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The Greenhouse Effect: An Evaluation of Arrhenius' Thesis and a New Energy Equilibrium Model

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ABSTRACT

In 1896, Svante Arrhenius proposed a model predicting that increased concentration of carbon dioxide and water vapour in the atmosphere would result in a warming of the planet. In his model, the warming effects of atmospheric carbon dioxide and water vapour in preventing heat flow from the Earth' s surface (now known as the "Greenhouse Effect") are counteracted by a cooling effect where the same gasses are responsible for the radiation of heat to space from the atmosphere. His analysis found that there was a net warming effect and his model has remained the foundation of the Enhanced Greenhouse Effect—Global Warming hypothesis. This paper attempts to quantify the parameters in his equations but on evaluation his model cannot produce thermodynamic equilibrium. A modified model is proposed which reveals that increased atmospheric emissivity enhances the ability of the atmosphere to radiate heat to space overcoming the cooling effect resulting in a net cooling of the planet. In consideration of this result, there is a need for greenhouse effect—global warming models to be revised.

1. INTRODUCTION

The origins of the greenhouse effect theory can be traced back to the Swedish scientist, Svante August Arrhenius (1859-1927). He received the Nobel Prize for Chemistry in 1903.

In 1896 Arrhenius proposed that changes in the levels of "carbonic acid" (carbon dioxide) in the atmosphere could substantially alter the surface temperature of the Earth. This has come to be known as the greenhouse effect. Arrhenius' paper, "*On the Influence of Carbonic Acid in the Air upon the Temperature of the Ground*", was published in *Philosophical Magazine*. Arrhenius asks in his introduction:

Is the mean temperature of the ground in any way influenced by the presence of heat absorbing gases in the atmosphere? [1]

In his book, Worlds in the Making, Arrhenius concludes:

If the quantity of carbonic acid in the air should sink to one-half its present percentage, the temperature would fall by about 4˚; a diminution to one-quarter would reduce the temperature by 8˚. On the other hand, any doubling of the percentage of carbon dioxide in the air would raise the temperature of the earth's surface by 4˚; and if the carbon dioxide were increased fourfold, the temperature would rise by 8˚ [2].

It is interesting to note that Arrhenius considered this greenhouse effect a positive thing if we were to avoid the ice ages of the past. Nevertheless, Arrhenius' theory has become the foundation of the enhanced greenhouse effect—global warming hypothesis in the 21st century. His model remains the basis for most modern energy equilibrium models.

2. ARRHENIUS' ENERGY EQUILIBRIUM MODEL

Arrhenius' proposed a two-part energy equilibrium model in which the atmosphere radiates the same amount of heat to space as it receives and, likewise, the ground transfers the same amount of heat to the atmosphere and to space as it receives. The model contains the following assumptions:

- Heat conducted from the center of the Earth is neglected.
- Heat flow by convection between the surface and the atmosphere and throughout the atmosphere remains constant.
- Cloud cover remains constant. This is questionable but allows the model to be quantified.

Following is a description of the two parts of his model. I have used many of the terms employed by Arrhenius.

Part 1: Equilibrium of the Air

The balance of heat flow to and from the air (or atmosphere) has four components as shown in **Figure 1**. The arrow labelled S_1 indicates the solar energy absorbed by the atmosphere. R indicates the infra-red radiation from the surface of the Earth to the atmosphere, M is the quantity of heat "conveyed" to the atmosphere by convection and Q_1 represents heat loss from the atmosphere to space by radiation. All quantities are measured in terms of energy per unit area per unit time (W/m^2).

For thermal equilibrium, these quantities must balance:

$$Q_1 = R + M + S_1 \tag{1}$$

In turn, the individual contributions can be analysed. Firstly, the radiation to space Q_1 can be described by the Stefan-Boltzmann law:

$$Q_1 = \varepsilon\sigma T_a^4 \tag{2}$$

Here

ε is the emissivity/absorptivity of the atmosphere (Arrhenius refers to this as the "emission-coefficient" and uses the symbol β),

T_a is the effective temperature of the atmosphere (θ),

σ is the Stefan-Boltzmann constant ("radiation constant", γ) = 5.67×10^{-8} W·m^{-2}·K^{-4}.

