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Existence and Uniqueness of Positive (Almost) Periodic Solutions for a Neutral Multi-Species Logarithmic Population Model with Multiple Delays and Impulses*

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ABSTRACT

In this paper, by using the contraction mapping principle and constructing a suitable Lyapunov functional, we established a set of easily applicable criteria for the existence, uniqueness and global attractivity of positive periodic solution and positive almost periodic solution of a neutral multi-species Logarithmic population model with multiple delays and impulses. The results improve and generalize the known ones in [1], as an application, we also give an example to illustrate the feasibility of our main results.

Keywords: Contraction Mapping Principle; Impulses; Lyapunov Functional; Global Attractivity; Uniqueness; Positive Periodic Solution; Almost Periodic Solution

1. Introduction

Recently, there are more works on the periodic solution of neutral type Logistic models or Lotka-Volterra models (see [2-7] for details). Only a little scholars considered the neutral Logarithmic model (see [1,8-10]). In [8], Li had studied the following single species neutral Logarithmic model:

$$N'(t)=N(t)\left[r(t)-a(t)\ln N(t-\sigma)-b(t)\left(\ln N(t-\eta)\right)'\right]. \tag{1.1}$$

He had established a set of easily applicable criteria for the existence of positive periodic solution of system (1.1) by applying the continuation theorem of the coincidence degree theory which proposed in [11] by Mawhin. In [9], Lu and Ge employed an abstract continuous theorem of k-set contractive operator to investigate the following equation:

$$N'(t)=N(t)\left[r(t)-\sum_{j=1}^{n}a_j(t)\ln N\left(t-\sigma_j(t)\right)-\sum_{i=1}^{m}b_i(t)\left(\ln N\left(t-\tau_i(t)\right)\right)'\right]. \tag{1.2}$$

They established some criteria to guarantee the existence of positive periodic solutions of system (1.2). In [10], Chen studied the following neutral multi-species Logarithmic population model:

$$\begin{aligned}\frac{\mathrm{d}N_i(t)}{\mathrm{d}t}=N_i(t)\Bigg[&r_i(t)-\sum_{j=1}^{n}a_{ij}(t)\ln N_j(t)\\&-\sum_{j=1}^{n}b_{ij}(t)\ln N_j\left(t-\tau_{ij}(t)\right)\\&-\sum_{j=1}^{n}c_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)\ln N_j((s))\,\mathrm{d}s\\&-\sum_{j=1}^{n}d_{ij}(t)\frac{\mathrm{d}\ln N_j\left(t-\eta_{ij}(t)\right)}{\mathrm{d}t}\Bigg].\end{aligned} \tag{1.3}$$

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By using the method of fixed point theory and constructing a suitable Lyapunov functional, a set of easily applicable criteria are established for the existence, uniqueness and global attractivity of positive periodic solution (positive almost periodic solution) for system (1.3).

In [1], Wang *et al.* had investigated the existence, uniqueness of the positive periodic solution of the following neutral multi-species Logarithmic population model:

$$N'(t)=N(t)\Big[r(t)-a(t)\ln N(t)-\sum_{j=1}^{n}b_j(t)\ln N(t-\tau_j(t))-\sum_{j=1}^{n}c_j(t)\int_{-\infty}^{t}k_j(t-s)\ln N((s))\mathrm{d}s-\sum_{j=1}^{n}d_j(t)\big(\ln N(t-\eta_j(t))\big)'\Big]. \quad (1.4)$$

By using an abstract continuous theorem of *k*-set contractive operator, the criteria is established for the existence, global attractivity of positive periodic solutions for model (1.4).

On the other hand, there are some other perturbations in the real world such as fires and floods that are not suitable to be considered continually. These perturbations bring sudden changes to the system. Systems with such sudden perturbations involving impulsive differential equations have attracted the interest of many researchers in the past twenty years [12-20], since they provide a natural description of several real processes subject to certain perturbations whose duration is negligible in comparison with the duration of the process. Such processes are often investigated in various fields of science and technology such as physics, population dynamics, ecology, biological systems, optimal control, etc. For details, see [21,22]. Recently, the corresponding theory for impulsive functional differential equations has been studied by many authors [23-25]. However there are few published papers discussing the impulsive neutral multi-species Logarithmic population model. Our method is different from that in [1,9].

In this paper, we investigate the existence, uniqueness of the positive periodic solution of the following neutral multi-species Logarithmic population system with multiple delays and impulses

$$\begin{cases}\dfrac{\mathrm{d}N_i(t)}{\mathrm{d}t}=N_i(t)\Big[a_i(t)-b_i(t)\ln N_i(t)-\displaystyle\sum_{j=1}^{n}c_{ij}(t)\ln N_j(t)-\sum_{j=1}^{n}d_{ij}(t)\ln N_j(t-\gamma_{ij}(t))\\ \qquad -\displaystyle\sum_{j=1}^{n}e_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)\ln N_j(s)\mathrm{d}s-\sum_{j=1}^{n}f_{ij}(t)\frac{\mathrm{d}\ln N_j(t-\delta_{ij}(t))}{\mathrm{d}t}\Big],\ i=1,2,\cdots,n,\ t\neq t_k,\\ \Delta N_i(t_k)=N_i(t_k^+)-N_i(t_k)=\theta_{ik}N_i(t_k),\ i=1,2,\cdots,n,\ k=1,2,\cdots.\end{cases} \quad (1.5)$$

where $i=1,2,\cdots,n,$ $a_i(t)$, $b_i(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$, $f_{ij}(t)\in C(R,(0,+\infty))$, $\gamma_{ij}(t)$, $\delta_{ij}(t)\in C(R,[0,+\infty))$ are all continuous functions with $\gamma=\max\{\gamma_{ij}(t),\delta(t)\}$, $\gamma'_{ij}(t)<1$, $\delta'_{ij}(t)<1$. And $1+\theta_{ik}>0$, $\int_0^{\infty}K_{ij}(s)\mathrm{d}s=1$, $\int_0^{\infty}sK_{ij}(s)\mathrm{d}s<+\infty$. We consider (1.5) together with the initial conditions

$$N_i(\xi)=\phi_i(\xi),\ N'_i(\xi)=\phi'_i(\xi),\ \xi\in[-\tau,0],\ \phi_i(0)>0,$$
$$\phi_i\in C([-\gamma,0),[0,+\infty))\cap C^1([-\gamma,0),[0,+\infty)), \quad (1.6)$$
$$i=1,2,\cdots,n.$$

For the ecological justification of (1.5) and the similar types refer to [1,8-10].

Throughout this paper, we make the following notations:

Let $\omega>0$ be a constant and

$$C_\omega=\left\{x(t)=(x_1(t),\cdots,x_n(t))^{\mathrm{T}}\,\middle|\,x_i(t)\in C(R,R),x_i(t+\omega)=x_i(t)\right\},$$

with the norm defined by $\|x\|_1=\max_{t\in[0,\omega]}\{|x_i(t)|\}$;

$$C_\omega^1=\left\{x(t)=(x_1(t),\cdots,x_n(t))^{\mathrm{T}}\,\middle|\,x_i(t)\in C^1(R,R),x_i(t+\omega)=x_i(t)\right\},$$

with the norm defined by $\|x\|_2 = \max_{t\in[0,\omega]}\{\|x\|_1, \|x'\|_1\}$.

Then $(C_\omega, \|x\|_1), (C_\omega^1, \|x\|_2)$ are Banach spaces.

