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Results on Generalized Quasi Contraction Random Operators

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Abstract

In this paper, we prove the existence of common random fixed point for two random operators under general quasi contraction condition in a complete *p*-normed space X (with whose dual separates the point of X). Also, the well-posedness problem of random fixed points is studied. Our results essentially cover special cases.

Subject Areas

Functional Analysis

Keywords

p-Normed Spaces, Common Random Fixed Point, Random Operators, Well-Posed Problem

1. Introduction

Let X be a linear space and $\|\ \|_p$ be a real valued function on X with $0 < p \le 1$.

The ordered pair $(X, \|\ \|_p)$ is called a p-normed space [1] if for all x, y in X and scalars λ:

1) $\|x\|_p \ge 0$ and $\|x\|_p = 0$ iff $x = 0$

2) $\|\lambda x\|_p = |\lambda|^p \|x\|_p$

3) $\|x + y\|_p \le \|x\|_p + \|y\|_p$

for more details about *p*-normed spaces, see [2] or [3]. Throughout this article, X will be complete *p*-normed space whose dual separates the points of it, $\varnothing \ne A \subseteq X$ be a separable closed, (Ω, Σ) be the measurable space with Σ a sigma algebra of subsets of Ω.

Definition (1.1): [4]

A mapping $F:\Omega\to X$ is called measurable if, for open subset B of,

$$F^{-1}(B)=\{\gamma\in\Omega:F(\gamma)\cap B\neq\varnothing\}\in\Sigma.$$

Definition (1.2): [4]

A mapping $h:\Omega\times X\to X$ is called a random operator if for any $x\in X$, $h(.,x)$ is measurable.

Definition (1.3): [5]

A measurable mapping $\lambda:\Omega\to A$ is called random fixed point of a random operator $h:\Omega\times X\to X$ if for every $\gamma\in\Omega$, $\lambda(\gamma)=h(\gamma,\lambda(\gamma))$.

Definition (1.4): [6]

A measurable mapping $\lambda:\Omega\to A$ is called common random fixed point of a random operator $h:\Omega\times A\to X$ and $G:\Omega\times A\to A$ if for all $\gamma\in\Omega$

$$\lambda(\gamma)=h(\gamma,\lambda(\gamma))=G(\gamma,\lambda(\gamma)).$$

Definition (1.5): [7]

A random operator $h:\Omega\times A\to X$ is called continuous (weakly continuous) if for each $\gamma\in\Omega$, $h(\gamma,.)$ is continuous (weakly continuous).

The stochastic generalization of fixed point theory is random fixed point theory. Many researchers are interesting in this subject and it's applications in best approximations, integral equations and differential equations such as [8]-[14].

Saluj [15] establish some common random fixed point theorems under contractive type condition in the framework of cone random metric spaces. Rashwan and Albaqeri [16] obtained common random fixed point theorems for six weakly compatible random operators defined on a nonempty closed subset of a separable Hilbert space. In 2013, Arunchaiand Plubtieng [17] proved some random fixed point theorem for some of weakly-strongly continuous random operators and nonexpansive random operators in Banach spaces. Singh, Rathore, Dubey and Singh [18] obtain a common random fixed point theorem for four continuous random operators in separable Hilbert spaces. Vishwakarme and Chauhan [19] proved common random fixed point theorems for weakly compatible random operators in symmetric spaces. Khanday, Jain and Badshah [20] proved the existence of common random fixed point theorems of two random multivalued generalized contractions by using functional expressions. Chanhan [21] obtained common random fixed point theorems for four continuous random operators satisfying certain contractive conditions in separable Hilbert spaces. In 2017 Abed, Ajeel, and Alsaidy [22] proved the existence of common random fixed point for two continuous random operators under quasi contraction condition in a complete p-normed space *X*. Also, the random coincidence point results are proved in [23] for $\varnothing$-weakly contraction condition under two pairs of random operators.