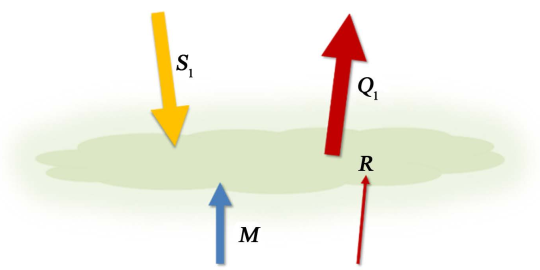

Figure 1. **Model of the energy balance of the atmosphere. The heat received by the atmosphere ($R + M + S_1$) equals the heat lost to space (Q_1). In this single layer atmospheric model, the absorbing and emitting layers are one and the same.**

The net radiation R from the Earths' surface at temperature T_g to the atmosphere at a temperature T_a is given by:

$$R = \varepsilon\sigma\nu\left(T_g^4 - T_a^4\right) \tag{3}$$

where:

ν is the emissivity of the Earth' s surface ≈ 1 and,

T_g is the mean surface temperature of the ground,

S_1 is the solar energy absorbed by the atmosphere.

Part 2: Thermal Equilibrium of the Ground

In the second part of his model, Arrhenius describes the heat flow equilibrium at the "ground" or surface of the Earth. There are four contributions to the surface heat flow as shown in **Figure 2**. S_2 is the solar energy absorbed by the surface, R is the infra-red radiation emitted from the surface and transferred to the atmosphere, N is the heat conveyed to the atmosphere by convection and Q_2 is the heat radiated to space from the surface. Note: Here Arrhenius uses the term N for the convective heat flow. It is equivalent to the term M used in the air equilibrium model. For heat flow equilibrium, these quantities must balance:

For equilibrium;

$$S_2 = R + N + Q_2 \tag{4}$$

The radiation to space Q_2 can be determined as follows:

$$Q_2 = (1-\varepsilon)\sigma\nu T_g^4 \tag{5}$$

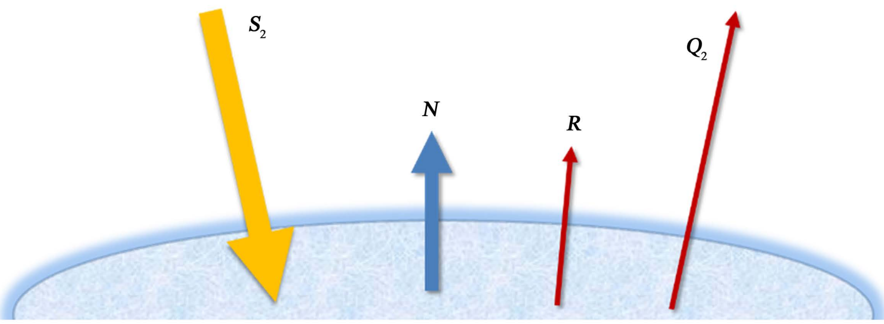

Figure 2. The energy balance at the surface of the Earth. The energy received by the ground is equal to the energy lost.

3. FINDING THE TEMPERATURE OF THE EARTH

Arrhenius combined these equations and, by eliminating the temperature of the atmosphere which according to Arrhenius "has no considerable interest", he arrived at the following relationship:

$$\Delta T_g = \sqrt[4]{\frac{2-\varepsilon_1}{2-\varepsilon_2}} \times T_0 - T_0$$

ΔT_g is the expected change in the temperature of the Earth for a change in atmospheric emissivity from ε_1 to ε_2. Arrhenius determined that the current transparency of the atmosphere was 0.31 and, therefore the emissivity/absorptivity $\varepsilon_1 = 0.69$. The current mean temperature for the surface of the Earth can be assumed to be $T_0 = 288$ K and the equation becomes:

$$T_g = \sqrt[4]{\frac{2-0.69}{2-\varepsilon}} \times 288\text{ K} \tag{6}$$

Arrhenius' model described by Equation (6) is plotted in **Figure 3**.

