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Designing mixed H_2/H_∞ structure specified controllers using Particle Swarm Optimization (PSO) algorithm

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ABSTRACT

This paper proposes an efficient method for designing accurate structure-specified mixed H_2/H_∞ optimal controllers for systems with uncertainties and disturbance using particle swarm (PSO) algorithm. It is designed to find a suitable controller that minimizes the performance index of error signal subject to an unequal constraint on the norm of the closed-loop system. Although the mixed H_2/H_∞ for the output feedback approach control is considered as a robust and optimal control technique, the design process normally comes up with a complex and non-convex optimization problem, which is difficult to solve by the conventional optimization methods. The PSO can efficiently solve design problems of multi-input-multi-output (MIMO) optimal control systems, which is very suitable for practical engineering designs. It is used to search for parameters of a structure-specified controller, which satisfies mixed H_2/H_∞ performance index. The simulation and experimental results show high feasibility, robustness and practical value compared with the conventional proportional-integral-derivative (PID) and proportional-Integral (PI) controller, and the proposed algorithm is also more efficient compared with the genetic algorithm (GA).

KEYWORDS

Mixed H_2/H_∞ Optimal Control; Particle Swarm Optimization Algorithm; Structure-Specified Controller

1. INTRODUCTION

Recently, mixed H_2/H_∞ optimal control problems have received a great deal of attention from the viewpoint of theoretical design because it is an advanced technique for designing robust and optimal controllers for systems associated with sources of uncertainties. It was firstly proposed by Bernstein [1], and has been further developed by many researchers [2,3]. Although GA is a useful tool for solving optimization problems and has been applied successfully in many control systems, it still has limitations due to its stochastic searching characteristic and complex computation that make it slow convergence to global optimum. PSO is a powerful method for solving complex and ill-defined optimization problems because of its oriented searching and simple computation search [4]. Many researchers have become increasingly interested in the use of PSO as a means to design various classes of control systems. Kao [5] used PSO to design a self-tuning PID controller for a slider-crank mechanism. Chang [6] used PSO to design a PID controller for chaotic synchronization. In this paper, we propose a method to design the structure-specified mixed H_2/H_∞ controllers by using PSO algorithm. The aims of this design are to obtain both robust stability and good performance, for instances, small tracking error, less control energy, etc. In the method, model uncertainty of the system is represented by multiplicative uncertainty, and the system is assumed to be affected by external unit step disturbances and then the structure-specified controller is defined. Finally, PSO is used to search for parameters of an admissible structure-specified controller that minimizes the integral of squared error (H_2 norm) subjected to robust stability constraints (H_∞ norm) against model uncertainty and external disturbances. The paper is organized as follows. Section 2, 3 and 4 explain a systematic procedure of the proposed controllers design algorithm. Simulation and results are presented in Section 5. Finally, Section 6 concludes the paper.

2. STRUCTURE-SPECIFIED MIXED H_2/H_∞ CONTROL

Consider a system with n_i inputs and n_o outputs controlled system as shown in **Figure 1**, where $P(s)$ is nominal plant model, $\Delta P(s)$ is plant perturbation, $C(s)$ is controller, $r(t)$ is reference input, $e(t)$ is trackingerror, $d(t)$ is external disturbance, and $y(t)$ is output of the system [7]. The plant perturbation $\Delta P(s)$ is assumed upper bounded by a known stable weighting function matrix $W_1(s)$

$$\bar{\sigma}(\Delta P(j\omega)) \le \bar{\sigma}(W_1(s)), \forall \omega \in [0,\infty) \tag{1}$$

where $\bar{\sigma}(A)$ denotes the maximum singular value of a matrix A.

It is proved that if a controller $C(s)$ is designed so that:

1) The nominal closed-loop system ($\Delta P(s)=0$ and $d(t)=0$) is asymptotically stable.

