

Peculiarities of Sound Wave in Moving Medium

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Abstract

Generalized equation for linear gravity waves in moving medium has been obtained. Sound wave is considered as a particular case and it is shown that in inhomogeneous medium at rest it is propagated in full concordance with the Doppler law and principle of motion relativity, *i.e.* these laws are invariant with reference to properties of medium (homogeneity or inhomogeneity). In moving medium they are fair only in the case of its homogeneity. In strongly inhomogeneous moving medium, propagation of sound is absolutely impossible.

Keywords

Speed of Sound, Atmosphere, Homogeneous Medium, Inhomogeneous Medium, Doppler's Effect, Principle of Relativity of Motion

1. Introduction

As is known, the law of conservation of mass being one of the fundamental laws of physics was formulated for homogeneous medium. However it is still considered universal and up to now has been applied to inhomogeneous medium like the Earth atmosphere [1]-[5] or the ocean [5]-[8]. In recent works [9] [10] it was demonstrated that allowing for the effect of the gravitational field of the Earth this equation needs to be perfected. It resulted in improvement of great number of known gas or hydrodynamic relations as well as to new understanding of the most important properties of medium like compressibility and incompressibility. From here on there has arisen logical necessity of verification of applicability to inhomogeneous medium of fundamental laws of physics like Doppler's effect and principle of relativity of motion. This theoretical work is dedicated exactly to this problem. In the second section, the equation of linear gravitational wave for moving medium is derived from which dispersive equation for sound wave is obtained as a particular case. Through analysis of this equation in the third section it is shown that if the medium is stationary, the properties of the medium have no influence on sound

propagation which occurs in full concordance with the Doppler's law and the relativity principle of motion. In the case of fixed source and moving medium the properties of medium play a certain role. In homogeneous medium everything remains as before. However in inhomogeneous medium, where the role of isobaric speed of sound is revealed, propagation of sound wave is impossible which can be explained by mechanism of propagation of isobaric sound.

2. Generalized Equation of Linear Gravitational Wave in Inhomogeneous Moving Medium

We deem it expedient to explain to the reader that under the term "gravity wave" we understand mechanical waves generated and propagated in inhomogeneous medium being under the influence of force gravitational field of the Earth. Therefore, these waves have nothing in common with the waves generated as a result of perturbation of gravitational field and can be propagated in vacuum. The equation for linear gravity wave in immovable inhomogeneous medium was first obtained in the paper [9] where it was shown that exactly this equation describes propagation of sound in the Earth atmosphere correctly. Thus, the generally received opinion that the gravitational field does not have any impact on the sound wave was invalidated. In the paper [11] it was shown that application of this equation to the surface gravitational waves is possible only in case of capillary waves and definition of phase speed of common surface gravitational wave, much less the Tsunami wave from the linear theory is incorrect. In this section we will obtain analogous equation for moving inhomogeneous medium.

Let's consider the generalized system of hydrodynamic equations for inhomogeneous medium in gravitational field of the Earth [9]

$$\begin{cases} \rho \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \nabla) \mathbf{V} \right] = -\nabla P + \rho \mathbf{g} \\ \frac{\partial \rho}{\partial t} + (\mathbf{V} \nabla) \rho = -\rho \nabla \mathbf{V} - \frac{\mathbf{V} \nabla P}{C_p^2} \end{cases} \quad (2.1)$$

Here P , ρ , and \mathbf{V} are pressure, density and speed of fluid, respectively, C_p is isobaric speed of sound.

Assume that the medium is moving with constant speed \mathbf{V}_0 in direction of wave propagation and let's present all its variables in the form of the sum of their stationary and disturbed values, *i.e.*

$$P = P_0 + P', \quad \mathbf{V} = \mathbf{V}_0 + \mathbf{V}', \quad \rho = \rho_0 + \rho' \quad (2.2)$$

Then, linearized system of Equations (2.1) will take the form of

$$\begin{cases} \rho_0 \left[\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}_0 \nabla) \mathbf{V} \right] = -\nabla P + \frac{\mathbf{g}}{C^2} P \\ \frac{1}{C^2} \frac{\partial P}{\partial t} + (\mathbf{V}_0 + \mathbf{V}) \nabla \rho_0 + \frac{\mathbf{V}_0}{C^2} \nabla P = -\rho_0 \nabla \mathbf{V} - \frac{(\mathbf{V}_0 + \mathbf{V}) \rho_0 \mathbf{g} + \mathbf{V}_0 \nabla P}{\tilde{C}_p^2} \end{cases} \quad (2.3)$$

The following equations are used here:

Condition of equilibrium of fluid in gravitational field of the Earth

$$\nabla P_0 = \rho_0 \mathbf{g} \quad (2.4)$$

The equation of state of the liquid

$$\rho = \frac{P}{\tilde{C}^2} \quad (2.5)$$

and marks of disturbed values are omitted. \tilde{C}^2 is the generalized meaning of squared velocity of sound towards moving medium $\tilde{C}^2 = (C - V_0)^2$, which is reduced from squares of adiabatic $\tilde{C}_s = C_s - V_0$ and isobaric $\tilde{C}_p = C_p - V_0$ speeds of sound in respect of the moving medium [12]

$$\tilde{C}^2 = \frac{\tilde{C}_s^2 \tilde{C}_p^2}{\tilde{C}_s^2 + \tilde{C}_p^2} \quad (2.6)$$

Here

$$C_s = \left[\left(\frac{\partial P_0}{\partial \rho_0} \right)_s \right]^{1/2} \quad (2.7)$$

$$C_p = (c_p/T)^{1/2} \frac{\rho_0}{(\partial \rho_0 / \partial T)_p} \quad (2.8)$$

are relevant speeds of sound in respect of immovable medium characterizing degree of its homogeneity or inhomogeneity [10]. c_p is the specific thermal capacity of medium at constant pressure and T is absolute temperature.

Let's apply operator ∇ to the first equation and operator $\partial/\partial t$ to the second one and then we will get:

$$\begin{cases} \nabla \left(\rho_0 \frac{\partial \mathbf{V}}{\partial t} \right) + \Delta P - \mathbf{g} \frac{\tilde{C}^2 \nabla P - 2\tilde{C} P \nabla \tilde{C}}{\tilde{C}^4} = -\nabla [\rho_0 (\mathbf{V}_0 \nabla) \mathbf{V}] \\ \frac{1}{\tilde{C}^2} \frac{\partial^2 P}{\partial t^2} + \nabla \left(\rho_0 \frac{\partial \mathbf{V}}{\partial t} \right) + \frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} \frac{\partial \mathbf{V}}{\partial t} = -\mathbf{V}_0 \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) \nabla P \end{cases} \quad (2.9)$$

Having expressed $\nabla [\rho_0 (\partial \mathbf{V} / \partial t)]$ from the first equation of system (2.9) and having substituted in the second one, we will get

$$\frac{1}{\tilde{C}^2} \frac{\partial^2 P}{\partial t^2} + \frac{\mathbf{g}}{\tilde{C}^2} \left(\nabla P - \frac{2\nabla \tilde{C}}{\tilde{C}} P \right) - \Delta P + \frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} \frac{\partial \mathbf{V}}{\partial t} = \nabla [\rho_0 (\mathbf{V}_0 \nabla) \mathbf{V}] - \mathbf{V}_0 \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) \frac{\partial}{\partial t} \nabla P \quad (2.10)$$

From the first equation of the system (2.1) we find that

$$\rho_0 \frac{\partial \mathbf{V}}{\partial t} = -\nabla P + \frac{\mathbf{g}}{\tilde{C}^2} P - \rho_0 (\mathbf{V}_0 \nabla) \mathbf{V} \quad (2.11)$$

Substituting (2.11) in (2.10) we obtain

$$\begin{aligned} & \frac{1}{\tilde{C}^2} \frac{\partial^2 P}{\partial t^2} + \mathbf{g} \left(\frac{1}{\tilde{C}^2} - \frac{1}{\tilde{C}_p^2} \right) \nabla P - \frac{\mathbf{g}}{\tilde{C}^2} \left(\frac{2\nabla \tilde{C}}{\tilde{C}} - \frac{\mathbf{g}}{\tilde{C}_p^2} \right) P - \Delta P \\ & = \frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} (\mathbf{V}_0 \nabla) \mathbf{V} + \nabla [\rho_0 (\mathbf{V}_0 \nabla) \mathbf{V}] - \mathbf{V}_0 \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) \frac{\partial}{\partial t} \nabla P \end{aligned} \quad (2.12)$$

Having taken into consideration that $1/\tilde{C}^2 - 1/\tilde{C}_p^2 = 1/\tilde{C}_s^2$ and ignoring the third term in the left part of Equation (2.12) due to its obvious smallness, we will finally get

$$\frac{1}{\tilde{C}^2} \frac{\partial^2 P}{\partial t^2} + \frac{\mathbf{g}}{\tilde{C}_s^2} \nabla P - \Delta P = \frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} (\mathbf{V}_0 \nabla) \mathbf{V} + \nabla [\rho_0 (\mathbf{V}_0 \nabla) \mathbf{V}] - \mathbf{V}_0 \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) \frac{\partial}{\partial t} \nabla P \quad (2.13)$$

