

A Time Dependent Inventory Model for Exponential Demand Rate with Constant Production Where Shelf-Life of the Product Is Finite

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Abstract

In this paper a time dependent inventory model is developed on the basis of constant production rate and market demands which are exponentially decreasing. It advances in quest of total average optimum cost considering those products which have finite shelf-life. The model also considers the small amount of decay. Without having any sort of backlogs, production starts. Reaching at the desired level of inventories, it stops production. After that due to demands along with the deterioration of the items it initiates its depletion and after certain periods the inventory gets zero. The decay of the products is level dependent. The objective of this paper is to find out the optimum inventory cost and optimum time cycle. The model has also been justified with proving the convex property and by giving a numerical example.

Keywords

Production Inventory, Shelf-Life, Time Dependent, Demand Class, Production Rate

1. Introduction

Inventory is an important ingredient of any business. It is a process and place by “proper and in time” utilization of which an enterprise can save a certain amount of production cost and the inventory cost has a vital role to reduce the production cost. Unless inventories are controlled, they are unreliable, inefficient and less cost effective. During our daily lives, generally we come across two types of materials; one is perishable items or items of

having decay and the other one is non perishable items or items of having decay or deterioration. Most of the necessary goods like cosmetics items, radio-active substances, fashion goods, pharmaceuticals, food items etc. decrease due to its finite or limited shelf-life. Due to the limited shelf-life and market demand, the stock level or inventory continuously decreases and thereby, deterioration occurs. To get the actual inventory cost, this deterioration must be taken into consideration. On the other hand, to make the inventory cost at optimum level *i.e.* to get the minimum inventory cost, a suitable inventory model is required which suits to meet the actual demand in the market. In minimizing inventory cost this paper proposes a time dependent inventory model with constant production rate and exponential demand of materials with small amount of decay, whereas the existing models very often ignore the production rate, instead those consider the instantaneous replenishment rate. As the production reaches at a certain level of inventory, the production stops. Satisfying the convex property and using a numerical example, the paper could justify that the objective of formulating this model is achieved. The objective of the proposed model is to get the optimum inventory cost and optimum time cycle by introducing a time dependent inventory model with constant production rate and exponential declining demand. The paper subsequently is unfolded with literature review, assumptions, notations used in the model, development of the model, numerical illustration, sensitivity analysis, conclusion and suggestions for future work in this field.

2. Literature Review

Sufficient numbers of works have already been done by a large number of researchers in the area of production inventory model to build the suitable inventory models. In last few years, many researchers have studied in this field and developed inventory models to solve the real life problems. In classical inventory models, the demand rate But on ground, it must not be always correct. There may be various type of demands. Demand may be linear, quadratic, exponential, time dependent, level or stock dependent, price dependent etc. Basing on the demand pattern, the firms decide how much to produce and when to produce. Harris [1] was the pioneer to study inventory model. Whitin [2] was the first researcher to develop the inventory model for fashion goods considering its little decay in the inventory. Ghare and Schrader [3] first pointed out the effect of decay in inventory analysis and discovered the economic order quantity (EOQ) model. They explained that the consumption of the deteriorating items was a negative exponential function of time. Shiraj [4] discussed the effect of just in time manufacturing system on EOQ model. Sivazlian and Stenfel [5] determined the optimum value of time cycle by using the graphical solution of the equation to obtain the economic order quantity model. Shah and Jaiswal [6] and Dye [7] established an inventory model by considering demand as a function of selling price and three parameters of Weibull rate of deterioration. Billington [8] discussed classic economic production quantity (EPQ) model without backorders or backlogs. Pakkala and Achary [9] established a deterministic inventory model for deteriorating items with two warehouses, while the replenishment rate was finite, demand was uniform and shortage was allowed. Bhaba R. Sarker, Subbasis Mukherjee and Chidambaram V. Balan [10] explained an inventory model where demand was a composite function consisting of a constant component and a variable component proportional to inventory level in a period in which decay was exponential and inventory was positive. Teng, Chern and Yang [11] developed the model with deteriorating items and shortages assuming that the demand function was positive and fluctuating with respect to time. Skouri and Papachristos [12] discussed a continuous review inventory model considering the five costs as deterioration, holding, shortage, opportunity cost due to the lost sales and the replenishment cost due to the linearly dependency on the lot size. Chund and Wee [13] developed an integrated two stages production inventory deteriorating model for the buyer and the supplier on the basis of stock dependent selling rate considering imperfect items and just in time (JIT) multiple deliveries. Applying inventory replenishment policy Mingbao Cheng and Guoqing Wang [14] expressed the inventory model for deteriorating items with trapezoidal type demand rate, where the demand rate is a piecewise linearly functions. Hassan Shavandi and Bita Bozorgi [15] developed the location of distribution centers with inventory. Sing and Pattanak [16]-[18] developed the model for deterioration and time dependent quadratic demand under permissible delay in payment. Amutha and Chandrasekaran [19] formulated the inventory model with deterioration items, quadratic demand and time dependent holding cost. Chang and Dye [20] expressed an inventory model with deteriorating items, with time varying demand and partial backlogging. Tripathy and Mishra [21] discuss the inventory model with ordering policy for weibull deteriorating items, quadratic demand and permissible delay in payments. Sarkar *et al.* [22] introduced and inventory model with finite replenishment rate, trade credit policy and price discount offer. Khieng *et al.* [23] presented a production model for the lot-size, order level inventory system with finite production rate and the effect of decay. In our model, we have used exponential decay with