For the sake of generality and convenience, we always make the following fundamental assumptions:

(H1) $a_i(t)$, $b_i(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$, are all positive periodic continuous functions with period ω, and $f_{ij}(t)$ are positive continuously differentiable ω-periodic functions. Furthermore, $\gamma_{ij}(t)$, $\delta_{ij}(t)$ are positive ω-periodic continuous functions such that $\gamma'_{ij}(t) < 1$, $\delta'_{ij}(t) < 1$, and $\delta''_{ij}(t)$ exists;

(H2) $0 < t_1 < t_2 < \cdots < t_k < \cdots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = +\infty$;

(H3) $\{\theta_{ik}\}$ is a real sequence such that $\theta_{ik} + 1 > 0$, $\prod_{0<t_k<t}(1+\theta_{ik})$ is an ω-periodic function;

(H4) $a_i(t)$, $b_i(t)$, $c_{ij}(t)$, $d_{ij}(t)$, $e_{ij}(t)$, are all almost periodic continuous functions with period ω on R, and $f_{ij}(t)$ are positive continuously differentiable almost periodic functions such that

$$a_i(t) \ge 0, b_i(t) \ge 0, c_{ij}(t) \ge 0, d_{ij}(t) \ge 0, e_{ij}(t) \ge 0,$$

$$f_{ij}(t) \ge 0, m(b_i(t) + d_{ii}(t)) > 0,$$

where

$$m(b_i(t) + d_{ii}(t)) = \lim_{T\to+\infty} \frac{1}{T}\int_t^{t+T} (b_i(\tau) + d_{ii}(\tau))\,\mathrm{d}\tau,$$

$$i, j = 1, 2, \cdots, n;$$

(H5) $\gamma_{ij}(t)$, $\delta_{ij}(t)$ are positive continuously differentiable almost periodic functions such that $\gamma'_{ij}(t) < 1$, $\delta'_{ij}(t) < 1$, and $\delta''_{ij}(t)$ exists, $0 < t_1 < t_2 < \cdots < t_k < \cdots$ are fixed impulsive points with $\lim_{k\to\infty} t_k = +\infty$;

(H6) $\{\theta_{ik}\}$ is a real sequence such that $\theta_{ik} + 1 > 0$, $\prod_{0<t_k<t}(1+\theta_{ik})$ is an almost periodic continuous function.

The outline of the paper is as follows. In the following section, some definitions and some useful lemmas are listed. In the third section, we first introduce a transformation, where some adjustable real parameters $\rho_i > 0$ are introduced. After that, by using contraction mapping principle, we derive some sufficient conditions which ensure the existence and uniqueness of positive periodic solution (positive almost periodic solution) of system (1.5) and (1.6). In the fourth section, we derive a set of easily verifiable criteria for the global attractivity of the positive periodic solution (almost periodic solution) of (1.5) and (1.6) by constructing a suitable Lyapunov functional. Finally, we give an example to show our results. Here, We must point out, the idea of introducing parameters is stimulated by the recent works of [1,26, 27]. However, to the best of the authors knowledge, this is the first time such a technique is applied to the impulsive neutral delays ecosystem.

2. Preliminaries

In order to obtain the existence and uniqueness of a periodic solution for system (1.1) and (1.2), we first give some definitions and lemmas:

Definition 2.1 ([21]) *A function* $N_i : R \to (0, +\infty)$ *is said to be a positive solution of* (1.5) *and* (1.6), *if the following conditions are satisfied*:

1) $N_i(t)$ is absolutely continuous on each (t_k, t_{k+1});

2) for each $k \in Z_+$, $N_i(t_k^+)$ and $N_i(t_k^-)$ exist and $N_i(t_k^-) = N_i(t_k)$;

3) $N_i(t)$ satisfies the first equation of (1.1) and (1.2) for almost everywhere (for short a.e.) in $[0,\infty]\setminus\{t_k\}$ and satisfies $N_i(t_k^+) = (1+\theta_{ik})N_i(t_k)$ for $t = t_k$, $k \in Z_+ = \{1, 2, \cdots\}$.

Definition 2.2 *Let* $N^*(t) = (N_1^*(t), \cdots, N_n^*(t))^{\mathrm{T}}$ *be a strictly positive periodic solution* (*almost periodic solution*) *of* (1.5) *and* (1.6). *We say* $N^*(t)$ *is globally attractive if any other solution* $N(t) = (N_1(t), \cdots, N_n(t))^{\mathrm{T}}$ *of* (1.5) *and* (1.6) *has the property*:

$$\lim_{t\to+\infty} |N_i^*(t) - N_i(t)| = 0, \ i = 1, 2, \cdots, n.$$

We can easily get the following Lemma 2.1.

Lemma 2.1 *The region* $R_+^n = \{N_i(t) : N_i(0) > 0, i = 1, 2, \cdots, n\}$ *is the positive invariable region of the system* (1.5).

Proof. In view of biological population,we obtain $N_i(0) > 0$. By the system (1.5), we have

$$N_i(t) = N_i(0)\exp\left\{\int_0^t \left[a_i(\eta) - b_i(\eta)\ln N_i(\eta) - \sum_{j=1}^n c_{ij}(\eta)\ln N_j(\eta) - \sum_{j=1}^n d_{ij}(\eta)\ln N_j(\eta - \gamma_{ij}(\eta)) - \sum_{j=1}^n e_{ij}(\eta)\int_{-\infty}^t K_{ij}(\eta - s)\ln N_j(s)\,\mathrm{d}s - \sum_{j=1}^n f_{ij}(\eta)\frac{\mathrm{d}\ln N_j(\eta - \delta_{ij}(\eta))}{\mathrm{d}\eta}\right]\mathrm{d}\eta\right\}, \ t \in [0, t_1], i = 1, 2, \cdots, n,$$

and

$$N_i(t) = N_i(t_k)\exp\left\{\int_{t_k}^t \left[a_i(\eta) - b_i(\eta)\ln N_i(\eta) - \sum_{j=1}^n c_{ij}(\eta)\ln N_j(\eta) - \sum_{j=1}^n d_{ij}(\eta)\ln N_j(\eta - \gamma_{ij}(\eta)) - \sum_{j=1}^n e_{ij}(\eta)\int_{-\infty}^t K_{ij}(\eta - s)\ln N_j(s)\,\mathrm{d}s - \sum_{j=1}^n f_{ij}(\eta)\frac{\mathrm{d}\ln N_j(\eta - \delta_{ij}(\eta))}{\mathrm{d}\eta}\right]\mathrm{d}\eta\right\}, \ t \in (t_k, t_{k+1}], i = 1, 2, \cdots, n,$$

$$N_i\left(t_k^+\right)=\left(1+\theta_{ik}\right)N_i\left(t_k\right)>0,\, k\in N, i=1,2,\cdots,n.$$

Then the solution of (1.5) is positive. □

Under the above hypotheses (H_1)-(H_3), I consider the neutral non-impulsive system

$$\begin{aligned}\frac{\mathrm{d}y_i}{\mathrm{d}t}=y_i(t)\Bigg[&a_i(t)-B_i(t)\ln y_i(t)-\sum_{j=1}^{n}C_{ij}(t)\ln y_j(t)-\sum_{j=1}^{n}D_{ij}(t)\ln y_j\left(t-\gamma_{ij}(t)\right)\\ &-\sum_{j=1}^{n}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)\ln y_j(s)\mathrm{d}s-\sum_{j=1}^{n}F_{ij}(t)\frac{\mathrm{d}\ln y_j\left(t-\delta_{ij}(t)\right)}{\mathrm{d}t}\Bigg],\end{aligned}\tag{2.1}$$

with initial conditions:

$$\begin{aligned}&y_i(\xi)=\varphi_i(\xi),\ y_i'(\xi)=\varphi_i'(\xi),\ \xi\in[-\tau,0],\ \varphi_i(0)>0,\\ &\varphi_i\in C\left([-\tau,0),R^+\right)\cap C^1\left([-\tau,0),R^+\right),\ i=1,2,3,\cdots,n,\end{aligned}\tag{2.2}$$

where

$$\begin{aligned}&B_i(t)=b_i(t)\prod_{0<t_k<t}\left(1+\theta_{ik}\right),\ C_{ij}(t)=c_{ij}(t)\prod_{0<t_k<t}\left(1+\theta_{ik}\right),\ D_{ij}(t)=d_{ij}(t)\prod_{0<t_k<t}\left(1+\theta_{ik}\right),\\ &E_{ij}(t)=e_{ij}(t)\prod_{0<t_k<t-\gamma_{ij}(t)}\left(1+\theta_{ik}\right),\ F_{ij}(t)=f_{ij}(t)\prod_{0<t_k<t-\delta_{ij}(t)}\left(1+\theta_{ik}\right).\end{aligned}\tag{2.3}$$

By a solution $y_i(t), i=1,2,\cdots,n,$ of (2.1) and (2.2), it means an absolutely continuous function $y_i(t)$, $i=1,2,\cdots,n$, defined on $[-\tau,0]$ that satisfies (2.1) a.e., for $t\ge 0$, and $y_i(\xi)=\varphi_i(\xi),\ y_i'(\xi)=\varphi_i'(\xi)$ on $[-\tau,0]$.