Now, we define a new type of random operators

Definition (1.6):

Let A be a nonempty subset of a p-normed space, let (Ω,Σ) be a measurable space and let $h,G:\Omega\times A\to A$ be tow random operators. The random operator h is called

1. Generalized quasi contraction (gqc) random operator if for each $\gamma \in \Omega$, the mapping $h(\gamma,.): A \to A$ satisfies the following condition

$$\begin{aligned}\left\|h(\gamma,x)-h(\gamma,y)\right\|_p \le k \max\Big\{&\left\|x-y\right\|_p, \left\|x-h(\gamma,x)\right\|_p, \left\|y-h(\gamma,y)\right\|_p, \left\|x-h(\gamma,y)\right\|_p, \\ &\left\|y-h(\gamma,x)\right\|_p, \left\|h^2(\gamma,x)-x\right\|_p, \left\|h^2(\gamma,x)-h(\gamma,x)\right\|_p, \left\|h^2(\gamma,x)-y\right\|_p, \left\|h^2(\gamma,x)-h(\gamma,y)\right\|_p\Big\}\end{aligned} \tag{1.1}$$

For all $x, y \in A$ and $0 \le k < 1/2$.

2. G-generalized quasi contraction (G-gqc) random operator if for each $\gamma \in \Omega$, the mappings $h(\gamma,.), G(\gamma,.): A \to A$ satisfies the following condition

$$\begin{aligned}\left\|h(\gamma,x)-G(\gamma,y)\right\|_p \le k \max\Big\{&\left\|x-y\right\|_p, \left\|x-h(\gamma,x)\right\|_p, \left\|y-G(\gamma,y)\right\|_p, \left\|x-G(\gamma,y)\right\|_p, \\ &\left\|y-h(\gamma,x)\right\|_p, \left\|h^2(\gamma,x)-x\right\|_p, \left\|h^2(\gamma,x)-h(\gamma,x)\right\|_p, \left\|h^2(\gamma,x)-y\right\|_p, \left\|h^2(\gamma,x)-G(\gamma,y)\right\|_p\Big\}\end{aligned} \tag{1.2}$$

For all $x, y \in A$ and $0 \le k < 1/2$.

2. Common Random Fixed Point Theorems

We begin with the following result

Theorem (2.1):

Let $\varnothing \ne A \subseteq X$, $G: \Omega \times A \to A$ be a continuous random operator and $h: \Omega \times A \to A$ be a nonexpansive random operator. If A be a separable closed subset of a complete p-Normed space X and h be G-gqc random operator, then h and G have a unique common random fixed point.

Proof:

Let $\lambda_\circ: \Omega \to A$ be arbitrary measurable mapping. We construct a sequence of measurable mappings $\langle \lambda_n \rangle$ on Ω to A as follows

Let $\lambda_1, \lambda_2: \Omega \to A$ be tow measurable mappings such that

$$h(\gamma, \lambda_\circ(\gamma)) = \lambda_1(\gamma) \quad \text{and} \quad G(\gamma, \lambda_1(\gamma)) = \lambda_2(\gamma)$$

By induction, we construct sequence of measurable mappings $\lambda_n: \Omega \to A$ such that

$$h(\gamma, \lambda_{2n-1}(\gamma)) = \lambda_{2n}(\gamma) \quad \text{and} \quad G(\gamma, \lambda_{2n}(\gamma)) = \lambda_{2n+1}(\gamma) \tag{2.1}$$

