated in **Table 1**.

The integration of the f_1 equation requires the evaluation of the derivatives $\frac{\partial f_0}{\partial \Lambda}$ and $\frac{\partial f_0'}{\partial \Lambda}$.

A method of parameter differentiation was employed for the evaluation of these derivatives.

The integration of the f_2 equation was done easily since it does not involve the Λ -derivatives. For the f_3 equation, the evaluation of the derivatives $\frac{\partial f_0}{\partial \Lambda}$, $\frac{\partial f_0'}{\partial \Lambda}$, $\frac{\partial f_1}{\partial \Lambda}$ and $\frac{\partial f_1'}{\partial \Lambda}$ was also carried out using the method of parameter differentiation.

The integration step size $\Delta\eta$ used in numerical computation of the f_i , (for $i = 1, 2, 3$) were the same as those for f_0 .

The above choice of $\Delta\eta$ led to wall derivatives of the θ_0 function which are accurate to six significant figures, based on a comparison with the already cited work of Chao and Fagbenle. This is equally demonstrated in **Table 2**.

The iterative routine for the computation of $\theta_0'(\Lambda, 0)$ is essentially the same as that for the f_0 function.

Table 1. Wall derivatives of universal velocity functions $f_0'(\Lambda, 0)$.

Λ	Chao and Fagbenle [1974]	Present Results
-0.15	0.216361405970	0.216362100000
-0.10	0.319269759903	0.319270000000
-0.05	0.400322595443	0.400323600000
0.00	0.469599988372	0.469600600000
0.05	0.531129630467	0.531129700000
0.10	0.587035219193	0.587034100000
0.20	0.686708181014	0.686707100000
0.25	0.731940848489	0.731939900000
0.30	0.774754580283	0.774753200000
0.35	0.815491778633	0.815490500000
0.40	0.854412131156	0.854420200000
0.45	0.891758591602	0.891757500000
0.50	0.927680039803	0.927680300000
0.60	0.995836440590	0.995835900000
0.70	1.05980777319	1.059807000000
0.80	1.12026765739	1.120268000000
0.85	1.14934554399	1.149346000000
0.90	1.17772781922	1.177728000000
0.95	1.20546125464	1.205462000000
1.00	1.23258765687	1.232588000000

Table 2. Wall derivatives of temperature function $\theta_0'(\Lambda, 0)$, $Pr = 1.0$.

Λ	Chao and Fagbenle [1974]	Present Results
-0.15	0.4093363128	0.409336900000
-0.10	0.4367975318	0.436797900000
-0.05	0.4553719553	0.455372800000
0.00	0.4695999884	0.469600600000
0.05	0.4811774669	0.481177600000
0.10	0.4909492790	0.490949300000
0.20	0.5068537821	0.506853800000
0.25	0.5135083784	0.513508600000
0.30	0.5195184185	0.519518400000
0.35	0.5249948751	0.524995000000
0.40	0.5300221727	0.530022100000
0.45	0.534660954	0.534666100000
0.50	0.5389789351	0.538979300000
0.60	0.5467728601	0.546773100000
0.70	0.5536608195	0.553661300000
0.80	0.5598232273	0.559823600000
0.85	0.5626748955	0.562675400000
0.90	0.5653917079	0.565392200000
0.95	0.5679851340	0.567986000000
1.00	0.5704652525	0.570466000000

3.1. Accuracy of the Numerical Program

We can actually assess the accuracy of the present numerical program for the two basic functions f_0 and θ_0 by comparing the results deduced from it with known accurate results. The data of Chao and Fagbenle [7] [8] is used since they are (obtained) accurate to twelve significant figures.

It is seen that for the entire range of Λ used, there is almost complete agreement between the present work for $f_0''(\Lambda, 0)$ and those of Chao and Fagbenle up to six significant figures (Table 1).

Computed values of $\theta_0'(\Lambda, 0)$ for all combinations of Pr and Λ used also shows excellent agreement with the Chao and Fagbenle data (Table 2). These tables show that the present numerical program is capable of producing very accurate data for the two basic functions f_0 and θ_0 .