The following lemmas will be used in the proofs of our results, The proof of the first lemma is similar to that of Theorem 1 in [20].

Lemma 2.2 *Suppose that* (H_1)-(H_3) *hold. Then*

1) if $y_i(t)\left(i=1,2,\cdots,n\right)$ is a solution of (2.1) and (2.2) on $[-\tau,+\infty)$, then

$$N_i(t)=\prod\nolimits_{0<t_k<t}\left(1+\theta_{ik}\right)y_i(t)\left(i=1,2,\cdots,n\right)$$

is a solution of (1.5) and (1.6) on $[-\tau,+\infty)$.

2) if $N_i(t)\left(i=1,2,\cdots,n\right)$ is a solution of (1.5) and (1.6) on $[-\tau,+\infty)$, then

$$y_i(t)=\prod\nolimits_{0<t_k<t}\left(1+\theta_{ik}\right)N_i(t)\left(i=1,2,\cdots,n\right)$$

is a solution of (2.1) and (2.2) on $[-\tau,+\infty)$.

Proof. Its proof is similar to that of Theorem 1 in [20], here we omit it. □

Lemma 2.3 ([28]) *Suppose* $\sigma\in C_\omega^1$ *and* $\sigma'(t)<1$, $t\in[0,\omega]$. *Then the function* $t-\sigma(t)$ *has a unique inverse* $\mu(t)$ *satisfying* $\mu\in C(R,R)$ *with* $\mu(a+\omega)=\mu(a)+\omega\quad\forall a\in R.$

Proof. Its proof is similar to that of Lemma 2.4 in [29], here we omit it. □

Lemma 2.4 (*Barbalat's Lemma* [30]) *Let* $f(t)$ *be a nonnegative function defined on* $[0,+\infty)$ *such that* $f(t)$ *is integrable and uniformly continuous on* $[0,+\infty)$, *then* $\lim_{t\to+\infty}f(t)=0$.

Lemma 2.5 *Assume that* $u(t)$, $\tau(t)$ *are all continuously differentiable* ω*-periodic functions,* $a(t)$, $b(t)$ *are both nonnegative continuous* ω*-periodic functions such that* $\int_0^\omega a(t)\mathrm{d}t>0$, *then*

$$\begin{aligned}&\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}b(s)u'\left(s-\tau(s)\right)\mathrm{d}s\\ &=c(t)u\left(t-\tau(t)\right)\\ &\quad-\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}\left[a(s)c(s)+c'(s)\right]u\left(s-\tau(s)\right)\mathrm{d}s,\end{aligned}$$

where $c(t)=\dfrac{b(t)}{1-\tau'(t)}.$

Proof. As

$$\begin{aligned}&\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}b(s)u'\left(s-\tau(s)\right)\mathrm{d}s\\ &=\int_{-\infty}^{t}\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}c(s)\mathrm{d}u\left(s-\tau(s)\right)\\ &=\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}c(s)u\left(s-\tau(s)\right)\Big|_{-\infty}^{t}\\ &\quad-\int_{-\infty}^{t}u\left(s-\tau(s)\right)\mathrm{d}\left(\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}c(s)\right)\\ &=\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}c(s)u\left(s-\tau(s)\right)\Big|_{-\infty}^{t}\\ &\quad-\int_{-\infty}^{t}u\left(s-\tau(s)\right)\left[a(s)c(s)+c'(s)\right]\mathrm{e}^{-\int_s^t a(\xi)\mathrm{d}\xi}\mathrm{d}s,\end{aligned}\tag{2.4}$$

Denote $m=\mathrm{e}^{-\int_0^\omega a(t)\mathrm{d}t}$, then from $a(t)\ge 0$, $\int_0^\omega a(t)\mathrm{d}t>0$, it follows $m<1$. Also, when $t\ge s$ without loss of generality, we may assume $s+n\omega\le t\le s+(n+1)\omega$, thus

$$\left|e^{-\int_s^t a(\xi)d\xi} c(s)u(s-\tau(s))\right| \le e^{-\int_s^t a(\xi)d\xi} \|c\|\|u\|$$
$$= e^{-\sum_{j=1}^{n-1}\int_{s+j\omega}^{s+(j+1)\omega} a(\xi)d\xi - \int_{s+n\omega}^t a(\xi)d\xi} \|c\|\|u\|$$
$$= k^n e^{-\int_{s+n\omega}^t a(\xi)d\xi} \|c\|\|u\| \le k^n \|c\|\|u\|.$$

Therefore

$$\lim_{s\to-\infty} e^{-\int_s^t a(\xi)d\xi} c(s)u(s-\tau(s)) = 0,$$

and so, from (2.9) it follows:

$$\int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} b(s)u'(s-\tau(s))ds$$
$$= c(t)u(t-\tau(t))$$
$$-\int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi}\left[a(s)c(s)+c'(s)\right]u(s-\tau(s))ds.$$

The proof is complete. □

Lemma 2.6 *Assume that* $u(t)$ *are all continuously differentiable almost periodic functions*, $a(t), b(t)$ *are both nonnegative continuous almost periodic functions such that* $m(a(t)) > 0$, τ *is positive number*, *then*

$$\int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi} b(s)u'(s-\tau(s))ds$$
$$= c(t)u(t-\tau(t))$$
$$-\int_{-\infty}^t e^{-\int_s^t a(\xi)d\xi}\left[a(s)b(s)+b'(s)\right]u(s-\tau(s))ds,$$

where $m(a(t)) = \lim_{T\to+\infty} \frac{1}{T}\int_t^{t+T} a(\tau)d\tau.$

Proof. Similar to the proof of Lemma 2.5, we omit it here. □

3. Main Theorem

Here, we take the transformation $y_i(t) = \exp\{\rho_i x_i(t)\}$, then (2.1) can be rewritten in the following form

$$\frac{dx_i(t)}{dt} = -\left[b_i(t)+C_{ij}(t)\right]x_i(t) - \sum_{j=1, j\ne i}^n \frac{\rho_j}{\rho_i} C_{ij}(t)x_j(t)$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} D_{ij}(t)x_j(t-\gamma_{ij}(t))$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} E_{ij}(t)\int_{-\infty}^t K_{ij}(t-s)x_j(s)ds$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} F_{ij}(t)(1-\delta'_{ij}(t))x'_j(t-\delta_{ij}(t)) + \frac{a_i(t)}{\rho_i},$$
$$i = 1, 2, \cdots, n. \tag{3.1}$$

Obviously, the existence, uniqueness and global attractivity of positive periodic solution (almost periodic solution) of system (1.5) is equivalent to the existence, uniqueness and global attractivity of periodic solution (almost periodic solution) of system (3.1).

For $u(t) = (u_1(t), u_2(t), \cdots, u_n(t))^T \in C_\omega^1$, let us consider the equation

$$\frac{dx_i(t)}{dt} = -\left[b_i(t)+C_{ii}(t)\right]x_i(t) - \sum_{j=1, j\ne i}^n \frac{\rho_j}{\rho_i} C_{ij}(t)u_j(t)$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} D_{ij}(t)u_j(t-\gamma_{ij}(t))$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} E_{ij}(t)\int_{-\infty}^t K_{ij}(t-s)u_j(s)ds$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} F_{ij}(t)(1-\delta'_{ij}(t))u'_j(t-\delta_{ij}(t)) + \frac{a_i(t)}{\rho_i},$$
$$i = 1, 2, \cdots, n. \tag{3.2}$$

