Believing Bernoulli

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ABSTRACT

Eight separate applications of Bernoulli's equation to fluid flows are reviewed. A possibility that Bernoulli's law was anticipated, some 300 years earlier, by Leonardo da Vinci is pointed out as well as Einstein's use of the equation for elementary explanations of several fluid motions. One example of how a classical mathematical technique applied to the standard theoretical description of the surface gravity wave inadvertently obliterated Bernoulli's physics is reported. In the atmosphere a proposal is made that Bernoulli's law helps to understand the low pressure inside a tornado.

1. INTRODUCTION

Where the speed is greatest, the pressure is least, and vice versa. Who does not believe that, or more likely, who thinks Bernoulli's law is ok as far as it goes, but then lacks the confidence to use it? A surprise or two are coming.

Many things have happened to Bernoulli's law, almost everything except outright controversy. It has been enhanced by the addition of some physics terms, including friction, surface tension, acceleration under gravitational attraction, and the centrifugal force. It has been utterly destroyed, shredded would be a good word, by the application of a standard mathematical technique in the historical and standard theory of the surface gravity wave. It may have been anticipated by an observation of Leonardo da Vinci some three hundred years earlier. And Einstein, in 1916, used it to explain in elementary terms certain observed fluid phenomena (bird flight, ocean wave propagation).

In an earlier review article promoting the usefulness of the centrifugal force for solving and understanding various problems in physics [1], I experienced a stumbling block in the publication process. The reviewer wanted the chapter on the Kepler problem taken out. No persuasive reasons were given, such as pointing out errors in the manuscript. After doing that, the paper was printed. Years later that omitted chapter was published anyway although in a different context.

Controversy surrounding the centrifugal force can surface in peculiar ways. Since every chapter in that review made use of the centrifugal force, singling out only one to be deleted still seems a bit strange.

With Bernoulli this is not a difficulty, and the writing up is altogether more fun and positive. Following the Leonardo segment the sections are not strictly chronological with respect to when they happened to come to my attention during my career. Some sense of relative "importance" has been superimposed on the ranking order, and also occasionally two separate segments have been combined into a single one for a bit of streamlining.

Some mornings I am reminded of Bernoulli at the break of day, when the pool circulation system begins running and water flows from the Jacuzzi through a narrow notch in the dam into the pool. Snug up against the side of the pool beside the notch, where the speed of flow is fastest, sits the floating thermometer for hours on end.

2. LEONARDO DA VINCI

There is a small amount of evidence that Leonardo da Vinci, through watching water flowing in rivers, came up with an observational form of Bernoulli's law. Start with a quote from a recent biography of da Vinci by Walter Isaacson, in his chapter 11 on birds and fight. "When he was compiling his bird treatise, Leonardo began a section of another notebook with a directive to put them into a broader context. 'To explain the science of the flight of birds, it is necessary to explain the science of the winds, which we shall prove by the motion of the waters,' he wrote. 'The understanding of this science of water will serve as a ladder to arrive at the knowledge of things flying in the air.' He not only got the basic principles of fluid dynamics correct, but he was able to turn his insights into rudimentary theories that foreshadowed those of Newton, Galileo, and Bernoulli."

In addition to this general assessment is a specific single line from one of Leonardo's notebooks: "where the water has more movement it is lighter if it is of the same height". That sounds very close to being a statement of Bernoulli's law. This sentence comes from a section of a book, The Nature of Water, in The Notebooks of Leonardo da Vinci, containing 135 pages taken from various notebooks of Leonardo and translated into English. There is no relation between this sentence and the ones that come before or after it in that volume.

3. WING'S LIFT

Although a completed theory of the lift force on a wing is not yet available, Bernoulli's law has almost always played a major role in understanding how it must work. Where the speed is greatest, the pressure is least. Since the speed is observed to be greatest at the top of the wing, the pressure is least there. Consequently an upward pressure gradient or force exists at the wing's top surface causing lift.

From observations also comes the fact that the speed of the fluid at the top of the wing, which is at its maximum there, monotonically decreases with increasing distance away from the wing. To predict the velocity shear over the wing is the job of theory, but this is just where the incompleteness occurs at the present time.

