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Deriving of the Generalized Special Relativity (GSR) by Using Mirror Clock and Lorentz Transformations

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Abstract

Einstein relativity theory shows its high capability of promoting itself to solve the long stand physical problems. The so-called generalized special relativity (GSR) was derived later, using the beautiful Einstein relation between field and space-time curvature. In this work we re-derive (GSR) expression of time by incorporating the field effect in it, and by using mirror clock and Lorentz transformations. This expression reduces to that of (GSR) the previous conventional one, besides reducing to special relativistic expression. It also shows that the speed of light is constant inside the field and is equal to C. This means that the observed decrease of light in matter and field is attributed to the strong interaction of photons with particles and mediates which causes successive absorption and reemission processes that lead to time delay. This absorption process makes some particles appear to move faster than light within the field or medium. This new expression, unlike that of GSR, can describe time and coordinate relativistic expressions for strong as well as weak fields at constant acceleration.

Keywords

Lorentz Transformations, Mirror Clock, Space-Time Curvature, Gravitational Field

1. Introduction

Einstein's special relativity and general relativity represent one of the biggest achievements that change radically the space-time concept [1]. Special relativity (SR) succeeds in explaining the constancy of light speed in va-

cuum, long-time meson decay, and mass-energy conversion [2].

SR shows that time, length, mass, and energy are velocity dependent [3]. Einstein extends his SR theory to the so-called general relativity theory (GR) which extends the space-time interval to the curved space GR succeeded in explaining a wide variety of astronomical phenomena. These include the Doppler red shift, which is interpreted as resulting from the universe expansion, the existence of relic microwave background, beside the gravitational red shift [4].

These remarkable successes of SR and GR motivate some authors to promote SR within the frame of GR to produce the so-called Generalized Special Relativity (GSR) [5].

Mubarak model, the matter energy-momentum tenser relation on the generalized Lorentz factor (r) derived from the space-time interval in the curve space, was utilized to construct the seminal EGSR model; time, length, mass and energy are dependent on the field potential as well as velocity [6].

In this work two mirrors of certain length, acting as time clock, are under the action of gravity. The motion of the two mirrors under gravity is utilized to find a useful expression of time in the presence of the gravitational field. The speed of light is assumed to be constant in the gravity field. This assumption is confirmed by obtaining the light speed in accursed space time [7].

This paper, which is concerned with the derivation of time dilation in the gravitational field, consists of 3 sections; apart from introduction, Section 2 is devoted for presenting EGSR theory. The derivation of time dilation in the gravitational field for any field is done in Section 3. The speed of light in the gravitational field is found in Section 4. Sections 5 and 6 are devoted for discussion and conclusion (Generalized Special Relativity [8]).

According to GSR the expressions of time t, length L, and mass m, are dependent on the velocity as well as field potential per unit mass and according to the relations.

$$t = \frac{t_0}{\sqrt{9_{00} - v^2/c^2}} \tag{1}$$

$$L = L_0 \sqrt{9_{00} - v^2/c^2} \tag{2}$$

$$m = \frac{9_{00} m_0}{\sqrt{9_{00} - v^2/c^2}} \tag{3}$$

where

$$9_{00} = 1 + \frac{2\phi}{c^2} \tag{4}$$

c stands for light speed in vacuum, while L_0 and m_0 represent the rest time, length and mass respectively.

2. Time Dilation and Length Contraction

The expression of time in the presence of gravitational field can be found by considering two as a clock, with time intervals to representing the travel between the two mirrors as shown in **Figure 1**.
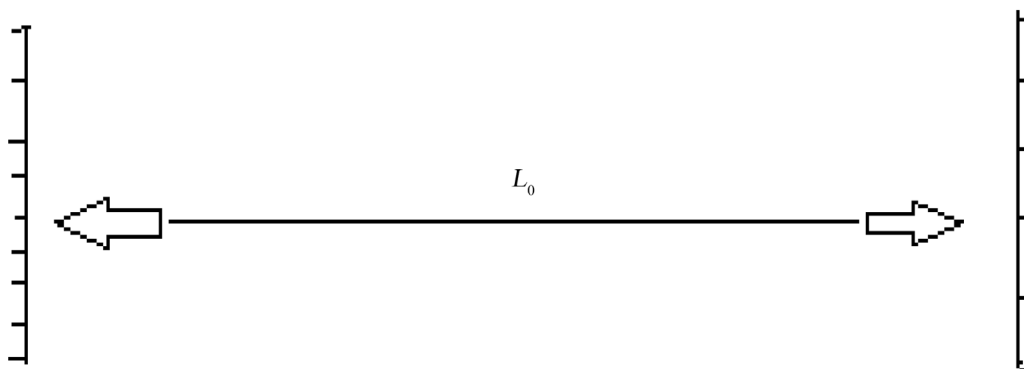

Figure 1. The mirror in free space.

$$L_0 = ct_0 \tag{5}$$

It moves with velocity up ward under gravity of acceleration g, the velocity of the lower mirror is given by

$$v^2 = v_0^2 - 2ay \tag{6}$$

where v_0 stands for the initial velocity, while y represents the displacement.