Since $b_i(t)+C_{ii}(t) > 0$, $\int_0^\omega \left[b_i(t)+C_{ii}(t)\right]dt > 0$, it follows that the linear system of system (3.2)

$$\frac{dx_i}{dt} = -\left[b_i(t)+C_{ii}(t)\right]x_i(t), \ i = 1, 2, \cdots, n, \tag{3.3}$$

admits exponential dichotomies on R, and so, system (3.3) has a unique continuous periodic solution $x_{iu}(t)$, which can be expressed as

$$x_{iu}(t) = \int_{-\infty}^t e^{-\int_\tau^t\left[b_i(\xi)+C_{ii}(\xi)\right]d\xi} f_{iu}(\tau)d\tau, \quad i = 1, 2, \cdots, n, \tag{3.4}$$

where

$$f_{iu}(\tau) = -\sum_{j=1, j\ne i}^n \frac{\rho_j}{\rho_i} C_{ij}(\tau)u_j(\tau)$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} D_{ij}(\tau)u_j(\tau-\gamma_{ij}(\tau))$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} E_{ij}(\tau)\int_{-\infty}^\tau K_{ij}(\tau-s)u_j(s)ds$$
$$-\sum_{j=1}^n \frac{\rho_j}{\rho_i} F_{ij}(\tau)(1-\delta'_{ij}(\tau))u'_j(\tau-\delta_{ij}(\tau)) + \frac{a_i(\tau)}{\rho_i},$$
$$i = 1, 2, \cdots, n. \tag{3.5}$$

Now, by using Lemma 2.5, $x_{iu}(t)$ can also be expressed as

$$x_{iu}(t) = -\sum_{j=1}^n \frac{\rho_j}{\rho_i} F_{ij}(t)u_j(t-\delta_{ij}(t))$$
$$+\int_{-\infty}^t e^{-\int_\tau^t\left[b_i(\xi)+C_{ii}(\xi)\right]d\xi} g_{iu}(\tau)d\tau, \tag{3.6}$$
$$i = 1, 2, \cdots, n,$$

where

$$g_{iu}(\tau)=-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\mathrm{d}s$$
$$+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F'_{ij}(\tau)\right\}u_j\left(\tau-\delta_{ij}(\tau)\right)+\frac{a_i(t)}{\rho_i},\quad i=1,2,\cdots,n. \tag{3.7}$$

Our main result on the global existence of a positive periodic solution of (1.5) and (1.6) is stated as follows.

Theorem 3.1 *In addition to* (H_1)-(H_3), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

(H_7) $\max_{t\in R}\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_i(\xi)+C_{ii}(\xi)]\mathrm{d}\xi}q_i(\tau)\mathrm{d}\tau<1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1$.

Then (1.5) has a unique positive ω-periodic solution with strictly positive components, say

$$N^*(t)=\left(N_1^*(t),N_2^*(t),\cdots,N_n^*(t)\right)^{\mathrm{T}},$$

where

$$q_i(\tau)=\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{D_{ij}(\tau)+E_{ij}(\tau)+F_{ij}(\tau)\left[b_i(\tau)+C_{ii}(\tau)\right]+\left|F'_{ij}(\tau)\right|\right\},$$

and

$$\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1=\max_{t\in[0,\omega]}\left\{\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right|\right\}.$$

Proof. For $u(t)=\left(u_1(t),u_2(t),\cdots,u_n(t)\right)^{\mathrm{T}}\in C_\omega$, from (3.6), we know that

$$x_{iu}(t)=-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_i(\xi)+C_{ii}(\xi)]\mathrm{d}\xi}g_{iu}(\tau)\mathrm{d}\tau,$$
$$i=1,2,\cdots,n, \tag{3.8}$$

where $g_{iu}(\tau)$ are defined by (3.7), is a continuous ω-periodic function, and so $x_u(t)=\left(x_{1u}(t),x_{2u}(t),\cdots,x_{nu}(t)\right)^{\mathrm{T}}\in C_\omega$. Now define the mapping $\psi:C_\omega\to C_\omega$ as follows:

$$\psi u(t)=x_u(t),\ u(t)\in C_\omega. \tag{3.9}$$

Following we will prove the mapping ψ is a contraction mapping. In fact, for any $u(t)=\left(u_1(t),u_2(t),\cdots,u_n(t)\right)^{\mathrm{T}}$ and $u^*(t)=\left(u_1^*(t),u_2^*(t),\cdots,u_n^*(t)\right)^{\mathrm{T}}$ from (3.8), (3.9) and the conditions of Theorem 3.1 it follows:

$$\begin{aligned}\left\|\psi u-\psi u^*\right\|_1&=\max_{t\in[0,\omega]}\Bigg\{\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_1}F_{1j}(t)\left[u_j\left(t-\delta_{1j}(t)\right)-u_j^*\left(t-\delta_{1j}(t)\right)\right]\right|\\&\quad+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_1(\xi)+C_{11}(\xi)]\mathrm{d}\xi}\left|g_{1u}(\tau)-g_{1u^*}(\tau)\right|\mathrm{d}\tau,\cdots,\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_n}F_{nj}(t)\left[u_j\left(t-\delta_{nj}(t)\right)-u_j^*\left(t-\delta_{nj}(t)\right)\right]\right|\\&\quad+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_n(\xi)+C_{nn}(\xi)]\mathrm{d}\xi}\left|g_{nu}(\tau)-g_{nu^*}(\tau)\right|\mathrm{d}\tau\Bigg\}\\&\le\max_{t\in[0,\omega]}\Bigg\{\left[\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_1}F_{1j}(t)\right\|_1+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_1(\xi)+C_{11}(\xi)]\mathrm{d}\xi}q(\tau)\mathrm{d}\tau\right]\left\|u-u^*\right\|_1,\\&\quad\left[\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_n}F_{nj}(t)\right\|_1+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_n(\xi)+C_{nn}(\xi)]\mathrm{d}\xi}q(\tau)\mathrm{d}\tau\right]\left\|u-u^*\right\|_1\Bigg\}\\&<\left\|u-u^*\right\|_1,\end{aligned} \tag{3.10}$$

where

$$\left\|g_{iu}(\tau)-g_{iu^*}(\tau)\right\|_1 \le \sum_{j=1,j\ne i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)\left|u_j(\tau)-u_j^*(\tau)\right|+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)\left|u_j\left(\tau-\gamma_{ij}(\tau)\right)-u_j^*\left(\tau-\gamma_{ij}(\tau)\right)\right|$$

$$+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)\left|u_j(s)+u_j^*(s)\right|\mathrm{d}s$$

$$+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F_{ij}'(\tau)\right\}\left|u_j\left(\tau-\delta_{ij}(\tau)\right)-u_j^*\left(\tau-\delta_{ij}(\tau)\right)\right|$$

$$\le \sum_{j=1,j\ne i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)\left\|u_j-u_j^*\right\|_1+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{D_{ij}(\tau)+E_{ij}(\tau)+F_{ij}(\tau)\left[b_i(\tau)+C_{ii}(\tau)\right]+\left|F_{ij}'(\tau)\right|\right\}\left\|u_j-u_j^*\right\|_1$$

$$=q_i(\tau)\left\|u_j-u_j^*\right\|_1,\quad i=1,2,\cdots,n.$$

That is

$$\left\|\psi u-\psi u^*\right\|_1<\left\|u-u^*\right\|_1. \tag{3.11}$$

This shows that ψ is a contraction mapping. Hence, there exists a unique fixed point $x(t)=\left(x_1(t),\cdots,x_n(t)\right)^{\mathrm{T}}\in C_\omega$ such that $\psi x=x$, that is

$$\begin{aligned}x_i(t)=&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)+\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}\left[-\sum_{j=1,j\ne i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau)\right.\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\mathrm{d}s\\&\left.+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F_{ij}'(\tau)\right\}u_j\left(\tau-\delta_{ij}(\tau)\right)+\frac{a_i(t)}{\rho_i}\right]\mathrm{d}\tau,\\&i=1,2,\cdots,n.\end{aligned} \tag{3.12}$$

Following, we prove $x(t)=\left(x_1(t),\cdots,x_n(t)\right)^{\mathrm{T}}\in C_\omega$ is the periodic solution of system (3.1). Noticing that (3.12) is equivalent to

$$\begin{aligned}&x_i(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)\\&=\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}\left[-\sum_{j=1,j\ne i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right)\right.\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\mathrm{d}s\\&\left.+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F_{ij}'(\tau)\right\}u_j\left(\tau-\delta_{ij}(\tau)\right)+\frac{a_i(t)}{\rho_i}\right]\mathrm{d}\tau,\\&i=1,2,\cdots,n.\end{aligned} \tag{3.13}$$

