Amartya Sen's Peasant Economies: A Review with Examples

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Abstract

This article provides partial mathematical analysis of Amartya Sen's published paper "Peasants and Dualism with or without Surplus Labor". This paper may provide useful illustrations of the applications of mathematics to economics. Here, three portions of Sen's paper "the simplest model, production for a market response and to withdrawal of labor" are discussed in some details. Results of the study are given in mathematical formulations with physical interpretations. An attempt is taken here to make the Sen's paper more interesting to the readers who have desire for detailed mathematical explanations with theoretical analysis.

Keywords

Peasant Economy, Output, Sen, Withdrawal of Labor

Subject Areas: **Mathematical Economics**

1. Introduction

Amartya Kumar Sen is the most important and prolific living philosopher-economist. At present, he is Thomas W. Lamont University Professor and Professor of Economics and Philosophy, Harvard University. He was born in Santiniketan (India) and studied at Calcutta and at Cambridge. He has influential contributions to economic science in the fields of social choice theory, welfare economics, feminist economics, political philosophy, feminist philosophy, identity theory and the theory of justice. He was awarded the Nobel Prize in Economic Sciences in1998 [1].

In this study, we have discussed peasant economies on the basis of Sen's published paper "Peasants and Dualism with or without Surplus Labor" [2]. In 1966, most of the peasants were very poor and some of them were landless. They used old technologies and traditional seeds for cultivation. Some laborers worked on the field only for a poor meal. They worked some cases in agriculture with little or no wages. On the other hand, in 2016, most of the peasants are solvent and use modern technologies. They are using new variety of seeds, insecticides and manure and finding proper irrigation facilities. As a result, they find maximum harvest.

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In this article, we explore elementary mathematical techniques in some details with displaying diagram where necessary. We have chosen this article of Sen for mathematical review because we have observed that we can do some work on it which will be beneficial for the modern peasants. We stress application of mathematics in the Sen's paper so that readers can realize it easily. Although Sen's paper was published in 1966, we thought its usefulness would remain same to some (but very few) peasant seven 50 years later in 2016. We consider here some explicit functions with the stated properties, such as the derivative being positive by Sen. In this review paper, we set two examples to examine various aspects, such as points of equilibrium clearly and in some details.

The objective of the study is to represent mathematical analysis of Sen's paper mentioned above. Although the paper was published in 1966, we thought its importance would remain present to few farmers even in 2016. We hope detailed mathematical analysis will be helpful to the readers those who want to work on peasant family. Main objective of this review paper is to help the peasants of Bangladesh those who are in backward and may be benefited from this study.

2. Literature Review

Amartya Kumar Sen has given peasants economies in his published paper in 1966, where he discusses the economic equilibrium of a peasant family, the effect of surplus labor and withdrawal of labor, dual equilibrium between peasant and capitalist, and efficiency of resource allocation in peasant agriculture [2]. Sen [3] has discussed that food security is based in turn on access to resources, production technologies, environmental and market conditions, non-market food transfers and accumulated food reserves. Dale W. Jorgenson has enlightened the surplus agricultural labor and the development of a dual economyfocusing on the relationship between the degrees of industrialization and the level of economic development [4]. A survey was conducted by Jagdish N. Bhagwati and Sukhamoy Chakravarty on: 1) planning theory and techniques; 2) agriculture, and; 3) foreign trade of Indian economy [5]. Mark R. Rosenzweighas shown that to capture the essential features of rural agriculture and to maintain tractability, a labor market composed of two types of labor, male and female, and three agricultural households; a landless household and two households with different size plots, small and large, of quality standardized land producing a homogeneous agricultural commodity [6]. Abhijit V. Banerjee and Andrew F. Newman has examined the interactions among different institutional arrangements in a general equilibrium model of a modernizing economy [7]. Scale efficiency of Indian farmers is studied by Atanu Sengupta and Subrata Kundu [8]. Haradhan Kumar Mohajan has discussed food, agriculture, nutrition and economic development of Bangladesh [9] [10].

Michael P. Todaro and Stephen C. Smith have revealed that the agricultural progress and rural development in developing nations and expressed the progressive improvement in rural levels through increases in small-farm incomes, output and productivity, along with genuine food security [11]. Paul Spicker, Sonia Alvarez Leguizamón and David Gordon analyzed the female-male wage ratio, and female labor-force participation rate in agriculture. They also discussed about lowland small and medium farm owners and cultivators [12]. Zipporah G. Glass worked on Amartya Sen's model of entitlement and food security which focuses from supply and demand economics towards a household unit of analysis and effect [13]. Mausumi Mahapatro examined the nexus between land, migration and rural differentiation within the context of two villages in rural Bangladesh [14]. M. N. Baiphethi and P. T. Jacobs highlighted that poor households of South Africa access their food from the market, subsistence production and transfers from public programmes [15]. Sophia Murphy has exposed that agriculture had historically not been a global matter, though food has been traded across borders for thousands of years [16].

3. Methodology of the Study

In this study we have used the secondary data and analyze on previous published papers. This is a review paper and discusses the mathematical analysis of Sen's paper "Peasants and Dualism with or without Surplus Labor". In this work we introduce two examples and try to give mathematical framework which (we think) Sen has not provided in detail. We have used techniques of the optimization of differential calculus. We also discussed the geometrical interpretation of mathematical results. In addition we have displayed diagrams where appropriate.