There is a way to single out and isolate the unknown feature of the problem of the lift force on a wing [2]. That much may prove to be helpful in the future, because at least one avenue out of many possible ones has been selected for following and searching for a solution. Leonardo da Vinci watched a rock in a stream that was wet on top. He noticed that water moved faster than average on the sides of the rock rather than going over the top of the rock. As far as I can tell from reading excerpts on water motion put together from his notebooks, he did not comment on how the perturbed velocity died away with increasing distance from the rock. Also he did not mention what happens in the case of two rocks in a stream where each rock was in the "influence zone" of the other one.

Erect a vertical axis at the top surface of a curved wing. Formulate two equations in two unknowns, pressure and velocity, that apply along this axis. First is Bernoulli's law and second is the force balance on a fluid particle traveling along a curved path: centrifugal force equals a pressure gradient. By itself each equation is non-linear. But when one equation is one unknown is obtained by elimination of a variable between the two equations, that equation is linear for either pressure or velocity!

A clear prediction comes from the two governing equations: the pressure perturbation dies away from the wing more rapidly than the velocity variation does. Exactly how rapidly the die-off rate is for each variable remains unknown at this time due to a lack of knowledge of the unknown non-constant coefficient in the governing equations: how the radius of curvature of the streamlines varies with distance from the wing. Some-day measurements may be good enough to provide this missing information, or another theory could turn up. In any case, a problem to solve in the future has been pinpointed.

4. GROOVED SPHERES AND CYLINDERS

Next in importance to increasing understanding the lift force on a wing is explaining an enhanced method of reducing friction when a solid moves through a fluid. Bernoulli's law plays a large part here too. Although nature probably started using the method eons ago, humans are apparently slow to catch on. Even an accidental discovery in the game of golf has not yet caught fire.

Originally golf balls were made out of the sap of a particular type of tree. During 50 years of experience it was noticed that the balls with nicks and scrapes often traveled a greater distance. Therefore, when the golf balls were manufactured, round dimples were put into the surface on purpose.

After this history became known to me not too many years ago, I selected two out of six identical (except for color) croquet balls from an old set to experiment with in the swimming pool. These balls came from the factory with small concentric grooves on their surfaces for some reason or other, perhaps style. One of the balls was made completely smooth by filling in the grooves with water proof wood putty and sanded. Their weights out of water were essentially the same. Being buoyant the balls would rise up through the water. Concern whether the path would be straight or wavy, and should the balls roll about an axis or not, caused me to have small holes drilled along diameters, passing through the centers of the circular grooves, so that the balls would slide up on taut wires. That concern turned out to be unnecessary, but it caused no harm. When released from the bottom of the pool simultaneously, the grooved ball reached the surface first every time for as many times as I was willing to do the trials [3].

A mechanical engineering friend watched some of the trials and made a few suggestions for further work. His point was that more evidence is always beneficial showing that grooves on solid bodies reduce friction, when the bodies move through fluids, and a different shape, specifically a cylinder, would be one step closer to potential applications than a sphere. Also he thought it would be a good idea to communicate any significant results to a professor in the mechanical engineering department of UC San Diego (that was done in good time). Therefore a neighbor with great woodworking skills was asked to make two cylinders, a foot long and two inches in diameter, out of solid buoyant hard wood, one with grooves and one smooth. This was accomplished very well with beautiful craftsmanship, and the weights in air were made equal by drilling a hole in the flat bottom surface of the smooth cylinder. Top surfaces were both rounded.

Meanwhile a single photograph with a single paragraph describing it were found in one of my eight or nine fairly standard fluid dynamics texts (the one in question was published by Oxford University Press). The streak photograph showed fluid flowing over a flat surface with a single groove in it. Fluid essentially jumped the groove, which had a nearly square cross-section. Self-evident is the fact that where the flow jumped the groove, there was no friction on that part of the wall. If two grooves are made in the wall, friction should be further reduced. A priori one cannot say at present what the optimum number of grooves is for a given wall and a given speed of flow in order to optimize the reduction of friction. A golf ball appears to have about as many dimples on the surface as could be put there, and in tournament play the maximum number of dimples allowed on a ball is specified; if this number is exceeded, the player will be disqualified.