2) The robust stability performance against plant perturbation satisfies the following inequality

$$J_a = \|W_1(s)T(s)\|_\infty < 1 \tag{2}$$

3) The disturbance attenuation performance satisfies the following inequality

$$J_b = \|W_2(s)S(s)\|_\infty < 1 \tag{3}$$

Then, the closed-loop system is also asymptotically stable with $\Delta P(s)$ and $d(t)$, where $W_2(s)$ is a stable weighting function matrix specified by the designers. $S(s)$ and $T(s)=I-S(s)$ are the sensitivity functions, and the complementary sensitivity of the system, respectively [8]

$$S(s) = (I+P(s)C(s))^{-1} \tag{4}$$

$$T(s) = P(s)C(s)(I+P(s)C(s))^{-1} \tag{5}$$

In many control systems, not only the robust stability against plant perturbation and external disturbances, but also small tracking error is also important. The problem of minimizing the tracking error of a system can be defined as minimizing the cost function, called the integral of the squared error (ISE)

$$J_2 = \int_0^\infty e^2(t)\,\mathrm{d}t = \|E(s)\|_2^2 \tag{6}$$

where $e(t)$ is the system error between input and output which can be obtain from inverse Laplace transformation of $E(s)$ with $\Delta P(s)=0$ and $d(t)=0$

$$E(s) = (I+P(s)C(s))^{-1} R(s) \tag{7}$$

The handling of constraints in Equation (2), and Equation (3) is to recast the constraints as objectives tobeminimized and, consequently, a weighted sum approach is conveniently used with suitable weightings u_1 and u_2, which can be calculated by the designer. Therefore, the objective function of the investigated problem of designing mixed H_2/H_∞ optimal controllers will be as follows

$$\min_C J = u_1 J_2 + u_2 J_\infty \tag{8}$$

In this paper, the value of u_1 and u_2 is chosen equal to 1, and a suitable structure specified PID controller will be chosen depending on the number of the inputs and the number of the outputs.

3. PSO ALGORITHM

Particle swarm optimization is a heuristic global optimization method and also an optimization algorithm, which is based on swarm intelligence. It comes from the research on the bird and fish flock movement behavior. In an initial moment, all the particles are positioned randomly in the searching space, in which the solution must be. The movement of each particle is the result of a vector sum of three distinct terms; the first contribution is related to the inertia of the particle (a particle's personal component), the second is related to the best position occupied by the particle (a personal component-memory) and the third is relative to the best position found by the group (group contribution-cooperation). Let the search space be N-dimensional, and the particle i is represented by an N-dimensional position vector

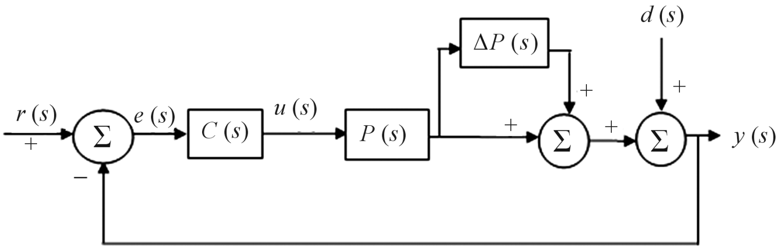

Figure 1. Control system with plant perturbation and external disturbance.

$x_i = (x_{i1}, x_{i2}, \cdots, x_{iN})$. The velocity is represented by an N-dimensional velocity vector $v_i = (v_{i1}, v_{i2}, \cdots, v_{iN})$. The fitness of particles is evaluated by the objective function of the optimization problem. The best previously function of the optimization problem. The best previously visited position of particle i is noted as its individual best position, $P_i = (P_{i1}, P_{i2}, \cdots, P_{iN})$. The position of the best individual of the whole swarm is noted as the global best position, $G = (g_1, g_2, \cdots, g_N)$. At each step of searching process, the velocity of particle and its new position are updated according to the following two equations [9]