4. Highlights on the Simplest Model

Here we have discussed basic assumptions of Sen's economic equilibrium of peasant model. Suppose a community of identical peasant families each with α working members and β total members $(\beta \geq \alpha)$. Each of the families has some stock of land and capital. The output of the family Q is only function of labor L, *i.e.*, $Q = Q(L)$, which is twice differentiable always and diminishing with marginal productivity of labor. Hence the derivative of $Q(L)$ yields;

$$\frac{\mathrm{d}Q(L)}{\mathrm{d}L} = Q'(L) \tag{1}$$

is the marginal productivity of labor. From our common sense,

$$Q'(L) > 0 \quad \text{and} \quad Q''(L) < 0 . \tag{2}$$

For the maximization output $\bar{Q}$, for $0 \leq L \leq \bar{L}$ and $Q'(\bar{L})$ vanishes (**Figure 1**), *i.e.*,

$$Q'(\bar{L}) = 0 . \tag{3}$$

On the other hand $Q'(L)$ approaches zero asymptotically, while $Q(L)$ approaches $\bar{Q}$ (**Figure 2**), *i.e.*,

$$\lim_{L\to\infty}\left(Q'(L)\right) = 0, \ Q(\infty) = \hat{Q} . \tag{4}$$

The total income (output) of the family, Q, is shared equally among all the members of the family, but the total labor L, is shared equally among all the working members. Let q is the individual income of any member and l is the amount of labor of any working member as,

$$Q = \beta q \quad \text{and} \quad L = \alpha l . \tag{5}$$

Again, every member of the family has a personal utility function $U(q)$ (*i.e.*, $U(q)$ is the same for all members), which is a function of individual income q and every working member has a personal disutility function $V(l)$ (*i.e.*, $V(l)$ is the same for all working members), which is a function of individual labor l. The "disutility" is roughly speaking, the "difficulty" or "inconvenience" of putting in labor of amount l. The function U and V satisfy the following properties:

$$U = U(q), U'(q) > 0, U''(q) \leq 0 \tag{6}$$

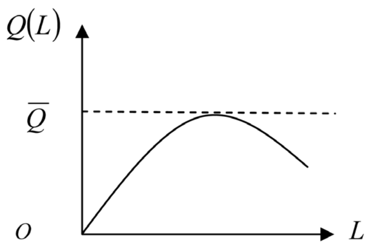

Figure 1. The function $Q(L)$ represents the curve of maximization output $\bar{Q}$.

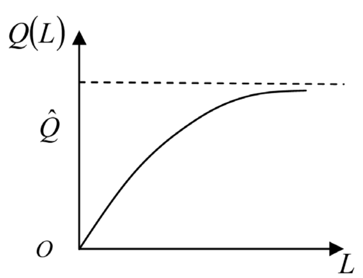

Figure 2. The function $Q(L)$ represents the asymptotic curve of output $\hat{Q}$.

$$V = V(l), V'(l) \ge 0, V''(l) \ge 0. \tag{7}$$

From (6) we see that the marginal utility from income is positive and non-increasing. From (7) we observe that the marginal disutility from labor is non-negative and non-decreasing [2].

Each person's notion of family welfare *W* in a suitable sense is given by the net utility from income and effort of all members taken together attaching the same weight to everyone's happiness. Let a subscript *i* represents the i*th* individual, then the family welfare *W* is given by;

$$W = \sum_{i=1}^{\beta} U_i - \sum_{i=1}^{\alpha} V_i. \tag{8}$$

If it is assumed that all the functions U_i and V_i are the same, then we have,

$$W = \beta U(q) - \alpha V(l) = W(q, l). \tag{9}$$

Each individual could equally well regard *W* as a function of *Q* and *L*, since, $q = \frac{Q}{\beta}$, $l = \frac{L}{\alpha}$ (by (5)). Further, since, *Q* is a function of *L*, we can conclude that *W* is also a function of *L*;

$$W = \beta U(q) - \alpha V(l) = \beta U\left(\frac{Q}{\beta}\right) - \alpha V\left(\frac{L}{\alpha}\right) = \beta U\left(\frac{Q(L)}{\beta}\right) - \alpha V\left(\frac{L}{\alpha}\right) \equiv \hat{W}(L), \text{ (say)}. \tag{10}$$

Assume welfare is maximized by $L = L_0$, then we can write;

$$\frac{\mathrm{d}\hat{W}}{\mathrm{d}L} \equiv \hat{W}'(L) = 0 \tag{11}$$

provided that $\hat{W}''(L) < 0$. Now we can write,

$$\frac{\mathrm{d}\hat{W}}{\mathrm{d}L} = \frac{\partial W}{\partial q} \cdot \frac{\partial q}{\partial L} + \frac{\partial W}{\partial l} \cdot \frac{\partial l}{\partial L} = U'(q) Q'(l) - V'(l), \tag{12}$$

since

$$\frac{\partial W}{\partial q} = \beta U'(q), \quad \frac{\partial q}{\partial L} = \frac{Q'(L)}{\beta}, \quad \frac{\partial W}{\partial l} = -\alpha V'(l) \quad \text{and} \quad \frac{\partial l}{\partial L} = \frac{1}{\alpha}. \tag{13}$$

From (11) and (12) we get;

$$Q'(L) = \frac{V'(l)}{U'(q)} \equiv x \quad \text{(say)}. \tag{14}$$

Sen defined *x* as the "real cost of labor" which indicates that labor is applied up to the point where its marginal product equals the real cost of labor.

5. Illustrative Examples

In the light of above discussion we consider two explicit examples as follows.

5.1. Example A

We make an ad hoc assumptions about the form of the functions $Q(L)$, $V(l)$, $U(q)$ and show that for suitable values of the parameters occurring in these functions, they satisfy the conditions stated above. We then proceed to calculate the maximization point $L = L_0$ etc.

We assume $Q(L)$, $U(q)$ and $V(l)$ to be given by the following expressions:

$$Q(L) = Q_0\left(1 - \mathrm{e}^{-aL}\right) \tag{15a}$$

$$U(q) = U_0 \ln(1 + bq) \tag{15b}$$

$$V(l) = V_0\left(\mathrm{e}^{kl} - 1\right) \tag{15c}$$

where Q_0, U_0, V_0, a, k and b are positive constants. Note that, $Q(0)=0$, $V(0)=0$ and $U(0)=0$. First and second order partial derivatives of (15a, b, c) give;

$$Q'(L)=Q_0 a\mathrm{e}^{-aL}>0,\ \ Q''(L)=-Q_0 a^2\mathrm{e}^{-aL}<0 \tag{16a}$$

$$U'(q)=\frac{U_0 b}{1+bq}>0,\ \ U''(q)=\frac{-U_0 b^2}{(1+bq)^2}<0 \tag{16b}$$

$$V'(l)=V_0 k\mathrm{e}^{kl}>0,\ \ V''(l)=V_0 k^2\mathrm{e}^{kl}>0. \tag{16c}$$

Also the conditions (2), (6) and (7) are all satisfied if all the constants are positive. Again, (15a) and (16a) give;

$$\lim_{L\to\infty}Q'(L)=\lim_{L\to\infty}\left(Q_0 a\mathrm{e}^{-aL}\right)=0,\ \ Q(\infty)=Q_0. \tag{17}$$

