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Neutron Pairing Energy of Finite Nuclei

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Abstract

Neutron pairing energies of some of the finite nuclei have been calculated to understand how the neutron pairing changes as the neutron and proton numbers change from odd to even values in isotopes. How its value changes for even (*N*) - even (*Z*), even (*N*) - odd (*Z*), odd (*N*) - even (*Z*) and odd (*N*) - odd (*Z*) nuclei has been brought out. The values of pairing energies (P_n) have been calculated for light nuclei ($20 \le A \le 55$), medium nuclei ($100 \le A \le 140$) and heavy nuclei ($190 \le A \le 238$) and found to lie between: $-16 \le P_n \le +14$ MeV, $-6 \le P_n \le +6$ MeV and $-5 \le P_n \le +5$ MeV respectively. Positive pairing energies only occurred in odd (*N*) - even (*Z*) nuclei and this indicated the most stable isotopic nuclei.

Subject Areas

Applied Physics, Mechanical Engineering

Keywords

Neutron Pairing

1. Introduction

One of the purposes of nuclear physics is to investigate the interaction between the nucleons in the nucleus. This will give insight into the strong nuclear forces and pairing among the nucleons. In this study, the neutron pairing energy for odd-odd (o-o), even-even (e-e), even-odd (e-o) and odd-even (o-e) nuclei has been explored.

Nuclear forces act between pairs of nucleons only, the presence of other nucleons nearby does not influence the force law between any given two nucleons. The force is always attractive and independent of the type of nucleons interacting with each other. The nuclear binding energies and forces are on the order of a million times greater than electron binding energies of light atoms like hydrogen. Nuclear force is a close-range force (its strongly attractive at a distance of 1.0 fm and becomes extremely small beyond a distance of 2.5 fm), and virtually no effect of this force is observed outside the nucleus [1] [2]. Binding energy of the nucleus is given by the Einstein's relation:

$$B = \Delta c^2 \tag{1}$$

where Δ is the mass defect discovered in 1905 by Albert Einstein and it's the difference between the mass of an object and the sum of the masses of its constituent particles [3].

In the periodic table of elements, the series of light elements from hydrogen up to sodium is observed to exhibit generally increasing binding energy per nucleon as atomic masses increases. This increase is generated by increasing forces per nucleon in the nucleus, as each additional nucleon is attracted by other nearby nucleons, and thus more tightly bound to the whole. At the peak of binding is nickel and iron nuclei which is the most stable and abundant [4].

The separation energy $S_a(X)$ is the energy necessary to remove a particle a to infinity from nucleus X in its ground state, leaving residual nucleus Y also in its ground state *i.e.*

$$X = Y + a \tag{2}$$

The particle could be a neutron or a proton. The separation energy in terms of masses can be written as:

$$S_a(X) = m_X c^2 = (m_Y + m_a)c^2 \tag{3}$$

In light nuclei the number of protons and neutrons is the same $(N = Z)$ but the number of neutrons increases fast as go up to balance the repulsive force between positively charges protons so that the nucleus remains a bound system. When the number of neutrons becomes larger than protons $(N > Z)$ then neutron excess parameter is given by:

$$\eta = N - Z/A \tag{4}$$

This study investigates the neutron pairing energy due to separation energy of neutrons based on if the number of neutron to proton in nuclei is e-e, o-o, o-e or e-o.

2. Theoretical Derivations

The pairing energy is estimated as the difference between the binding energy of the second neutron N (proton Z) minus that of the first neutron N (proton Z) in the case when two neutrons (protons) are successively added to the nucleus with even (neutron) or (proton) number [5]. So far it is known that for the even (N) - even (Z) nucleus in general, the pairing energy depends on the kind of particles that compose the pair, and the state occupied by the pair; and for odd—A nucleus it is smaller compared to the even—even nucleus by a factor of 1/2 to 2/3 when the pair is in the same shell and the mass number A values very close to one another [6]. The difference between two kinds of particles pairing energy may depend on the character of the neutron-neutron, neutron-proton, or the so-called nucleon-nucleon potential.