Let's assume that $\mathbf{V}_0 = V_0 \mathbf{e}_x$, where $V_0 > 0$, \mathbf{e}_x is the unit vector directed along the axis X and $\rho_0 = \rho_0(z)$. Then it is easy to check, that

$$\frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} (\mathbf{V}_0 \nabla) \mathbf{V} = -\frac{\rho_0 V_0 \mathbf{g}}{\tilde{C}_p^2} \frac{\partial V_z}{\partial x} \quad (2.14)$$

$$\nabla [\rho_0 (\mathbf{V}_0 \nabla) \mathbf{V}] = V_0 \left[\rho_0 \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial}{\partial z} \left(\rho_0 \frac{\partial V_z}{\partial x} \right) \right] \quad (2.15)$$

Substituting both (2.14) and (2.15) into (2.13), we will obtain the following expression:

$$\frac{1}{\tilde{C}^2} \frac{\partial^2 P}{\partial t^2} - \frac{\mathbf{g}}{\tilde{C}_s^2} \frac{\partial P}{\partial z} - \Delta P = V_0 \left[\rho_0 \frac{\partial^2 V_x}{\partial x^2} + \frac{\partial}{\partial z} \left(\rho_0 \frac{\partial V_z}{\partial x} \right) - \frac{\rho_0 \mathbf{g}}{\tilde{C}_p^2} \frac{\partial V_z}{\partial x} - \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) \frac{\partial^2 P}{\partial x \partial t} \right] \quad (2.16)$$

Let's find all variables in the form of plane wave $f(\mathbf{r}, t) = f_a(z) \exp[i(kx - \omega t)]$ where $f_a(z)$ is the amplitude. Then, we will obtain from (2.16) the following

$$\begin{aligned} & \frac{\omega^2}{\tilde{C}^2} P_a(z) + \frac{g}{\tilde{C}_s^2} \frac{dP_a(z)}{dz} + \frac{d^2 P_a(z)}{dz^2} - k^2 P_a(z) \\ & = V_0 \left[k^2 \rho_0 V_{ax}(z) - ik \frac{d}{dz} (\rho_0 V_{az}(z)) + ik \frac{\rho_0 g}{\tilde{C}_p^2} V_{az}(z) + \omega k \left(\frac{1}{\tilde{C}^2} + \frac{1}{\tilde{C}_p^2} \right) P_a(z) \right] \end{aligned} \quad (2.17)$$

For x and z components of the first Equation of system (2.1) we will easily find

$$\rho_0 V_{ax}(z) = \frac{k P_a(z)}{\omega - k V_0} \quad (2.18)$$

$$\rho_0 V_{az}(z) = -\frac{i}{\omega - k V_0} \left[\frac{dP_a(z)}{dz} + \frac{g}{\tilde{C}^2} P_a(z) \right] \quad (2.19)$$

Differentiating Equation (2.19) with respect to z we will get

$$\frac{d}{dz} [\rho_0 V_{az}(z)] = -\frac{i}{\omega - k V_0} \left[\frac{d^2 P_a(z)}{dz^2} + \frac{g}{\tilde{C}^2} \frac{dP_a(z)}{dz} - \frac{2g}{\tilde{C}^3} \frac{d\tilde{C}(z)}{dz} P_a(z) \right] \quad (2.20)$$

Having substituted (2.18), (2.19) and (2.20) in (2.17) and having ignored the same summand as in (2.12), we will obtain equation for amplitude of linear gravitational wave in moving inhomogeneous medium in the form of

$$\frac{d^2 P_a(z)}{dz^2} + \frac{g}{\tilde{C}_s^2} \frac{dP_a(z)}{dz} + \left[\frac{(\omega - k V_0)^2}{\tilde{C}^2} - \frac{k V_0 (\omega - k V_0)}{\tilde{C}_p^2} - k^2 \right] P_a(z) = 0 \quad (2.21)$$

3. Influence of Properties of Moving Medium on Propagation of Sound Wave

For the sake of simplicity let's consider sound wave when $P_a(z) = \text{const}$. In this case the dispersive Equation (2.21) takes the form of

$$\frac{(\omega - k V_0)^2}{\tilde{C}^2} - \frac{k V_0 (\omega - k V_0)}{\tilde{C}_p^2} - k^2 = 0 \quad (3.1)$$

When $V_0 = 0$ Equation (3.1) results in

$$\omega^2 = k^2 \tilde{C}^2 = k^2 C^2 \quad (3.2)$$

and thus homogeneity (compressibility, $(C = C_s)$) or in homogeneity (incompressibility, $(C = C_p)$) of the medium at rest have no influence on propagation of sound wave. If medium is immovable and the source of sound is moving towards the receiver with speed of U_0 , then, as a result of change of length of the wave $k \rightarrow \tilde{k} = \omega / (C - U_0)$, the wave frequency perceived by the receiver will equal to

$$\tilde{\omega} = \tilde{k} C = \frac{\omega}{1 - U_0 / C} \quad (3.3)$$

i.e. frequency shift in any medium occurs according to Doppler's law [13]. Substitution of U_0 by $-U_0$ in (3.3) corresponds to motion of the source from the receiver. Consequently, Doppler's law and the principle of motion relativity for sound waves are invariant in respect of properties of immovable medium.