From (10) we get (15a, b, c) for the welfare function W as;

$$\begin{aligned} W &= \beta U(q)-\alpha V(l)=\beta U_0\ln(1+bq)-\alpha V_0\left(\mathrm{e}^{kl}-1\right) \\ &= \beta U_0\ln\left(1+\frac{b}{\beta}\right)Q_0\left(1-\mathrm{e}^{-aL}\right)-\alpha V_0\left(\mathrm{e}^{kL/\alpha}-1\right)\equiv\hat{W}(L). \end{aligned} \tag{18}$$

Now we get,

$$\frac{\mathrm{d}\hat{W}}{\mathrm{d}L}=\frac{ab\beta Q_0U_0\mathrm{e}^{-aL}}{\beta+bQ_0\left(1-\mathrm{e}^{-aL}\right)}-V_0k\mathrm{e}^{kL/\alpha}=0 \tag{19}$$

which is the same as the explicit form as (11). Now from (19) we get;

$$\begin{aligned} &ab\beta Q_0U_0\mathrm{e}^{-aL}-V_0\beta k\mathrm{e}^{kL/\alpha}-bV_0Q_0k\mathrm{e}^{kL/\alpha}+bV_0Q_0k\mathrm{e}^{-aL+kL/\alpha}=0 \\ &ab\beta Q_0U_0\mathrm{e}^{-aL-kL/\alpha}+bV_0Q_0k\mathrm{e}^{-aL}-kV_0\left(\beta+bQ_0\right)=0. \end{aligned} \tag{20}$$

Now we define a new variable X in terms of L and choose the constants a and k as;

$$X=\mathrm{e}^{-aL},\ \ k=a\alpha. \tag{21}$$

Using (21), Equation (20) becomes the quadratic equation for X as;

$$\left(ab\beta Q_0U_0\right)X^2+bkV_0Q_0X-kV_0\left(\beta+bQ_0\right)=0. \tag{22}$$

Solution of (22) becomes;

$$\begin{aligned} X &= \frac{-bkV_0Q_0\pm\left\{\left(bkV_0Q_0\right)^2+4abk\beta Q_0U_0V_0\left(\beta+bQ_0\right)\right\}^{1/2}}{2ab\beta Q_0U_0} \\ &= \frac{kV_0}{2a\beta U_0}\left[-1\pm\left\{1+\frac{4a\beta U_0}{kV_0}\left(1+\frac{\beta}{bQ_0}\right)\right\}^{1/2}\right]. \end{aligned} \tag{23}$$

For the relevant solution we should consider only positive sign of (23), then we get,

$$X=\frac{kV_0}{2a\beta U_0}\left[-1+\left\{1+\frac{4a\beta U_0}{kV_0}\left(1+\frac{\beta}{bQ_0}\right)\right\}^{1/2}\right]. \tag{24}$$

$$X=\frac{2}{A}\left[-1+\left\{1+A\left(1+\frac{\beta}{bQ_0}\right)\right\}^{1/2}\right]$$

$$\text{where}\ \ A\equiv\frac{4\beta U_0}{\alpha V_0}. \tag{25}$$

For real solution we get from (21);
$0 < X < 1$ and the constants α, b, U_0 and Q_0 must satisfy the following inequality;

$$bQ_0U_0 > \alpha V_0 \tag{25a}$$

Inequality (25a) is free of β, it has a wider meaning than simply facilitating the derivation of an exact and explicit solution which is not clear at this stage. The solution can be studied in detail by taking specific, reasonable sets of numerical values of the constants occurring in the solution.

5.2. Example B

Here we made ad hoc assumptions about the form of the function, and show that these satisfy the relevant conditions, and then proceed via the corresponding welfare function, to obtain the value of L which maximizes this function at $L = \bar{L}$ and $0 \le L \le \bar{L}$. We consider the functions $Q(L)$, $V(l)$, $U(q)$ be as follows:

$$Q(L) = a(2\bar{L} - L)L \tag{26a}$$

$$U(q) = U_0 \ln(1 + bq) \tag{26b}$$

$$V(l) = V_0 l \big/ (\bar{l} - l) = V_0 L \big/ (\bar{L} - L), \tag{26c}$$

since $l = L/\alpha$ and $\bar{l} = \bar{L}/\alpha$. Throughout this example we confine ourselves to the interval $0 \le l \le \bar{l}$ or $0 \le L \le \bar{L}$. Here (26b) is same as (15b) of example A. From (26c) we get;

$$V'(l) = V_0 \bar{l} \big/ (\bar{l} - l)^2 > 0, \ V''(l) = 2V_0 \bar{l} \big/ (\bar{l} - l)^3 > 0. \tag{27}$$

We observe that if l tends to $\bar{l}$, the three functions $V(l)$, $V'(l)$ and $V''(l)$ tend to infinity. Hence it is reasonable that it is difficult for an individual to reach the amount of labor given by $l = \bar{l}$. For this reason we confine the values of l are confined to the interval $0 \le l \le \bar{l}$ and also those of L are to the interval $0 \le L \le \bar{L}$ and $\bar{L} = \alpha \bar{l}$.

From (26a) we get;

$$Q'(L) = 2a(\bar{L} - L) > 0, \ Q''(L) = -2a < 0 \tag{28}$$

for $0 \le L \le \bar{L}$ and $Q'(L) = 0$ at $L = \bar{L}$ as we expect. Maximum of Q is given by $\bar{Q} = Q(\bar{L})$; relate L with a as follows:

$$\bar{Q} = Q(\bar{L}) = a\bar{L}^2. \tag{29}$$

Now we can express the function $V(l)$ in **Figure 3**. Here we have measured the output Q in terms of money but it is not the case, because Q can be measured in some other units, such as, in kg, liter etc. (e.g., if the peasants themselves consume their own products and calculate in such units). If Q is measured in money, then q must be in money also. If $U(q)$ is measured in some units, then U_0 must be measured in the same unit, so that $\ln(1+bq)$ must be dimensionless, that is, a pure number. Obviously bq must be a pure number, which