constant production rate. Ekramol [24] [25] considered various production rate assuming the demand is constant, whereas in this model we have used constant production with varying demand. Ouyang and Cheng [26] explained the inventory model for deteriorating items with exponential declining demand and partial backlogging. In this model, we have proposed exponential declining demand along with constant production rate instead of instantaneous replenishment.

3. Assumptions

- a. Production rate is constant at any time.
- b. Production starts when inventory level is zero and it stops when inventory level is highest.
- c. Inventory level is highest at $\theta = t_1$. Since, the production stops while inventory is highest, inventory depletes quickly due to demand and decay.
- d. Demand rate exponentially decreases.
- e. Deteriorating or decay rate is constant and very small.
- f. Decreasing rate of demand is also constant and less than decay rate for unit inventory
- g. Shortages are not allowed.
- h. Lead time is zero.

4. Development of the Model

The model is developed on the basis of exponential market demands and constant production capacity of the organization. The model is suitable for the products which have finite shelf-life and ultimately causes the products decay. At the beginning, *i.e.* at time $\theta = 0$, the production starts with zero inventory. In this model, the production rate λ remains constant for entire production cycle. But the demands exponentially decrease time to time, which is shown in the **Figure 1**.

During time $\theta = 0$ to t_1 , the inventory decreases at the rate of $\lambda - Q_0 e^{-\gamma\theta} - \mu I(\theta)$, as $Q_0 e^{-\gamma\theta}$ is the market demand. Here, $\mu I(\theta)$ is the decay of $I(\theta)$ inventories at instant θ . By using the above arguments we can have the following differential equations,

$$\frac{d}{d\theta} I(\theta) + \mu I(\theta) = \lambda - Q_0 e^{-\gamma\theta}$$

The general solution of the differential equation is,

$$I(\theta) = \frac{\lambda}{\mu} - \frac{Q_0 e^{-\gamma\theta}}{\mu - \gamma} + A e^{-\mu\theta}$$

We now apply the following boundary condition, at $\theta = 0$, we get, $I(\theta) = 0$

By solving we get, $A = \frac{Q_0}{\mu - \gamma} - \frac{\lambda}{\mu}$,

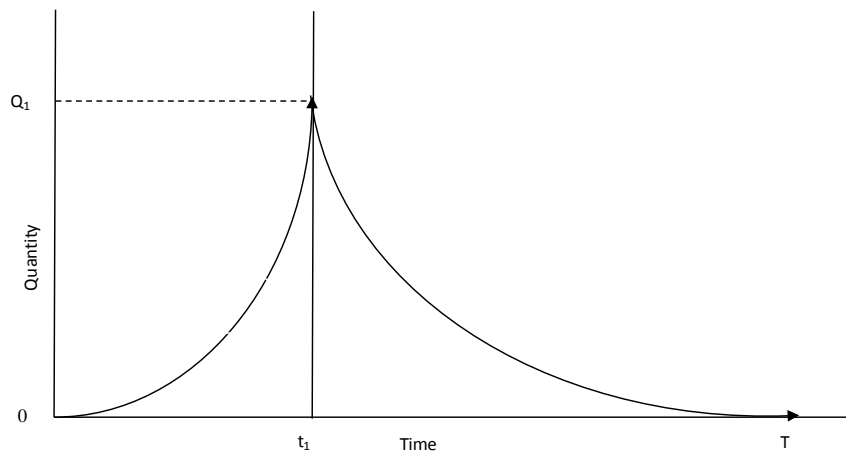


Figure 1. Time verses quantity.

