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The Dispersion Method for Estimating Non-Linearity of Electro-Acoustic Systems in the Presence of Additive Noise

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Abstract

It is shown that the estimation of nonlinear distortions in the various circuits based on the measurement of the ratio of the dispersion and correlation functions does not depend on the level of additive noise acting on the input (or output) of nonlinear circuit. The proposed theoretical method is confirmed by experimental measurements.

Keywords

Nonlinear Circuits and Networks, Nonlinear Distortions, Dispersion Function, Correlation Function, Correlation Ratio

1. Introduction

Nonlinear distortion of signals occurs in all circuits of analog or digital form and especially in electro-acoustical Hi-Fi systems. Classical methods for evaluating distortion are based on the so-called harmonic factor, which is very convenient if harmonic signal affects circuit [1]. More sophisticated methods based on spectral analysis of the output signal are developed if the input signal is a wideband [2]-[5]. However, the problem of estimating the nonlinear distortions becomes much more complicated besides if a broadband signal additive noise affects the input of the nonlinear circuit. Even if the additive noise is independent, the spectrum of the output signal can contain not only higher frequency components and intermodulation components in the original input signal. In other words, all the known methods for evaluating nonlinear distortion are very sensitive to the level of additive noise and rather inaccurate. To increase the accuracy in these evaluations, various methods have been proposed, in particular methods of regularization [6], introduction of parametric feedback [7] and others.

Below we will show that, based on the methods of regression and dispersion analysis, it is possible to estimate the value of non-linear distortions which will be invariant to the level of the input additive noise. The proposed evaluation of the nonlinearity is valid for signals of any type and based on measurement of joint (two-dimensional) probability density functions of the input and output signals, which allows calculating the dispersion and regression functions and other necessary statistical parameters identifying any functional relationships of stochastic signals in inertialess circuits of any type.

2. Method

If two random processes are related by certain non-linear dependence, which can be represented as a power series (regression), the coefficient of the linear term of this series describes the degree of linear correlation between the input and output processes. The coefficients of the terms with higher powers are expressed thru statistical moments of higher orders [8]. The degree of nonlinearity of the regression equation can be derived from the general theory of correlation and one of such assessments based on the difference of the correlation ratio and cross-correlation coefficient was first obtained in [9], namely

$$n(\tau)=\sqrt{\eta_{yx}^{2}(\tau)-\rho_{yx}^{2}(\tau)} \tag{1}$$

where

$\eta_{yx}(\tau)$—correlation ratio of output process to the input process

$\rho_{yx}(\tau)$—cross-correlation coefficient of the output and input processes.

Note that the order of indices in the correlation ratio is important because this function is not symmetric, *i.e.* in general case $\eta_{xy}\neq\eta_{yx}$, The formula (1) has been applied in [9] for the study of nonlinear dynamical systems of automatic control and has been called "the degree of nonlinearity of the object." Its meaning is that it contains the difference of two functions, one of which characterizes an arbitrary correlation connectivity of stochastic processes *y*(*t*) and *x*(*t*), and the other—only their linear dependence.

Let us evaluate the applicability of (1) for measuring the non-linear distortion in the presence of additive noise. To do this it is necessary to consider the effect of additive noise on the coefficient of correlation ratio and cross-correlation coefficient. Suppose that the known signal *s*(*t*)and independent additive noise *n*(*t*) act on the input of inertialess nonlinear circuit. According [9]

$$\eta=\sqrt{\frac{\theta_{f(s+n)/s}}{D\left[f(s+n)\right]}} \tag{2}$$

where

$\theta_{f(s+n)/s}=M\left\{\left[M\left(f(s+n)/s\right)-M\left(f(s+n)\right)\right]^{2}\right\}$—non-normalized dispersion function

$D\left[f(s+n)\right]$—dispersion of signal on the output of nonlinear circuit.

M—symbol of mathematical expectation

The function *f*(*s*+*n*) can be decomposed into a McLaurin series ($Ms = Mn = 0$) and $\theta_{f(s+n)/s}$ can be written as $\theta_{f(s+n)/s}=M\left\{\left[\sum_{k=0}^{\infty}B_kM\left[(s+n)^k/s\right]-\sum_{k=0}^{\infty}B_kM(s+n)^2\right]^2\right\}$, where $B_k=\frac{f^k(0)}{k!}$.