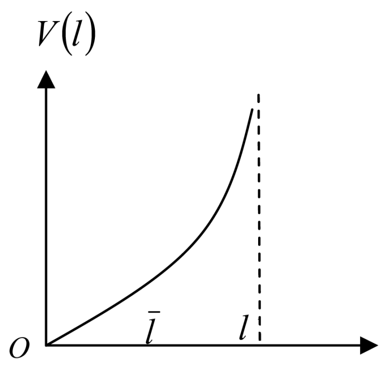

Figure 3. The function $V(l)$ is asymptote at $l = \bar{l}$.

indicates that b must have dimension of inverse money. Similarly, if labor L is measured in hours, then the constant "a" has the dimension of money/hour^2, etc. To avoid the different form of dimension we avoid dimension in our calculations. The welfare function W is given by;

$$W = \beta U(q) - \alpha V(l) = \beta U_0 \ln\left[1 + \frac{ba}{\beta}\left(2\bar{L} - L\right)L\right] - \alpha V_0 \frac{L}{\bar{L} - L} \equiv \hat{W}(L). \tag{30}$$

From (30) derivative of $\hat{W}(L)$ with respect to L gives;

$$\frac{\mathrm{d}\hat{W}}{\mathrm{d}L} = \frac{2abU_0\left(\bar{L} - L\right)}{\left(1 + \frac{ab}{\beta}\right)\left(2\bar{L} - L\right)L} - \alpha V_0 \frac{\bar{L}}{\left(\bar{L} - L\right)^2}. \tag{31}$$

For maximum welfare (*i.e.*, $\frac{\mathrm{d}\hat{W}}{\mathrm{d}L} = 0$) we get from (31);

$$2abU_0\left(\bar{L} - L\right)^3 - \alpha V_0 \bar{L}\left(1 + \frac{ab}{\beta}\right)\left(2\bar{L} - L\right)L = 0$$

$$L^3 - \left(3 + \frac{\alpha V_0}{2\beta U_0}\right)\bar{L}L^2 + \left(3 + \frac{\alpha V_0}{2\beta U_0}\right)\bar{L}^2 L + \left(\frac{\alpha V_0}{2abU_0}\left(\bar{L}L - L^2\right) - \bar{L}^2\right)\bar{L} = 0. \tag{32}$$

We know that a cubic equation can be solved in radicals in terms of the coefficients. We observe that solution of (32) will be complicated, so that we cannot find exact and necessary information from it. In this situation we proceed in an indirect way. First, we introduce some preliminary remarks.

The property of a cubic equation that it has three roots, all real, or one real and two complex. In this example we are confined to find a root in the interval $0 \le L \le \bar{L}$ that must satisfy that second order derivative of welfare function will be negative, since it must maximize W.

From (30) we see that welfare function $\hat{W}(L)$ vanishes at $L = 0$ and $\hat{W}(0) = 0$. It is reasonable in the present situation, since if there is no labor, there is no welfare (income). As L increases from zero, one expects welfare to rise from value zero. We can proceed if the first derivative of (31) is positive at $L = 0$;

$$\left(\frac{\mathrm{d}\hat{W}}{\mathrm{d}L}\right)_{L=0} = 2abU_0\bar{L} - \frac{\alpha V_0}{\bar{L}} > 0. \tag{33}$$

i.e., $2bU_0\left(a\bar{L}^2\right) - \alpha V_0 > 0$

$$2bU_0\bar{Q} > \alpha V_0, \text{ (by (29), } a\bar{L}^2 = \bar{Q}\text{)}. \tag{34}$$

Here the constant U_0 is a sort of measure of the utility to the individual and hence to the family, while V_0 is a measure of disutility of labor to the working members. For any fixed U_0, the inequality (34) will not be valid if V_0 become too large, in such a situation welfare will not rise from the value zero. This happen if the potential working members have some chronic illness, so that labor becomes prohibitively difficult for large V_0. Finally, we conclude that inequality (33) is satisfied when L increases from zero, and the welfare function also increase from zero.

Now the second derivative of $\hat{W}$ gives;

$$\frac{\mathrm{d}^2\hat{W}}{\mathrm{d}L^2} = -\frac{2abU_0}{1 + \frac{ab}{\beta}\left(2\bar{L} - L\right)L} - \frac{4a^2b^2U_0\left(\bar{L} - L^2\right)^2}{\beta\left\{1 + \frac{ab}{\beta}\left(2\bar{L} - L\right)L\right\}^2} - \frac{2aV_0L}{\left(\bar{L} - L\right)^3}. \tag{35}$$

We observe that this function is negative for all values of L in our expected interval $0 \le L \le \bar{L}$. Thus, as $\hat{W}(L)$ increases from zero at $L = 0$, its rate of increase diminishes and there will be a point in $0 \le L \le \bar{L}$ at which $\hat{W}(L)$ reaches its maximum value. We assume that the $\hat{W}(L)$ is maximum at $L = \frac{1}{2}\bar{L}$. We will also examine if reasonable parameter values can be found that will achieve this circumstance. Now we write (32) using (25) and (29) as follows:

$$L^3-\left(3+\frac{2}{k}\right)\bar{L}L^2+\left(3+\frac{4}{k}\right)\bar{L}^2L-\frac{1}{a}\left(\bar{Q}-\frac{2\beta}{bk}\right)\bar{L}=0. \tag{36}$$