Therefore,

$$I(\theta) = \frac{\lambda}{\mu}(1 - e^{-\mu\theta}) + \frac{Q_0}{\mu - \gamma}(e^{-\mu\theta} - e^{-\gamma\theta}) \quad (1)$$

Applying the other boundary condition, we get $I(\theta) = Q_1$ at $\theta = t_1$. Then Equation (1) gives us,

$$Q_1 = \frac{\lambda}{\mu}(1 - e^{-\mu t_1}) + \frac{Q_0}{\mu - \gamma}(e^{-\mu t_1} - e^{-\gamma t_1}) \quad (2)$$

Hence, the total un-decayed inventory during $\theta = 0$ to t_1 ,

$$\begin{aligned} I_1 &= \int_0^{t_1} I(\theta) d\theta \\ &= \int_0^{t_1} \left[\frac{\lambda}{\mu}(1 - e^{-\mu\theta}) + \frac{Q_0}{\mu - \gamma}(e^{-\mu\theta} - e^{-\gamma\theta}) \right] d\theta \\ &= \frac{\lambda}{\mu} \left(t_1 + \frac{e^{-\mu t_1} - 1}{\mu} \right) - \frac{Q_0}{\mu - \gamma} \left(\frac{e^{-\mu t_1} - 1}{\mu} - \frac{e^{-\gamma t_1} - 1}{\gamma} \right) \\ &= \frac{\lambda}{\mu} t_1 + \frac{\lambda}{\mu^2} (e^{-\mu t_1} - 1) - \frac{Q_0}{\mu(\mu - \gamma)} (e^{-\mu t_1} - 1) + \frac{Q_0}{\gamma(\mu - \gamma)} (e^{-\gamma t_1} - 1) \\ &= \frac{\lambda}{\mu} t_1 + \frac{\lambda}{\mu^2} \left(-\mu t_1 + \frac{\mu^2 t_1^2}{2} \right) - \frac{Q_0}{\mu(\mu - \gamma)} \left(-\mu t_1 + \frac{\mu^2 t_1^2}{2} \right) + \frac{Q_0}{\gamma(\mu - \gamma)} \left(-\gamma t_1 + \frac{\gamma^2 t_1^2}{2} \right) \end{aligned}$$

Neglecting the higher power of μ and γ , it becomes,

$$\begin{aligned} &= \frac{\lambda}{\mu} t_1 - \frac{\lambda}{\mu} t_1 + \frac{\lambda}{2} t_1^2 + \frac{Q_0}{\mu - \gamma} t_1 - \frac{Q_0 \mu}{2(\mu - \gamma)} t_1^2 - \frac{Q_0}{\mu - \gamma} t_1 + \frac{Q_0 \gamma}{2(\mu - \gamma)} t_1^2 \\ &= \left(\frac{\lambda}{2} - \frac{Q_0 \mu}{2(\mu - \gamma)} + \frac{Q_0 \gamma}{2(\mu - \gamma)} \right) t_1^2 = \frac{1}{2} (\lambda - Q_0) t_1^2 \end{aligned} \quad (3)$$

Again during $\theta = t_1$ to T , the inventory decreases at the rate of $Q_0 e^{-\gamma\theta}$. Thereby, following the process of forming first differential equation, we get the second differential equation as below:

$$\frac{d}{d\theta} I(\theta) + \mu I(\theta) = -Q_0 e^{-\gamma\theta}$$

Applying the boundary condition at $\theta = T$, we consider, $I(\theta) = 0$. Then we get the general solution as below:

$$I(\theta) = -\frac{Q_0 e^{-\gamma\theta}}{\mu - \gamma} + \frac{Q_0 e^{(\mu - \gamma)T - \mu\theta}}{\mu - \gamma} \quad (4)$$

We get the following conditions, if we put the other boundary condition in equation no. (4), i.e. at $\theta = t_1$, $I(\theta) = Q_1$.

$$Q_1 = \frac{Q_0}{\mu - \gamma} e^{(\mu - \gamma)T - \mu t_1} - \frac{Q_0}{\mu - \gamma} e^{-\gamma t_1} \quad (5)$$

