Chern-Simons-Matter Theory in Superspace Formalism

Ashaq Hussain Sofi[1], Sajad Ul Majeed[2]

[1]Department of Physics, National Institute of Technology, Srinagar, India
[2]Department of Physics, University of Kashmir, Srinagar, India
Email: shifs237@gmail.com, mohammadsajadullah99@gmail.com

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Abstract

In this letter, we will study the Chern-Simons-matter theory in Harmonic superspace. It will be shown that this superspace is well suited to write theories with high amount of supersymmetry. This will be done using harmonic variables. The harmonic superspace will have $\mathcal{N}=3$ supersymmetry. It will be argued that it will be possible to analyse this theory in non-anticommutative superspace. The non-anticommutative superspace for this theory will be explicitly constructed.

Keywords

Chern-Simons-Matter Theory, Harmonic Superspace, Supersymmetry, Analytic Superspace

Subject Areas: Applied Physics, Modern Physics

1. Introduction

Harmonic superspace is well suited for analysing theories that have eight real generators of supersymmetry [1]. After complexification eight generators of supersymmetry correspond to the tensor product of a four dimensional Dirac spinors with the fundamental representation of $SU(2)$. The quotient space $SU(2)/U(1) \approx S^2 \simeq CP^1$ is a 2-sphere. This is because $SU(2) \approx S^3$, and we get S^2 after a projection over $U(1) \approx S^1$. Harmonic superspace describes theories with $\mathcal{N}=2$ supersymmetry in four dimensions, in a manifestly covariant manner [2]-[4]. It also describes theories with $\mathcal{N}=1$ supersymmetry in five dimensions, in a manifestly covariant manner [5]-[8]. In three dimensions it can be used to describe theories with $\mathcal{N}=3$ supersymmetry [9] [10]. If we view $SU(2) \approx S^3$ as a $U(1)$ principle bundle over S^2 with nonzero first Chern class, then the fields over S^2 are characterized by an integral charge. Thus, harmonic variables, $u^{i\pm}$ parameterizing the coset $SU(2)/U(1)$, satisfy the the following constraints $u^{+i}u_i^- = 1$, $u^{+i}u_i^+ = u^{-i}u_i^- = 0$. Now the coordinates of harmonic superspace can be written as $z = (x^{ab}, \theta_a^{++}, \theta_a^{--}, \theta_a^0, u_i^\pm)$, where $\theta_a^\pm = \theta_a^{ij}u_i^\pm u_j^\pm$ and $\theta_a^0 = \theta_a^{ij}u_i^+u_j^-$. Analytic superfields, $\Phi_A = \Phi_A(\zeta_A)$ are independent of the θ_a^{--}, and thus satisfy, $D_a^{++}\Phi_A = 0$. The coordinates for the analytic subspace are given by

$$\zeta_A = \left(x_A^{ab}, \theta_a^{++}, \theta_a^0, u_i^{\pm}\right) \tag{1}$$

where

$$x_A^{ab} = \left(\gamma_m\right)^{ab} x_A^m = x^{ab} + i\left(\theta^{++a}\theta^{--b} + \theta^{++b}\theta^{--a}\right) \tag{2}$$

We will now construct a harmonic superspace suitable for dealing with three dimensional theories. It will be shown that this harmonic superspace has $\mathcal{N}=1$ supersymmetry. Then we will impose non-anticommutation of this superspace. It is know that non-anticommuativity breaks some part of the supersymmetry of theory. We will use this non-anticommutative superspace to study a Chern-Simons theory. We will also analyse the gauge transformations of this theory.

2. Harmonic Superspace

We need to define harmonic superspace derivatives using harmonic variables, $u^{i\pm}$ parameterizing the coset $SU(2)/U(1)$. Now the following derivatives are defined,

$$\begin{aligned}
\mathcal{D}^{++} &= \partial^{++} + 2i\theta^{++a}\theta^{0b}\partial_{ab}^A + \theta^{++a}\frac{\partial}{\partial\theta^{0a}} + 2\theta^{0a}\frac{\partial}{\partial\theta^{--a}}, \\
D^{--} &= \partial^{--} - 2i\theta^{--a}\theta^{0b}\partial_{ab}^A + \theta^{--a}\frac{\partial}{\partial\theta^{0a}} + 2\theta^{0a}\frac{\partial}{\partial\theta^{++a}}, \\
D^0 &= \partial^0 + 2\theta^{++a}\frac{\partial}{\partial\theta^{++a}} - 2\theta^{--a}\frac{\partial}{\partial\theta^{--a}},
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
D_a^{--} &= \frac{\partial}{\partial\theta^{++a}} + 2i\theta^{--b}\partial_{ab}^A, \\
D_a^0 &= -\frac{1}{2}\frac{\partial}{\partial\theta^{0a}} + i\theta^{0b}\partial_{ab}^A, \\
D_a^{++} &= \frac{\partial}{\partial\theta^{--a}},
\end{aligned} \tag{4}$$

where the derivatives ∂^{++}, ∂^{--} and ∂^0 are given by

$$\begin{aligned}
\partial^{++} &= u_i^+\frac{\partial}{\partial u_i^-}, \\
\partial^{--} &= u_i^-\frac{\partial}{\partial u_i^+}, \\
\partial^0 &= u_i^+\frac{\partial}{\partial u_i^+} - u_i^-\frac{\partial}{\partial u_i^-}.
\end{aligned} \tag{5}$$