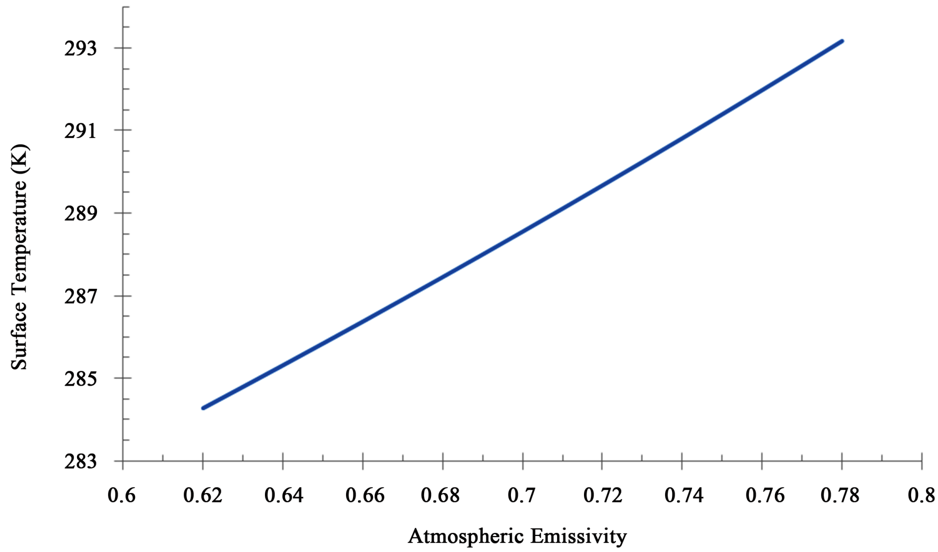

Figure 3. **Arrhenius' model is used to determine the mean surface temperature of the Earth as a function of atmospheric emissivity ε. For initial conditions, $\varepsilon = 0.69$ and the surface temperature is 288 K. An increase in atmospheric emissivity produces an increase in the surface temperature of the Earth.**

Arrhenius estimated that a doubling of carbon dioxide concentration in the atmosphere would produce a change in emissivity from 0.69 to 0.78 raising the temperature of the surface by approximately 6 K. This value would be considered high by modern climate researchers, however, Arrhenius' model has become the foundation of the greenhouse–global warming theory today. Arrhenius made no attempt to quantify the specific heat flow values in his model. At the time of his paper there was little quantitative data available relating to heat flow for the Earth.

4. EVALUATION OF ARRHENIUS' MODEL UNDER PRESENT CONDITIONS

More recently, Kiehl and Trenberth (K & T) [3] and others have quantified the heat flow values used in Arrhenius' model. K & T's data are summarised in **Figure 4**.

The reflected solar radiation, which plays no part in the energy balance described in this model, is ignored. R is the net radiative transfer from the ground to the atmosphere derived from K & T's diagram. The majority of the heat radiated to space originates from the atmosphere ($Q_1 > Q_2$). And the majority of the heat lost from the ground is by means of convection to the atmosphere ($M > R + Q_2$).

Equations (2) and (3) can be used to determine the present atmospheric emissivity ε and the effective atmospheric temperature T_a:

$$Q_1 = \varepsilon\sigma T_a^4 \tag{2}$$

$$R = \varepsilon\sigma\left(T_g^4 - T_a^4\right) \tag{3}$$

Substituting $Q_1 = 195\ \mathrm{W/m^2}$, $R = 26\ \mathrm{W/m^2}$ and $T_g = 288$ K and solving the equations simultaneously the effective atmospheric temperature T_a and emissivity ε can be found:

$$T_a = 279.1\ \mathrm{K} \tag{7a}$$

$$\varepsilon = 0.567 \tag{7b}$$

Note: Arrhenius calculated the emissivity of the atmosphere to be approximately 0.69.

Q_2 is the heat radiated to space and can be determined from Equation (5).

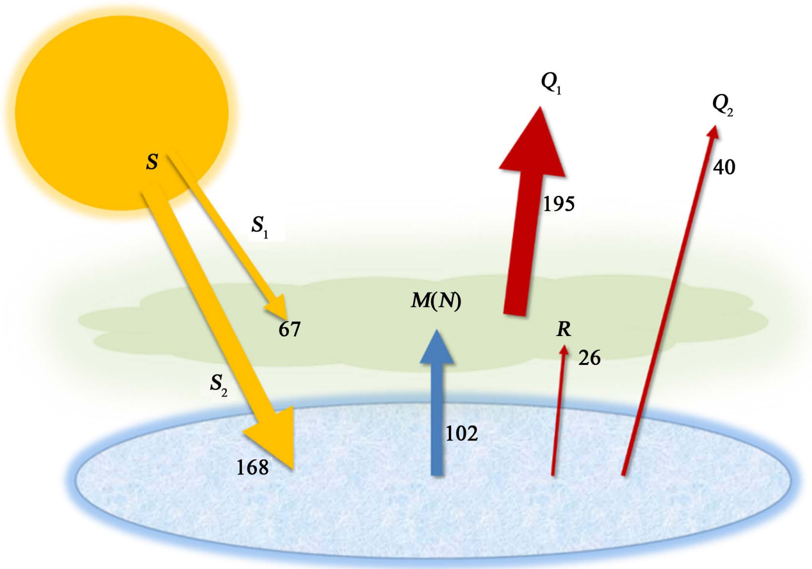

Figure 4. **Model of the mean energy budget of the earth as determined by Kiehl and Trenberth.**