Hence, with the help of equation no (4), we get the un-decayed inventory during $\theta = t_1$ to T ,

$$\begin{aligned}
 I_2 &= \int_{t_1}^T I(\theta) d\theta = \int_{t_1}^T \left[\frac{Q_0 e^{-\gamma\theta}}{\mu - \gamma} + \frac{Q_0 e^{(\mu-\gamma)T - \mu\theta}}{\mu - \gamma} \right] d\theta \\
 &= \frac{Q_0}{\mu - \gamma} \left[\frac{e^{-\gamma T} - e^{-\gamma t_1}}{\gamma} \right] - \frac{Q_0}{\mu - \gamma} \left[\frac{e^{(\mu-\gamma)T - \mu T} - e^{(\mu-\gamma)T - \mu t_1}}{\mu} \right] \\
 &= \frac{Q_0}{\gamma(\mu - \gamma)} (e^{-\gamma T} - e^{-\gamma t_1}) - \frac{Q_0}{\mu(\mu - \gamma)} (e^{-\gamma T} - e^{(\mu-\gamma)T - \mu t_1}) \\
 &= \frac{Q_0}{\mu(\mu - \gamma)} e^{(\mu-\gamma)T - \mu t_1} + \frac{Q_0}{\mu\gamma} e^{-\gamma T} - \frac{Q_0}{\gamma(\mu - \gamma)} e^{-\gamma t_1} \\
 &= \frac{Q_0}{\mu(\mu - \gamma)} \left[1 + (\mu - \gamma)T - \mu t_1 + \frac{1}{2} \{(\mu - \gamma)T - \mu t_1\}^2 \right] + \frac{Q_0}{\mu\gamma} \left[1 - \gamma T + \frac{1}{2} \gamma^2 T^2 \right] \\
 &\quad - \frac{Q_0}{\mu(\mu - \gamma)} \left[1 - \gamma t_1 + \frac{1}{2} \gamma^2 t_1^2 \right] \quad \text{[neglecting the higher power of } \mu \text{ and } \gamma] \\
 &= \frac{Q_0}{\mu(\mu - \gamma)} + \frac{Q_0}{\mu} T - \frac{Q_0}{\mu - \gamma} t_1 + \frac{Q_0(\mu - \gamma)}{2\mu} T^2 + \frac{Q_0\mu}{2(\mu - \gamma)} t_1^2 - Q_0 T t_1 + \frac{Q_0}{\mu\gamma} \\
 &\quad - \frac{Q_0}{\mu} T + \frac{Q_0\gamma}{2\mu} T^2 - \frac{Q_0}{\gamma(\mu - \gamma)} + \frac{Q_0}{\mu - \gamma} t_1 - \frac{Q_0\gamma}{2(\mu - \gamma)} t_1^2 \\
 &= -\frac{Q_0}{\mu\gamma} + \frac{Q_0\mu}{2\mu} T^2 + \frac{Q_0}{2} t_1^2 - Q_0 T t_1 + \frac{Q_0}{\mu\gamma} = \frac{1}{2} Q_0 (T - t_1)^2
 \end{aligned} \tag{6}$$

And, the deteriorating items during $\theta = t_1$ to T ,

$$\begin{aligned}
 D_2 &= Q_1 - \int_{t_1}^T R(\theta) d\theta = Q_1 - \int_{t_1}^T Q_0 e^{-\gamma\theta} d\theta \\
 &= \frac{Q_0}{\mu - \gamma} e^{(\mu-\gamma)T - \mu t_1} - \frac{Q_0}{\mu - \gamma} e^{-\gamma t_1} + \frac{Q_0}{\gamma} (e^{-\gamma T} - e^{-\gamma t_1})
 \end{aligned}$$

Neglecting the higher power of μ and γ , it becomes,

$$\begin{aligned}
 &= \frac{Q_0}{\mu - \gamma} \left[1 + (\mu - \gamma)T - \mu t_1 + \frac{1}{2} \{(\mu - \gamma)T - \mu t_1\}^2 \right] - \frac{Q_0\mu}{\gamma(\mu - \gamma)} \left[1 - \gamma t_1 + \frac{1}{2} \gamma^2 t_1^2 \right] \\
 &\quad + \frac{Q_0}{\gamma} \left[1 - \gamma T + \frac{1}{2} \gamma^2 T^2 \right] \\
 &= \frac{Q_0}{\mu - \gamma} + Q_0 T - \frac{Q_0\mu}{\mu - \gamma} t_1 + \frac{Q_0 T^2 (\mu - \gamma)}{2} + \frac{Q_0 \mu^2 t_1^2}{2(\mu - \gamma)} - Q_0 \mu T t_1 - \frac{Q_0\mu}{\gamma(\mu - \gamma)} + \frac{Q_0\mu}{\mu - \gamma} \\
 &\quad - \frac{Q_0\mu\gamma t_1^2}{2(\mu - \gamma)} + \frac{Q_0}{\gamma} - Q_0 T + \frac{Q_0\gamma T^2}{2} \\
 &= \frac{Q_0}{\mu - \gamma} - \frac{Q_0\mu}{\gamma(\mu - \gamma)} + \frac{Q_0}{\gamma} + \left[\frac{Q_0(\mu - \gamma)}{2} + \frac{Q_0\gamma}{2} \right] T^2 + \left[\frac{Q_0\mu^2}{2(\mu - \gamma)} - \frac{Q_0\mu\gamma}{2(\mu - \gamma)} \right] t_1^2 - Q_0 \mu T t_1 \\
 &= \frac{Q_0\mu}{2} T^2 + \frac{Q_0\mu}{2} t_1^2 - Q_0 \mu T t_1 = \frac{Q_0\mu}{2} (T - t_1)^2
 \end{aligned} \tag{7}$$