Expanding $(s+n)^k$ by the binomial theorem and performing consistently averaging, squaring and another averaging, we can obtain that

$$\theta_{f(s+n)/s}=M\left\{\left[\sum_{k=0}^{\infty}B_ks^k\right]^2\right\}-\left\{M\sum_{k=0}^{\infty}B_ks^k\right\}^2=D\left[f(s)\right] \tag{3}$$

From (3) it follows that the non-normalized dispersion function is independent of the additive noise. Cross correlation coefficient between input and output processes is equal to, [5]

$$\rho_{yx}=\frac{f(\sigma_s)\rho_{ss}(\tau)}{\sqrt{D\left[f(s+n)\right]}} \tag{4}$$

where $f(\sigma_s)$ is a factor characterizing the reduction of cross-correlation coefficient because of the nonlinear transformation, and

$$f(\sigma_s)=1,\ \text{if}\ M\left[f(\sigma_s)-a\sigma_s\right]^2\le\varepsilon$$

$$f(\sigma_s)<1,\ \text{if}\ M\left[f(\sigma_s)-a\sigma_s\right]^2>\varepsilon$$

where σ_s^2—dispersion of input process, ε—a given measure of approximation error, ρ_{ss}—autocorrelation function of $s(t)$. By substituting (3) and (4) to (1)

$$n(\tau)=\sqrt{\frac{D^2\left[f(s)\right]-f^2(\sigma_s)\rho_{ss}^2(\tau)}{D\left[f(s+n)\right]}} \tag{5}$$

Equation (5) shows that the degree of nonlinearity [4] depends on the additive noise and therefore formula (1) is not suitable for decision of our task. Taking into account the properties of the correlation ratio and the coefficient of cross-correlation another evaluation of nonlinear distortion can be offered, namely

$$m(\tau)=1-\frac{\left|\rho_{yx}(\tau)\right|}{\eta_{yx}(\tau)} \tag{6}$$

It follows from (6) that this relative evaluation of non linearity is invariant to noise and has limits similar to limits of $\eta(\tau)$; *i.e.* if nonlinearity is absent then $m_{\min}=0$ and vice versa $m_{\max}=1$ if nonlinear distortions are maximal; in this case $\left|\rho_{yx}(\tau)\right|=0$. In private case if processes $x(t)$ and $y(t)$ are functionally related to each other $\eta_{yx}=1$ and formula (6) can be presented as

$$m^*(\tau)=1-\left|\rho_{yx}(\tau)\right| \tag{7}$$

Because the equality (7) is true only at functional connection of processes $x(t)$ and $y(t)$, the estimate of distortion is reduced to the measurement of cross-correlation coefficient and is substantially similar to the previously proposed methods [4].

Let's discuss how to measure dispersion function and correlation ratio. The simplest way is to measure two-dimensional probability density function $W(x, y)$ of input and output signals and having the estimation of $W(x, y)$ then to calculate all mentioned statistical parameters.

As an illustration the examples of two-dimensional probability density functions for some types of circuits and signals are shown in **Figure 1(a)**, **Figure 1(b)**.

Each section of $W(x, y)$ presents conditional probability density function $W(x/y)$ and it is clearly seen that maxima of these sections, which correspond to the conditional mathematical expectations, are located along a certain curve which is the regression line. Note that for quadratic circuit cross correlation coefficient $\rho_{x,yy}$ is equal to zero, though the random processes $Y(t)$ and $S(t)=X(t)+N(t)$ are connected functionally, not statistically. The distribution on **Figure 1(b)** is $W(y,x)$ for linear circuit and input signal as sum of quasi-harmonic signal and additive Gaussian noise. This distribution has typical "two peaks" form. Having the measured evaluation of $W^*(y,x)$ all necessary statistical parameters can be calculated using standard formulas [8] [9].

(a) (b)

Figure 1. (a) Two-dimensional probability density function $W(x, y)$ for quadratic nonlinearity; (b) Two-dimensional probability density function $W(x, y)$ of quasi-harmonic signal and undependable Gaussian noise, circuit is linear.