$$v_i(k+1) = w \cdot v_i(k) + c_1 \cdot r_1 \cdot (P_i(k) - x_i(k)) + c_2 \cdot r_2 \cdot (G(k) - x_i(k)) \tag{9}$$

$$x_i(k+1) = x_i(k) + v_i(k) \tag{10}$$

where *w*, called inertia weight, controls the impact of previous velocity of the particle. r_1, r_2 are random variables in the range of [0, 1]. c_1, c_2 are positive constant parameters called acceleration coefficients. The value of each component in v is limited to the range $[-v_{\max}, v_{\max}]$ to control excessive roaming of particles outside the search space.

4. PSO-BASED STRUCTURE-SPECIFIED MIXED H_2/H_∞ CONTROL

A procedure for designing PSO-based structure-specified mixed H_2/H_∞ controllers for the problem defined in Section 3.1 is presented below.

***Step* 1:** Define a structure-specified controller of the form

$$C(s) = \frac{N_c(s)}{D_c(s)} = \frac{B_m s^m + B_{m-1} s^{m-1} + \cdots + B_0}{s^n + a_{n-1} s^{n-1} + \cdots + a_0} \tag{11}$$

and specify the upper bound of plant uncertainty, $W_1(s)$, weighting function for disturbance rejection, $W_2(s)$.

***Step* 2:** Set particle i to $x_i = (x_{i1}, x_{i2}, \cdots, x_{iN}) = (B_0, B_1, \cdots, a_0, a_1, \cdots)$, the number of parameters of the controller in Equation (11) is the dimension of particle, $N = m + n + 1$.

***Step* 3:** Initialize a random swarm of H particles as $[x_1, x_2, \cdots, x_H]$ when the swarm size is set to H.

***Step* 4:** For each generation, evaluate objective function of each particle using the objective function expressed in (6), and also evaluate the constraints (2) and (3). The cost function can then be calculated as following:

- If $E(s)$ has right half-plane poles, then set $J_2 = \infty$.
- If $Max(J_{\infty,a}, J_{\infty,b}) \ge 1$ then set $J_2 = \infty$ else $J_2 = E(s)$. Determine the individual best, and the global best.

***Step* 5:** Update the velocity of particle and its new position using (9) and (10).

***Step* 6:** When the maximum number of iterations is arrived, stop the algorithm. Otherwise, go to Step 4.

5. SIMULATION RESULTS

In this section, an PID and PI examples are given to illustrate the proposed design procedures and a comparison study with GA algorithm is carried out to illustrate the effectiveness. Consider a highly coupled distillation column model [8]

$$P(s) = \begin{bmatrix} \dfrac{-33.98}{(98.02s+1)(0.42s+1)} & \dfrac{32.63}{(99.6s+1)(0.35s+1)} \\ \dfrac{-18.85}{(75.43s+1)(0.3s+1)} & \dfrac{34.84}{(110.5s+1)(0.03s+1)} \end{bmatrix} \tag{12}$$

The bound $W_1(s)$ of the plant uncertainties $\Delta P(s)$ is

$$W_1(s) = \begin{bmatrix} \dfrac{100s+1}{s+1000} & 0 \\ 0 & \dfrac{100s+1}{s+1000} \end{bmatrix} \tag{13}$$

To attenuate disturbance, the stable weighting function $W_2(s)$ is chosen as

$$W_2(s) = \begin{bmatrix} \dfrac{s+1000}{1000s+1} & 0 \\ 0 & \dfrac{s+1000}{1000s+1} \end{bmatrix} \tag{14}$$

5.1. PI Problem

Since there are 2-inputs and 2-outputs, the structured-specified PI controller will be

$$C(s) = \frac{\begin{bmatrix} B_1 & B_3 \\ B_5 & B_7 \end{bmatrix} s + \begin{bmatrix} B_2 & B_4 \\ B_6 & B_8 \end{bmatrix}}{s}$$