We now derive an expression for the pairing energy of the neutron pair, denoted by: $P_n(A,Z)$ in terms of the separation energy S_n of the neutron and the binding energy $B(A,Z)$ of the nucleus. Consider a nucleus $X(A+1,Z)$ and its separation energy for the neutron can be written as $S_n(A+1,Z)$ *i.e.*

$$S_n(A+1,Z) = B(A+1,Z) - B(A,Z) \tag{5}$$

Similarly the separation energies $S_n(A,Z)$ and $S_n(A-1,Z)$ can be written as,

$$S_n(A,Z) = B(A,Z) - B(A-1,Z) \tag{6}$$

$$S_n(A-1,Z) = B(A-1,Z) - B(A-2,Z) \tag{7}$$

Now, by definition, the pairing energy $P_n(A,Z)$ of the neutron pair can be written as,

$$P_n(A,Z) = S_n(A+1,Z) - 2S_n(A,Z) + S_n(A-1,Z) \tag{8}$$

Equations (5)-(7) in Equation (8) gives

$$P_n(A,Z) = B(A+1,Z) - 3B(A,Z) + 3B(A-1,Z) - B(A-2,Z) \tag{9}$$

Equation (9) is used to calculate $P_n(A,Z)$ for different isotopes.

3. Results

Tables 1-3 are obtained by using Equation (9) and the values of binding energies were obtained from [7] [8] [9].

Table 1. Neutron pairing energy for $20 \le A \le 55$.

Z	Nucleus	A	N	P_n (MeV)	Type of Pairing (N - Z)
10	Ne	20	10	−15.333	e − e
10	Ne	21	11	+13.706	o − e
10	Ne	22	12	−8.767	e − e
11	Na	23	12	−6.811	e − o
12	Mg	24	12	−12.587	e − e
12	Mg	25	13	+12.963	o − e
12	Mg	26	14	−8.412	e − e
13	Al	27	14	−7.025	e − o
14	Si	28	14	−12.570	e − e
14	Si	29	15	+10.842	o − e
14	Si	30	16	−6.158	e − e
15	P	31	16	−5.367	e − o
16	S	32	16	−8.392	e − e
16	S	33	17	+9.178	o − e

Continued

16	S	34	18	−7.207	e − e
16	S	36	20	−8.489	e − e
17	Cl	35	18	−5.202	e − o
17	Cl	37	20	−5.934	e − o
18	Ar	36	18	−8.984	e − e
18	Ar	58	20	−8.291	e − e
18	Ar	40	22	−7.055	e − e
19	K	39	20	−6.936	e − o
19	K	40	21	+7.585	o − o
19	K	41	22	−5.075	e − o
20	Ca	40	20	−9.612	e − e
20	Ca	42	22	−7.665	e − e
20	Ca	43	23	+16.746	o − e
20	Ca	44	24	−6.914	e − e
20	Ca	46	26	−6.106	e − e
21	Sc	45	24	−1.194	e − o
21	Ti	46	24	−7.965	e − e
22	Ti	47	25	+7.054	o − e
22	Ti	48	26	−6.238	e − e
22	Ti	49	27	+6.281	o − e
22	Ti	50	28	−7.419	e − e
23	V	50	27	+3.938	o − o
23	V	51	28	−5.452	e − o
24	Cr	50	26	−6.158	e − e
24	Cr	52	28	−6.879	e − e
24	Cr	53	27	+1.727	o − e
24	Cr	54	30	−5.253	e − e

Table 2. Neutron pairing energy for $100 \le A \le 140$.

Z	Nucleus	A	N	P_n (MeV)	Type of Pairing
44	Ru	100	50	−5.073	e − e
44	Ru	101	57	+5.290	o − e
44	Ru	102	58	−5.406	e − e
44	Ru	104	60	−5.534	e − e
45	Rh	103	58	−4.198	e − o
46	Pd	102	56	−5.170	e − e
46	Pd	104	58	−5.300	e − e
46	Pd	105	59	+5.382	o − e

Continued

46	Pd	106	60	−5.490	e – e
46	Pd	108	62	−5.755	e – e
46	Pd	110	64	−5.711	e – e
47	Ag	107	60	−3.833	e – o
47	Ag	101	62	−4.287	e – o
48	Cd	106	58	−5.373	e – e
48	Cd	108	60	−5.414	e – e
45	Cd	110	62	−5.531	e – e
48	Cd	111	63	+5.350	o – e
48	Cd	112	64	−5.273	e – e
48	Cd	113	65	+5.396	o – e
48	Cd	114	56	−5.405	e – e
48	Cd	116	68	−5.480	e – e
49	In	113	64	−3.952	e – o
49	In	115	66	−4.016	e – o
50	Sn	112	62	−5.664	e – e
50	Sn	114	64	−5.316	e – e
50	Sn	115	65	+4.780	o – e
50	Sn	116	66	−4.639	e – e
50	Sn	117	67	+5.005	o – e
50	Sn	118	68	−5.234	e – e
50	Sn	119	69	+5.466	o – e
50	Sn	120	70	−5.557	e – e
50	Sn	122	72	−5.490	e – e
50	Sn	124	74	−5.299	e – e
51	Sb	121	70	−4.688	e – o
51	Sb	123	72	−4.646	e – o
52	Te	120	68	−5.712	e – e
52	Te	122	70	−5.505	e – e
52	Te	123	71	+5.408	o – e
52	Te	124	72	−5.352	e – e
52	Te	125	73	+5.400	o – e
52	Te	126	74	−5.369	e – e
52	Te	128	76	−5.196	e – e
52	Te	130	78	−4.825	e – e
53	I	127	74	−4.319	e – e
54	Xe	124	70	−5.399	e – e
54	Xe	126	72	−5.201	e – e

