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Chaos Anti-Synchronization between Chen System and Genesio System

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Abstract

In this paper, anti-synchronization of two different chaotic systems is investigated. On the basis of Lyapunov theory, adaptive control scheme is proposed when system parameters are unknown or uncertain; sufficient conditions for the stability of the error dynamics are derived, where the controllers are designed by using the sum of the state variables in chaotic systems. Numerical simulations are performed for the Chen system and Genesio system to demonstrate the effectiveness of the proposed control strategy.

Keywords

Chaotic System, Anti-Synchronization, Adaptive Control

Subject Areas: **Chaotic System, Anti-Synchronization, Adaptive Control**

1. Introduction

Since the pioneering work by Pecora and Carroll [1], chaos synchronization, a very active topic in nonlinear science, has received increasing attention. The concept of synchronization has been extended in scope, for example to generalized synchronization [2]-[4], phase synchronization [5], lag synchronization [6], and even anti-phase synchronization (APS) [7]-[9]. APS can also be interpreted as anti-synchronization (AS), which is a phenomenon in which the state variables of the synchronized systems have the same amplitude as but opposite signs to those of the driving system. Therefore, the sums of two signals are expected to converge to zero when AS appears. Recently, Singh and Roy generalized active control to AS for two systems [10]. However, it seems that there are fewer previous results on AS between two different systems with unknown parameters using adaptive control. In this paper, we will focus on the AS of two different systems. Adaptive control methods will be employed; a sufficient condition for anti-synchronization is derived rigorously. Numerical simulations on Chen system and Genesio system are performed, which demonstrate the effectiveness and feasibility of the proposed control technique.

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The layout of the rest of the paper is as follows. Section 2 describes the systems and their mathematical models; in Section 3, adaptive anti-synchronization between Chen and Genesio systems is presented; numerical simulation results are given for illustration and verification. Finally, conclusions are drawn in Section 4.

2. Systems Description and Mathematical Models

Consider nonlinear chaotic system as follows.

$$\begin{cases} \dot{x} = f(t,x) \\ \dot{y} = g(t,y) + u(t,x,y) \end{cases} \tag{1}$$

where $x, y \in R^n$, $f, g \in R \times R^n \to R^n$ are differentiable functions, the first equation in (1) is the drive system, and the second one is the response system, $u(t,x,y)$ is the control input. Let $\mathbf{e} = \mathbf{y} + \mathbf{x}$ be the anti-synchronization error, our goal is to design controllers $\mathbf{u}$ such that the trajectory of the response system with initial conditions y_0 can asymptotically approach the drive system with initial conditions x_0 reversely and implement anti-synchronization finally, in the sense that

$$\lim_{t\to\infty}\|e\| = \lim_{t\to\infty}\|y(t,y_0) + x(t,x_0)\| = 0$$

where $\|\cdot\|$ is the Euclidean norm.

The Genesio system, proposed by Genesio and Tesi [11], is one of paradigms of chaos since it captures many features of chaotic systems. It includes a simple square part and three simple ordinary differential equations that depend on three negative real parameters. The dynamic equations of the system is given by

$$\begin{cases} \dot{x} = y \\ \dot{y} = z \\ \dot{z} = ax + by + cz + x^2 \end{cases} \tag{2}$$

where x, y, z are state variables, when $a = -6, b = -2.92, c = -1.2,$ system (2) is chaotic.

Chen system is described by

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) \\ \dot{y}_1 = (c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 \end{cases} \tag{3}$$

When $a_1 = 35, b_1 = 3, c_1 = 28$, system (3) is chaotic.

In the next section, we will study chaos anti-synchronization between Chen and Genesio systems with known or unknown parameters using adaptive control.

