Scientific Research

The Relationship between Stock Returns and Volatility in the Seventeen Largest International Stock Markets: A Semi-Parametric Approach

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Abstract

We empirically investigate the relationship between expected stock returns and volatility in the twelve EMU countries as well as five major out of EMU international stock markets. The sample period starts from December 1992 until December 2007 *i.e.* up to the recent financial crisis. Empirical results in the literature are mixed with regard to the sign and significance of the mean – variance tradeoff. Based on parametric GARCH in mean models we find a weak relationship between expected returns and volatility for most of the markets. However, using a flexible semi-parametric specification for the conditional variance, we unravel significant evidence of a negative relationship in almost all markets. Furthermore, we investigate a related issue, the asymmetric reaction of volatility to positive and negative shocks in stock returns confirming a negative asymmetry in almost all markets.

1. Introduction

An important topic in asset valuation research is the tradeoff between volatility and return (typically log price changes). The theoretical asset pricing models [1-5] link returns of an asset to its own variance or to the covariance between the returns of stock and market portfolio. However, whether such a relationship is positive or negative has been controversial. As summarized in [6], most asset-pricing models [1-4] suggest a positive tradeoff for expected returns and volatility. On the other hand, there are many empirical studies that confirm a negative relationship between returns and volatility: [7-10]. Reference [11] indicates that there is a strong positive relationship between risk and excess return while [12,13] as well as [9] found negative relationship in a variety of USA market indices. Reference by [14] using data from the S&P index, find a negative relationship between unexpected volatility and excess returns. Similar are the findings of [15] which use a two regime model. According to [14] argue that the observed negative relationship provides indirect evidence of a positive relationship between expected risk premium and exante volatility. In

other words, if expected risk premiums are positively related to predictable volatility, then a positive unexpected change in volatility increases future expected risk premiums and lowers current stock prices. [16] also points to a positive relationship between risk and return for USA monthly and daily returns over the period 1926-1988. [17,18] in various assets of USA market find a negative relationship before 1990. [19] reports no significant relationship in USA stock market at the same period. Also, [20] suggests that the relationship between risk and return may be time varying. These conflicting empirical results in the literature warrant further examination using different and probably more appropriate econometric techniques.

In this work we utilize a flexible functional form to model conditional variance. We favour this approach, given that estimation via a parametric GARCH-M model is prone to model misspecifications. Consistent estimation of a GARCH-M model requires that the full model is correctly specified [21]. Indeed, the problem that inferences drawn on the basis of GARCH-M models may be susceptible to model misspecification is well known to applied researchers. [18] argues that parameter restric-

tions imposed by GARCH models may unduly restrict the dynamics of the conditional variance process. In contrast, a semi-parametric specification of the conditional variance allows flexible functional forms, and therefore can lead potentially to more reliable estimation and inference. In this paper, we propose three parametric GARCH-M models and a semi-parametric model for testing the null hypothesis of zero GARCH in mean effect. We then apply the above models to daily empirical data of seventeen largest international stock markets and two world indices. We show some evidence that a significant negative relationship between stock market returns and market volatility prevails in most major stock markets.

Moreover the paper investigates the asymmetric impact of positive and negative shocks on volatility using two parametric models: EGARCH-M and AGARCH-M. [21-23] provide thorough surveys in this area.

The rest of this paper is organized as follows. Section two discusses the parametric and semi-parametric models used in the present empirical analysis, section three presents the data, in section four we test the forecasting power of each model, in section five we interpret the empirical results and in section six we conclude the paper.