Continued

54	Xe	128	74	−4.967	e – e
54	Xe	129	75	+4.752	o – e
54	Xe	130	76	−4.701	e – e
54	Xe	131	77	+4.995	o – e
54	Xe	134	78	−4.834	e – e
54	Xe	136	80	−4.313	e – e
54	Xe		82	−5.792	e – e
55	Cs	133	78	−3.922	e – o
56	Ba	130	74	−5.289	e – e
56	Ba	132	76	−4.962	e – e
56	Ba	134	78	−4.774	o – e
56	Ba	135	79	+4.548	e – e
56	Ba	136	80	−4.412	o – e
56	Ba	137	81	+3.908	e – e
56	Ba	138	82	−5.598	o – o
57	La	138	81	+3.045	e – o
57	La	139	82	−4.945	e – e
58	Ce	136	78	−4.592	e – e
58	Ce	138	80	−4.519	e – e
58	Ce	140	82	−5.655	e – e

Table 3. Neutron pairing energy for $190 \le A \le 238$.

Z	Nucleus	A	N	P_n (MeV)	Type of Pairing
76	Os	190	114	−3.905	e – e
77	Os	192	116	−3.775	e – e
77	Ir	191	114	−3.480	e – o
78	Ir	193	116	−3.279	e – o
78	Pt	190	112	−4.933	e – e
78	Pt	192	114	− 4.597	e – e
78	Pt	194	116	−4.335	e – e
78	Pt	195	117	+4.064	o – e
78	Pt	196	118	−3.924	e – e
79	Au	198	120	−3.709	e – e
80	Hg	197	118	−2.990	e – o
80	Hg	196	116	−4.080	e – e
80	Hg	198	118	−3.521	e – e
80	Hg	199	119	+3.187	o – e
80	Hg	200	120	−3.162	e – e

Continued

80	Hg	201	121	+3.320	e – e
80	Hg	202	122	−3.282	o – e
81	Hg	204	124	−3.320	e – e
81	Tl	203	122	−2.177	e – o
82	Tl	205	124	−2.249	e – o
82	Pb	204	122	−3.137	e – e
82	Pb	206	124	−2.704	e – e
82	Pb	207	125	+1.979	o – e
82	Pb	208	126	−4.068	e – e
83	Bi	209	126	−3.430	e – o
84	Po	209	125	+2.118	e – o
85	At	210	125	+1.911	e – o
86	Rn	222	136	−4.075	e – e
87	Fr	223	136	−2.455	e – o
88	Ra	226	138	−3.326	e – e
89	Ac	227	138	−2.273	e – o
90	Th	232	142	−2.976	e – e
91	Pa	231	140	−2.296	e – o
92	U	234	142	−2.632	e – e
92	U	235	143	+2.795	o – e
92	U	238	146	−2.374	e – e

Table 1 gives the calculated values of pairing energies for light nuclei $20 \le A \le 55$ with values of pairing energies between −16 MeV and +14 MeV.

Table 2 gives the calculated values of pairing energies for medium nuclei $100 \le A \le 140$ with values of pairing energies between −6 MeV and +6 MeV.

Table 3 gives the calculated values of pairing energies for heavy nuclei $190 \le A \le 238$ with values of pairing energies between −5 MeV and +5 MeV.

4. Discussion and Conclusion

In summary, the neutron pairing energies of finite nuclei $20 \le A \le 238$ are calculated and the following are conclusions:

1) For light nuclei ($20 \le A \le 55$), the P_n was found to lie between: $-16 \le P_n \le +14$ MeV.

2) For medium nuclei ($100 \le A \le 140$), the P_n was found to lie between: $-6 \le P_n \le +6$ MeV.

3) For heavy nuclei ($190 \le A \le 238$), the P_n was found to lie between: $-5 \le P_n \le +5$ MeV.

4) Positive pairing energies only occurred in odd (N) - even (Z) nuclei that indicate the most stable isotopic nuclei.

5) Negative pairing energies occurred in even (N) - even (Z), odd (N) - odd (Z) and even (N) - odd (Z).

6) The magnitude of the values of pairing energies decreases as you move from light to heavy nuclei.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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