$$Q_2 = (1-\varepsilon)\sigma\nu T_e^4 \tag{5}$$

Substituting $\varepsilon = 0.567$, $\nu = 1.0$ and $T_g = 288$ K we get:

$$Q_2 = 149.2\ \text{W/m}^2$$

Using Arrhenius value of 0.69 for the atmospheric emissivity Q_2 = 120.9 W/m^2. Both values are significantly more than the 40 W/m^2 determined by K & T. The equation will not balance, something is clearly wrong. Figure 5 illustrates the problem.

Equation (5) is based on the Stefan-Boltzmann law which is an empirical relationship which describes the amount of radiation from a hot surface passing through a vacuum to a region of space at a temperature of absolute zero. This is clearly not the case for radiation passing through the Earth's atmosphere and as a result the amount of heat lost by radiation has been grossly overestimated. No amount of adjusting parameters will allow this relationship to produce sensible quantities and the required net heat flow of 40 W/m^2. This error affects the equilibrium heat flow values in Arrhenius' model and the model is not able to produce a reasonable approximation of present day conditions as shown in Table 1. In particular, the convective heat flow takes on very different values from the two parts of the model. The values M and N in the table should be equivalent.

5. A NEW ENERGY EQUILIBRIUM MODEL

A modified model is proposed which will determine the change in surface temperature of the Earth caused by a change in the emissivity of the atmosphere (as would occur when greenhouse gas concentrations change). The model incorporates the following ideas:

1) The total heat radiated from the Earth ($Q_1 + Q_2$) will remain constant and equal to the total solar radiation absorbed by the Earth ($S_1 + S_2$).