A typical controller $C(s)$ obtained from the POS-based method with 10 runs

$$C(s) = \frac{\begin{bmatrix} -18.5612 & 16.4037 \\ 2.4486 & 21.6076 \end{bmatrix} s + \begin{bmatrix} -25.5642 & 15.4331 \\ -25.5703 & 15.8784 \end{bmatrix} s}{s}$$

With the following value of performance indices $J_2 =$ 0.5835, $J_a = 0.636$, and $J_b = 0.0.4599$. The step response, the disturbance response, and Step response with the following value uncertainty for the resultant system using PSO is shown in **Figures 2(a)-(d)**.

5.2. PID Problem

Since there are 2-inputs and 2-outputs the structured-specified PI controller will be

$$C(s)=\frac{\begin{bmatrix}B_1 & B_4\\ B_7 & B_{10}\end{bmatrix}s^2+\begin{bmatrix}B_2 & B_5\\ B_8 & B_{11}\end{bmatrix}s+\begin{bmatrix}B_3 & B_6\\ B_9 & B_{12}\end{bmatrix}}{s}$$

A typical controller $C(s)$ obtained from the PSO-based method with 10 runs (equation at the end of the page).

The step response, the disturbance response, and step response with the following value uncertainty of the resultant system using PSO is shown in **Figures 3(a)-(d)**. **Table 1** shows the statistical results from the 10 runs of problems 1 and 2. **Figure 4** shows the convergence of the best trial from both PSO-based and GA-based algorithms. It can be seen that PSO-based algorithm is convergent at about 30 generations whereas GA-based algorithm takes about 45 generations.

6. CONCLUSION

This paper proposed a PSO-based algorithm for designing structure-specified mixed H_2/H_∞ MIMO controllers with robust stability and disturbance attenuation. The high performance and validity of the proposed method are demonstrated by a MIMO system with PI and