For the cylinder trials in the pool a selection had to be made of the number and spacing of the grooves as well as the size of a groove. In fact, grooves where chosen instead of dimples. Seventeen grooves covered the surface of the one cylinder with a constant spacing of 3/8 inch between grooves. All grooves had a square cross-section with 1/8 inch on all three sides. Since the cylinders rose up vertically through the water, no guide wires were used. Swimmers took turns taking the cylinders down to the bottom of the deep end of the pool and letting them go simultaneously. Every time the top of the grooved cylinder reached the air before the top of the smooth cylinder did, and by an average distance of one cylinder length. Clearly the grooved cylinder experienced less friction [4].

To explain this result Bernoulli's law comes into play. Water is trapped in the grooves and does not go in or out of a groove. Because there is a lack of motion inside a groove, the pressure is relatively high there. Flow passing by the opening of the groove has lower pressure (where the speed is high the pressure is low). An outward pressure gradient holds the outside flow from entering the groove.

Nature probably already anticipated, by many millions of years, this method of decreasing the force of friction in the form of scales on a fish. First of all, the sides of most fish are not polished to be smooth like are the surfaces of racing boats (rowing shells) of various kinds. A pattern the fish scales exhibit appears to be two intersecting sets of grooves that make equal angles to the vertical when the fish swims horizontally.

Combining the results of the comparisons between grooved and smoothed surfaces of spheres and cylinders moving through water leads to greater confidence in suggesting a few practical applications of a method of friction reduction on solid bodies. One is to put grooves on the inside surface of pipes. For a constant pressure head the pipe with interior grooves is predicted to transport more fluid per unit time. A ship with vertical grooves below the waterline is expected to use less fuel to cross a given distance. After the grooves are in place, the talk will not be about savings in nickels and dimes. Plenty of room exists that can be filled with more effort in the future.

5. AIR-CONDITIONING FLOW

Another logical name for this section is Upside-down Flow, as will be shown below through examples that are non-intuitive but are a direct consequence of Bernoulli's law [5]. How can cold air stick to the ceiling of a room in a house? How can water cling to the underside of an overhanging ledge without falling down through the air beneath it? Movement is the key. And basically nothing more than Bernoulli's law is required. No extra physics terms need to be added.

In designing a pitcher or coffee pot so that it will not leak or spill when liquid is poured from it, is Bernoulli's law ever consulted? Probably not. If nothing else works, try pouring very slowly. This follows directly from Bernoulli's law. A second suggestion, to make the exiting stream thicker, is based on experiences related next.

On vacation in Mexico at a very nice hotel, two swimming pools at different levels were connected by wide and smooth stone steps with a thin layer of water running down them. Each step overhung the one below with a ledge extending out about six inches and having a rounded end. Startling to me was the fact that the water flowed upside down on the underside of the ledge and did not fall off. Water is about 800 times denser than air. How could it hang on to the ledge? While trying to figure out what was happening, I asked my grandson to jump into the pool near the top step to make a wave. He did and caused a thicker layer of water to flow over the step. This thicker layer could not negotiate the curve at the end of the step; it fell off making a small water fall which was repeated at the step below and the one below that. Evidently thickness of the layer is a significant parameter of the problem. For example, the hydrostatic pressure acting normal to the upside-down flow will cause the flow to break away from the solid surface above if the layer is thick enough.

Back at home I told this experience to a friend of mine, a retired mechanical engineer with a specialty in air conditioning. He said that in his profession an analogous thing is done on purpose. Cold air comes out of the ceiling through a vent in the middle of the room. Small vanes surrounding the vent direct the cold air parallel to the ceiling. The thin layer of flowing cold air sticks to the ceiling and makes its way to the nearest wall. Had I not been told, I probably never would have found out about this fluid flow phenomenon, since air is transparent. Remaining is the question: why do air-conditioning people want to cool a room this way?

A carpenter was making a new threshold for the front door of our house out of one piece of hardwood and I happened to come along when he had it upside down cutting a small groove parallel to the outside edge and a few inches in. My curiosity got the better of me. He explained that rain water would flow along the top surface, which had a gentle slope to it, out away from the front door, go around two almost right angle bends and then flow upside down along the bottom surface back toward the house. Wood rot would eventually occur inside the house as a result. The groove's purpose was to break the Bernoulli suction of the flow so that the water fell down to the ground before entering the house. He just knew to do that; Bernoulli's name would have fallen on deaf ears.