From (2.1) and (1.2), we have

$$\begin{aligned}
&\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p = \left\|h(\gamma,\lambda_{2n-1}(\gamma))-G(\gamma,\lambda_{2n}(\gamma))\right\|_p \\
&\le k \max\Big\{\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n-1}(\gamma)-h(\gamma,\lambda_{2n-1}(\gamma))\right\|_p, \left\|\lambda_{2n}(\gamma)-G(\gamma,\lambda_{2n}(\gamma))\right\|_p, \\
&\quad \left\|\lambda_{2n-1}(\gamma)-G(\gamma,\lambda_{2n}(\gamma))\right\|_p, \left\|\lambda_{2n}(\gamma)-h(\gamma,\lambda_{2n-1}(\gamma))\right\|_p, \left\|h^2(\gamma,\lambda_{2n-1}(\gamma))-\lambda_{2n-1}(\gamma)\right\|_p, \\
&\quad \left\|h^2(\gamma,\lambda_{2n-1}(\gamma))-h(\gamma,\lambda_{2n-1}(\gamma))\right\|_p, \left\|h^2(\gamma,\lambda_{2n-1}(\gamma))-\lambda_{2n}(\gamma)\right\|_p, \left\|h^2(\gamma,\lambda_{2n-1}(\gamma))-G(\gamma,\lambda_{2n}(\gamma))\right\|_p\Big\} \\
&= k \max\Big\{\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p, \left\|\lambda_{2n-1}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p, \\
&\quad \left\|\lambda_{2n}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n+1}(\gamma)-\lambda_{2n-1}(\gamma)\right\|_p, \left\|\lambda_{2n+1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \\
&\quad \left\|\lambda_{2n+1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n+1}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p\Big\} \\
&= k \max\Big\{\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p, \left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p, \left\|\lambda_{2n-1}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p\Big\}
\end{aligned}$$

using triangle inequality, we get

$$\begin{aligned}\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p &\le k\max\Big\{\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p,\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p,\\ &\quad \left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p+\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p\Big\}\\ &= k\left[\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p+\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p\right]\end{aligned}$$

hence,

$$\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p \le \lambda\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p$$

where $\lambda=(k/1-k)<1$.

By similar way, we have

$$\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p \le \lambda\left\|\lambda_{2n-2}(\gamma)-\lambda_{2n-1}(\gamma)\right\|_p$$

therefore,

$$\begin{aligned}\left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p &\le \lambda\left\|\lambda_{2n-1}(\gamma)-\lambda_{2n}(\gamma)\right\|_p \le \lambda^2\left\|\lambda_{2n-2}(\gamma)-\lambda_{2n-1}(\gamma)\right\|_p\\ &\vdots\\ \left\|\lambda_{2n}(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p &\le \lambda^{2n}\left\|\lambda_0(\gamma)-\lambda_1(\gamma)\right\|_p\end{aligned}$$

To prove $\langle\lambda_n\rangle$ is Cauchy sequence, for $n,m\in\mathbb{N}, n>m$

$$\begin{aligned}&\left\|\lambda_n(\gamma)-\lambda_m(\gamma)\right\|_p\\ &\le \left\|\lambda_n(\gamma)-\lambda_{n-1}(\gamma)\right\|_p+\left\|\lambda_{n-1}(\gamma)-\lambda_{n-2}(\gamma)\right\|_p+\cdots+\left\|\lambda_{m+1}(\gamma)-\lambda_m(\gamma)\right\|_p\\ &\le \left(\lambda^{n-1}+\lambda^{n-2}+\cdots+\lambda^m\right)\left\|\lambda_0(\gamma)-\lambda_1(\gamma)\right\|_p \le \left(\lambda^m/1-\lambda\right)\left\|\lambda_0(\gamma)-\lambda_1(\gamma)\right\|_p\end{aligned}$$

Let $\epsilon>0$ be given, choose a natural number *K* large enough such that $\lambda^m\left\|\lambda_1(\gamma)-\lambda_0(\gamma)\right\|_p<\epsilon$ for every $m\ge K$.

Hence $\left\|\lambda_n(\gamma)-\lambda_m(\gamma)\right\|_p<\epsilon$ for every $n>m\ge K$.

So, $\{\lambda_n(\gamma)\}$ is a Cauchy sequence in, and completeness of *X* implise that there exists $\lambda(\gamma)\in X$ such that $\lambda_n(\gamma)\to\lambda(\gamma)$ as $n\to\infty$.