3. Numerical Example

Let's demonstrate the application of introduced evaluation of non-linear distortion for the circuit with nonlinearity of the form

$$y = f(x) = \exp(\alpha x), \quad -\infty < x < +\infty \tag{8}$$

This type of nonlinearity is typical for many analog electronic circuits and networks. Let's propose that input random process $x(t)$ affects the input of this nonlinear circuit

$$x(t) = s(t) + n(t) \tag{9}$$

where $s(t)$ and $n(t)$ are independent and centered Gaussian signals with variances σ_s^2 and σ_n^2, $s(t)$ is a useful known signal and $n(t)$ is noise. Now let's derive an analytical expression for the proposed evaluation of nonlinearity.

Not normalized cross-correlation function is equal to

$$B_{sy}(\tau) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} s \cdot f(x) W(s, x, \tau) \mathrm{d}s\mathrm{d}x \tag{10}$$

where $W(s,x,\tau)$ is the joint probability density function of $s(t)$ and $n(t)$. In our case $W(s,n) = W(s) * W(n)$.

Introducing a new variable $x = s + n$, we can write

$$W(s, x) = D\exp\left(-as^2 + bsx - cx^2\right) \tag{11}$$

where

$$D = \frac{1}{2}\pi\sigma_s\sigma_n,\ a = \frac{1}{2}\sigma_\xi^2,\ b = \frac{1}{2}\sigma_s^2,\ c = \frac{1}{2}\sigma_n^2,\ \sigma_\xi^2 = \frac{\sigma_n^2\sigma_s^2}{\sigma_s^2 + \sigma_n^2}$$

Substituting Formula (11) into Formula (10) and integrating over x and s after algebraic transformations we obtain that

$$B_{sy}(\tau) = \alpha\sigma_s^2 \exp\left[\frac{\alpha^2}{2}\left(\sigma_s^2 + \sigma_n^2\right)\right] \cdot \rho_{ss}(\tau) \tag{12}$$

Mathematical expectation is equal to

$$M[y] = \int_{-\infty}^{\infty} f(x) W(x) \mathrm{d}x = \exp\left[\frac{\alpha^2}{2}\left(\sigma_s^2 + \sigma_n^2\right)\right] \tag{13}$$

and the second initial moment is equal to

$$M_2[y] = \int_{-\infty}^{\infty} f^2(x) W(x) \mathrm{d}x = \exp\left[2\alpha^2\left(\sigma_s^2 + \sigma_n^2\right)\right] \tag{14}$$

From expressions (13) and (14) the variance can be written as

$$D[y] = \exp\left[\alpha^2\left(\sigma_s^2 + \sigma_n^2\right)\right]\left\{\exp\left[\alpha^2\left(\sigma_s^2 + \sigma_n^2\right)\right] - 1\right\} \tag{15}$$

Therefore cross correlation function of input and output signals will be written as

$$\rho_{sy}(\tau) = \frac{\alpha\rho_{ss}(\tau)\sigma_s}{\sqrt{\exp\left[\alpha^2\left(\sigma_y^2 + \sigma_n^2\right)\right] - 1}} \tag{16}$$

Acting similarly we obtain an expression for the correlation ratio

$$\eta_{ys}(\tau) = \frac{\sqrt{\exp\left[\alpha^2\sigma_{ss}^2(\tau)\right] - 1}}{\sqrt{\exp\left[\alpha^2\left(\sigma_s^2 + \sigma_n^2\right)\right] - 1}} \tag{17}$$

Substituting (16) and (17) into the expression (7)

$$m^*(\tau) = \frac{\alpha\sigma_s\rho_{ss}(\tau)}{\sqrt{\exp\left[\alpha^2\sigma_s^2\rho_{ss}^2(\tau)\right]-1}} \tag{18}$$

And finally according (6) we will have

$$m(\tau) = 1 - \frac{\alpha\sigma_s\rho_{ss}(\tau)}{\sqrt{\exp\left[\alpha^2\sigma_s^2\rho_{ss}^2(\tau)\right]-1}} \tag{19}$$

From the last expression it follows that the assessment of non-linearity does not depend on the noise variance.