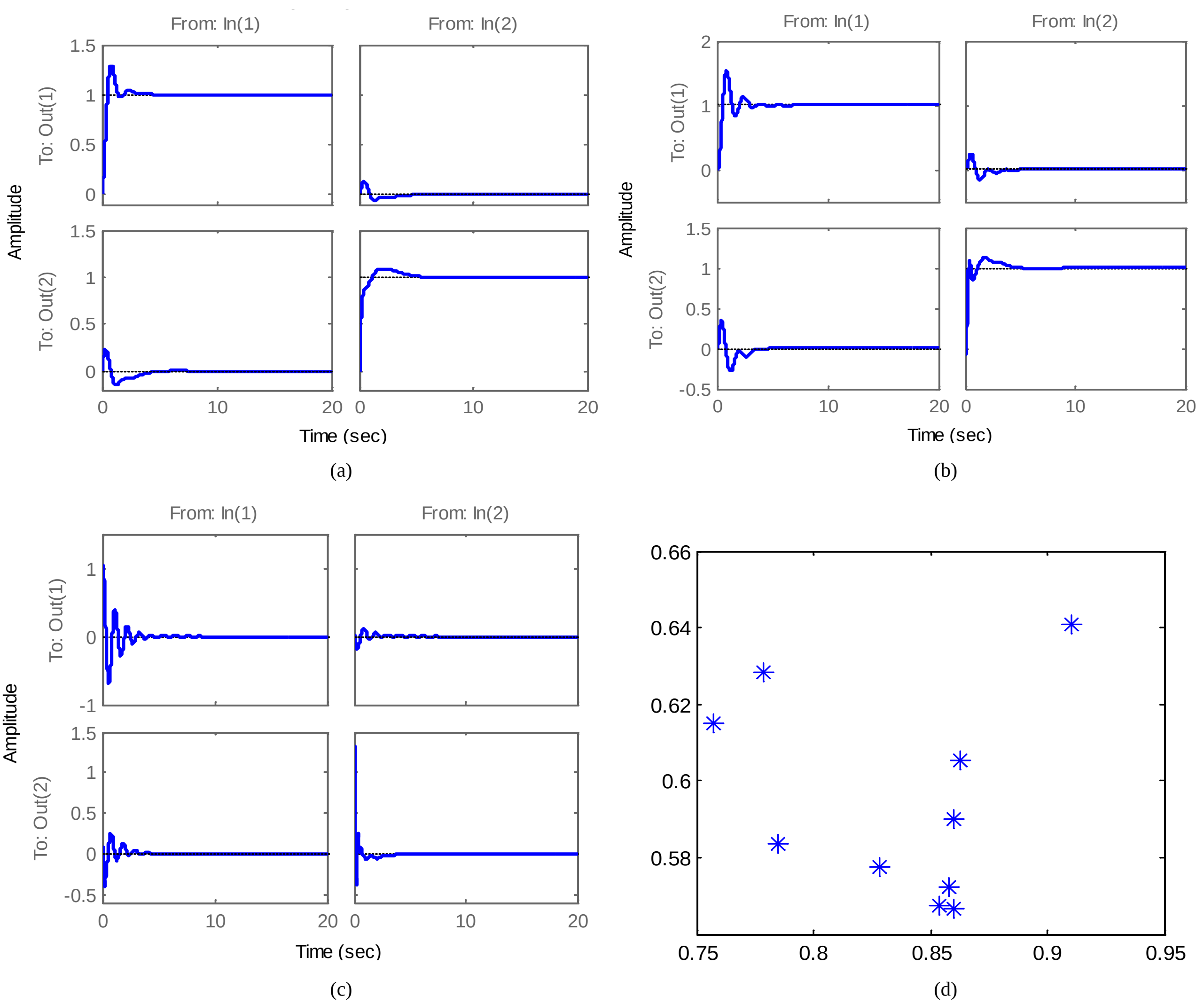

Figure 2. Output response for example 1. (a) PI step response; (b) PI disturbance response; (c) PI Step response with uncertainty; (d) Distribution of the ten PI controllers.

$$C(s)=\frac{\begin{bmatrix}-5.7086 & -1.5980\\ -0.3567 & -0.1178\end{bmatrix}s^2+\begin{bmatrix}-40.8379 & 10.8117\\ -17.3910 & 12.0784\end{bmatrix}s+\begin{bmatrix}-1.1010 & 1.8873\\ -1.1966 & 25.6294\end{bmatrix}}{s}$$

