

Dark Energy Stars and the Cosmic Microwave Background

George F. Chapline

Lawrence Livermore National Laboratory, Livermore, CA, USA

Email: chapline1@llnl.gov

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Abstract

Cosmological models with a large cosmological constant are unstable due to quantum critical fluctuations at the de Sitter horizon. Due to this instability the space-time in these models will quickly evolve into a Friedmann-like expanding universe containing dark energy stars and radiation. In this paper it is pointed out that this provides a simple explanation for both the observed radiation entropy per gram of dark matter and the level of temperature fluctuations in the cosmic microwave background. A novel prediction is that large dark energy stars provide the seeds for the formation of galaxies.

Keywords

Cosmology, Dark Matter, CMB

Subject Areas: Modern Physics, Quantum Mechanics

1. Introduction

It was suggested some time ago by Eddington and Lemaitre [1] [2] that the observable universe may not have had a singular beginning, but instead may have evolved from a non-singular quasi-static space-time that Lemaitre called the "primeval atom". Lemaitre further suggested that this initial state of the universe was a macroscopic quantum state [3]. More recently it has been suggested [4]-[6] that apart from event horizons the vacuum state of space-times in general can be represented as a macroscopic quantum state. Representing the vacuum state of space-time as a macroscopic quantum state has the pleasing consequence that it immediately suggests an attractive explanation for the coexistence of dark energy and dark matter; namely, dark matter consists of gravitationally stable droplets of quantum fluid—for which the author has previously proposed the name "dark energy stars" [7]—whose interior has a space-time structure that resembles de Sitter space with a large vacuum energy density. Thus we visualize cosmological space-time as consisting of a homogeneous vacuum plus dark energy stars in much the same way that steam typically consists of a mixture of water vapor and water droplets.

As is the case for a vapor near the liquid-gas transition, it would be quite natural for the average mass-energy densities of the homogeneous vacuum and the primordial dark energy stars to be comparable.

Our picture for the "cosmic seed" differs from the primeval atom of the Eddington and Lemaitre in that we imagine that initially the vacuum energy is very much larger than the Einstein critical value, so that any matter that is initially present would rapidly expand as in inflationary cosmological models. However, in contrast with inflationary cosmological models we assume that the initial de Sitter phase is immediately terminated by the quantum instability of the de Sitter horizons [6] [8]. This instability allows for the conversion of the bulk of the vacuum energy into dark energy stars, which can then emit γ -rays, which we identify as the precursor of the CMB. Thus in our model the cosmic seed rapidly evolves into a Friedmann-like universe containing dark energy stars and radiation which in fact strongly resembles the standard cosmological model [9]. After the breakup of the cosmic seed the vacuum energy is unimportant until z-1, when the vacuum energy of the " Λ sea" begins to be felt again. Overall, we picture the evolution of the universe as a sequence of flat (k=0) Lemaitre models [10].

In Section 2 we show that whereas the radiation, dark matter, and dark energy densities are independent parameters in a general Robertson-Walker cosmological model, these parameters are linked in our model. In particular we predict that a value for the radiation entropy per gram of dark matter is close to the observed value. This is a significant advance over standard cosmological models where no such prediction is possible. Another attractive feature of our model is that fossilized remains of the initial quantum instability of the cosmic seed remain today in the form of temperature fluctuations of the cosmic microwave background. In Section 3 we show that our model in fact provides a simple explanation for the observed 10⁻⁵ level of mean temperature fluctuations. To our knowledge this is the first time that a simple explanation has been provided for this parameter. In Section 4 we briefly comment on the connection between our model and previous efforts to understand what underlies the standard cold dark matter cosmology.

2. Entropy of the CMB

We envision that the average vacuum energy density inside this cosmic seed will be on the order of 10 times the energy equivalent of the mass density of closely packed nucleons; *i.e.* $\approx 10^{16}$ gm/cc. This is also on the order of the maximum possible central density for a neutron star [11], and therefore we adopt this mass density as a reasonable guess as to the minimum density where ordinary matter can easily get converted into the vacuum energy. Indeed during the final stages of the gravitational collapse the mass-energy of stellar matter inside a massive star must get converted into vacuum energy density. In this note we will show that this idea provides a simple explanation for the existence of both dark matter and the CMB. In addition our guess for the vacuum energy density inside the cosmic seed leads us to estimates for the present day average dark matter density and level of CMB temperature fluctuations very close to the values inferred from CMB observations [12].