From the right-hand sides of (3.13), we know that

$$x_i(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)$$

is differentiable. And so, from (3.13) it follows that

$$\begin{aligned}
&\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F'_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta'_{ij}(t)\right)u_j\left(t-\delta_{ij}(t)\right)\\
&=\frac{\mathrm{d}}{\mathrm{d}t}\left[x_i(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)\right]\\
&=-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)u_j(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)u_j\left(t-\gamma_{ij}(t)\right)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)u_j(s)\mathrm{d}s\\
&\quad+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(t)\left[b_i(t)+C_{ij}(t)\right]+F'_{ij}(t)\right\}u_j\left(t-\delta_{ij}(t)\right)+\frac{a_i(t)}{\rho_i}\\
&\quad-\left[b_i(t)+C_{ii}(t)\right]\int_{-\infty}^{t}\mathrm{e}^{-\int_{\tau}^{t}\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}\left[-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right)\right.\\
&\quad\left.-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\mathrm{d}s+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F'_{ij}(\tau)\right\}u_j\left(\tau-\delta_{ij}(\tau)\right)+\frac{a_i(t)}{\rho_i}\right]\mathrm{d}\tau\\
&=-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)u_j(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)u_j\left(t-\gamma_{ij}(t)\right)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)u_j(s)\mathrm{d}s\\
&\quad+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(t)\left[b_i(t)+C_{ij}(t)\right]+F'_{ij}(t)\right\}u_j\left(t-\delta_{ij}(t)\right)+\frac{a_i(t)}{\rho_i}\\
&\quad-\left[b_i(t)+C_{ii}(t)\right]\left[x_i(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right)\right],\ i=1,2,\cdots,n,
\end{aligned}$$

here using the equality (3.13) again. That is

$$\begin{aligned}
\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}&=-\left[b_i(t)+C_{ii}(t)\right]x_i(t)-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)u_j(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)u_j\left(t-\gamma_{ij}(t)\right)\\
&\quad-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)u_j(s)\mathrm{d}s-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta'_{ij}(t)\right)u'_j\left(t-\delta_{ij}(t)\right)+\frac{a_i(t)}{\rho_i},i=1,2,\cdots,n.
\end{aligned}\tag{3.14}$$

This shows that $x(t)=\left(x_1(t),\cdots,x_n(t)\right)^{\mathrm{T}}$ is continuously differentiable ω-periodic function and satisfies Equation (3.1). Therefore, $x(t)=\left(x_1(t),\cdots,x_n(t)\right)^{\mathrm{T}}$ is the unique continuously differentiable ω-periodic solution of system (3.1), and so,

$$y(t)=\left(\exp\left\{\rho_1x_1(t)\right\},\cdots,\exp\left\{\rho_nx_n(t)\right\}\right)^{\mathrm{T}}$$

is the unique positive ω-periodic solution of system (2.1), from Lemma 2.2,

$$N(t)=\left(\prod_{0<t_k<t}\left(1+\theta_{ik}\right)\exp\left\{\rho_1x_1(t)\right\},\cdots,\prod_{0<t_k<t}\left(1+\theta_{ik}\right)\exp\left\{\rho_nx_n(t)\right\}\right)^{\mathrm{T}}$$

is the unique positive ω-periodic solution of system (1.5). The proof is complete. □

As a direct corollary of Theorem 3.1, one has

Corollary 3.1 *In addition to* (H_1)-(H_3), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

$$\begin{aligned}
q_i(t)&=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{D_{ij}(t)+E_{ij}(t)+F_{ij}(t)\left[b_i(t)+C_{ii}(t)\right]+\left|F'_{ij}(t)\right|\right\}\\
&<\left[b_i(t)+C_{ii}(t)\right]\left(1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1\right).
\end{aligned}$$

Then (1.5) has a unique positive ω-periodic solution with strictly positive components.

Our next theorem concerned with the existence of unique positive almost periodic solution of systems (1.5) and (1.6).

Let $u(t)=\left(u_1(t),u_2(t),\cdots,u_n(t)\right)^{\mathrm{T}}$ be any continuously differentiable almost periodic function, and consider equation,

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\left[b_i(t)+C_{ii}(t)\right]x_i(t) - \sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)u_j(t)$$
$$-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)u_j\left(t-\gamma_{ij}(t)\right) - \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)u_j(s)\,\mathrm{d}s \tag{3.15}$$
$$-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta_{ij}'(t)\right)u_j'\left(t-\delta_{ij}(t)\right) + \frac{a_i(t)}{\rho_i},\quad i=1,2,\cdots,n.$$

Since $m\left(b_i(t)+d_{ii}(t)\right)>0$, it follows that the linear system of system (3.15)

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = -\left[b_i(t)+C_{ii}(t)\right]x_i(t),\ \ i=1,2,\cdots,n, \tag{3.16}$$

admits exponential dichotomies on R, and so, system (3.16) has a unique continuous almost periodic solution $x_{iu}(t)$, which can be expressed as

$$x_{iu}(t) = \int_{-\infty}^{t}\mathrm{e}^{-\int_{\tau}^{t}\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}f_{iu}(\tau)\,\mathrm{d}\tau,\quad i=1,2,\cdots,n, \tag{3.17}$$

where

$$\begin{aligned} f_{iu}(\tau) = &-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau) - \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right) - \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\,\mathrm{d}s \\ &-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(\tau)\left(1-\delta_{ij}'(\tau)\right)u_j'\left(\tau-\delta_{ij}(\tau)\right) + \frac{a_i(\tau)}{\rho_i},\ i=1,2,\cdots,n. \end{aligned} \tag{3.18}$$

Now, by using Lemma 2.5, $x_{iu}(t)$ can also be expressed as

$$x_{iu}(t) = -\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)u_j\left(t-\delta_{ij}(t)\right) + \int_{-\infty}^{t}\mathrm{e}^{-\int_{\tau}^{t}\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}g_{iu}(\tau)\,\mathrm{d}\tau,\quad i=1,2,\cdots,n, \tag{3.19}$$

where

$$\begin{aligned} g_{iu}(\tau) = &-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau)u_j(\tau) - \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(\tau)u_j\left(\tau-\gamma_{ij}(\tau)\right) - \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(\tau)\int_{-\infty}^{t}K_{ij}(\tau-s)u_j(s)\,\mathrm{d}s \\ &+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{F_{ij}(\tau)\left[b_i(\tau)+C_{ij}(\tau)\right]+F_{ij}'(\tau)\right\}u_j\left(\tau-\delta_{ij}(\tau)\right) + \frac{a_i(t)}{\rho_i},\quad i=1,2,\cdots,n. \end{aligned} \tag{3.20}$$

Then, we have

Theorem 3.2 *In addition to* (H_4)-(H_6), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

(H_8) $\max_{t\in R}\int_{-\infty}^{t}\mathrm{e}^{-\int_{\tau}^{t}\left[b_i(\xi)+C_{ii}(\xi)\right]\mathrm{d}\xi}q_i(\tau)\,\mathrm{d}\tau < 1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1$.

Then (1.5) has a unique positive almost periodic solution with strictly positive components, say

$$N^*(t) = \left(N_1^*(t), N_2^*(t), \cdots, N_n^*(t)\right)^{\mathrm{T}},$$

where

$$q_i(\tau) = \sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(\tau) + \sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{D_{ij}(\tau)+E_{ij}(\tau)+F_{ij}(\tau)\left[b_i(\tau)+C_{ii}(\tau)\right]+\left|F_{ij}'(\tau)\right|\right\},$$

and

$$\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1 = \max_{t\in[0,\omega]}\left\{\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right|\right\}.$$

Proof. Set

$$C = \left\{v(t) = \left(v_1(t),\cdots,v_n(t)\right)^{\mathrm{T}} \text{ is continuous almost periodic function}\right\}$$

with the norm $\|v\|_1 = \sup\left\{\|u(t)\|_1 : t\in R\right\}$, obviously, C is a Banach space. For any continuously almost periodic function $u(t)=(u_1(t),\cdots,u_n(t))^{\mathrm{T}}$ we know that $x_{iu}(t)$ defined by (3.19) is also a continuously almost periodic function. Now define the mapping $\varphi: C\to C$ as follows:

$$\varphi u(t)=x_u(t),\ u(t)\in C. \tag{3.21}$$

Then similarly to the prove of Theorem 3.1, we could prove that under the assumptions of Theorem 3.2, the mapping φ is a contract mapping, and so system (3.19) has a unique fixed point $x(t)=(x_1(t),\cdots,x_n(t))^{\mathrm{T}}$. and so,

$$y(t)=\left(\exp\{\rho_1x_1(t)\},\cdots,\exp\{\rho_nx_n(t)\}\right)^{\mathrm{T}}$$

is the unique positive almost periodic solution of system (2.1), from Lemma 2.2,

$$N(t)=\Big(\prod_{0<t_k<t}(1+\theta_{ik})\exp\{\rho_1x_1(t)\},\cdots,$$
$$\prod_{0<t_k<t}(1+\theta_{ik})\exp\{\rho_nx_n(t)\}\Big)^{\mathrm{T}}$$

is the unique positive almost periodic solution of system (1.5). The proof is complete. □