$$Q_1 + Q_2 = S_1 + S_2 = S$$

$$Q_2 = S - Q_1 \tag{8}$$

$$S = 235\ \text{W/m}^2 \quad \text{from K \& T}$$

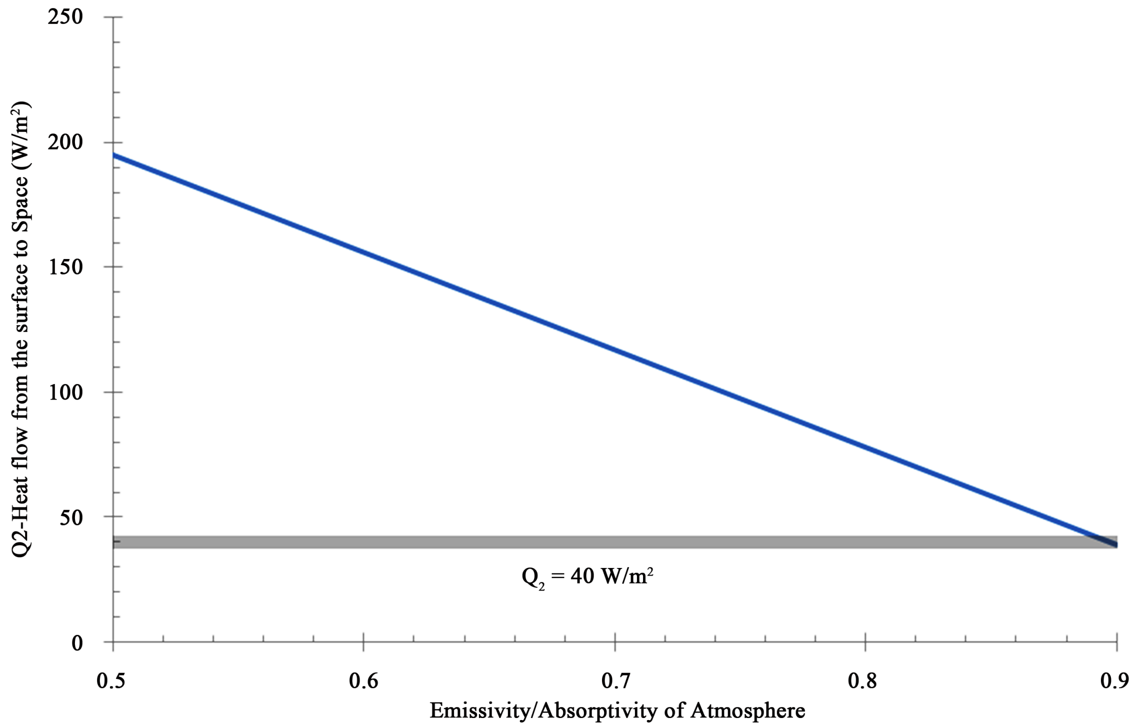

Figure 5. **The heat flow from the surface of the Earth, calculated from Equation (5) as a function of atmospheric emissivity/absorptivity for a surface temperature of 288 K. The heat flow is expected to be 40 W/m^2 as shown by the horizontal line but this can only be achieved at (impossibly) high values of emissivity.**

Table 1. **Equilibrium values calculated from Arrhenius model compared to those given from K & T.**

	T_g (1)	T_a (2)		R (3)	Q_1 (4)	Q_2 (5)	M (6)	N (7)
Arrhenius	288	265.6	0.690	74.6	195.0	121.1	53.4	−27.7
K & T	288	279.1	0.566	26.0	195.0	40.0	102.0	102.0

Notes: 1. T_g is the mean surface temperature of the Earth' s surface = 288 K; 2. T_a is adjusted to produce a radiative heat flow $Q_1 = 195\ \text{W/m}^2$; 3. $R = \varepsilon\sigma\left(T_g^4 - T_a^4\right)$; 4. $Q_1 = \varepsilon\sigma T_a^4$; 5. $Q_2 = (1-\varepsilon)\sigma T_g^4$; 6. $M = Q_1 - R - S_1$ where $S_1 = 67\ \text{W/m}^2$; 7. $N = S_2 - R - Q_2$ where $S_2 = 168\ \text{W/m}^2$.

2) Convective heat flow M remains constant. Convective heat flow between two regions is dependent on their temperature difference, as expressed by Newton's Law of cooling[1]. The temperature difference between the atmosphere and the ground is maintained at 8.9 K (see Equation 7(a)). M = 102 W/m^2 (K & T).

3) A surface temperature of 288 K and an atmospheric emissivity of 0.567 (Equation (7b)) is assumed for initial or present conditions.

4) The energy balance model can now be derived from Equation (1) as follows:

$$Q_1 = R + M + S_1 \tag{1}$$

$$\varepsilon\sigma T_a^4 = \varepsilon\sigma\nu\left(T_g^4 - T_a^4\right) + M + S_1 \quad (\nu = 1)$$

[1]Convective heating or cooling can be described by Newton's law of cooling: The rate of heat loss of a body is proportional to the difference in temperatures between the body and its surroundings.

$$\varepsilon\sigma T_a^4 - \varepsilon\sigma T_g^4 + \varepsilon\sigma T_a^4 = M + S_1$$

$$\varepsilon\sigma\left(2T_a^4 - T_g^4\right) = M + S_1$$

$$\varepsilon = \frac{M + S_1}{\sigma\left(2T_a^4 - T_g^4\right)} \tag{9}$$

Equation (9) represents the new model relating the emissivity of the atmosphere ε to the surface temperature T_g. Results from this model are shown in Table 2. The table shows the individual heat flow quantities and the temperature of the surface of the Earth that is required to maintain equilibrium:

The table shows that as the value of the atmospheric emissivity ε is increased less heat flows from the Earth's surface to space, Q_2 decreases. This is what would be expected. As well, more heat is radiated to space from the atmosphere; Q_1 increases. This is also expected. The total energy radiated to space $Q_1 + Q_2 = 235\ \text{W/m}^2$. A plot of the resultant surface temperature T_g versus the atmospheric emissivity ε is shown below Figure 6.

This model proposes that an increase in atmospheric emissivity which would occur with an increase in greenhouse gas concentrations, will lead to a DECREASE in global surface temperature! The warming effects of increased atmospheric carbon dioxide and water vapour concentrations, inhibiting heat flow from the Earth's surface is counter-acted by a cooling effect where the same gasses are responsible for increased radiation of heat from the Earth's atmosphere. Most of the Earths heat loss to space originates in the atmosphere. It is not surprising therefore that the second effect is greater than the first.