The mass M_* of the quantum droplets initially produced in the breakup of the cosmic seed is related to the energy density ρ_* of the cosmic seed by

$$M = \left[2 \times 10^{16} \text{ gm} \cdot \text{cm}^{-3} / \rho_* \right]^{1/2} M_O$$
 (1)

We are immediately faced with the puzzle though that expansion of a cloud of dark energy stars with mass M_* would lead to a present day density of dark matter that is many orders of magnitude larger than the observed dark matter density. Evidently almost all the mass-energy of the quantum droplets initially produced gets converted into radiation energy. In order to understand how this could be, let us first try to understand how the radiation in a collapsing universe of dark energy stars and radiation might get converted into vacuum energy. A curious property of dark energy stars is that they are very good at absorbing and thermalizing radiation only if the typical quantum energies of the elementary particles are greater than a certain cutoff [13]:

$$h\nu_c \approx 100 \left(M_{\odot}/M\right)^{1/2} \text{ MeV},$$
 (2)

where M is the mass of the dark energy star. This means that if dark matter consists of dark energy stars with a characteristic mass $M_{\rm DM}$ radiation in the collapsing cloud can be efficiently absorbed by these dark energy stars if the ambient radiation temperature exceeds the cutoff for $M = M_{\rm DM}$. For example, if $M_{\rm DM} = 10^3~M_{\rm O}$ then the radiation in the collapsing cloud will get absorbed when the radiation temperature exceeds a few MeV. The energy storage capacity of a dark energy star with mass M is (4):

$$U = 50.6N_* T_{\text{keV}}^3 \left(\frac{M}{M_{\odot}}\right)^3 M_{\odot} c^2,$$
 (3)

where $T_{\rm keV}$ is the temperature of the dark energy star in units of keV and N_* is the number of different kinds of microscopic degrees of freedom inside the dark energy star. The heat capacity (3) exceeds the heat capacity of the ordinary vacuum by a factor $\approx m_p c^2/k_B T$, where m_P is the Planck mass. As indicated by Equation (3), the interior temperatures of primordial dark energy stars will be below -0.1 keV. However, these equilibrium temperatures may never be reached because of the slowing of relaxation rates for quantum energies below the cutoff $h \nu_c$.

Formally the thermal energy, Equation (3), can exceed the zero temperature mass. However, increasing the internal energy density of a dark energy star increases its size, and at some point this would cause the dark energy star to become unstable due to the appearance of internal quantum critical layers, which then cause the dark energy star to break-up into smaller dark energy stars with interior energy densities that are similar to the increased energy density of the larger dark energy star. As time goes on and the ambient temperature exceeds the cutoff frequency, Equation (2), for the smaller dark energy stars this process would repeat itself. The process of converting radiation into the mass-energy of dark energy stars will cease when all the radiation has been used up. The cloud of dark energy stars will continue to collapse until they overlap. We now argue that this process can also work in reverse.

Using the cross-section $s = 27\pi \left(GM/c^2 \right)^2$ for binary collisions between dark energy stars one can show that the collision rate between quantum droplets in the initial matter dominated regime is approximately the same as the expansion time. However, on a similar time scale collections of the initially formed quantum droplets can coalesce to form compact objects via gravitational collapse. The general conditions for this to happen are well known [14]. The most widely studied case is when the density fluctuation spectrum has the Harrison-Zeldovich form [15] [16]:

$$\frac{\delta \rho}{\rho} = \varepsilon \left(\frac{ct_0}{R_0}\right)^2 \tag{4}$$

where $\delta\rho/\rho$ is the fractional density fluctuation within a sphere of radius R_0 , ε is the rms metric fluctuation, and ct_0 is the horizon radius. If ε is independent of R_0 then Equation (4) becomes the Harrison-Zeldovich-Peebles spectrum $\delta\rho/\rho-k^2$ [15]-[17]. One can show [14] that, independently of the value of ε , as a result of even a small increase in density within a sphere of radius R_0 at the time of the break-up of the cosmic seed $t_0 = \left(3/8\pi G\rho_*\right)^{1/2}$, the matter inside this sphere will not expand indefinitely. When the speed of sound within the expanding volume is zero the radius will reach a maximum:

$$R_{\text{max}} \approx R_0 \left(\delta \rho / \rho \right)^{-1},$$
 (5)

and the density contrast at that time will be ≈ 3 . In all cases the volume at the time of maximum expansion will lie inside the horizon. Also in all cases the subsequent collapse time will be less than the expansion time. However only for $\varepsilon - 1$ will R_{max} be close to the Schwarzschild radius. In this case compact objects with a large range of masses will be formed. If $\varepsilon \leq 1$ compact objects will continue to form only as long as the speed of sound remains small; *i.e.* the expanding cloud remains matter dominated.

When large collections of the initially formed dark energy stars begin to condense there is going to be a large mismatch between their internal energy density and the vacuum energy density inside any much large dark energy star that might be formed as a result of the condensation. In the reverse of what happens in a collapsing universe of dark energy stars and radiation this mismatch is resolved by radiating away most of their massenergy. As it happens the transition between storing energy as the mass-energy of the initial dark energy stars and radiation can occur very rapidly. If a dark energy star were in thermal equilibrium, its luminosity would be:

$$L \approx 0.4 \left(\frac{2GM}{c^2}\right)^2 \frac{\left(k_B T\right)^4}{\hbar^3 c^2} \left[\left(\frac{m_P c^2}{k_B T}\right)^{3/2} \left(\frac{h \nu_c}{k_B T}\right)^2 e^{-h \nu_c / k_B T} \right]. \tag{6}$$

The factor in front of the brackets is on the same order the familiar Stefan-Boltzmann thermal luminosity of a sphere with radius $2GM/c^2$. The first factor inside the brackets represents an enhancement of this luminosity by a factor -10^{38} , due to the enormous density of states near to the surface of a dark energy star (this is the dark energy star realization of the "holographic picture" of a black hole [18]). As a result of this enhancement in the luminosity, a condensation of dark energy stars with mass M can radiate away its internal energy on the same time scale, $-10(M/M_*)$ µsec, that it takes for the condensation to collapse provided the Wien factor

 $\exp(-hv_c/k_BT)$ is not smaller than about $10^{-23}(M_*/M)^2$. This condition is satisfied for condensation masses $M > 10^8 M_*$. As a consequence, after a time which would allow closely packed condensations of quantum droplets with masses $M > 10^8 M_*$ to form essentially all the mass-energy of the quantum droplets will be converted into radiation, leaving only residual dark energy stars with approximately the same radii as the condensation radii.

As a simple model for the transition between the regime where the dark matter and radiation are decoupled and the high temperature regime where there is strong coupling between the dark matter and radiation we will simply assume that for red shifts greater than a certain red shift, $1 + z_r$, the radiation energy is stored as the mass-energy of quantum droplets with mass M_* . For $1 + z < 1 + z_r$, we will assume that the mass-energy of the quantum droplets has been converted into radiation and dark energy stars with mass $M_{\rm DM}$. For the red shift separating these two regimes we will use the value

$$1 + z_r = h v_c / k_B T_{\text{CMB}} , \qquad (7)$$

where $T_{\rm CMB} = 2.73$ K is the present day temperature of the CMB. The radiation energy density as a function of red shift is given by:

$$\rho_{rad}\left(z\right) = \rho_* \frac{N\left(z\right)}{N\left(z_r\right)} \left(\frac{1+z_*}{1+z_r}\right) \left(\frac{1+z}{1+z_*}\right)^4, \tag{8}$$

where $1 + z_* \approx 10^{13}$ is the red-shift of the initial break-up of the cosmic seed corresponding to the origin of the observable universe. The temperature of the radiation is obtained from the usual radiation equation of state (11):

$$\rho_{rad} = N(T) \frac{\pi^2}{30} \left(\frac{\left(k_B T \right)^4}{\left(\hbar c \right)^3} \right), \tag{9}$$

where N(T) is the effective number of elementary particle species present in the radiation. Equations (7)-(9) yield a present day radiation temperature that is very close to the measured temperature of the CMB if one assumes $\rho_* = 10^{16}$ gm/cc and $M_{\rm DM} = 1500 M_{\rm O}$. This model predicts that the CMB originates at a red shift $1 + z_r = 1.3 \times 10^{10}$. The radiation temperature at this time was -3 MeV, so the cosmological production of helium and other light elements should be approximately the same as in the standard model.