Since $L=\frac{1}{2}\bar{L}$ is a root of Equation (36), putting $L=\frac{1}{2}\bar{L}$ we get;

$$\frac{1}{8}\bar{L}^3-\frac{1}{4}\left(3+\frac{2}{k}\right)\bar{L}^3+\frac{1}{2}\left(3+\frac{2}{k}\right)\bar{L}^3-\frac{1}{\bar{Q}}\left(\bar{Q}-\frac{2\beta}{bk}\right)\bar{L}^3=0$$

$$\frac{1}{8}-\frac{1}{4}\left(3+\frac{2}{k}\right)+\frac{1}{2}\left(3+\frac{2}{k}\right)-1+\frac{1}{\bar{Q}}\frac{2\beta}{bk\bar{Q}}=0$$

$$\frac{3}{2k}+\frac{2\beta}{bk\bar{Q}}=\frac{1}{8}. \tag{37}$$

From (34) we get;

$$\alpha V_0<2b\bar{Q}U_0$$

$$\frac{2\beta}{b\bar{Q}}\cdot\frac{\alpha V_0}{4\beta U_0}<1$$

$$\frac{2\beta}{b\bar{Q}k}<1,\ \text{by (23)}. \tag{38}$$

Since $k>0$, so that $\frac{3}{2k}>1$. Hence from (37) we get the strict inequality,

$$\frac{2\beta}{b\bar{Q}k}<\frac{1}{8}. \tag{39}$$

But we need a more consistent value and we choose (39) for our convenience way as follows:

$$\frac{2\beta}{b\bar{Q}k}<\frac{1}{32}. \tag{40}$$

Using (37) to (40) we can write (36) in a more convenience way as solvable form as follows:

$$L^3-\frac{25}{8}\bar{L}L^2+\frac{26}{8}\bar{L}^2L-\frac{31}{32}\bar{L}^3=0. \tag{41}$$

Since $L=\frac{1}{2}\bar{L}$ is a solution of (41) we get;

$$L^2\left(L-\frac{1}{2}\bar{L}\right)-\frac{21}{8}L\bar{L}\left(L-\frac{1}{2}\bar{L}\right)+\frac{31}{16}\bar{L}^2\left(L-\frac{1}{2}\bar{L}\right)=0$$

$$L=\frac{1}{2}\bar{L},\ L^2-\frac{21}{8}L\bar{L}+\frac{31}{16}\bar{L}^2=0. \tag{42}$$

Solution of 2nd equation of (42) is;

$$L=\frac{1}{16}\left(21\pm i\sqrt{55}\right)\bar{L}. \tag{42a}$$

Hence, $L=\frac{1}{2}\bar{L}$ is the only real root at which the welfare function $\hat{W}(L)$ is maximum. We can write welfare function (30) as follows:

$$\hat{W}(L)=W_1(L)-W_2(L)$$

where,

$$W_1(L) = \beta U_0 \ln\left[1+\frac{ab}{\beta}(2\bar{L}-L)L\right];\ W_2(L) = \frac{\alpha V_0 L}{\bar{L}-L}. \tag{43}$$

Since $\hat{W}(0)=0$, so that $W_1(0)=W_2(0)=0$, which implies that there is no welfare if there is no labor. Derivative of (43) gives,

$$\frac{\mathrm{d}W_1}{\mathrm{d}L} = \frac{2abU_0(\bar{L}-L)}{1+\frac{ab}{\beta}(2\bar{L}-L)L} \text{ and } \frac{\mathrm{d}W_2}{\mathrm{d}L} = \frac{aV_0\bar{L}}{(\bar{L}-L)^2}. \tag{44}$$

For $L=0$ (44) becomes;

$$\left(\frac{\mathrm{d}W_1}{\mathrm{d}L}\right)_{L=0} = 2abU_0\bar{L},\ \left(\frac{\mathrm{d}W_2}{\mathrm{d}L}\right)_{L=0} = \frac{\alpha V_0}{\bar{L}}. \tag{45}$$

From (44) and (45) we have two properties as;

$$\left(\frac{\mathrm{d}W_1}{\mathrm{d}L}\right)_{L=0} > \left(\frac{dW_2}{\mathrm{d}L}\right)_{L=0};\ \left(\frac{\mathrm{d}W_1}{\mathrm{d}L}\right)_{L=0} < \left(\frac{dW_2}{\mathrm{d}L}\right)_{L=0}. \tag{46}$$

$$i.e.,\ 2abU_0\bar{L} > \frac{\alpha V_0}{\bar{L}};\ 2abU_0\bar{L} < \frac{\alpha V_0}{\bar{L}}. \tag{47}$$

We represent (47) in **Figure 4**, which indicates $\theta_1 > \theta_2$ and **Figure 5**, which indicates $\theta_1 < \theta_2$ respectively. The broken lines are tangents to the curves $W_1(L)$ and $W_2(L)$ at $L=0$, if they make angles θ_1 and θ_2 respectively with the positive *L*-axis, then clearly,

$$\tan\theta_1 = 2abU_0\bar{L};\ \tan\theta_2 = \frac{\alpha V_0}{\bar{L}}. \tag{48}$$

Again $\frac{\mathrm{d}^2W_2}{\mathrm{d}L^2} = 2\alpha V_0\frac{\bar{L}}{(\bar{L}-L)^3}$ is positive throughout the interval $0 \le L \le \bar{L}$ and tends to infinity as $\bar{L}$ is

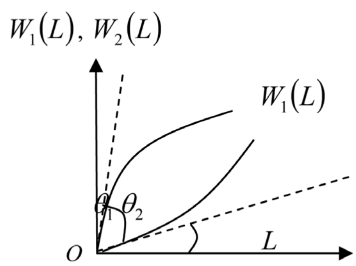

Figure 4. The behavior of the functions $W_1(L)$ and $W_2(L)$ for $\theta_1 > \theta_2$.

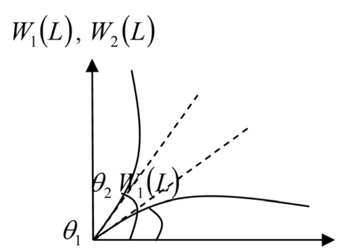

Figure 5. The behavior of the functions $W_1(L)$ and $W_2(L)$ for $\theta_1 < \theta_2$.

approached from below. Since $W_2(L)$ occurs with a negative sign in the expression for $\hat{W}(L)$ in (43), it is this property of $\frac{\mathrm{d}^2W_2}{\mathrm{d}L^2}$ that makes $\frac{\mathrm{d}^2\hat{W}}{\mathrm{d}L^2}$ negative throughout the interval $0 \le L \le \bar{L}$, as noted earlier. Again vanishes at $L = \bar{L}$ at which $\frac{\mathrm{d}^2W_1}{\mathrm{d}L^2} < 0$ (by first two terms of (35)), so that $L = \bar{L}$ is a maximum of $W_1(L)$ (**Figure 6**). The above analysis has some intrinsic, wider interest, since the welfare function generally consists of a positive term representing the utility of the whole family, and a negative term incorporating the disutility of the working members. Another reason for carrying out the above analysis in some detail is to display a mildly pathological situation which nevertheless can be given a reasonable interpretation.