3. Adaptive Anti-Synchronization between Chen and Genesio System with Unknown Parameters

We assume that Genesio system (2) is the drive system, and the controlled Chen system (4) is the response system.

$$\begin{cases} \dot{x}_1 = a_1(y_1 - x_1) + u_1 \\ \dot{y}_1 = (c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1 + u_2 \\ \dot{z}_1 = x_1 y_1 - b_1 z_1 + u_3 \end{cases} \tag{4}$$

We add (2) from Equation (4) and yield

$$\begin{cases} \dot{e}_1 = a_1(y_1 - x_1) + y + u_1 \\ \dot{e}_2 = (c_1 - a_1)x_1 - x_1 z_1 + c_1 y_1 + z + u_2 \\ \dot{e}_3 = x_1 y_1 - b_1 z_1 + ax + by + cz + x^2 + u_3 \end{cases} \tag{5}$$

Our goal is to find the proper controllers u_i $(i = 1,2,3)$ and parameter update laws, such that system (4) globally anti-synchronizes system (2) asymptotically. *i.e.*

$$\lim_{t\to\infty}\|\mathbf{e}\|=0$$

where $\mathbf{e}=[e_1,e_2,e_3]^{\mathrm{T}}$.

Theorem: If the controllers are chosen as

$$\begin{cases} u_1=-\hat{a}_1(x_1-y_1)-y-k_1e_1 \\ u_2=-(\hat{c}_1-\hat{a}_1)x_1+x_1z_1-\hat{c}_1y_1-z-k_2e_2 \\ u_3=-x_1y_1+\hat{b}_1z_1-\hat{a}x-\hat{b}y-\hat{c}z-x^2-k_3e_3 \end{cases} \tag{6}$$

and the update laws of parameters are chosen as

$$\begin{cases} \dot{\hat{a}}=xe_3 \\ \dot{\hat{b}}=ye_3 \\ \dot{\hat{c}}=ze_3 \\ \dot{\hat{a}}_1=(y_1-x_1)e_1-x_1e_2 \\ \dot{\hat{b}}_1=-z_1e_3 \\ \dot{\hat{c}}_1=(x_1+y_1)e_2 \end{cases} \tag{7}$$

Then system (4) globally anti-synchronizes system (2) asymptotically, where $k_i\ (i=1,2,3)$ are positive constants, $\hat{a},\hat{b},\hat{c},\hat{a}_1,\hat{b}_1,\hat{c}_1$ are estimate values of a,b,c,a_1,b_1,c_1, respectively.

Proof: Applying control laws (6) to (5) yields the resulting error dynamics as follows.

$$\begin{cases} \dot{e}_1=\tilde{a}_1(x_1-y_1)-k_1e_1 \\ \dot{e}_2=\tilde{a}_1x_1-\tilde{c}_1(x_1+y_1)-k_2e_2 \\ \dot{e}_3=-\tilde{a}x-\tilde{b}y-\tilde{c}z+\tilde{b}_1z_1-k_3e_3 \end{cases} \tag{8}$$

where $\tilde{a}=\hat{a}-a,\tilde{b}=\hat{b}-b,\tilde{c}=\hat{c}-c,\tilde{a}_1=\hat{a}_1-a_1,\ \tilde{b}_1=\hat{b}_1-b_1,\tilde{c}_1=\hat{c}_1-c_1$.

Consider the following Lyapunov function

$$V=\frac{1}{2}\left(e^{\mathrm{T}}e+\tilde{a}^2+\tilde{b}^2+\tilde{c}^2+\tilde{a}_1^2+\tilde{b}_1^2+\tilde{c}_1^2\right)$$