When medium is moving (fixed source and receiver moving with medium) the picture essentially changes. From (3.1) it is clear that in moving inhomogeneous medium frequency shift occurs according to the law different for Doppler's law.

$$(\omega - kV_0) \left[\omega - kV_0 \left(1 + \frac{\tilde{C}^2}{\tilde{C}_p^2} \right) \right] = k^2 \tilde{C}^2 \quad (3.4)$$

Equation (3.4) is fair for adiabatic medium representing the troposphere [9] [12], where speed of the wind is $V_0 \leq 30$ m/sec and since $V_0 \ll C_s \leq C_p$, the following inequality is fair

$$\frac{\tilde{C}}{\tilde{C}_p} \approx \frac{C}{C_p} = \frac{1}{\sqrt{1 + C_p^2/C_s^2}} \leq 1 \quad (3.5)$$

Equality sign in (3.5) is fair only for strongly inhomogeneous medium, when $C_s \gg C_p$ and $C \approx C_p$. When $C_s \ll C_p$ and $C \approx C_s$, relation $\tilde{C}^2/\tilde{C}_p^2$ in (3.4) can be ignored and then we will have

$$\tilde{\omega}^2 = (\omega - kV_0)^2 = \omega^2 (1 - V_0/C_s)^2 = k^2 (C_s - V_0)^2 = k^2 \tilde{C}_s^2 \quad (3.6)$$

As we can see, in weakly inhomogeneous medium frequency shift occurs according to Doppler's law when the receiver moves in the direction towards the source and the wave length does not change. Substitution in (3.6) of V_0 by $-V_0$, which is equivalent to the case when the receiver moves in direction to the source, results in the following

$$\tilde{\omega}^2 = (\omega + kV_0)^2 = \omega^2 (1 + V_0/C_s)^2 = k^2 (C_s + V_0)^2 = k^2 \tilde{C}_s^2 \quad (3.7)$$

Thus, for weakly inhomogeneous moving medium ($C_s \ll C_p$) Doppler's law and the principle of relativity of motion are fair. This case is realized in the Earth atmosphere for instance up to the altitude of approximately $z = 3$ km, where $C_s \approx 328.6$ m/sec and $C_p \approx 1355.3$ m/sec [12] and consequently $C^2/C_p^2 \approx C_s^2/C_p^2 \approx 0.006$. At the upper boundary of troposphere, where $C_p = C_s$ [9] [12] and $C^2 = 0.5C_p^2$ the Equation (3.4) results in

$$(\omega - kV_0)(\omega - 0.5kV_0) = k^2 (C - V_0)^2 \quad (3.8)$$

While for strongly inhomogeneous media we have

$$(\omega - kV_0)(\omega - 2kV_0) = k^2 (C - V_0)^2 \quad (3.9)$$

We see that equality of (3.8) and (3.9) describes sound wave only when $V_0 = 0$. The said means that in moving inhomogeneous medium, where isobaric speed of sound is at least to some extent significant, propagation of sound is impossible. This is explained by the mechanism of propagation of isobaric sound whereby perturbation of density (sound) is spread as a result of change of the volume of specified mass of liquid or gas. During motion of inhomogeneous medium the concept of volume of specified mass loses its meaning. This is evidenced by the formula for change of the mass of liquid (gas) in inhomogeneous medium obtained in the work [10]

$$\frac{\partial m}{\partial t} = -\oint \rho \mathbf{V}_0 d\mathbf{f} - \frac{1}{C_p^2} \int \mathbf{V}_0 \nabla p dV \quad (3.10)$$

For incompressible (strongly inhomogeneous) medium surface integral in (3.10) equals to zero identically, *i.e.* even when $V_0 \neq 0$. Therefore, mass constancy in given volume in strongly inhomogeneous medium can be considered only when $V_0 = 0$.

4. Conclusion

The obtained theoretical results give an evidence that the fundamental laws of physics like frequency shift law of Doppler and the relativity principle of motion are applicable to sound waves in inhomogeneous immovable medium like the Earth atmosphere. Thus it is necessary the sound is to be propagated in horizontal direction. In case of wind these laws are fair up to the height above which in homogeneity effect becomes noticeable. Impossibility of propagation of sound wave in moving inhomogeneous medium proves existence of isobaric mechanism of generation and propagation of sound. Experimental verification of this result will greatly contribute to exploration of the Earth atmosphere as well as to many applied tasks related to air navigation, alpinism and mil-

itary actions in high-mountain conditions.

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