Let us fix the values of a, b, α, β, U_0 and choose two values of V_0 denoted by $V_0^{(1)}, V_0^{(2)}$, such that;

$$2abU_0\bar{L} > \frac{\alpha V_0^{(1)}}{\bar{L}};\ 2abU_0\bar{L} < \frac{\alpha V_0^{(2)}}{\bar{L}} \tag{49}$$

and set $W_1(L)$ as defined by (43) as above. Now we define the corresponding welfare functions as follows:

$$\hat{W}^{(1)}(L) = W_1(L) - W_2^{(1)}(L);\ \hat{W}^{(2)}(L) = W_2(L) - W_2^{(2)}(L). \tag{50}$$

For $V = V_0^{(1)}$, the welfare function $\hat{W}^{(1)}(L)$ begins to rise with L from the value zero at $L = 0$, and for $V = V_0^{(2)}$, the welfare function $\hat{W}^{(2)}(L)$ begins to diminish from the value zero as L increase from $L = 0$. We have,

$$W_1(L) = \beta U_0 \ln\left\{1 + \frac{ab}{\beta}\left(2\bar{L} - L\right)L\right\}. \tag{51}$$

Let, $h(L) = 1 + \frac{ab}{\beta}\left(2\bar{L} - L\right)L$, then,

$$W_1(L) = \beta U_0 \ln\left(h(L)\right). \tag{52}$$

The quadratic $h(L)$ vanishes at $L = L_1$ and $L = L_2$ is given by;

$$L_1 = \bar{L} - \left(\bar{L}^2 + \frac{\beta}{ab}\right)^{1/2};\ L_1 = \bar{L} + \left(\bar{L}^2 + \frac{\beta}{ab}\right)^{1/2} \tag{53}$$

with $L_1 < 0$ and $L_2 > 2\bar{L}$ (**Figure 6**). Moreover $h(L) = 1$ at $L = 0, 2\bar{L}$, where vanishes, $h(L)$ is maxi-

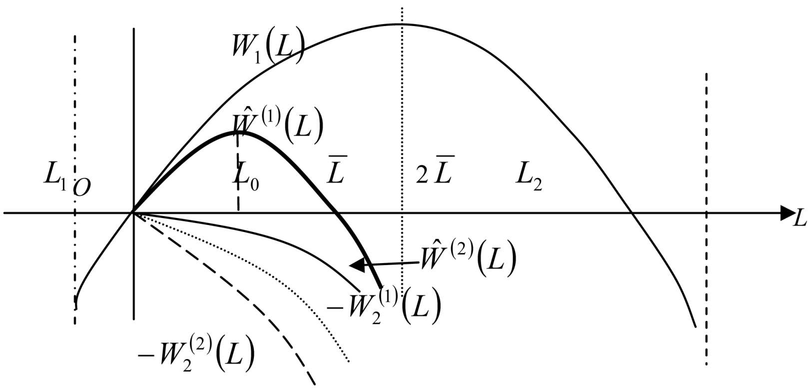

Figure 6. Nature of the functions $W_1(L)$, $\hat{W}^{(1)}(L)$, $\hat{W}^{(2)}(L)$, $-W_2^{(1)}(L)$ and $-W_2^{(2)}(L)$ are displayed. The welfare function $\hat{W}^{(1)}(L)$ has a positive maximum at $L = L_0$, but $\hat{W}^{(2)}(L)$ decreases from the value zero at $L = 0$ to negative values and therefore has no maximum in $0 \le L \le \bar{L}$.

mum at $L=\bar{L}$, with $h(L)=1+\frac{ab}{\beta}\bar{L}^2=1+\frac{\bar{Q}b}{\beta}>0$, so that, $W_1(L)>0$. Since $h(L)$ vanishes at $L=L_1,L_2$; $\ln(h(L))$ and $W_1(L)$ tends to infinity at these values.

Now consider the mild pathological situation. For this we consider (47) the in equation as equation,

$$2abU_0\bar{L}=\frac{\alpha V_0}{\bar{L}}$$

$$2bU_0\bar{Q}=\alpha V_0, \text{ (by (29), } \bar{Q}=a\bar{L}^2\text{)}. \tag{54}$$

Using (54) in (32) we get;

$$L\left(L^2-\left(3+\frac{b\bar{Q}}{\beta}\right)\bar{L}L\right)+\left(3+\frac{2b\bar{Q}}{\beta}\right)\bar{L}^2=0. \tag{55}$$

The solutions of (55) are;

$$L=0,\quad L=\frac{1}{2}\left[\left(3+\frac{b\bar{Q}}{\beta}\right)\pm\left\{\left(3+\frac{b\bar{Q}}{\beta}\right)^2-4\left(3+\frac{2b\bar{Q}}{\beta}\right)\right\}^{1/2}\right]$$

$$L=\frac{1}{2}\left[\left(3+\frac{b\bar{Q}}{\beta}\right)\pm\left\{\left(\frac{b\bar{Q}}{\beta}-3\right)\left(\frac{b\bar{Q}}{\beta}+1\right)\right\}^{1/2}\right]. \tag{56}$$

As we have seen earlier that two roots of (56) are complex, let us now choose $b,\bar{Q},\beta$ as, $0<\frac{b\bar{Q}}{\beta}<3$. Again since $\frac{\mathrm{d}^2\hat{W}}{\mathrm{d}L^2}<0$ at $L=0$ (from (35)), the point $L=0$ in this case is only maximum in the range $0\le L\le\bar{L}$ of the function $\hat{W}(L)$. Hence no welfare is a genuine maximum at no labor (**Figure 7**).