To show that λ is a common random fixed point of *h* and *G*, coinsider the following by using triangle inequality, (2.1) and (1.2)

$$\begin{aligned}&\left\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p \le \left\|\lambda(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p+\left\|\lambda_{2n+2}(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p\\ &=\left\|\lambda(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p+\left\|h(\gamma,\lambda(\gamma))-G(\gamma,\lambda_{2n+1}(\gamma))\right\|_p\\ &\le\left\|\lambda(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p+k\max\Big\{\left\|\lambda(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p,\left\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|\lambda_{2n+1}(\gamma)-G(\gamma,\lambda_{2n+1}(\gamma))\right\|_p,\left\|\lambda(\gamma)-G(\gamma,\lambda_{2n+1}(\gamma))\right\|_p,\left\|\lambda_{2n+1}(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|h^2(\gamma,\lambda(\gamma))-\lambda(\gamma)\right\|_p,\left\|h^2(\gamma,\lambda(\gamma))-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|h^2(\gamma,\lambda(\gamma))-\lambda_{2n+1}(\gamma)\right\|_p,\left\|h^2(\gamma,\lambda(\gamma))-G(\gamma,\lambda_{2n+1}(\gamma))\right\|_p\Big\}\\ &=\left\|\lambda(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p+k\max\Big\{\left\|\lambda(\gamma)-\lambda_{2n+1}(\gamma)\right\|_p,\left\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|\lambda_{2n+1}(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p,\left\|\lambda(\gamma)-\lambda_{2n+2}(\gamma)\right\|_p,\left\|\lambda_{2n+1}(\gamma)-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|h^2(\gamma,\lambda(\gamma))-\lambda(\gamma)\right\|_p,\left\|h^2(\gamma,\lambda(\gamma))-h(\gamma,\lambda(\gamma))\right\|_p,\\ &\quad\left\|h^2(\gamma,\lambda(\gamma))-\lambda_{2n+1}(\gamma)\right\|_p,\left\|h^2(\gamma,\lambda(\gamma))-\lambda_{2n+2}(\gamma)\right\|_p\Big\}\end{aligned}$$

taking the limit as $n \to \infty$ in the above inequality, getting that

$$\lambda(\gamma)-h(\gamma,\lambda(\gamma))_p \le k\max\Big\{\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p, \|h^2(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p, \|h^2(\gamma,\lambda(\gamma))-h(\gamma,\lambda(\gamma))\|_p\Big\}$$

By using triangle inequality and non-expansive of *h*, we have

$$\begin{aligned}
&\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p \\
&\le k\max\Big\{\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p, \|h^2(\gamma,\lambda(\gamma))-h(\gamma,\lambda(\gamma))\|_p \\
&\quad +\|h(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p, \|h^2(\gamma,\lambda(\gamma))-h(\gamma,\lambda(\gamma))\|_p\Big\} \\
&\le k\max\Big\{\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p, \|h(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p \\
&\quad + h(\gamma,\lambda(\gamma))-\lambda(\gamma)_p, \|h(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p\Big\} \\
&\le 2k\|h(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p
\end{aligned}$$

this implies that

$$(1-k)\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p \le 0 \tag{2.2}$$

since $0 \le k < 1/2$, (2.1.4) must be true only $\|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p = 0$, thus

$$\lambda(\gamma) = h(\gamma,\lambda(\gamma)) \tag{2.3}$$

Similarly, we can show that

$$\lambda(\gamma) = G(\gamma,\lambda(\gamma)) \tag{2.4}$$

hence $\lambda:\Omega\to A$ is a common random fixed point of *h* and *G*.

For uniqueness, let $\alpha(\gamma)$ be another common random fixed point of *S* and *T*, that is for all $\gamma\in\Omega$, $\alpha(\gamma) = h(\gamma,\alpha(\gamma)) = G(\gamma,\alpha(\gamma))$.