As a direct corollary of Theorem 3.2, one has

Corollary 3.2 *In addition to* (H$_4$)-(H$_6$), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

$$q_i(t)=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{D_{ij}(t)+E_{ij}(t)+F_{ij}(t)\left[b_i(t)+C_{ii}(t)\right]+\left|F'_{ij}(t)\right|\right\}$$
$$<\left[b_i(t)+C_{ii}(t)\right]\left(1-\left\|-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\right\|_1\right).$$

Then (1.5) has a unique positive almost periodic solution with strictly positive components.

Consider the following equation:

$$\frac{\mathrm{d}N_i(t)}{\mathrm{d}t}=N_i(t)\Bigg[a_i(t)-b_i(t)\ln N_i(t)-\sum_{j=1}^{n}c_{ij}(t)\ln N_j(t)-\sum_{j=1}^{n}d_{ij}(t)\ln N_j\left(t-\gamma_{ij}(t)\right)$$
$$-\sum_{j=1}^{n}e_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)\ln N_j(s)\mathrm{d}s-\sum_{j=1}^{n}f_{ij}(t)\frac{d\ln N_j\left(t-\delta_{ij}(t)\right)}{dt}\Bigg],\quad i=1,2,\ldots,n \tag{3.22}$$

which is a special case of system (1.5) and (1.6) without impulse. Similarly, we can get the following results.

Theorem 3.3 *In addition to* (H$_1$), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

(H$_9$) $\max_{t\in R}\int_{-\infty}^{t}\mathrm{e}^{-\int_\tau^t[b_i(\xi)+C_{ii}(\xi)]\mathrm{d}\xi}q_i(\tau)\mathrm{d}\tau<1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right\|_1$.

Then (1.5) has a unique positive ω-periodic solution with strictly positive components, say

$$N^*(t)=\left(N_1^*(t),N_2^*(t),\cdots,N_n^*(t)\right)^{\mathrm{T}}.$$

where

$$q_i(\tau)=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}c_{ij}(\tau)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{d_{ij}(\tau)+e_{ij}(\tau)+f_{ij}(\tau)\left[b_i(\tau)+c_{ii}(\tau)\right]+\left|f'_{ij}(\tau)\right|\right\},$$

and

$$\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right\|_1=\max_{t\in[0,\omega]}\left\{\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right|\right\}.$$

Proof. Similar to the proof of Theorem 3.1, we omit it here. □

As a direct corollary of Theorem 3.3, one has

Corollary 3.3 *In addition to* (H$_1$), *assume further that there exist positive constants* $\rho_i\ (i=1,2,\cdots,n)$, *such that*

$$q_i(t)=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}c_{ij}(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{d_{ij}(t)+e_{ij}(t)+f_{ij}(t)\left[b_i(t)+c_{ii}(t)\right]+\left|f'_{ij}(t)\right|\right\}$$
$$<\left[b_i(t)+c_{ii}(t)\right]\left(1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right\|_1\right).$$

Then (1.5) has a unique positive ω-periodic solution with strictly positive components.

Theorem 3.4 *In addition to* (H_4), *assume further that there exist positive constants* $\rho_i \ (i=1,2,\cdots,n)$, *such that*

$$(H_{10})\quad \max_{t\in R}\int_{-\infty}^{t} e^{-\int_\tau^t [b_i(\xi)+C_{ii}(\xi)]d\xi} q_i(\tau)d\tau < 1-\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right\|_1 .$$

Then (1.5) has a unique positive almost periodic solution with strictly positive components, say

$$N^*(t)=\left(N_1^*(t),N_2^*(t),\cdots,N_n^*(t)\right)^{T}.$$

where

$$q_i(\tau)=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}c_{ij}(\tau)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{d_{ij}(\tau)+e_{ij}(\tau)+f_{ij}(\tau)\left[b_i(\tau)+c_{ii}(\tau)\right]+\left|f'_{ij}(\tau)\right|\right\},$$

and

$$\left\|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(\tau)\right\|_1=\max_{t\in[0,\omega]}\left\{\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(\tau)\right|\right\}.$$

As a direct corollary of Theorem 3.4, one has

Corollary 3.4 *In addition to* (H_4), *assume further that there exist positive constants* $\rho_i \ (i=1,2,\cdots,n)$, *such that*

$$\begin{aligned} q_i(t)&=\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}c_{ij}(t)+\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left\{d_{ij}(t)+e_{ij}(t)+f_{ij}(t)\left[b_i(t)+c_{ii}(t)\right]+\left|f'_{ij}(t)\right|\right\}\\ &<\left[b_i(t)+c_{ii}(t)\right]\left(1-\left|\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}f_{ij}(t)\right|_1\right).\end{aligned}$$

Then (1.5) has a unique positive almost periodic solution with strictly positive components.

4. Global Asymptotic Stability

In this section, we devote ourselves to the study of the global attractivity of periodic solutions (almost periodic solutions) of system (1.5), (1.6) and (3.22) (which is a special case of system (1.5) and (1.6) without impulse). Now, we state our main results of this section as follows:

Theorem 4.1. Assume that the conditions in Theorem 3.1 hold. Suppose further the following conditions hold:

(H_{11}) There is a positive constant M such that

$$\sum_{j=1}^{n}\left|\frac{\rho_j}{\rho_i}F_{ij}(t)\right|+\int_0^t\left\{\sum_{j=1}^{n}\left[\left|\frac{\rho_j}{\rho_i}H_{ij}(u)\right|+\left|\frac{\rho_j}{\rho_i}D_{ij}(u)\right|+\left|\frac{\rho_j}{\rho_i}E_{ij}(u)\right|\right]+\sum_{j=1,j\neq i}^{n}\left|\frac{\rho_j}{\rho_i}C_{ij}(u)\right|\right\}e^{-\int_u^t[b_i(\xi)+C_{ii}(\xi)]d\xi}du\le M<1;$$

$$(H_{12})\quad \exp\left\{-\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)d\xi\right\}\to 0,\ \text{as}\ t\to+\infty, i=1,2,\cdots,n.$$

Then system (1.5) and (1.6) has a unique periodic solution which is globally attractive.

Proof. Let $N^*(t)=\left(N_1^*(t),N_2^*(t),\cdots,N_n^*(t)\right)^{T}$ be the unique positive periodic solution of system (1.5) and (1.6), whose existence and uniqueness are guarantee by Theorem 2.1, and $N(t)=\left(N_1(t),N_2(t),\cdots,N_n(t)\right)^{T}$ be any other solution of system (1.5) and (1.6). Let

$$N_i^*(t)=\prod_{0<t_k<t}\left(1+\theta_{ik}\right)\exp\left\{\rho_i x_i^*(t)\right\}, N_i(t)=\prod_{0<t_k<t}\left(1+\theta_{ik}\right)\exp\left\{\rho_i x_i(t)\right\}$$

then, similar to Equation (3.1), we have

$$\begin{aligned}\frac{dx_i^*(t)}{dt}&=-\left[b_i(t)+C_{ii}(t)\right]x_i^*(t)-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)x_j^*(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)x_j^*\left(t-\gamma_{ij}(t)\right)\\ &-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)x_j^*(s)ds-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta'_{ij}(t)\right)\left(x_j^*\left(t-\delta_{ij}(t)\right)\right)'+\frac{a_i(t)}{\rho_i},\end{aligned}\quad (4.1)$$

and,

$$\begin{aligned}\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}=&-\left[b_i(t)+C_{ii}(t)\right]x_i(t)-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)x_j(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)x_j\left(t-\gamma_{ij}(t)\right)\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)x_j(s)\mathrm{d}s-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta_{ij}'(t)\right)x_j'\left(t-\delta_{ij}(t)\right)+\frac{a_i(t)}{\rho_i},\end{aligned}\tag{4.2}$$