Total Cost Function: Total average inventory cost per unit time per cycle can be expressed as below,

$$TC(t_1, T) = \frac{K + hI_1 + \eta(I_2 + D_2)}{T}$$

By using the Equation (3), (6) and (7), we now have,

$$TC(t_1, T) = \frac{K}{T} + \frac{h}{T} \left[\frac{\lambda - Q_0}{2} t_1^2 + \frac{Q_0 (T - t_1)^2}{2} \right] + \frac{\eta}{T} \left[\frac{Q_0 \mu (T - t_1)^2}{2} \right] \quad (8)$$

Now, the objective is to minimize the total inventory cost TC . For the minimum average inventory cost TC the optimum values of time t_1 and T are the solutions of the following convex property:

$$(i) \quad \frac{\partial}{\partial t_1} TC(t_1, T) = 0 \quad \text{and} \quad \frac{\partial}{\partial T} TC(t_1, T) = 0$$

and

$$(ii) \quad \left(\frac{\partial^2 TC}{\partial t_1^2} \right) \left(\frac{\partial^2 TC}{\partial T^2} \right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T} \right)^2 > 0$$

The cost function will be convex if these well recognized criteria are satisfied. Thereby, we can determine the total optimum cost TC , optimum time interval t_1^* , total time cycle T^* and the optimum order quantity Q^* . Now,

$$\frac{\partial TC}{\partial t_1} = \frac{h}{T} [(\lambda - Q_0)t_1 - Q_0(T - t_1)] - \frac{\eta}{T} [Q_0 \mu (T - t_1)] \quad (9)$$

$$\frac{\partial^2 TC}{\partial t_1^2} = \frac{h\lambda}{T} + \frac{\eta Q_0 \mu}{T} = \frac{1}{T} (h\lambda + \eta Q_0 \mu) \quad (10)$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial T \partial t_1} &= \frac{h}{T} (-Q_0) - \frac{h}{T^2} (\lambda t_1 + Q_0 - Q_0 T) - \frac{\eta}{T} (Q_0 \mu) + \frac{\eta}{T^2} (Q_0 \mu) (T - t_1) \\ &= \frac{-hQ_0}{T} - \frac{h(\lambda t_1 + Q_0)}{T^2} + \frac{hQ_0}{T} - \frac{\eta Q_0 \mu}{T} + \frac{\eta Q_0 \mu}{T} - \frac{\eta Q_0 \mu t_1}{T^2} \\ &= -\frac{1}{T^2} (hQ_0 + h\lambda t_1 + \eta Q_0 \mu t_1) = -\frac{1}{T^2} [hQ_0 + (h\lambda + \eta Q_0 \mu) t_1] \end{aligned} \quad (11)$$

$$\begin{aligned} \frac{\partial TC}{\partial T} &= -\frac{K}{T^2} + \frac{h}{T} [Q_0 (T - t_1)] - \frac{h}{T^2} \left[\frac{\lambda - Q_0}{2} t_1^2 + Q_0 t_1 + \frac{Q_0 (T - t_1)^2}{2} \right] \\ &\quad - \frac{\eta}{T^2} \left[\frac{Q_0 \mu (T - t_1)^2}{2} \right] + \frac{\eta}{T} [hQ_0 \mu (T - t_1)] \\ &= -\frac{K}{T^2} + hQ_0 - \frac{hQ_0 t_1}{T} - \frac{h(\lambda - Q_0) t_1^2}{2T^2} - \frac{hQ_0 t_1}{T^2} - \frac{hQ_0 (T - t_1)^2}{2T^2} \\ &\quad - \frac{\eta Q_0 \mu (T - t_1)^2}{2T^2} + \eta Q_0 \mu - \frac{\eta Q_0 \mu t_1}{T} \end{aligned} \quad (12)$$