Then for all $\gamma\in\Omega$, we have

$$\|\lambda(\gamma)-\alpha(\gamma)\|_p = \|h(\gamma,\lambda(\gamma))-G(\gamma,\alpha(\gamma))\|_p$$

From (1.2), (2.3) and (3.2), we have

$$\begin{aligned}
\|\lambda(\gamma)-\alpha(\gamma)\|_p \le k\max\Big\{&\|\lambda(\gamma)-\alpha(\gamma)\|_p, \|\lambda(\gamma)-h(\gamma,\lambda(\gamma))\|_p, \\
&\|\alpha(\gamma)-T(\gamma,\alpha(\gamma))\|_p, \lambda(\gamma)-G(\gamma,\alpha(\gamma))_{pp}, \\
&\|\alpha(\gamma)-h(\gamma,\lambda(\gamma))\|_p, \|h^2(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p, \\
&\|h^2(\gamma,\lambda(\gamma))-\lambda(\gamma)\|_p, \|h^2(\gamma,\lambda(\gamma))-\alpha(\gamma)\|_p, \\
&\|h^2(\gamma,\lambda(\gamma))-G(\gamma,\alpha(\gamma))\|_p\Big\} \\
= k\max\Big\{&\lambda(\gamma)-\alpha(\gamma)_p, 0\Big\} \\
= k\lambda(\gamma)&-\alpha(\gamma)_p < \lambda(\gamma)-\alpha(\gamma)_p
\end{aligned}$$

Which is contraction. Hence $\lambda:\Omega\to A$ is a unique common random fixed point of *h* and. ■

Corollary (2.2):

If A and h as in theorem (2.1) and for each $\gamma \in \Omega$, $h(\gamma,.): A \to A$ is (gqc):

$$\|h(\gamma,x)-h(\gamma,y)\|_p$$
$$\leq k\max\{\|x-y\|_p, \|x-h(\gamma,x)\|_p, \|y-h(\gamma,y)\|_p, \|x-h(\gamma,y)\|_p,$$
$$\|y-h(\gamma,x)\|_p, \|h^2(\gamma,x)-x\|_p, \|h^2(\gamma,x)-h(\gamma,x)\|_p,$$
$$\|h^2(\gamma,x)-y\|_p, \|h^2(\gamma,x)-h(\gamma,y)\|_p\}$$

Then there is a random fixed point of h.

Corollary (2.3):

If A, h, G as in theorem (2.1) and for each $\gamma \in \Omega$, $h(\gamma,.), G(\gamma,.): A \to A$ satisfies one of the following conditions:

1. $$\|h(\gamma,x)-G(\gamma,y)\|_p \leq k\max\{\|x-y\|_p, \|x-h(\gamma,x)\|_p, \|y-G(\gamma,y)\|_p, \|x-T(\gamma,y)\|_p, \|y-h(\gamma,x)\|_p\}.$$
2. $$\|h(\gamma,x)-G(\gamma,y)\|_p \leq k\max\{\|x-y\|_p, \|x-h(\gamma,x)\|_p, \|y-G(\gamma,y)\|_p\}.$$
3. $$\|h(\gamma,x)-G(\gamma,y)\|_p \leq k\max\{\|x-h(\gamma,x)\|_p, \|y-G(\gamma,y)\|_p\}.$$
4. $$\|h(\gamma,x)-G(\gamma,y)\|_p \leq \max\{\|x-y\|_p, \|x-h(\gamma,x)\|_p, \|y-G(\gamma,y)\|_p, 1/2[\|x-G(\gamma,y)\|_p + \|y-h(\gamma,x)\|_p]\}.$$
5. $$\|h(\gamma,x)-G(\gamma,y)\|_p \leq k\max\{\|x-y\|_p, 1/2[\|x-h(\gamma,x)\|_p + \|y-G(\gamma,y)\|_p], 1/2[x-G(\gamma,y)_p + y-h(\gamma,x)_p]\}.$$

For all $x, y \in X; 0<k<1/2$. Then h and G have a unique common random fixed point.