The time derivative of *V* along the solution of error dynamical system (8) gives that

$$\begin{aligned}\dot{V}&=e^{\mathrm{T}}\dot{e}+\tilde{a}\dot{\tilde{a}}+\tilde{b}\dot{\tilde{b}}+\tilde{c}\dot{\tilde{c}}+\tilde{a}_1\dot{\tilde{a}}_1+\tilde{b}_1\dot{\tilde{b}}_1+\tilde{c}_1\dot{\tilde{c}}_1\\ &=e_1(\tilde{a}_1x_1-\tilde{a}_1y_1-k_1e_1)+e_2(\tilde{a}_1x_1-\tilde{c}_1x_1-\tilde{c}_1y_1-k_2e_2)\\ &\quad+e_3\left(-\tilde{a}x-\tilde{b}y-\tilde{c}z+\tilde{b}_1z_1-k_3e_3\right)+\tilde{a}(xe_3)+\tilde{b}(ye_3)\\ &\quad+\tilde{c}(ze_3)+\tilde{a}_1((y_1-x_1)e_1-x_1e_2)+\tilde{b}_1(-z_1e_3)+\tilde{c}_1(x_1+y_1)e_2\\ &=-k_1e_1^2-k_2e_2^2-k_3e_3^2=-e^{\mathrm{T}}Pe\le 0\end{aligned}$$

where $P=diag\{k_1,k_2,k_3\}$.

Since V is positive definite and $\dot{V}$ is negative semi-definite in the neighborhood of zero solution of system (5), it follows that $e_1,e_2,e_3\in L_\infty$, $\tilde{a},\tilde{b},\tilde{c}\in L_\infty$, and $\tilde{a}_1,\tilde{b}_1,\tilde{c}_1\in L_\infty$, from (8), we have $\dot{e}_1,\dot{e}_2,\dot{e}_3\in L_\infty$, since $\dot{V}=-e^{\mathrm{T}}Pe$, we obtain

$$\int_0^t\lambda_{\min}(P)\|e\|^2\,\mathrm{d}t\le\int_0^t e^{\mathrm{T}}Pe\mathrm{d}t=\int_0^t-\dot{V}\mathrm{d}t=V(0)-V(t)\le V(0)$$

where $\lambda_{\min}(P)$ is the minimal eigenvalue of the positive definite matrix *P*. Thus, $\dot{e}_1,\dot{e}_2,\dot{e}_3\in L_2$, by Barbalat's lemma, we have $\lim_{t\to\infty}\|e(t)\|=0$. Therefore, response system (4) can globally anti-synchronize drive system (2) asymptotically. This completes the proof.

In simulation, Fourth order Runge-Kutta integration method is used to solve the systems of differential Equations (2) and (4) with the controllers (6) and the parameter update laws (7). We select the parameters of Genesio system as $a=-6, b=-2.92, c=-1.2$ and the parameters of Chen system as $a_1=35, b_1=3, c_1=28$, respectively, and $k_i\ (i=1,2,3)=1$. The initial values of drive and response systems are $x(0)=0.2, y(0)=0.4$, $z(0)=0.6$ and $x_1(0)=-2, y_1(0)=2, z_1(0)=5$, respectively, while the initial errors of system (5) are $e_1(0)=-1.8, e_2(0)=2.4, e_3(0)=5.6$ and the initial values of estimate parameters are $\hat{a}(0)=\hat{b}(0)=\hat{c}(0)=1$, $\hat{a}_1(0)=\hat{b}_1(0)=\hat{c}_1(0)=2$. The anti-synchronization errors between Chen system and Genesio system are shown in **Figure 1**, the estimate values of parameters a,b,c and a_1,b_1,c_1 are shown in **Figure 2** and **Figure 3**, respectively. Obviously, the anti-synchronization errors converge asymptotically to zero and two different systems are indeed achieved chaos anti-synchronization. Furthermore, the estimates of parameter converge to their true values.