Return to the pitcher or coffee pot spout. Typically the shape is triangular as viewed from above and from both sides, or like a pyramid cut in half, causing the exiting fluid to be squeezed in two directions: horizontally and vertically. Consequently the speed is greatest at the spout's tip. Speed is the enemy in this case because the lowest pressure will be at the tip of the spout. Low pressure can enable the thin flow to stick to the outside of the container and go down to the counter or table.

6. UPWELLING BY OCEAN WAVES

All properties of the surface gravity wave are known from observations to monotonically decrease in magnitude with increasing depth below the air-water interface, variations in pressure and particle velocity in particular, and to vanish at depths greater than or equal to about one wavelength. What is not known for sure theoretically is the exact decay rate or whether pressure variations decrease more rapidly than velocity variations do or if both decay at the same (exponential) rate. Two different theories exist, an old one and a new one.

Take a straight length of rigid pipe, open at both ends, and hold it fixed in the vertical position below the waves so that the upper end never becomes exposed to the air. At any time the Bernoulli suction at the top opening will exceed that at the bottom opening when waves are passing by, no matter which direction they travel. Flow inside the pipe will always be upward on the average therefore [6].

Proof of concept was obtained on a small scale in a swimming pool. A glass tube was held fixed vertically with the lower end in a container of blue dye. When waves were made on the surface by oscillating a wooden paddle, blue dye shot up the tube.

Among the ideas for larger scale applications, there is one for farming the ocean. Deploy rigid vertical pipes and let them drift with the currents. Wind waves will overtake them because they travel faster than the currents. They can be held in the vertical position by floats pulling upward on the tops of the pipes and weights pulling down on the bottom ends. So called "desert" areas of the open oceans are the logical places to put these pipes, because there is not much life going on there. Plankton sink when they die and dissolve when they sink. Thus nutrients are deposited below the light zone and can no longer be used in growing new plankton. By wave action the nutrients would be pumped back up into the sunlit waters to start a food chain. My understanding is that something like this idea is catching on with a few Japanese scientists.

7. SHREDDING BERNOULLI'S LAW

Before discussing how the applications of Bernoulli's law can be widened by adding certain physical terms to it, one example of how it can be utterly destroyed by mathematics is presented.

In the early theoretical history of the surface gravity wave, the equations of fluid motion were subjected to a solution technique that had been successfully applied before in other fields, such as astronomy. Perturbation expansion is the name of the method. For my PhD dissertation I got a full exposure to these perturbation expansions in order to solve a problem (wave-wave interactions) during a two or three year period, which did not lead to one of my fondest memories from graduate school.

In all the governing equations each variable, such as pressure, velocity,…, is expanded in a series of terms of what is presumed to be ever decreasing magnitude: zeroth order, first order, second order,…etc. [The constant is actually not a variable but it is called "zeroth order" here for illustration purposes, which does not affect the main thrust of the argument.] Then a new set of equations is set up which contain only all terms of the same order. Solutions are sought to these new equations and then they are added together to get the total solution. A priori there is no guarantee that the end solution will make any sense or that the series formed will even converge. But when faced with nonlinear equations to start with, there are not very many options to choose from.

It is very easy to explain how the perturbation crunching machine can shred Bernoulli's equation: pressure equals a constant minus one half the fluid density times the square of the fluid speed. Mathematically each term in this equation has a different order. The constant is zeroth order, pressure is first order, the square of the speed is second order. Therefore, by using the perturbation technique each term of Bernoulli's equation would end up in a different new equation. In so doing a beautiful expression of physics has been completely ripped apart. This was not done with malicious intent in the beginning. It was just that belief in the mathematics was so strong, and the motivation to get around the non-linearity was always there, of course.

As it turns out now, there is a pathway to understanding several characteristics of the surface gravity involving linear equations [7]. And the new theory preserves the integrity of Bernoulli's law. For example, pressure variations are predicted to decay with depth faster than velocity variations do. Classically both pressure and velocity variations decay exponentially and at the same rate. Observations will eventually sort this all out.

8. FLOW PAST ONE OR TWO CYLINDERS

Flow past a single cylinder is a text book problem in hydrodynamics. Here the light of Bernoulli's law will be shined on this configuration [8]. In the text books velocity all by itself is quickly brought out for discussion from Laplace's equation, using the potential flow method that was previously useful in electromagnetic theory. But any mention of pressure is hard to find in these books. Bernoulli's law couples together pressure and velocity, which would seem to be a more reasonable starting position to approach the phenomenon at hand. What follows is the same procedure described above for the wing's lift force. It is believed to be worth saying again.