The vertical displacement y is given by

$$y = v_0 t - \frac{1}{2} a t^2$$

The speed v is also given by $v = v_0 - at$

Thus

$$y = \frac{(2v_0 - at)t}{2} = \frac{(v_0 + v_0 - at)t}{2} = \frac{(v_0 + v)t}{2} = v_a t \tag{7}$$

where: $v_a = \frac{(v_0 + v)}{2} \rightarrow (8)$ is the average speed; a is any arbitrary acceleration in general. For the force F the potential v is given by

$$F = ma, \quad V = may = m\phi$$

where ϕ is the potential per unit mass and is given to be

$$\phi = ay = gy \tag{9}$$

Inserting (9) in (6) yields as shown in **Figure 2**.

$$v^2 = v_0^2 - 2\phi \tag{10}$$

Since the point p at which the photon hits the mirror is displaced y meter vertically down ward, it follows that the light photon hits p as shown in **Figure 3**.

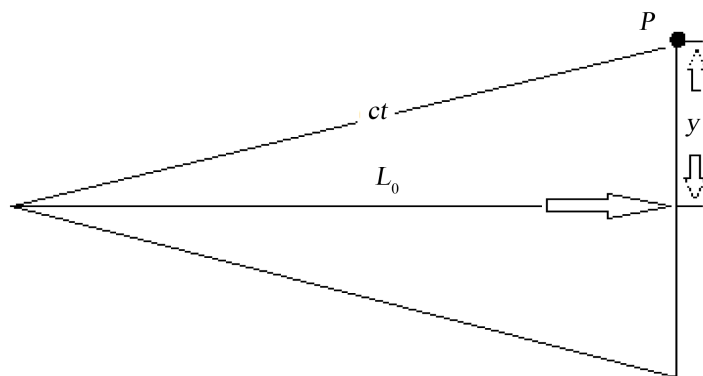**Figure 2.** The mirror at $t = 0$.

Figure 3. The mirror displaced y meter vertically down ward.

Speed c in the gravitational field the distance travelled by light is given indicates that

$$(ct)^2 = L_0^2 + y^2$$

In view of Equations (7)-(10), one gets

$$c^2 t^2 = ct_0^2 + v_m^2 t^2$$

$$\left(1 - \frac{v_m^2}{c^2}\right) t^2 = t_0^2$$

$$t = \frac{t_0}{\sqrt{1 - \frac{v_a^2}{c^2}}} = \frac{t_0}{\sqrt{1 - \frac{(v_0 + v)^2}{4c^2}}} = \gamma t_0 \tag{11}$$

Thus:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_a^2}{c^2}}} = \frac{1}{\sqrt{1 - \frac{(v_0 + v)^2}{4c^2}}} \tag{12}$$

Using relation (10)

$$\gamma = \frac{1}{\sqrt{1 - \frac{v_0^2 + 2vv_0 + v^2}{4c^2}}},$$

$$\gamma = \frac{1}{\sqrt{1 - \frac{\left(2v_0^2 - 2\phi + 2v_0\sqrt{v_0^2 - 2\phi}\right)}{4c^2}}}. \tag{13}$$

If one consider v and v_0 as standing for effective speed which is related to the maximum speed through the relations

$$v_0 = \frac{1}{\sqrt{2}} v_{0_{\max}}, \quad v = \frac{1}{\sqrt{2}} v_{\max} \tag{14}$$

Then relation (10) becomes

$$v_m^2 = v_{0_{\max}}^2 - 4\phi \tag{15}$$

And relation (13) becomes

$$\gamma = \frac{1}{\sqrt{1 - \frac{\left(2v_{0_{\max}}^2 - 4\phi + 2v_{0_{\max}}\sqrt{v_{0_{\max}}^2 - 4\phi}\right)}{4c^2}}}$$

For weak field

$$2v_0\left(v_{0_{\max}}^2 - 4\phi\right)^{\frac{1}{2}} = 2v_{0_{\max}}^2\left(1 - \frac{4\phi}{v_{0_{\max}}^2}\right)^{\frac{1}{2}} \approx 2v_{0_{\max}}^2 - 4\phi$$

Thus

$$\gamma = \frac{1}{\sqrt{1 - \frac{\left(2v_{0_{\max}}^2 - 4\phi + 2v_{0_{\max}}^2 - 4\phi\right)}{4c^2}}} = \frac{1}{\sqrt{1 + \frac{2\phi}{c^2} - \frac{v_{0_{\max}}^2}{c^2}}} \tag{16}$$

Which coincides completed with the expression of time in the presence of the gravitational field obtain within the framework of GSR. Relation (14) it is important to note that of the factor $1/2$ in Einstein SR energy expression where

$$E = \frac{1}{2}mv_m^2 = m\left(\frac{v_m}{\sqrt{2}}\right)^2 = mv^2 \tag{17}$$

According to Equations (9) and (10) this relation holds for any field other than the gravitational field.