$$\begin{aligned} \frac{\partial^2 TC}{\partial T^2} &= \frac{2K}{T^3} + \frac{hQ_0 t_1}{T^2} + \frac{h(\lambda - Q_0) t_1^2}{T^3} + \frac{2hQ_0 t_1}{T^3} + \frac{hQ_0 (T - t_1)^2}{T^3} - \frac{hQ_0 (T - t_1)}{T^2} \\ &\quad + \frac{\eta Q_0 \mu (T - t_1)^2}{T^3} - \frac{\eta Q_0 \mu (T - t_1)}{T^2} + \frac{\eta Q_0 \mu t_1}{T^2} \\ &= \frac{2K}{T^3} + \frac{hQ_0 t_1}{T^2} + \frac{h\lambda t_1^2}{T^3} - \frac{hQ_0 t_1^2}{T^3} + \frac{2hQ_0 t_1}{T^3} - \frac{hQ_0 (T - t_1) t_1}{T^3} - \frac{\eta Q_0 \mu (T - t_1) t_1}{T^3} + \frac{\eta Q_0 \mu t_1}{T^2} \\ &= \frac{2K}{T^3} + \frac{h\lambda t_1^2}{T^3} + \frac{\eta Q_0 \mu t_1^2}{T^3} + \frac{2hQ_0 t_1}{T^3} \end{aligned} \quad (13)$$

Now, using the Equations (10), (11) and (13), we get,

$$\begin{aligned}
& \left(\frac{\partial^2 TC}{\partial t_1^2} \right) \left(\frac{\partial^2 TC}{\partial T^2} \right) - \left(\frac{\partial^2 TC}{\partial t_1 \partial T} \right)^2 \\
&= \frac{1}{T} (h\lambda + \eta Q_0 \mu) \left[\frac{2K}{T^3} + \frac{h\lambda t_1^2}{T^3} + \frac{\eta Q_0 \mu t_1^2}{T^3} + \frac{2hQ_0 t_1}{T^3} \right] - \frac{1}{T^4} (hQ_0 + h\lambda t_1 + \eta Q_0 \mu t_1)^2 \\
&= \frac{1}{T^4} \left[2Kh\lambda + h^2 \lambda^2 t_1^2 + h\eta \lambda Q_0 \mu t_1^2 + 2h^2 \lambda Q_0 t_1 + 2K\eta Q_0 \mu + h\eta \lambda Q_0 \mu t_1^2 + \eta^2 Q_0^2 \mu^2 t_1^2 \right] \\
&+ \frac{1}{T^4} \left[2h\eta Q_0^2 \mu t_1 - h^2 Q_0^2 - h^2 \lambda^2 t_1^2 - \eta^2 Q_0^2 \mu^2 t_1^2 - 2h^2 \lambda Q_0 t_1 - 2h\eta Q_0^2 \mu t_1 - 2h\eta \lambda Q_0 \mu t_1^2 \right] \\
&= \frac{1}{T^4} \left[2Kh\lambda + 2K\eta Q_0 \mu - h^2 Q_0^2 \right]
\end{aligned}$$

This term will be greater than zero, *i.e.* convex property (i) will be satisfied, if $2Kh\lambda + 2K\eta Q_0 \mu - h^2 Q_0^2$ is greater than zero. Hence, $2Kh\lambda + 2K\eta Q_0 \mu - h^2 Q_0^2 > 0$ is to be satisfied for the minimum optimum cost. Now putting the values of (9) and (12) in the convex property (i), *i.e.* in $\frac{\partial}{\partial t_1} TC(t_1, T) = 0$ and $\frac{\partial}{\partial T} TC(t_1, T) = 0$ respectively and thereby we get the optimum time interval. From the equation $\frac{\partial TC}{\partial t_1} = 0$, we get the value of T as below,

$$T = \frac{hQ_0 + (h\lambda + \eta Q_0 \mu)t_1}{Q_0(h + \eta\mu)} \quad (14)$$

Again from $\frac{\partial TC}{\partial T} = 0$, the value of T can be obtained as follows,

$$T = \pm \sqrt{\frac{(h\lambda + \eta Q_0 \mu)t_1^2 + 2hQ_0 t_1 + 2K}{Q_0(h + \eta\mu)}} \quad (15)$$

Now from the equation no (9) and (10) with the positive value of T from (10), we get,

$$\frac{hQ_0 + (h\lambda + \eta Q_0 \mu)t_1}{Q_0(h + \eta\mu)} = \sqrt{\frac{(h\lambda + \eta Q_0 \mu)t_1^2 + 2hQ_0 t_1 + 2K}{Q_0(h + \eta\mu)}}$$