Next, add to Bernoulli's law, which is one equation in two unknowns, a second equation in the same two unknowns, pressure and velocity. That second equation is the balance of forces on a fluid particle traveling along a curved streamline: centrifugal force equals a pressure gradient. By itself each equation is nonlinear. However, if one of the unknowns is eliminated between the two equations, a linear equation in one unknown is obtained. This is a rather striking result, and it works for pressure as well as velocity! One draw-back is that there is an unknown constant in the governing equation, the radius of curvature of the streamlines as a function of distance from the cylinder. But that inconvenience is minor compared to the great simplicity of linearity. And if there is no other way, the constant can be obtained from a good set of measurements in the future, streak photographs, for example. Then the equation can simply be integrated numerically.

One theoretical result comes out without detailed knowledge of the radius of curvature function. Perturbations of the flow pressure, due to the presence of the cylinder, die away at a faster rate than do perturbations of velocity. Such a prediction will not be found in the fluid dynamics texts. They forecast that velocity variations fall off as the inverse square of the distance, but they are silent about the die off rate of the pressure variations. Also apparently the inverse square law for velocity has never been held up against measurements.

What happens if two cylinders are held fixed and parallel in a uniform flow [9]? Without guidance from any theory, one can guess what will probably happen. Nothing if the cylinders are very far apart. When the cylinders are within the perturbation zones of each other, then the perturbed velocity between them is expected to be larger than it would be if there were only one cylinder present because the two perturbed velocities should add together to some extent. That would mean the pressure between them would be lower than if only one cylinder were present by Bernoulli's law and also lower than on the opposite sides of the cylinders. Consequently the cylinders will feel a net force trying to pull them together.

On the other hand, suppose the two cylinders touch together such that no fluid can flow between them. Then there will be a net force attempting to pull them apart. If they are not rigidly fixed in position and begin to move apart, and the space between them widens sufficiently, the net force will pull them back together. Under the right conditions there is a possibility of an oscillation being set up, at least in theory. The rope snapping against the flag pole when the wind blows and there is no flag flying might be a familiar example.

9. VENA CONTRACTA

English for the Latin heading is "contracted vein", and the Latin name suggests the observation is pretty old, going back at least to Roman times. Make a circular hole in the side and near the bottom of a large container of water, wine or whatever. A steady stream of fluid comes out and gets narrower as it goes. What causes the narrowing? Simple question. Answers given in the texts are not completely satisfactory to me. There is no way to calculate the amount of fluid that will be collected in a given period of time unless this contraction can be understood. Several fields of study are still interested in this problem including fluid mechanics, some branches of engineering and even medicine (because of strokes).

My attempts to explain the narrowing stream occurred off and on over a number of years, and I was completely stuck for an answer, until I got an e-mail from my younger nephew. In order to cool the surface of his grinding wheel, he squashed the end of a copper tube, which had a smaller diameter than the width of the wheel, thinking the water exiting an oval shaped opening in the tube would then flare out and cover the whole surface of the wheel. When the opposite thing happened, his frustration led to the e-mail. Instead of widening the stream immediately narrowed. It occurred to me later that surface tension might be having an important influence, because the effects of it increase as the radius of curvature decreases. At the two ends of the oval opening in the horizontal plane the radius of curvature of the tube was the smallest, so surface tension would expect to be greater at those two places than at the top and bottom of the tube.

For some reason this idea never occurred to me for fluid exiting a round hole. However, it makes sense that surface tension acting uniformly around the surface of a round stream could make it become narrower as it went by squeezing. To help along that explanation I added a surface tension term to Bernoulli's law, which I don't think had ever been done before, and the result got published [10].

10. ROLL WAVES

Anybody who has watched rain falling on a sloping street has seen roll waves. I did not know they were called that until I was told of it at a pretty advanced age. Even most of those educated in fluid dynamics will not have run across the name because reading about the phenomenon is usually encountered in the field of hydraulic engineering.

Since friction is such an important part of understanding these waves, and friction and Bernoulli's law do not mix very well, then this discussion of problems related to Bernoulli's law will be unexpected by many readers. Strictly speaking Bernoulli's law is derived from the equations of motion in fluid mechanics books by first assuming there is no friction.