The length contraction can be obtained by considering a clock falling by sliding on a rod of a height L_0. For the observer moving with a clock the average speed is given by

$$v = \frac{L}{t_0} \tag{18}$$

where the rod is moving and accelerated w.r.t him, thus his length is L. But for an observer at rest w.r.t the rod, the rod length is L_0 for him and the clock time is it. Thus the speed v is given by

$$v = \frac{L_0}{t} \tag{19}$$

Thus, with the aid of Equations (18) (19) and (13).

$$\frac{L}{t_0} = \frac{L_0}{t}$$

$$L = L_0 \frac{t_0}{t} = \gamma^{-1} L_0 \tag{20}$$

This is in agreement with the corresponding expression in GSR.

3. Derivation of Time and Coordinate Expressions by Using Lorentz Transform

Using Lorentz transformation, the event at point (x_1, t_1) in the frame x at a point x is given to be according to Equation (7) by replacing y by x

$$x = \gamma\left(x' + v_a t'\right) \tag{21}$$

Since the space is homogeneous it follows that

$$x' = \gamma\left(x + v_a t\right) \tag{22}$$

Consider ampoule of light emitted from a source when the origins 0 and 01 are in coincidence at

$$t = t' = 0$$

In this care

$$\begin{align} x &= ct \\ x' &= ct' \end{align} \tag{23}$$

Substituting Equation (23) in Equations (22) and (21) yields

$$ct = \gamma\left(ct' + v_a t'\right) = \gamma\left(c + v_a\right)t' \tag{24}$$

$$ct' = \gamma\left(ct - v_a t\right) = \gamma\left(c - v_a\right)t \tag{25}$$

Inserting Equation (25) in Equation (24) yields

$$ct = \frac{\gamma^2}{c}\left(c + v_a\right)\left(c - v_a\right)t$$

$$c^2 = \gamma^2\left(c^2 - v_a^2\right)t$$

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v_a^2}{c^2}}} \tag{26}$$

With the aid of Equation (8) and Equation (10)

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{\left(v_0^2 + 2vv_0 + v^2\right)}{4c^2}}} = \frac{1}{\sqrt{1 - \dfrac{\left(v_0^2 - \phi + v_0\sqrt{v_0^2 - 2\phi}\right)}{2c^2}}} \tag{27}$$

When one consider the expression for the maximum speed in Equation (15), one gets

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{\left(v_{0_{max}}^2 - 2\phi + v_{0_{max}}\sqrt{v_{0_{max}}^2 - 4\phi}\right)}{2c^2}}} \tag{28}$$

It is very striking to observe that when no field exist, *i.e.* $\phi = 0$

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v_{0_{max}}^2}{c^2}}} \tag{29}$$

This is the usual SR expression.
For weak field

$$\frac{\phi}{v_{0_{max}}^2} \prec\prec 1$$

Therefore

$$\left(v_{0_{max}}^2 - 4\phi\right)^{\frac{1}{2}} = v_{0_{max}}\left(1 - \frac{2\phi}{v_{0_{max}}^2}\right)$$

Thus Equation (25) becomes

$$\gamma = \sqrt{1 - \frac{\left(v_{0_{max}}^2 - 2\phi + v_{0_{max}}^2 - 2\phi\right)}{2c^2}} \tag{30}$$

This is the usual expression of GSR for a weak field.

4. Discussion

In Section 3 time dilation expression in the presence of any field has been derived by using mirror, see Equations (11) and (14). The only assumption which causes a limitation to this expression is the constancy of acceleration. The mirror motion is described by using Newton's Law of linear motion with constant acceleration.

The speed of light is assumed to be constant. The resulting expression for t reduced to that of SR in the absence of a field as shown by Equation (16) where

$$\gamma = \frac{1}{\sqrt{1 - \dfrac{v_{0_{max}}^2}{c^2}}}$$

It also reduced to GSR form for a weak field as shown in Equation (16).

The expression of length contraction has been obtained as well. Again this expression reduced to SR and GSR also [9].

Lorentz transformation is also utilized to find a relativistic expression for t and x. The expression shown in

Equations (27)-(29) indicates that these expressions are similar to that obtained by the mirror method and are reduced to the corresponding SR and GSR expressions.

5. Conclusion

The expressions for time and length obtained by using Lorentz transformation and using mirror clock, for fields at constant acceleration, and by assuming the speed of light to be constant indicate that GSR rests on a solid ground. It also indicates that space and time are affected by any field, not gravity only. Unlike the curved space-time derivation, where the field is assumed to be week, this derivation holds for strong fields as well. By reducing to SR and GSR for a weak field, it indicates its self-consistency.

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