They satisfy the following algebra

$$\begin{aligned}
&\left\{D_a^{++}, D_b^{--}\right\} = 2i\partial_{ab}^A, && \left\{D_a^0, D_b^0\right\} = -i\partial_{ab}^A, \\
&\left[D^{\mp\mp}, D_a^{\pm\pm}\right] = 2D_a^0, && \left[D^0, D_a^{\pm\pm}\right] = \pm 2D_a^{\pm\pm}, \\
&\partial^0 = \left[\partial^{++}, \partial^{--}\right], && \left[D^{++}, D^{--}\right] = D^0. \\
&\left\{D_a^{\pm\pm}, D_b^0\right\} = 0, && \left[\mathcal{D}^{\pm\pm}, D_a^0\right] = D_a^{\pm\pm}.
\end{aligned} \tag{6}$$

The conjugation in the harmonic superspace is defined by

$$\widetilde{\left(u_i^{\pm}\right)} = u^{\pm i}, \quad \widetilde{\left(x_A^{m}\right)} = x_A^m,$$
$$\widetilde{\left(\theta_a^{\pm\pm}\right)} = \theta_a^{\pm\pm}, \quad \widetilde{\left(\theta_a^{0}\right)} = \theta_a^0. \tag{7}$$

The measure in full harmonic superspace is given by

$$\mathrm{d}^9 z = -\frac{1}{16}\mathrm{d}^3 x\left(D^{++}\right)^2\left(D^{--}\right)^2\left(D^0\right)^2 \tag{8}$$

and the measure in analytic superspace is given by

$$\mathrm{d}\zeta^{(-4)} = \frac{1}{4}\mathrm{d}^3 x_A \mathrm{d}u\left(D^{--}\right)^2\left(D^0\right)^2 \tag{9}$$

So, the analytic superspace measure is real $\widetilde{\mathrm{d}\zeta^{(-4)}} = \mathrm{d}\zeta^{(-4)}$ and the full superspace measure is imaginary $\widetilde{\mathrm{d}^9 z} = -\mathrm{d}^9 z$.

3. Deformation

It is now possible to break a part of this supersymmetry by imposing the following anticommutation relationship, $\left\{\theta_a^{--}, \theta_b^{--}\right\} = C_{ab}$. If we do that, we will have to replace the product of all the fields with star product given by

$$V^{--}(z) \star V^{--}(z) = \exp -\frac{1}{2}\Delta V^{++}(z_1) \star V^{++}(z_2)\Big|_{z_1 = z_2 = z} \tag{10}$$

where

$$\Delta = C^{ab}\partial_{1a}^{--}\partial_{2b}^{--} \tag{11}$$

Here this start product maps the non-anticommutative superspace to the usual harmonic superspace. This is a standard technique in non-anticommutativity and it is like the superspace version of Moylar star product. This will break a part of the supersymmetry of the theory. This could have been imposed by a background field, $H = dC$, where $C_{ab} \sim \left(H_{\mu\nu\tau\rho}\gamma^\mu\gamma^\nu\gamma^\tau\gamma^\rho\right)_{ab}$. We could also combine this deformation generated by $C_{a\mu}$. This will modify the add the addition term to the star product by the inclusion of $\Delta' = C^{a\mu}\left(\partial_{1a}^{--}\partial_{2\mu} - \partial_{1\mu}\partial_{2a}^{--}\right)$, apart from the previous factor. However, this new term does not break any supersymmetry.

We will study the Chern-Simons-matter theory in the harmonic superspace. Let the gauge fields corresponding to $SU(N)_k$ be denoted by $\left(V^{++}\right)$. Then, the covariant derivative can be defined as

$$\nabla^{++} q^+ = D^{++} q^+ + V^{++} \star q^+ \tag{12}$$

The action for the Chern-Simons-matter theory can now be written as

$$S = \frac{ik}{4\pi}\mathrm{tr}\sum_{n=2}^{\infty}\frac{(-1)^n}{n}\int \mathrm{d}^3 x \mathrm{d}^6\theta \mathrm{d}u_1 \cdots \mathrm{d}u_n \frac{V^{++}(z,u_1) \star V^{++}(z,u_2) \star \cdots \star V^{++}(z,u_n)}{\left(u_1^+ u_2^+\right)\cdots\left(u_n^+ u_1^+\right)} + \mathrm{tr}\int \mathrm{d}^3 x \mathrm{d}\zeta^{(-4)}\bar{q}^+ \star \nabla^{++} \star q^+ \tag{13}$$

Not all the degrees of freedom of this theory are physical as it is invariant under gauge transformations [11]

$$\begin{aligned}
\delta q^+ &= \Lambda \star q^+, \\
\delta \bar{q}^+ &= -\bar{q}^+ \star \Lambda, \\
\delta V^{++} &= -D^{++}\Lambda_L - \left[V^{++}, \Lambda\right]_\star .
\end{aligned} \tag{14}$$

4. Conclusion

We analysed a Chern-Simons theory in harmonic superspace. This superspace had $\mathcal{N} = 3$ supersymmetry. We also constructed a non-anticommutative harmonic superspace, and analysed this theory using that non-anticommutative harmonic superspace. This broke some of the supersymmetry of this theory. We studied the gauge transformations of this theory in harmonic superspace. It may be noted that it will be interesting to give a vacuum expectation value to one of the scalars in the theory. It is known that if we do that for ABJM theory, we expect that the gauge part of the action to reduce to a deformed super-Yang-Mills theory. We expect that the ABJM theory action transform to an action whose gauge part will be proportional to $W^{++} \star W^{++}$. It would be interesting to analyse what thing happens to Chern-Simons-matter theory, in this context. It may be noted various application of deformed quantum field theories have been analysed, it will thus be interesting to analyse such quantum field theories using the deformation analysed in this paper [12]-[98]. Thus, it will be possible to analyse such a deformation of both field theories and string theory inspired models. It will also be possible to study such deformation of quantum gravity inspired models [99]-[113]. It will be interesting to perform this analysis.

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