Anyway, rain is falling on a sloping street and waves are going downhill. First impressions are that there are wave crests but no troughs, and the crests are strangely far apart with an approximately constant separation. How hard it rains and how steep the slope of the road is do not seem to matter. A straightforward question is: why can't the water flow down in a layer of constant thickness? Since water in the crests is farther away from the effects of friction with the road, that water can travel faster than the water if the thickness of the layer were constant, and it may very well be that the total downslope flux of water is larger with waves than without them. There appears to be no law of physics that says this should be so, such as maximizing the flux of mass.

My attempt to understand the roll wave began with Bernoulli's law and added in a friction term [11]. A colleague told me that such a thing had been done before and in one of my nine fluid dynamics texts (by Milne-Thompson) I found a single page devoted to the topic. New was my application of this technique to the roll wave. Studying this problem analytically is fairly complicated, the details of which are not of interest here. Nevertheless, Bernoulli's law is very helpful in this case also.

11. TORNADO AND DOWNBURST

Both phenomena are misunderstood for logical reasons. The downburst occurs very infrequently so data on them are extremely scarce. Tornadoes are more frequent but they usually overpower and destroy most instruments that occur in or are put in their path. Tornadoes have rotation but the down burst does not. Bernoulli's law will be featured in both, although modified by the addition of a vertical acceleration term due to gravity and density differences.

One successful tornado measurement event took place in South Dakota when a rugged squashed cone shaped housing, containing several instruments, was placed on a road in the path of a tornado and the tornado went right over it without any destruction, or even movement of the contraption along the road. Reported to the public immediately afterward, in Weatherwise magazine for May/June 2004, was the remarkable drop in pressure as the tornado went by. Other properties were also measured, the article said, temperature and relative humidity, but they have not yet reached the public after so many years. That is a puzzle to me. My conjecture was that the air inside the tornado was cold and dry, but I have not found out if I am right. Cold and dry air is denser than the typically warm humid air of the environment and so it would fall down to the ground.

In spite of the downburst being such a rare happening, the occurrence can leave a footprint lasting for forty years. And if my thinking is correct, some people, once they find out, might decide not to fly in planes anymore.

In the mail one day, at least 15 years ago, arrived a photograph from a friend living in southwestern Colorado. Later I saw the scene first hand from my car on a drive north on a paved road. Within a forest of green trees was a roughly circular brown area of dead trees lying on the ground. This area was considerably larger than that of my friend's house, which was only a few miles to the south.

What is one to conclude from this photograph? Probably there were no witnesses to the event, and nothing written up about it of either a scientific or popular nature. Extreme observations imply extreme explanations. A force strong enough to knock to the ground a bunch of live trees apparently took place. Tornadoes are capable of doing this, but they are not known to occur in southwestern Colorado. Also obvious evidence of rotary motion was not present which all tornadoes have apparently.

Suppose a blast of air comes straight down out of the sky with enough velocity to do the damage that exists on the ground according to the photograph. To get such a high velocity, in the range of several hundred miles per hour probably, linear acceleration must have been involved over a large vertical distance with no horizontal rotation at all. Very cold dry air, denser than its surroundings, presumably did just that. An analytical description for this explanation is Bernoulli's equation plus an acceleration under gravity term, which I don't think has been done before [12].

Although the tornado kills people, so there is an urgency in the dealings with it, the physical understanding is still in a confused state. For example, generally it is believed that at the core of a tornado rising motion takes place. My conviction is just the opposite: falling motion should exist starting when the funnel cloud moves from the cloud base to the ground and afterwards as well. Admittedly my training in oceanography, in spite of being in one of the better graduate schools (Scripps), did not include a single lecture on meteorology (the 100 mile trip to LA was an impenetrable barrier). A most elementary model, predicated on the downward movement of relatively cold and dry air, has been offered in which Bernoulli's law is an integral part [13].

12. CONCLUSION

Eight different applications and adaptations of Bernoulli's equation within the field of fluid dynamics for air and water motions have been summarized. My involvement in all of them took place in the past 50 years or so. More work can be done on several of the applications to build up the earlier efforts. Particularly experimental data are needed. Speculations suggest that extended explorations involving the interactions between fluid flows and one or two cylinders could lead to practical results, perhaps deriving energy from the natural movements of the atmospheric winds and ocean waves or currents.

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