3. Well-Posed Problem

Definition (3.1):

Let $(X, \|\|_p)$ be a p-normed space and $T: \Omega \times X \to X$ a random mapping. the random fixed point problem of T is said to be well-posed if:

i. T has a unique random fixed point $\lambda: \Omega \to X$;

ii. for any sequence $\{\lambda_n(\gamma)\}$ of measurable mappings in X such that $\lim_{n\to\infty}\|T(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p = 0$, we have $\lim_{n\to\infty}\|\lambda_n(\gamma)-\lambda(\gamma)\|_p = 0$.

Definition (3.2):

Let $(X, \|\|_p)$ be a p-normed space and let $\mathcal{T}$ be a set of a random operators in X. The random fixed point of $\mathcal{T}$ is said to be well-posed if:

i. $\mathcal{T}$ has a unique random fixed point $\lambda:\Omega\to X$;

ii. for any sequence $\{\lambda_n(\gamma)\}$ of measurable mappings in X such that $\lim_{n\to\infty}\|T(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p=0$, $\forall T\in\mathcal{T}$ we have $\lim_{n\to\infty}\|\lambda_n(\gamma)-\lambda(\gamma)\|_p=0$.

Theorem (3.3):

If A, h, G as in theorem (2.1) and for each $\gamma\in\Omega$, $h(\gamma,.),G(\gamma,.):A\to A$ satisfies (1.2), then the common random fixed point for the set of random operators $\{h,G\}$ is well-posed.

Proof:

By theorem (2.1), the random operators h and G have a unique common random fixed point $\lambda:\Omega\to A$. Let $\{\lambda_n(\gamma)\}$ be a sequence of measurable mappings in A such that

$$\lim_{n\to\infty}\|h(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p=\lim_{n\to\infty}\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p=0$$

By the triangle inequality, (1.2), (2.3) and (2.4), we have

$$\begin{aligned}
&\|\lambda(\gamma)-\lambda_n(\gamma)\|_p\\
&\le\|h(\gamma,\lambda(\gamma))-G(\gamma,\lambda_n(\gamma))\|_p+\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p\\
&\le h\max\Big\{\|\lambda(\gamma)-\lambda_n(\gamma)\|_p,\|\lambda_n(\gamma)-G(\gamma,\lambda_n(\gamma))\|_p,\|\lambda(\gamma)-G(\gamma,\lambda_n(\gamma))\|_p\\
&\quad\|\lambda_n(\gamma)-h(\gamma,\lambda(\gamma))\|_p,\|h^2(\gamma,\lambda(\gamma))-\lambda_n(\gamma)\|_p,\\
&\quad h^2\|(\gamma,\lambda(\gamma))-G(\gamma,\lambda_n(\gamma))\|_p\Big\}+\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p\\
&\le h\Big[\|\lambda(\gamma)-G(\gamma,\lambda_n(\gamma))\|_p+\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p\Big]+\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p\\
&=h\|\lambda(\gamma)-G(\gamma,\lambda_n(\gamma))\|_p+(1+h)\|\theta\|_p
\end{aligned}$$

By the triangle inequality, we get

$$\begin{aligned}
&\|\lambda(\gamma)-\lambda_n(\gamma)\|_p\\
&\le h\Big[\|\lambda(\gamma)-\lambda_n(\gamma)\|_p+\|\lambda_n(\gamma)-G(\gamma,\lambda_n(\gamma))\|_p\Big]\\
&\quad+(1+h)\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p\\
&=h\|\lambda(\gamma)-\lambda_n(\gamma)\|_p+(1+2h)\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p
\end{aligned}$$

$$(1-h)\|\lambda(\gamma)-\lambda_n(\gamma)\|_p\le(1+2h)\|G(\gamma,\lambda_n(\gamma))-\lambda_n(\gamma)\|_p$$

thus we have, $\lim_{n\to\infty}\|\lambda(\gamma)-\lambda_n(\gamma)\|_p=0$, it follows that the common random fixed point for the set of random operators $\{h,G\}$ is well-posed. ■

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