6. Review on "Production for a Market"

A. K. Sen has considered the circumstance when the product Q is not directly useable by the peasants, so it is exchanged for goods directly enjoyable by the peasants. Also it may happen that part of the product Q is used while the rest is exchanged for other goods. If the whole amount C of the new product, the individual share being $c=\frac{C}{\beta}$, we can define as a (section 5) a utility function of the same type that is, a function of c;

$$U=U(c),\ U'(C)>0,\ U''(C)\le 0. \tag{57}$$

The price of output Q in terms of C is p per unit;

$$C=Qp=\beta c. \tag{58}$$

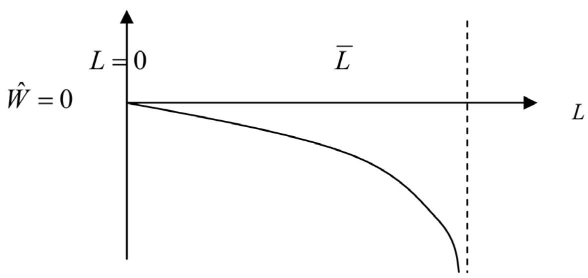

Figure 7. Zero welfare (no welfare) is a genuine maximum at no labor ($L=0$), of the welfare function $\hat{W}(L)$.

So that the maximum of the family welfare is given by;

$$Q'(L) = \frac{V'(l)}{pU'(c)}. \tag{59}$$

Let us now consider a situation in which a part of the product Q is sold and a part is consumed. Individually, C amount of the purchased commodity and q of the self product one is enjoyed per member. Let y be the properties of output that is marketed. Sen defines a utility function with the following properties;

$$U = U(c,q);\ U_q > 0,\ U_c > 0,\ U_{qq} \le 0,\ U_{cc} \le 0;$$

$$\left.\begin{aligned} U_q(c,q) &\ge U_q(\lambda c, \lambda q), \\ U_c(c,q) &\ge U_c(\lambda c, \lambda q) \end{aligned}\right\} \lambda > 1. \tag{60}$$

Again we have;

$$Q(1-y) = \beta q,\ C = Qyp = \beta c \tag{61}$$

with allocation rules;

$$U_q = pU_c;\ Q'(L) = \frac{V'(L)}{U_q}. \tag{62}$$

We have also used the same form of the utility function for both of the examples, A and B. Now we consider the utility function,

$$U(c,q) = U_0 \ln\{1 + b(c + pq)\}. \tag{63}$$

Taking derivatives of (63) with respect to q we get;

$$U_q(c,q) = \frac{bpU_0}{1 + b(c + pq)};\ U_q(\lambda c, \lambda q) = \frac{bpU_0}{1 + b\lambda(c + pq)},\ \text{ if } \lambda > 1. \tag{64}$$

Hence from (64) we have;

$$U_q(c,q) \ge U_q(\lambda c, \lambda q). \tag{65}$$

We observe that (65) agrees with (60) and also agrees with examples A and B.

7. Discussion on Response to Withdrawal of Labor

Sen also discusses the problem of surplus labor and response of peasant output to withdrawal of labor. The surplus labor is defined as that part of the labor force in this peasant economy that can be removed without reducing the total amount of output produced, even when the amount of other factors is not changed [2]. Now from (13) in slightly different form we get;

$$V'(L) = Q'(L)U'(q) \tag{66a}$$

$$V'\left(\frac{L}{\alpha}\right) = Q'(L)U'(Q(L)/\beta) \tag{66b}$$

where (66a) is an equation but not identity. Here maximization of welfare function occurs at $L = L_0$. We assume (66a) to be valid for all $l = \frac{L}{\alpha}$, $q = Q(L)/\beta$, etc. Taking derivatives of both sides of (66a) with respect to l we get;

$$\begin{aligned} V''(l) &= \frac{\mathrm{d}Q'(L)}{\mathrm{d}l}U'(q) + Q'(L)\frac{\mathrm{d}U'(q)}{\mathrm{d}l} \\ &= \alpha Q''(L)U'(q) + \frac{\alpha}{\beta}(Q'(L))^2 U''(q). \end{aligned} \tag{67}$$

Now if $Q''(L)\le 0$, $U'(q)>0$, $U''(q)\le 0$, then (67) gives $V''(l)\le 0$, which violates (7), unless $Q''(L)=V''(L)=V''(l)=0$, which is not necessarily the case. Hence (66a) is indeed an equation. Sen envisages a situation, in which the ratio of total number of members to working members is constant, denoted by *K*;

$$\beta = K\alpha . \tag{68}$$

So that when one working member leaves, he provides support for *K* members (including himself) and so the peasant family is left with one less working member and *K* less consuming ones.

Taking derivatives of (14) with respect to α we get;

$$\frac{\mathrm{d}x}{\mathrm{d}\alpha}=\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(\frac{V'(l)}{U'(q)}\right)=\frac{\mathrm{d}V'(l)}{\mathrm{d}\alpha}\cdot\frac{1}{U'(q)}-V'(l)\cdot\frac{1}{\left(U'(q)\right)^2}\cdot\frac{\mathrm{d}U'(q)}{\mathrm{d}\alpha}. \tag{69}$$

We have,

$$\frac{\mathrm{d}V'(l)}{\mathrm{d}\alpha}=\frac{\mathrm{d}V'(l)}{\mathrm{d}l}\cdot\frac{\mathrm{d}l}{\mathrm{d}\alpha}=V''(l)\left(\frac{\alpha\dfrac{\mathrm{d}L}{\mathrm{d}\alpha}-L}{\alpha^2}\right). \tag{70}$$

$$\begin{aligned}\frac{\mathrm{d}U'(q)}{\mathrm{d}\alpha}&=\frac{\mathrm{d}U'(q)}{\mathrm{d}Q}\cdot\frac{\mathrm{d}Q}{\mathrm{d}\alpha}=U''(q)\cdot\frac{\mathrm{d}q}{\mathrm{d}Q}\cdot\frac{\mathrm{d}Q}{\mathrm{d}\alpha}=U''(q)\cdot\frac{\mathrm{d}(Q/\beta)}{\mathrm{d}Q}\cdot\frac{\mathrm{d}Q}{\mathrm{d}\alpha}\\&=U''(q)\cdot\frac{\mathrm{d}(Q/K\alpha)}{\mathrm{d}Q}\cdot\frac{\mathrm{d}Q}{\mathrm{d}\alpha}=U''(q)\cdot\left(\frac{1}{\beta}\frac{\mathrm{d}Q}{\mathrm{d}\alpha}-\frac{Q}{\beta\alpha}\right)\\&=U''(q)\cdot\left(\beta Q'(L)\frac{\mathrm{d}L}{\mathrm{d}\alpha}-KQ\right)\Big/\beta^2.\end{aligned} \tag{71}$$