Solving this equation we now get the value of t_1 which is mentioned here,

$$\begin{aligned}
t_1 &= \frac{-2h^2 Q_0 (\lambda - Q_0) \pm \sqrt{4h^4 Q_0^2 (\lambda - Q_0)^2 - 4hQ_0 (\lambda - Q_0) (h\lambda + \eta Q_0 \mu) (h^2 Q_0 - 2hK - 2\eta K \mu)}}{2h(\lambda - Q_0)(h\lambda + \eta Q_0 \mu)} \\
&= \frac{-2h^2 Q_0 (\lambda - Q_0) + \sqrt{4h^4 Q_0^2 (\lambda - Q_0)^2 + 4hQ_0 (\lambda - Q_0) (h\lambda + \eta Q_0 \mu) (2\eta K \mu + h(2K - hQ_0))}}{2h(\lambda - Q_0)(h\lambda + \eta Q_0 \mu)}
\end{aligned} \quad (16)$$

5. Numerical Illustration

Here, we provide a numerical illustration to justify the optimum inventory cost and the optimum order cycle. Let us consider, the inventory system has the following parameters, $K = 100, h = 1, \eta = 0.5, \lambda = 15, Q_0 = 2, \mu = 0.4, \gamma = 0.1$. With these parameters we check the condition, $2Kh\lambda + 2K\eta Q_0 \mu - h^2 Q_0^2 = 3076 > 0$. Now, putting all the values in equation no (5), (16), (14) and (8) we get the optimum order quantity $Q_1^* = 68.62$ units, optimum order interval $t_1^* = 1.418$, total time cycle $T^* = 9.932$ units and total optimum inventory cost $TC^* = 20.428$ units. The **Figure 2** and **Figure 3** show the inventory cost function verses order interval and inventory cost function verses total time cycle respectively.

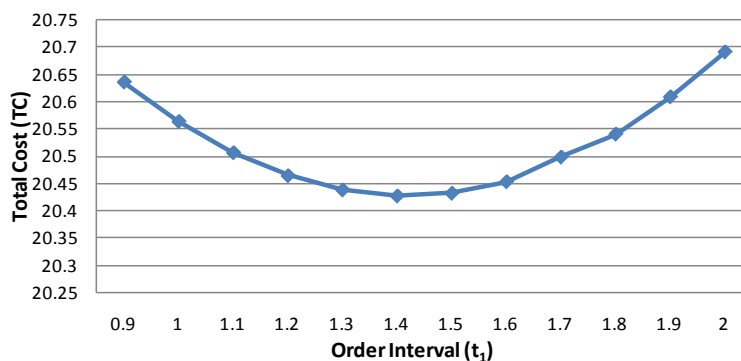


Figure 2. Order interval versus total cost.

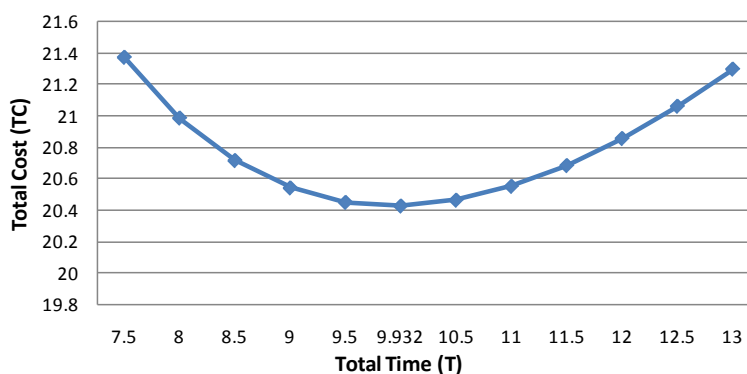


Figure 3. Total time versus total cost.

6. Sensitivity Analysis

Now we study the effects of changes of parameters $K, \lambda, Q_0, \mu, \gamma, h$ and η on the optimal length of ordering cycle t_1 , optimal time cycle T , optimal ordering quantity Q_1 and the minimum average total cost TC per unit time in the model. We performed the sensitivity analysis by changing each of the parameters by +50%, +25%, +10%, -10%, -25% and -50% taking one parameter at a time while keeping other parameters unchanged. The details are shown in **Table 1**.

Analyzing the results in the above table we can summarize the following observations:

- 1) t_1^* decreases and T^* as well as TC^* increase while Q_1^* remain unchanged with increase in the value of the parameter λ . Here λ is moderately sensitive to all the values.
- 2) t_1^*, Q_1^* and TC^* increase while T^* decreases with increase in the value of the parameter Q_0 and μ . Here Q_0 and μ are moderately sensitive to t_1^*, T^* and TC^* and highly sensitive to Q_1^* .
- 3) Q_1^* decreases and t_1^*, T^* and TC^* remain unchanged with increase in the value of the parameter γ . Here γ is moderately sensitive to Q_1^* .
- 4) t_1^* decreases and T^* as well as TC^* increases while Q_1^* remain unchanged with increase in the value of the parameter h . Here h is moderately sensitive to all the values.
- 5) t_1^* and TC^* increase and T^* decreases while Q_1^* remain unchanged with increase in the value of the parameter η . Here η is moderately sensitive to all the values are to changes.