For red shifts smaller than the red shift $1 + z_r$, which marks the beginning of the radiation dominated era of our observable universe, the density of dark matter will be given by

$$\rho_{DM} = \rho_* \left(\frac{M_{\bullet}}{M_{DM}} \right)^2 \left(\frac{1+z}{1+z_*} \right)^3 \tag{10}$$

Again assuming $\rho_* = 10^{16}$ gm/cc and $M_{\rm DM} = 1500 M_{\rm O}$ we find that the present day density of the primordial dark energy stars is 2×10^{-30} gm/cc, which is exactly the present day dark matter density inferred from WMAP [12]. The value of $M_{\rm DM}$ is close to the value that we estimated would survive after most of mass-energy of the initially produced quantum droplets were converted into radiation. Of course a cynic might argue that we have merely chosen $M_{\rm DM}$ to give the observed dark matter density, so this is not a priori prediction. However, the masses of the primordial dark energy stars that are formed by coalescence of the initially formed quantum droplets could in principle be calculated from first principles using a detailed model for the coupled gravitational and radiation dynamics of the quantum droplets in an expanding universe with density fluctuations of the form (4). Thus there is a hope that our model will lead to an a priori theory for the present day ratio of dark matter density to radiation density. It might be noted in this connection that usual way of expressing the specific entropy of the CMB as the number of photons per baryon has no fundamental significance in our theory. Instead what our theory yields is the number of CMB photons per gram of dark matter.

Our simple model does lead to one dramatic independent prediction: dark matter should be clumped on a

mass scale $1000M_{\rm O}$. This prediction can perhaps be checked in the near future. For example, when strong gravitational lensing leads to multiple images of the same object, the variations in brightness from image to image is sensitive to the clumpy nature of dark matter [19]. In fact variations in brightness already seen in examples of gravitational lensing with multiple images is suggestive that dark matter may indeed be clumped on some mass scale. Extension of the techniques used in the MACHO project to search for transient micro-lensing by compact objects [20] might also yield direct evidence for our dark matter dark energy stars.

3. Why Is $\Delta T/T \approx 10^{-5}$?

As discussed in the previous section, metric fluctuations in the cosmic seed eventually lead to the production of primordial dark energy stars with masses $-10^3 M_{\odot}$. The size of these dark energy stars is many orders of magnitude smaller than the overall size of the cosmic seed, which accounts for the overall isotropy and homogeneity of the observable universe. However, density fluctuations during the process of break-up of the cosmic seed will lead to large variations in the densities of the quantum droplets for length scales small compared to the horizon radius. There are two obvious sources of the metric fluctuations in the cosmic seed: 1) density and velocity fluctuations in the collapsing cloud that was the precursor to the cosmic seed would have left an imprint in the cosmic seed, and 2) fluctuations associated with the quantum critical layers. We believe that the second possibility is the more important. Furthermore because the thickness of the quantum critical layers is small compared to the horizon size, the spectrum of metric fluctuations generated by these quantum critical layers should be approximately independent of k for length scales larger than the horizon size ct_0 . By the well know arguments of Harrison and Zeldovich [15] [16] this leads to a density fluctuation spectrum of the form (4). Furthermore, the space-time inside a horizon surface is approximately described by the interior de Sitter metric, where g_{00} decreases from -1 farthest away from the horizon to nearly zero near to the quantum critical layers. Thus we expect that the spectrum of fluctuations in the density of quantum droplets at the time of break-up of the cosmic has the form (4) with $\varepsilon_0 - 1$; i.e. $\delta \rho / \rho \approx (ct_0/R_0)^2$. On the other hand observations of variations in the temperature of the CMB [21] as well as theories of the evolution of the large scale structures found in the present day universe [22] are consistent with $\varepsilon \approx 10^{-5}$. Remarkably our theory offers a simple explanation for this difference.