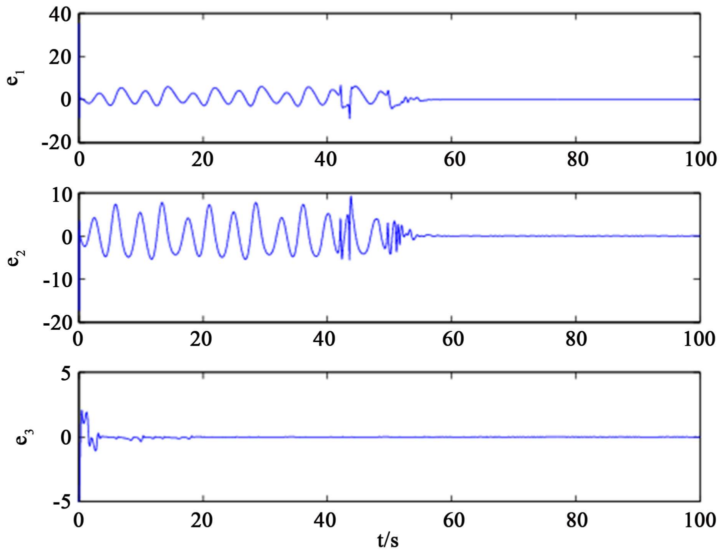

Figure 1. Anti-synchronization errors between Chen and Genesio systems via adaptive control.

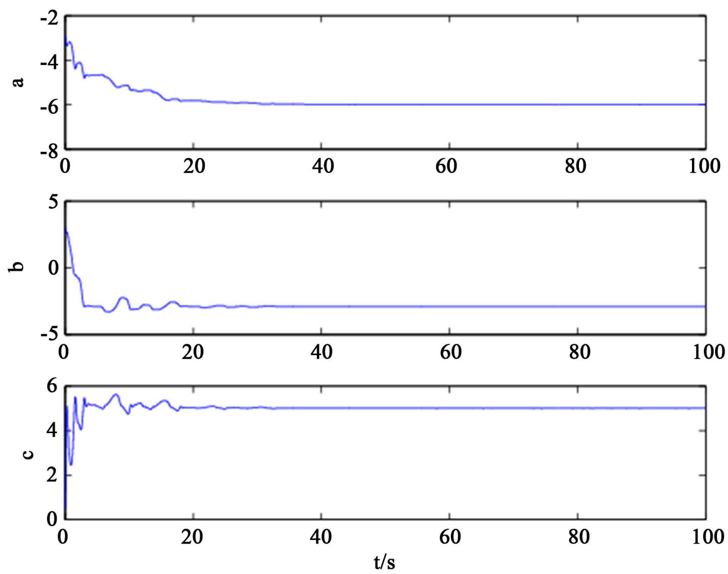

Figure 2. Estimate values of parameters *a*, *b*, *c* of drive system.

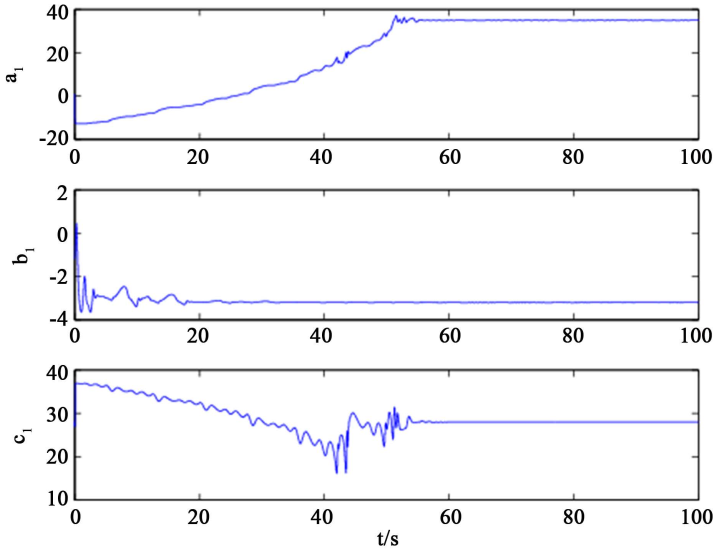

Figure 3. Estimate values of parameters a_1, b_1, c_1 of response system.

4. Conclusion

In this paper, chaos anti-synchronization between two different chaotic systems with different structures using adaptive control is presented. Chen system and Genesio system are taken as an illustrative example to verify the effectiveness of the proposed methods.

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