Then, from (4.1) and (4.2), we have

$$\begin{aligned}\frac{\mathrm{d}x_i^*(t)}{\mathrm{d}t}-\frac{\mathrm{d}x_i(t)}{\mathrm{d}t}=&-\left[b_i(t)+C_{ii}(t)\right]\left(x_i^*(t)-x_i(t)\right)-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)\left(x_j^*(t)-x_j(t)\right)\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)\left(x_j^*\left(t-\gamma_{ij}(t)\right)-x_j\left(t-\gamma_{ij}(t)\right)\right)\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)\left(x_j^*(s)-x_j(s)\right)\mathrm{d}s\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta_{ij}'(t)\right)\left(x_j^*\left(t-\delta_{ij}(t)\right)-x_j\left(t-\delta_{ij}(t)\right)\right)',\quad i=1,2,\cdots,n.\end{aligned}\tag{4.3}$$

Let $x_i^*(t)-x_i(t)=z_i(t)$, then

$$\begin{aligned}\frac{\mathrm{d}z_i(t)}{\mathrm{d}t}=&-\left[b_i(t)+C_{ii}(t)\right]z_i(t)-\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(t)z_j(t)-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}D_{ij}(t)z_j\left(t-\gamma_{ij}(t)\right)\\&-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)z_j(s)\mathrm{d}s-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta_{ij}'(t)\right)z_j'\left(t-\delta_{ij}(t)\right),\,i=1,2,\cdots,n.\end{aligned}\tag{4.4}$$

Multiply both sides of (4.4) with $\exp\left\{\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}$ and then integrate from 0 to t to obtain

$$\begin{aligned}&\int_0^t\left[z_i(u)\exp\left\{\int_0^u\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\right]'\mathrm{d}u\\&=-\int_0^t\left[\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(u)z_j(u)+\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_i}D_{ij}(u)z_j\left(u-\gamma_{ij}(u)\right)+\frac{\rho_j}{\rho_i}E_{ij}(u)\int_{-\infty}^{u}K_{ij}(u-s)z_j(s)\mathrm{d}s\right.\right.\\&\quad\left.\left.+\frac{\rho_j}{\rho_i}F_{ij}(u)\left(1-\delta_{ij}'(u)\right)z_j'\left(u-\delta_{ij}(u)\right)\right)\right]\exp\left\{\int_0^u\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u,\,i=1,2,\cdots,n,\end{aligned}\tag{4.5}$$

then

$$\begin{aligned}&z_i(t)\exp\left\{\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\\&=z_i(0)-\int_0^t\left[\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(u)z_j(u)+\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_i}D_{ij}(u)z_j\left(u-\gamma_{ij}(u)\right)+\frac{\rho_j}{\rho_i}E_{ij}(u)\int_{-\infty}^{u}K_{ij}(u-s)z_j(s)\mathrm{d}s\right.\right.\\&\quad\left.\left.+\frac{\rho_j}{\rho_i}F_{ij}(u)\left(1-\delta_{ij}'(u)\right)z_j'\left(u-\delta_{ij}(u)\right)\right)\right]\exp\left\{\int_0^u\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u,\,i=1,2,\cdots,n,\end{aligned}\tag{4.6}$$

thus

$$\begin{aligned}z_i(t)=&\,z_i(0)\exp\left\{-\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\\&-\int_0^t\left[\sum_{j=1,\,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(u)z_j(u)+\sum_{j=1}^{n}\left(\frac{\rho_j}{\rho_i}D_{ij}(u)z_j\left(u-\gamma_{ij}(u)\right)+\frac{\rho_j}{\rho_i}E_{ij}(u)\int_{-\infty}^{u}K_{ij}(u-s)z_j(s)\mathrm{d}s\right.\right.\\&\quad\left.\left.+\frac{\rho_j}{\rho_i}F_{ij}(u)\left(1-\delta_{ij}'(u)\right)z_j'\left(u-\delta_{ij}(u)\right)\right)\right]\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u,\,i=1,2,\cdots,n.\end{aligned}\tag{4.7}$$

Let $F_{ij}^{*}(u)=F_{ij}(u)\left(1-\delta_{ij}'(u)\right)$, by Lemma 2.3, we obtain

$$\begin{aligned}
&\int_0^t\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)\left(1-\delta_{ij}'(t)\right)z_j'\left(t-\delta_{ij}(t)\right)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u\\
&=\int_0^t\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}^{*}(u)z_j'\left(u-\delta_{ij}(u)\right)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u\\
&=\int_0^t\sum_{j=1}^{n}\frac{F_{ij}^{*}(u)z_j'\left(u-\delta_{ij}(u)\right)\left(1-\delta_{ij}'(u)\right)}{1-\delta_{ij}'(u)}\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u\\
&=\int_0^t\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left[\frac{F_{ij}^{*}(u)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}}{1-\delta_{ij}'(u)}\right]\left[z_i'\left(u-\delta_{ij}(u)\right)\left(1-\delta_{ij}'(u)\right)\right]\mathrm{d}u\\
&=\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left[\frac{F_{ij}^{*}(t)}{1-\delta_{ij}'(t)}z_j\left(t-\delta_{ij}(t)\right)-\frac{F_{ij}^{*}(0)}{1-\delta_{ij}'(0)}z_j\left(-\delta_{ij}(0)\right)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\right.\\
&\quad\left.-\int_0^t H_{ij}(u)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}z_j\left(u-\delta_{ij}(u)\right)\mathrm{d}u\right]\\
&=\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}\left[F_{ij}(t)z_j\left(t-\delta_{ij}(t)\right)-F_{ij}(0)z_j\left(-\delta_{ij}(0)\right)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\right.\\
&\quad\left.-\int_0^t H_{ij}(u)\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}z_j\left(u-\delta_{ij}(u)\right)\mathrm{d}u\right],
\end{aligned}\tag{4.8}$$

where

$$H_{ij}(u)=\frac{\left[F_{ij}'(u)+F_{ij}(u)\left(b_i(u)+C_{ii}(u)\right)\right]\left(\left(1-\delta_{ij}'(u)\right)+D_{ij}(u)\delta_{ij}''(u)\right)}{\left(1-\delta_{ij}'(u)\right)^2}.$$

Thus,

$$\begin{aligned}
z_i(t)&=\left[z_i(0)+\frac{\rho_j}{\rho_i}F_{ij}(0)z_j\left(-\delta_{ij}(0)\right)\right]\exp\left\{-\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\\
&\quad+\int_0^t\left\{\sum_{j=1}^{n}\left[\frac{\rho_j}{\rho_i}H_{ij}(u)z_j\left(u-\delta_{ij}(u)\right)-\frac{\rho_j}{\rho_i}D_{ij}(t)z_j\left(t-\gamma_{ij}(t)\right)-\frac{\rho_j}{\rho_i}E_{ij}(t)\int_{-\infty}^{t}K_{ij}(t-s)z_j(s)\mathrm{d}s\right]\right.\\
&\quad\left.-\sum_{j=1,j\neq i}^{n}\frac{\rho_j}{\rho_i}C_{ij}(u)z_j(u)\right\}\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u-\sum_{j=1}^{n}\frac{\rho_j}{\rho_i}F_{ij}(t)z_j\left(t-\delta_{ij}(t)\right),\ i=1,2,\cdots,n,
\end{aligned}\tag{4.9}$$

then

$$\begin{aligned}
\left|z_i\right|&\le\left|z_i(0)+\frac{\rho_j}{\rho_i}F_{ij}(0)z_j\left(-\delta_{ij}(0)\right)\right|\exp\left\{-\int_0^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\\
&\quad+\int_0^t\left\{\sum_{j=1}^{n}\left[\left|\frac{\rho_j}{\rho_i}H_{ij}(u)\right|+\left|\frac{\rho_j}{\rho_i}D_{ij}(u)\right|+\left|\frac{\rho_j}{\rho_i}E_{ij}(u)\right|\right]+\sum_{j=1,j\neq i}^{n}\left|\frac{\rho_j}{\rho_i}C_{ij}(u)\right|\right\}\exp\left\{-\int_u^t\left(b_i(\xi)+C_{ii}(\xi)\right)\mathrm{d}\xi\right\}\mathrm{d}u\left|z_j\right|\\
&\quad+\sum_{j=1}^{n}\left|\frac{\rho_j}{\rho_i}F_{ij}(t)\right|\left|z_j\right|,\ i=1,2,\cdots,n,
\end{aligned}\tag{4.10}$$