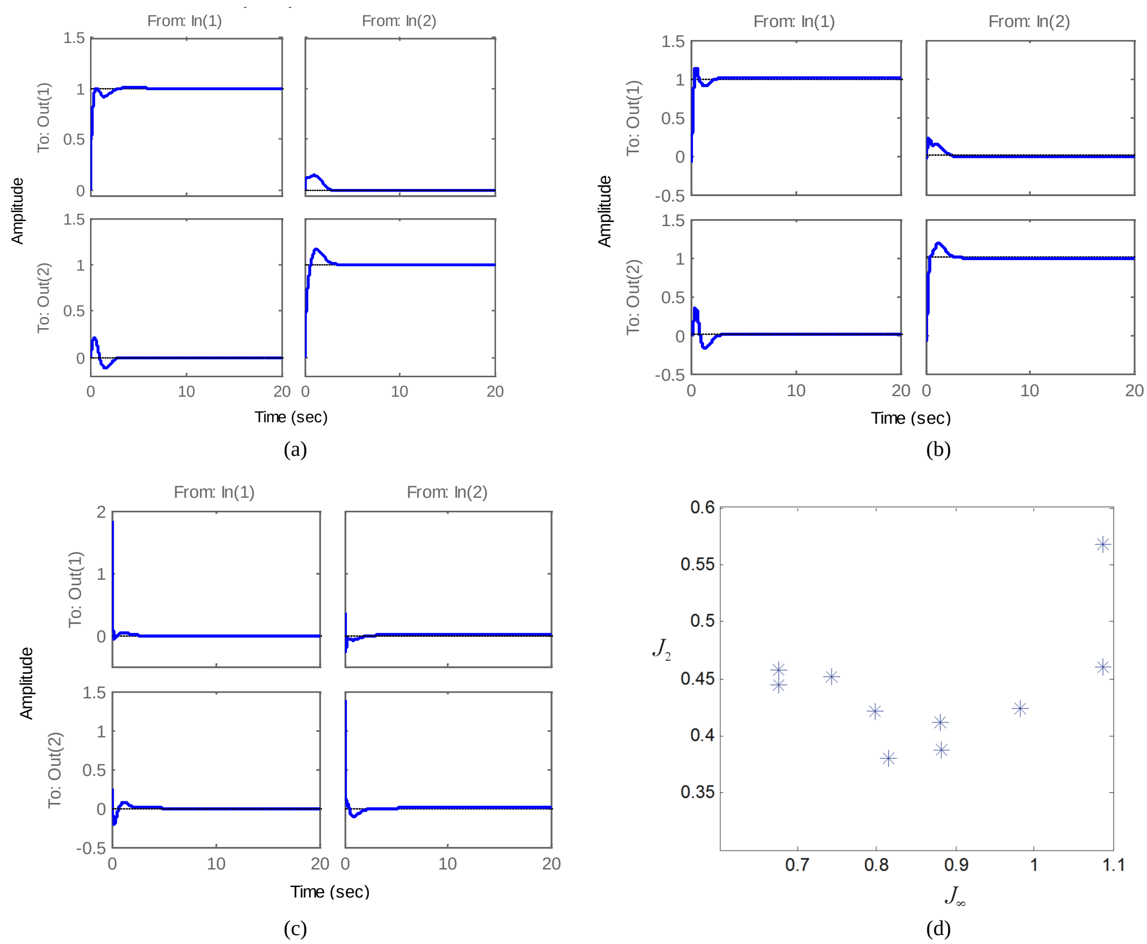

Figure 3. Output response for example 2. (a) PID step response; (b) PID disturbance response; (c) PID Step response with uncertainty; (d) Distribution of the ten PID controllers.

Table 1. Performance of the pso-based controller from 10 runs.

Controller	J_2			J_∞			J		
	Best	Avg.	Std.	Best	Avg.	Std.			
Problem 1 (PI)	0.5665	0.5947	0.0264	0.7574	0.8355	0.0475	1.3239	1.4302	0.0739
Problem 2 (PID)	0.3801	0.4405	0.0526	0.6749	0.8625	0.1517	1.055	1.303	0.2043

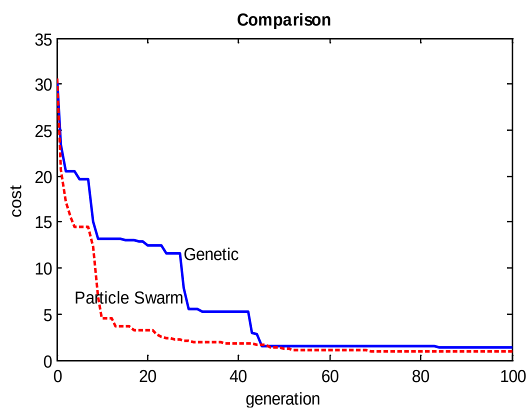

Figure 4. Convergence of the cost for PSO-based and GA-based optimization algorithms.

PID controllers. It shown empirically that the performance of the proposed method is more superior than that of existing GA due to the presence of very good speed and fitness with a few numbers of iteration and less complexity to reach the optima simulation and experimental results show the robustness and efficiency of the proposed controller. The PSO-based method can be most widely used for designing high-performance optimal controllers.

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