3.2. Tabulation of Significant Wall Derivatives and Other Relevant Functions

Computer printouts of the velocity functions f_0, f_1, f_2 and f_3 for Λ ranging from -0.15 to 1.0 and those of the temperature functions $\theta_0, \theta_1, \theta_2$ and θ_3 for the same range of Λ and for Prandtl numbers (Pr) = $0.01, 0.1, 0.7, 1.0, 5.0, 10.0$ and 100.0 are available but are too bulky to be included in this paper.

In design and other technological applications, it is, in most cases, the surface characteristics, such as the local frictional drag and the heat and mass transfer coefficients, which are of interest. The prediction of these quantities requires only the information on the wall derivatives:

$$f_i''(\Lambda, 0) \text{ and } \theta_i'(\Lambda, 0): i = 0, 1, 2, 3, \text{ etc.}$$

It should be emphasized that the Merk-Chao-Fagbenle's method as employed in this work is strictly applicable to incompressible, uniform property laminar boundary layer flows.

Since all tabulated data are available for various values of Λ it is convenient to plot ξ and Λ versus x/L and determine the values of x/L and ξ corresponding to these values of Λ given in the tables.

At the forward stagnation point, $x = \xi = 0$ and $\Lambda = 1$ or $\frac{1}{2}$, corresponding respectively to the two-dimensional or axisymmetrical boundary layers.

With the aid of the universal functions the local heat (or mass) transfer coefficient can be obtained as follows:

$$NuR^{-\frac{1}{2}} = \frac{r}{L} \frac{U(x)}{U_\infty} (2\xi)^{-\frac{1}{2}} \left[\theta'_0(\Lambda, 0) + 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta'_0(\Lambda, 0) + \gamma \theta'_2(\Lambda, 0) + \gamma 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta'_0(\Lambda, 0) + 4\xi^2 \left(\frac{d^2\Lambda}{d\xi^2} \right) \theta'_0(\Lambda, 0) + \dots \right] \quad (38)$$

where

$$Nu = \frac{hL}{K} = \frac{qwL}{K(T_w - T_\infty)} \quad (39)$$

and

$$\frac{T - T_w}{T_\infty - T_w} = \theta(\Lambda, 0) = \left[\theta_0(\Lambda, \eta) + 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta_1(\Lambda, \eta) + \gamma \theta_2(\Lambda, \eta) + \gamma 2\xi \left(\frac{d\Lambda}{d\xi} \right) \theta_3(\Lambda, \eta) + \left(4\xi^2 \frac{d^2\Lambda}{d\xi^2} \right) \theta_1(\Lambda, \eta) + \dots \right] \quad (40)$$

for various values of the Prandtl number.

The universal functions θ_i 's depend on the Prandtl number as a parameter.

4. Conclusion

The Chao and Fagbenle's modification of Merk series was employed for the analysis of forced convection thermal boundary layer transfer for non-isothermal surfaces. The sequence of the differential equations governing the universal functions associated with the method has been presented together with the tabulation of these functions.

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Nomenclature

D —Diameter of circular cylinder

h —Local heat transfer coefficient

k —Thermal conductivity

L —Reference length

Nu —Nusselt number defined in Equation (38)

Pr —Prandtl number

R —Radial distance from a surface element to the axis of a rotationally symmetrical body

Re —Reynolds number appropriately defined where used

T —Temperature

u —Velocity component in x -direction

U —Velocity at outer edge of boundary layer

v —Velocity component in Y -direction

x —Stream wise co-ordinate measured along surface from front stagnation point

y —Co-ordinate normal to surface

α —Thermal diffusivity

ξ —Dimensionless co-ordinate defined in Equation (7)

Λ —Wedge variable defined in Equation (10)

μ —Dynamic viscosity

η —Dimensionless co-ordinate defined in Equation (7)

ν —Kinematic viscosity

ρ —Mass density

T —Shear stress at wall

γ —Temperature parameter defined in Equation (20)

Subscript

w —Wall condition

∞ —Free stream condition

Other symbols are defined in the text.