7. Conclusion

In a real market, it is very unlikely that the rate of demand always remains same. At times, demand goes very high and at times it goes very low. These types of cases fit the exponential rate of demand. This proposed model expects exponential declining demand. We have developed a time dependent inventory model for the items which have finite shelf-life and time dependent demand with exponential decay. The production rate and the decay are constant all through. Total cost, order interval and total time cycle do not depend on the decreasing

Table 1. Effects of changes in various parameters.

Parameters	Change in %	Value of			
		t_1^*	T^*	Q^*	TC^*
λ	+50	0.923	14.363	68.620	21.187
	+25	1.118	12.148	68.620	20.808
	+10	1.280	10.818	68.620	20.580
	-10	1.588	9.046	68.620	20.276
	-25	1.937	7.717	68.620	20.049
	-50	3.066	5.501	68.620	19.669
Q_0	+50	1.766	6.978	102.930	24.849
	+25	1.597	8.160	85.774	22.638
	+10	1.491	9.126	75.481	21.312
	-10	1.342	10.917	61.757	19.544
	-25	1.222	12.886	51.465	18.218
	-50	0.996	18.794	34.310	16.007
μ	+50	1.472	9.277	241.587	21.158
	+25	1.445	9.592	126.405	20.793
	+10	1.429	9.793	87.184	20.574
	-10	1.406	10.077	54.397	20.282
	-25	1.389	10.302	38.958	20.063
	-50	1.360	10.706	23.308	19.698
γ	+50	1.418	9.932	47.872	20.428
	+25	1.418	9.932	57.231	20.428
	+10	1.418	9.932	63.793	20.428
	-10	1.418	9.932	73.845	20.428
	-25	1.418	9.932	82.506	20.428
	-50	1.418	9.932	99.468	20.428
H	+50	1.102	10.433	68.620	24.878
	+25	1.234	10.226	68.620	22.653
	+10	1.336	10.063	68.620	21.318
	-10	1.514	9.777	68.620	19.538
	-25	1.699	9.484	68.620	18.203
	-50	2.208	8.716	68.620	15.978
η	+50	1.472	9.272	68.620	21.158
	+25	1.445	9.592	68.620	20.793
	+10	1.429	9.793	68.620	20.574
	-10	1.406	10.077	68.620	20.282
	-25	1.389	10.302	68.620	20.063
	-50	1.360	10.706	68.620	19.698

rate of demand as the rate is considered very small. The model develops an algorithm to determine the optimal demand, optimal order interval, optimal time cycle and the optimum total cost. In this model, the initial demand Q_0 is highly sensitive to only optimum order quantity Q_1^* . All other values are moderately sensitive to changes in all the parameters. The model could be further developed considering the backlogs, varying production rate and price dependent demand rate for the perishable items. Even it could consider the non-perishable items as well.

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Notations

- λ = Production rate.
 $R(\theta) = Q_0 e^{-\gamma\theta}$ = The decreasing exponential demand rate at any instant θ .
 $d\theta$ = Vary small portion of instant θ .
 Q_0 = Initial demand, which is any positive quantity, *i.e.* $Q_0 > 0$.
 μ = Very small amount of constant deteriorating or decay rate for unit inventory.
 γ = Decreasing rate and $0 < \gamma < \mu$.
 $I(\theta)$ = Inventory level at any instant θ which is equal to 0, Q_1 and 0 respectively at $\theta = 0$, t_1 and T .
 Here Q_1 as the highest inventory level depicts the order quantity as well.
 A, B = Arbitrary constants.
 I_1 = Un-decayed inventory at $T = 0$ to t_1 .
 I_2 = Un-decayed inventory at $T = t_1$ to T .
 D_2 = Deteriorating inventory at $T = t_1$ to T .
 K = Set up or ordering cost.
 h = Average holding cost per unit inventory at $\theta = 0$ to t_1 .
 η = Average holding cost per unit inventory at $\theta = t_1$ to T .
 $TC(t_1, T)$ = Total Cost in terms of t_1 and T .
 Q_1^* = Optimum Order Quantity.
 t_1^* = Optimum Order Interval.
 T^* = Total time.