Differentiating (14), $x=Q'(L)$, with respect to α we get;

$$\frac{\mathrm{d}x}{\mathrm{d}\alpha}=Q''(L)\frac{\mathrm{d}L}{\mathrm{d}\alpha}. \tag{72}$$

Using (70) to (72) in (69) we get;

$$Q''(L)\frac{\mathrm{d}L}{\mathrm{d}\alpha}=\frac{1}{U'(q)}\left\{V''(l)\left(\frac{\alpha\dfrac{\mathrm{d}L}{\mathrm{d}\alpha}-L}{\alpha^2}\right)\right\}-V'(l)\cdot\frac{1}{\left(U'(q)\right)^2}\left[U''(q)\cdot\left(\beta Q'(L)\frac{\mathrm{d}L}{\mathrm{d}\alpha}-KQ\right)\Big/\beta^2\right]. \tag{73}$$

Simplifying (73) we get;

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}\left[-Q''(L)+\frac{1}{\alpha}\frac{V''(l)}{U'(q)}-\frac{1}{\beta}\frac{V'(l)Q'(L)}{\left(U'(q)\right)^2}U''(q)\right]=\frac{L}{\alpha^2}\cdot\frac{V''(l)}{U'(q)}-\frac{K}{\beta^2}\cdot Q\cdot\frac{V'(l)U''(q)}{\left(U'(q)\right)^2}. \tag{74}$$

Using (14), $U'(q)=\dfrac{V'(l)}{Q'(l)}$ and multiplying by $\dfrac{\alpha\beta}{Q'(l)}$ we get from (74);

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}=\frac{KL\cdot\dfrac{V''(l)}{V'(l)}-Q\cdot\dfrac{U''(q)}{U'(q)}}{\beta\dfrac{V''(l)}{V'(q)}-\alpha\beta\dfrac{Q''(L)}{Q'(L)}-\alpha Q'(L)\dfrac{U''(q)}{U'(q)}}. \tag{75}$$

This is Sen's Equation (31) but we have derived the equation more detailed than Sen has. Sen introduces some elasticities as follows [2].

E is the elasticity of output with respect to the number of working members, *m* is the absolute value of the elasticity of the marginal utility of income with respect to individual income, *n* is the elasticity of marginal dis-utility from work with respect to individual hours of work, *G* is the elasticity of output with respect to hours of labor, g is the absolute value of the elasticity of the marginal product of labor with respect to hours of labor. These quantities are defined by the following relations:

$$E=\frac{\alpha}{Q}\cdot\frac{\mathrm{d}Q}{\mathrm{d}\alpha};\ \ n=l\cdot\frac{V''(l)}{V'(l)};\ \ m=-q\frac{U''(q)}{U'(q)};\ \ G=L\cdot\frac{Q'(L)}{Q};\ \ g=-L\cdot\frac{Q''(L)}{Q'(L)}. \tag{76}$$

$$\text{Also we have, } \frac{\mathrm{d}Q}{\mathrm{d}\alpha}=Q'(L)\frac{\mathrm{d}L}{\mathrm{d}\alpha}. \tag{77}$$

Using (5), (76) and (77) in (75) we get the response equation;

$$E\cdot\frac{1}{\alpha}\cdot\frac{Q}{Q'}=\frac{Kn\frac{L}{l}+\frac{Q}{q}m}{\frac{\beta}{l}\cdot n+\frac{\alpha}{L}\cdot\beta g+\alpha m\frac{Q'}{q}}$$

$$E=G\left(\frac{m+n}{n+Gm+g}\right). \tag{78}$$

Now we consider the example A. From (15a) we get;

$$\frac{\mathrm{d}Q}{\mathrm{d}\alpha}=Q_0a\mathrm{e}^{-aL}\frac{\mathrm{d}L}{\mathrm{d}\alpha}. \tag{79}$$

From (76), using (15a) we get;

$$E=\frac{\alpha a\mathrm{e}^{-aL}}{1-\mathrm{e}^{-aL}}\frac{\mathrm{d}L}{\mathrm{d}\alpha}. \tag{80}$$

From (76) and (15a), (16a, b, c) we get;

$$G=\frac{aL\mathrm{e}^{-aL}}{1-\mathrm{e}^{-aL}};\quad n=lK;\ \ m=\frac{qb}{1+qb};\ \ g=aL. \tag{81}$$

Using (79) to (81) in (78) we get;

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}=l\frac{lK+\frac{qb}{1+qb}}{lK+\frac{qbaL}{1+qb}\cdot\frac{\mathrm{e}^{-aL}}{1-\mathrm{e}^{-aL}}+aL}. \tag{82}$$

If $E=G$ we get from (76);

$$\frac{\mathrm{d}L}{\mathrm{d}\alpha}=\frac{L}{\alpha}=l. \tag{83}$$

Using (83) and (21), (82) becomes;

$$\frac{qb}{1+qb}\cdot\frac{1-2X}{1-X}=aL. \tag{84}$$

In this case L be very large to satisfy (84) and marginal disutility schedule approach to the vertical position, which of course will tend to toward constancy of the change in labor hours proportional to the change in the number of working people [2].

Now we consider a special case for $n=0$ and $E=G$ in (78) we get;

$$m=\frac{g}{1-G}. \tag{85}$$

Using (76) and (21), (85) becomes;

$$\frac{qb}{1+qb}=\frac{aL(1-X)}{1-(1+aL)X} \tag{86}$$

Equation (86) implies $\frac{\mathrm{d}l}{\mathrm{d}\alpha}=0$ and $\frac{\mathrm{d}}{\mathrm{d}\alpha}\left(Q'(L)\cdot\left(U'(q)\right)\right)=0$. Moreover (86) represents that when some people are withdrawn from the peasant economy, with an unchanged number of hours of work per person, the marginal physical return work will increase [2].

8. Conclusion

In this study, we have analyzed some parts of Sen's paper "Peasants and Dualism with or without Surplus Labor" with detail mathematical calculations. We have tried to give the physical interpretations of the mathematical results clearly (as far as possible). We hope the readers will feel comport when they study this article. We have not discussed all the portions of the paper of Sen. So that readers can take the opportunity to discuss the parts which we have not tried. In their study, they can set new examples to discuss the paper of Sen.

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