Table 2. Values obtained from the modified energy equilibrium model.

T_g (a)	T_a (b)	ε (c)	R (d)	Q_1 (e)	Q_2 (f)
282	273.1	0.619	26.7	195.7	39.3
283	274.1	0.610	26.6	195.6	39.4
284	275.1	0.601	26.4	195.4	39.6
285	276.1	0.592	26.3	195.3	39.7
286	277.1	0.583	26.2	195.2	39.8
287	278.1	0.574	26.1	195.1	39.9
288	279.1	0.566	26.0	195.0	40.0
289	280.1	0.557	25.9	194.9	40.1
290	281.1	0.549	25.8	194.8	40.2
291	282.1	0.541	25.7	194.7	40.3
292	283.1	0.533	25.5	194.5	40.5
293	284.1	0.525	25.4	194.4	40.6
294	285.1	0.518	25.3	194.3	40.7

Notes: (a) Surface temperature values for the range $T_g = 288 \pm 6$ K; (b) $T_a = T_g - 8.9$ K; (c) Atmospheric emissivity ε is calculated from Equation (9); (d) $R = \varepsilon\sigma\left(T_g^4 - T_a^4\right)$ from Equation (3); (e) $Q_1 = \varepsilon\sigma T_a^4$ from Equation (2); (f) $Q_2 = S - Q_1$ from Equation (8).

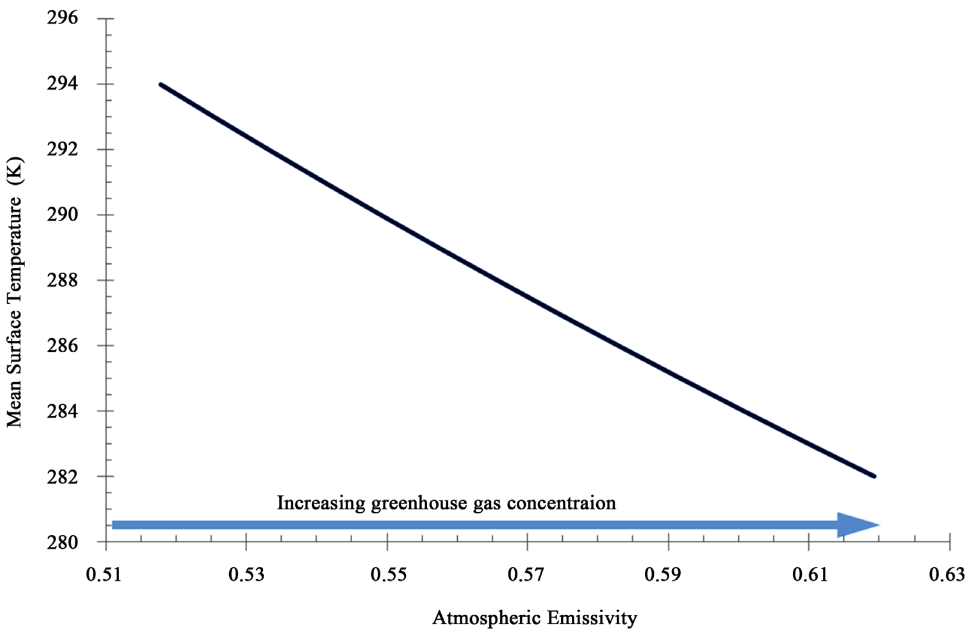

Figure 6. Plot of the Earth's mean surface temperature as a function of the atmospheric emissivity. This model predicts that the temperature of the Earth will decrease as the emissivity of the atmosphere increases.

6. CONCLUSION

Arrhenius identified the fact that the emissivity/absorptivity of the atmosphere increased with increasing greenhouse gas concentrations and this would affect the temperature of the Earth. He understood that infra-red active gases in the atmosphere contribute both to the absorption of radiation from the Earth's surface and to the emission of radiation to space from the atmosphere. These were competing processes; one trapped heat, warming the Earth; the other released heat, cooling the Earth. He derived a relationship between the surface temperature and the emissivity of the atmosphere and deduced that an increase in emissivity led to an increase in the surface temperature of the Earth.