That is

$$\|z\|_1 \le \left| z_i(0) + \frac{\rho_j}{\rho_i} F_{ij}(0) z_j(-\delta_{ij}(0)) \right| \exp\left\{ -\int_0^t \left(b_i(\xi) + C_{ii}(\xi) \right) d\xi \right\}$$
$$+ \int_0^t \left\{ \sum_{j=1}^n \left[\left| \frac{\rho_j}{\rho_i} H_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} D_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} E_{ij}(u) \right| \right] + \sum_{j=1, j\ne i}^n \left| \frac{\rho_j}{\rho_i} C_{ij}(u) \right| \right\} \exp\left\{ -\int_u^t \left(b_i(\xi) + C_{ii}(\xi) \right) d\xi \right\} du \|z\|_1 \quad (4.11)$$
$$+ \sum_{j=1}^n \left| \frac{\rho_j}{\rho_i} F_{ij}(t) \right| \|z\|_1, \ i = 1, 2, \cdots, n.$$

From (H_3), we have

$$\|w\|_1 \le \frac{\left| z_i(0) + \frac{\rho_j}{\rho_i} F_{ij}(0) z_j(-\delta_{ij}(0)) \right| e^{-\int_0^t (b_i(\xi) + C_{ii}(\xi)) d\xi}}{1 - \sum_{j=1}^n \left| \frac{\rho_j}{\rho_i} F_{ij}(t) \right| - \int_0^t \left\{ \sum_{j=1}^n \left[\left| \frac{\rho_j}{\rho_i} H_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} D_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} E_{ij}(u) \right| \right] + \sum_{j=1, j\ne i}^n \left| \frac{\rho_j}{\rho_i} C_{ij}(u) \right| \right\} e^{-\int_u^t (b_i(\xi) + C_{ii}(\xi)) d\xi} du}$$
$$\le \frac{\left| z_i(0) + \frac{\rho_j}{\rho_i} F_{ij}(0) z_j(-\delta_{ij}(0)) \right| e^{-\int_0^t (b_i(\xi) + C_{ii}(\xi)) d\xi}}{1 - M}, \quad i = 1, 2, \cdots, n. \quad (4.12)$$

From (H_4), we have

$$\|z\|_1 = \max_{t\in[0,\omega]} |z_i(t)| = \max_{t\in[0,\omega]} |x_i^*(t) - x_i(t)| = 0, \text{ as } t \to +\infty, i = 1, 2, \cdots, n, \quad (4.13)$$

thus, $x_i(t) \to x_i^*(t)$, as $t \to +\infty, i = 1, 2, \cdots, n,$ that is the positive ω-periodic solution of (3.1) is globally attractive,

$$N_i(t) = \prod_{0<t_k<t} (1+\theta_{ik}) \exp\{\rho_i x_i(t)\}$$
$$\to \prod_{0<t_k<t} (1+\theta_{ik}) \exp\{\rho_i x_i^*(t)\} = N_i^*(t) \text{ as } t \to +\infty, i = 1, 2, \cdots, n,$$

by Definition 2.2, the positive ω-periodic solution of (1.5) is globally attractive. The proof is completed.

Theorem 4.2. Assume that the conditions in Theorem 3.2 hold. Suppose further the following conditions hold:

(H_{13}) There is a positive constant m such that

$$\sum_{j=1}^n \left| \frac{\rho_j}{\rho_i} F_{ij}(t) \right| + \int_0^t \left\{ \sum_{j=1}^n \left[\left| \frac{\rho_j}{\rho_i} H_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} D_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} E_{ij}(u) \right| \right] + \sum_{j=1, j\ne i}^n \left| \frac{\rho_j}{\rho_i} C_{ij}(u) \right| \right\} e^{-\int_u^t (b_i(\xi) + C_{ii}(\xi)) d\xi} du \le m < 1;$$

(H_{14}) $\exp\left\{ -\int_0^t \left(b_i(\xi) + C_{ii}(\xi) \right) d\xi \right\} \to 0, \text{ as } t \to +\infty, i = 1, 2, \cdots, n.$

Then system (1.5) and (1.6) has a unique almost periodic solution which is globally attractive.

Proof. Similar to the proof of Theorem 4.1, we omit it here.

Theorem 4.3. Assume that the conditions in Theorem 3.3 (or Theorem 3.4) hold. Suppose further the following conditions hold:

(H_{15}) There is a positive constant λ such that

$$\sum_{j=1}^n \left| \frac{\rho_j}{\rho_i} f_{ij}(t) \right| + \int_0^t \left\{ \sum_{j=1}^n \left[\left| \frac{\rho_j}{\rho_i} H_{ij}^*(u) \right| + \left| \frac{\rho_j}{\rho_i} d_{ij}(u) \right| + \left| \frac{\rho_j}{\rho_i} e_{ij}(u) \right| \right] + \sum_{j=1, j\ne i}^n \left| \frac{\rho_j}{\rho_i} c_{ij}(u) \right| \right\} e^{-\int_u^t (b_i(\xi) + C_{ii}(\xi)) d\xi} du \le \lambda < 1;$$

(H_{16}) $\exp\left\{ -\int_0^t \left(b_i(\xi) + c_{ii}(\xi) \right) d\xi \right\} \to 0, \text{ as } t \to +\infty, i = 1, 2, \cdots, n.$

Then system (3.22) has a unique periodic solution (almost periodic solution) which is globally attractive, where

$$H_{ij}^*(u) = \frac{\left[f_{ij}'(u) + f_{ij}(u)\left(b_i(u) + c_{ii}(u) \right) \right] \left(\left(1 - \delta_{ij}'(u) \right) + d_{ij}(u) \delta_{ij}''(u) \right)}{\left(1 - \delta_{ij}'(u) \right)^2}.$$

Proof. Similar to the proof of Theorem 4.1, we omit it here.

5. An Example

Now, we give an example to demonstrate our result. Let us consider the following equation:

$$\begin{aligned}\frac{\mathrm{d}N(t)}{\mathrm{d}t} = N(t)\Big[&2+\cos t-\frac{1}{4}\ln N(t)-\frac{2+\sin t}{20}\ln N(t-2\pi-0.01-0.5\sin t)\\&-\frac{1-\sin t}{20}\int_{-\infty}^{t}K_j(t-s)\ln N(s)\mathrm{d}s-\frac{1}{10}\ln N(t-0.1+0.5\sin t)\Big]\end{aligned} \tag{5.1}$$

Compare with (3.22), we get $i = j = 1$, ,

$a_1(t) = 2+\cos t$, $b_1(t) = \frac{1}{4}$, $c_{11}(t) = 0$,

$d_{11}(t) = \frac{2+\sin t}{20}$, $e_{11}(t) = \frac{1-\sin t}{20}$, $f_{11}(t) = \frac{1}{10}$,

$\gamma_{11}(t) = 2\pi+0.01+0.5\sin t$, $\delta_{11}(t) = 0.1-0.5\sin t$.

So, $\gamma'_{11}(t) = 0.5\cos t < 1$, $\delta'_{11}(t) = -0.5\cos t < 1$, $\delta''_{11}(t) = 0.5\sin t$, $f_{11}(t) = 0$ and

$$\begin{aligned}q_1(t) &= d_{11}(t)+e_{11}(t)+f_{11}(t)b_1(t)+\left|f'_{11}(t)\right|\\&=\frac{2+\sin t}{20}+\frac{1-\sin t}{20}+\frac{1}{10}\times\frac{1}{4}\\&=\frac{7}{40}<b_1(t)\left(1-\left\|f_{11}(t)\right\|_1\right)=\frac{9}{40}.\end{aligned} \tag{5.2}$$

According to Corollary 3.3, we see that system (5.1) has at least one positive 2π-periodic solution.

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