As described in the previous section the CMB originates not at the time of the break-up of the cosmic seed, but at a somewhat later time when the dark matter dark energy stars are being formed. Because the radiation becomes freely streaming when it emerges from the confines of the quantum droplet condensations, the initial density fluctuations will tend to get smoothed out inside the horizon. A reasonable way to approximately evaluate this smoothing is to simply assume that ϵ_0 is renormalized by the average of the density fluctuations within a light sphere whose radius is the collapse time

$$c\tilde{t} = \frac{2}{3\sqrt{3}} \left(\frac{1+z_*}{1+z_r} \right)^{3/2} ct_0 \tag{11}$$

Using the weel known Lifshitz result that the density fluctuations during the matter dominated period $z_* > z_. > z_r$ would have grown by a factor $(1+z_*)/(1+z_r)$ independent of length scale we find that the renormalized value of ε is given by

$$\varepsilon = \varepsilon_0 \frac{1 + z_*}{1 + z_r} \left(\frac{4\pi}{3} \left(c\tilde{t} \right)^3 \right)^{-1} \int_{ct_0}^{c\tilde{t}} \left(\frac{ct_0}{R} \right)^2 4\pi R^2 dr = \frac{81}{4} \left(\frac{1 + z_r}{1 + z_*} \right)^2.$$
 (12)

Using the values for $1 + z_*$ and $1 + z_r$ implied by Equations (7)-(10) we obtain a renormalized metric fluctuation coefficient $\varepsilon \approx 10^{-5}$. This is an a priori prediction since there are only 2 (and maybe only 1) parameters in our model. Indeed to the author's knowledge this is the first time a cosmological model of any sort has yielded an a priori prediction of this quantity.

4. Retrospective

One of the conundrums of Friedmann cosmology is how does the early universe know that the universe is supposed to be flat? In fact in our model the space-time inside both the cosmic seed and the expanding cloud of dark energy stars and radiation is not flat, but has positive curvature. On the other hand, both the dark matter and

radiation in our theory evolve smoothly into the conditions seen today. In our theory the flatness problem is sidestepped in a simple way. Although the spatial curvature inside our cosmic seed is locally positive, the overall curvature of the universe remains flat if we imagine that the cosmic seed was created by gravitational collapse because the average density of dark matter in the overall universe is unchanged. After the time corresponding to redshift $z = z_r$, the densities of both dark matter and radiation in our theory vary with time in a way that is essentially the same as in the standard cosmological model. It is of course an interesting question whether there are any signatures in the observable universe from a finite cosmic seed. It may be worth noting in this connection that the possibility of converting vacuum energy into ordinary matter was discussed some time ago by Feynman [23], and this may play a role in observable phenomena such as radiations from the massive compact objects at the centers of galaxies.

Some time ago Zelodvich suggested [16] that the specific entropy of the CMB might be related to the metric fluctuations that for very large scales led to large-scale structures such as galactic clusters. Zeldovich imagined that the initial state was a zero entropy state of highly compressed baryons. Thus we share with Zeldovich a conviction that specific entropy of the CMB is intimately related to the level of primordial metric fluctuations and that the energy density of highly compressed baryons is of fundamental significance. However, there are important differences between Zeldovich's suggestions and our theory. The pressure inside Zeldovich's baryonic fluid is positive whereas the pressure inside our cosmic seed is negative. Also Zeldovich assumes that the CMB arises from the dissipation of sound waves whose wavelength is comparable to the inter-particle separation in his baryonic fluid. In our theory the CMB arises from the conversion of the mass-energy of dark energy stars into radiation. The metric fluctuations associated with this conversion on the scale of the initial separation between primordial dark energy stars are the seed for metric fluctuations at much larger distances, and are naturally scale-invariant. Thus in our theory we can not only relate the level of metric fluctuations responsible for structure formation to the specific entropy of the CMB, but also explain how these metric fluctuations were created. Zeldovich, on the other hand, didn't attempt to explain how the metric fluctuations were created. The formation of dark energy stars in our model which then act as seeds for the large scale structures observed in today's universe bears some similarity to the proposal by Khlopov, Rubin, and Sakharov [24] that topological defects and "false vacua" in an inflationary universe give rise to massive primordial black holes, which then act as the seeds for galaxy formation. However in our model it is really the collective density fluctuations of the primordial dark energy stars which give rise to these structures.

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