However, his model is unable to produce sensible results for the heat flow quantities as determined by K & T and others. In particular, his model and all similar recent models, grossly exaggerate the quantity of radiative heat flow from the Earth's surface to space. A new energy equilibrium model has been proposed which is consistent with the measured heat flow quantities and maintains thermal equilibrium. This model predicts the changes in the heat flow quantities in response to changes in atmospheric emissivity and reveals that Arrhenius' prediction is reversed. Increasing atmospheric emissivity due to increased greenhouse gas concentrations will have a net cooling effect. It is therefore proposed by the author that any attempt to curtail emissions of CO_2 will have no effect in curbing global warming.

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ADDENDA

The "Black-body" temperature of the Earth

One way to estimate the temperature of a planet is to find its hypothetical black body temperature. A black body does not reflect or transmit radiation; it has an absorptivity of one and an emissivity of one. The heat from solar radiation incident on a black body planet is equal to the radiative heat loss emitted from the planet as illustrated in the following Figure A1.

Using the Stefan-Boltzmann law and the value for the solar irradiance of 342 W/m^2 as determined by K & T the black body temperature of the Earth T_B can be calculated as follows:

$$Q_{\text{solar}} = Q_{\text{out}} = \sigma T_B^4$$

$$T_B^4 = \sqrt[4]{\frac{Q_{\text{solar}}}{\sigma}} \approx 279 \text{ K}$$

This temperature value of 279 K refers to the mean temperature of the radiating surfaces. In a two-layer model the radiating surfaces are the atmosphere and the ground. Approximately 71% of the radiation emitted from the Earth originates from the atmosphere and only 21% is emitted from the ground. Interestingly, 279 K is the atmospheric temperature found in the model describe previously. On average, this temperature represents a region in the atmosphere at an altitude of ≈1.5 kilometres.

The value of 255 K is often quoted in the literature but this value arises from some dubious assertions. Instead of using the actual solar irradiance, it is based on the amount of heat absorbed by the Earth, neglecting the energy reflected. This treatment immediately assumes that the Earth is NOT a blackbody. To be more rigorous, the Earth can be treated as a grey-body and its temperature found as follows:

$$Q_{\text{absorbed}} = Q_{\text{emitted}}$$

$$\alpha S_{\text{tot}} = \varepsilon \sigma T^4$$

Here S_{tot} is the total solar irradiance = 342 W/m^2

αS_{tot} is the energy absorbed by the Earth ≈ 235 W/m^2

α is the absorptivity of the Earth ≈ 0.7 (235/342)

Earlier it was shown that the Earth has an emissivity of ≈0.57 for the atmosphere (accounting for 71% of the radiation heat loss) and ≈1.0 for the surface of the Earth (21%). An approximate calculation would indicate that the overall emissivity of the Earth is:

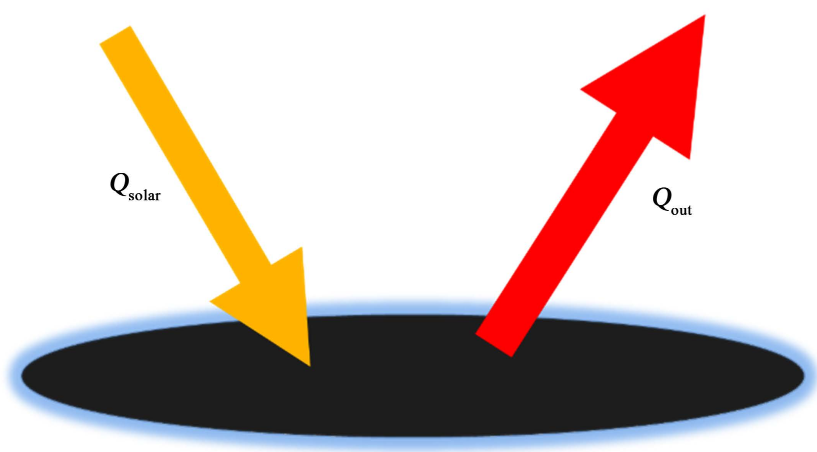

Figure A1. Black-body model of the Earth. 100% of the solar irradiation is absorbed (absorptivity = 1.0). This absorbed energy is then radiated back to space from a surface of emissivity = 1.0. At equilibrium, the heat flow in equals the heat flow out: $Q_{\text{solar}} = Q_{\text{out}}$.

$$\varepsilon_{tot} = 0.71 \times 0.57 + 0.29 \times 1.0 \approx 0.7$$

The absorptivity of the Earth as a whole is approximately equal to its emissivity and thus the mean temperature of the Earth is close to the black-body temperature ≈279 K. These values support the model proposed.