4. Experimental Check

Proposed method has been tested experimentally on a standard amplifier circuit according to the scheme shown in **Figure 2**.

Through-amplitude characteristic of the nonlinear circuit (6) is shown in **Figure 3**. Analog circuit had dynamic range about 75 dB and 16 bit ADC was used for digitizing. The value of non-linear distortion of the circuit measured by the standard method of harmonics does not exceed 20%. All calculations have been performed in PC.

Typical example of input harmonic signal and output distorted signal with additive noise is presented in **Figure 4**.

The form of function *W*(*x*, *y*) for circuit with amplitude characteristic (8) is shown on **Figure 5**. Each section of *W*(*x*, *y*) presents conditional probability density function *W*(*x*/*y*) and it is clearly seen that maxima of these sections, which correspond to the conditional mathematical expectations (regression line), are located along a

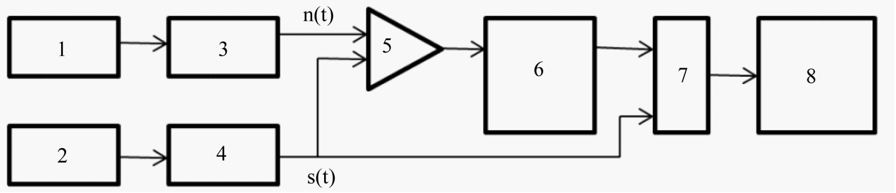

Figure 2. Scheme of experiments. 1, 2 Two generators of noise, 3, 4 Similar filters. 5. Adder, 6 Nonlinear circuit, 7. ADC, 8. PC.

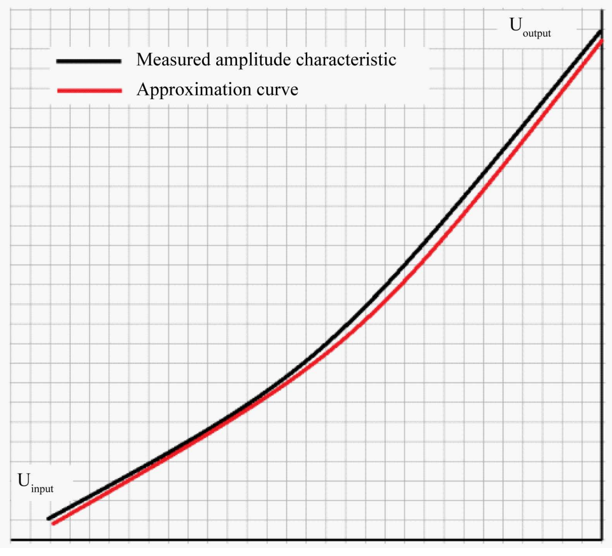

Figure 3. Measured amplitude characteristic of circuit and its approximation by function $y = exp(\alpha x)$.

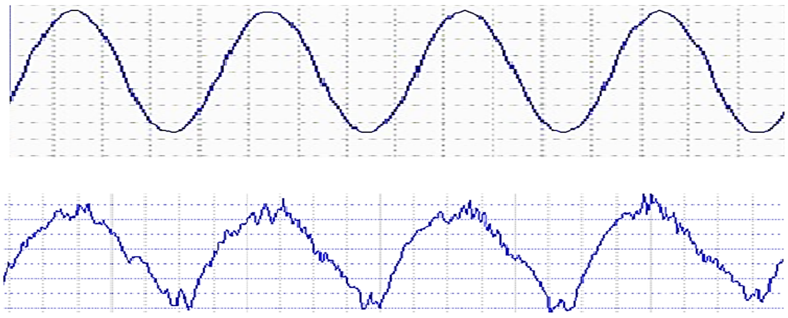

Figure 4. Input harmonic signal and distorted output signal with additive noise.

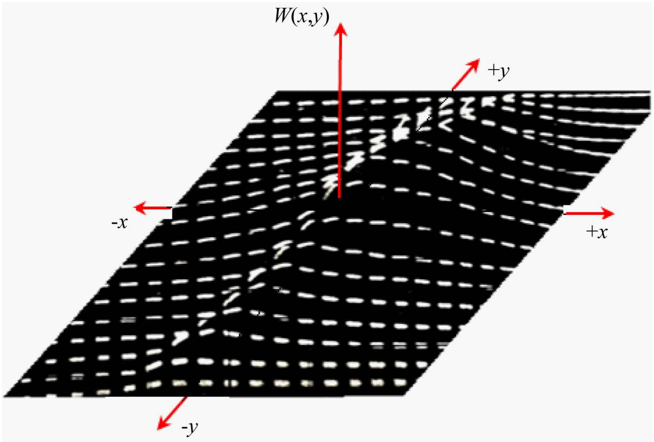

Figure 5. Two-dimensional probability density function $W(x, y)$ of input and output signals for circuit nonlinearity (8).

certain curve, the form of which is determined by the type of circuit nonlinearity (8).

The results of measurements of cross-correlation coefficients and correlation ratio are presented in **Figure 6**. The curves (1-6) are the results of calculation of cross-correlation coefficient for different values of signal/noise ratio. Curve 7 is calculated on the base of the formula (18). In this formula, the coefficient α, which characterizes the angle of the approximating curve, was chosen 1.1. Autocorrelation coefficient of the input signal is equal to one. Experimental results (green dots) are presented with confidence intervals with probability 0.68, which evaluation was performed on 10 independent measurements.

Influence of nonlinearity is clear from the behavior of the curve 1 in the absence of additive noise: with an increase of the level of input signal cross-correlation coefficient drops to 0.4. Impact of noise changes the behavior of cross-correlation coefficient. It becomes dependent both on the level of the additive noise and the degree of nonlinearity. These curves have extrema, whose position depends on the noise level and the degree of nonlinearity. Therefore in the presence of noise cross-correlation coefficient gives incorrect evaluation of nonlinearity.

The most interesting is the behavior of the curve 7, calculated by formula (18) and this curve is in the inverse to curve 1, which means that measured degree of nonlinearity does not depend on the level of additive noise.

5. Conclusion

A comparison of theoretical calculations with experimental data leads to the conclusion about the possibility of the suggested evaluation of nonlinear distortion of networks in the presence of additive noise. Nevertheless a more rigorous mathematical analysis shows that the variance of this evaluation depends on the level of additive noise, although this dependence is weak for input signal/noise ratio more than 10 - 12 dB. In other words, the accuracy of estimation of nonlinear distortions depends on the level of additive noise. More detail analysis of accuracy of proposed method will be discussed in a separate article.

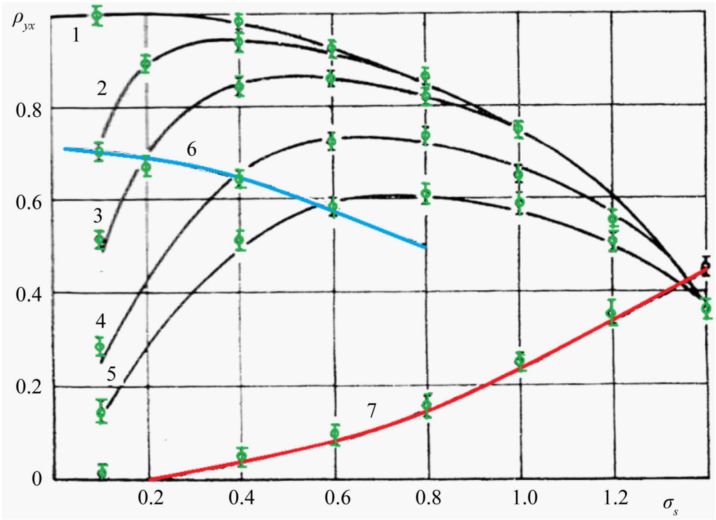

Figure 6. Comparison of calculated and measured data. Solid curves (1-5)—calculated dependencies of cross-correlation functions ρ_{yx}, formula (16) at different values of noise $\sigma_n = 0$; 0.1; 0.2; 0.4; 0.6. Curve 6 (blue) is ρ_{yx} for $\sigma_s = \sigma_n$. Solid curve 7 (red) is $m^*(\tau)$ calculated by formula (18). Green dots with confidence intervals are experimental data.

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