2. The Parametric GARCH-M Model with *t* Distributed Innovations

The model is specified as follows:

$$y_t = \mu + \sum_{s=1}^{k} b_s y_{t-s} + \delta \sigma_t^2 + u_t \tag{1}$$

$$u_t = \varepsilon_t \sigma_t \tag{2}$$

$$\varepsilon_t \big| \Omega_{t-1} \sim t.d.(0, \sigma_t, v), \tag{3}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}. \tag{4}$$

The dependent variable y_t denotes the stock market returns, σ_t^2 their conditional variance and u_t the disturbance term. We assume that the random term ε_t is distributed as the student's t with ν degrees of freedom. Equation (1) represents dynamic changes in the mean returns, while Equation (4) describes time variation in the conditional variance. The information available at period *t*-1 is denoted by Ω_{t-1}. The conditional variance σ_t^2 in (4), is modelled according to ARCH – GARCH (q, p) specification of [24]. Among all the parameters to be estimated, the most relevant for this study is the parameter *δ*. The sign and significance of the parameter defines the relationship between stock market returns and conditional variance. Setting $a_i + \beta_i = 1$ implies a

highly persistent conditional variance. This model is known as the integrated GARCH or IGARCH which is more likely for daily data series.

2.1. Parametric EGARCH-M Model

In this case the model is specified as follows:

$$y_t = \mu + \sum_{s=1}^{k} b_s y_{t-s} + \delta \sigma_t^2 + u_t \tag{5}$$

$$u_t = \varepsilon_t \sigma_t, \tag{6}$$

$$\varepsilon_t \big| \Omega_{t-1} \sim n.d.(0,1), \tag{7}$$

$$\log \sigma_t^2 = a_0 + \sum_{i=1}^{p} a_i \left\{ \theta_1 \varepsilon_{t-i} + \theta_2 \left(\left| \varepsilon_{t-i} \right| - E \left| \varepsilon_t \right| \right) \right\} + \sum_{i=1}^{p} \beta_i \log \sigma_{t-i}^2. \tag{8}$$

The EGARCH specification allows the conditional variance process to respond asymmetrically to positive and negative shocks. This is reflected in the value of the parameter product $\alpha_i \theta_1$. When $\alpha_i \vartheta_1 < 0 \ (> 0)$, the variance tends to rise (fall) when the shock is negative (positive).

2.2. The Asymmetric GARCH-M (GJR) Model

In order to capture the asymmetric impact of new information on volatility [25] suggested the GJR-GARCH (q, p) model. It is specified by the following equations:

$$y_t = \mu + \sum_{s=1}^{k} b_s y_{t-s} + \delta \sigma_t^2 + u_t \tag{9}$$

$$u_t = \varepsilon_t \sigma_t, \tag{10}$$

$$\varepsilon_t \big| \Omega_{t-1} \sim t.d.(0, \sigma_t, v), \tag{11}$$

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} a_i \varepsilon_{t-1}^2 + \kappa_2 D_{t-1} \varepsilon_{t-1}^2 + \sum_{i=1}^{p} \beta_i \sigma_{t-1}. \tag{12}$$

The conditional variance equation (12) includes an extra term: the dummy variable D_{t-1}, it is equal to one when ε_{t-1} is negative (good news), and equal to zero when ε_{t-1} is positive (bad news). A statistically significant parameter κ_2 indicates volatility clustering. In case $\kappa_2 > 0$ there are leverage effects while when $\kappa_2 = 0$ the news impact curve is symmetric, *i.e.* past positive shocks have the same impact as past negative shocks on today's volatility.

2.3. A Semi-Parametric GARCH-M Specification

We consider the following semi-parametric GARCH-M

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model

$$y_t = a_0 + a_1 y_{t-1} + \delta \sigma_t^2 + u_t \equiv x_t a + u_t, \qquad (13)$$

where y_t is the stock market returns, $x_t = \left(1, y_{t-1}, \sigma_t^2\right)$, $a = \left(a_0, a_1, \delta\right)'$ is a vector of parameters to be estimated, $\sigma_t^2 = \mathrm{var}\left(y_t \mid \Omega_{t-1}\right)$ is the conditional on Ω_{t-1} variance of y_t, Ω_{t-1} is the information set available up to time $t-1$. The error term is a martingale difference process, *i.e.*, $E\left(u_t \mid \Omega_{t-1}\right) = 0$. We are interested in testing the null hypothesis of $H_0 : \delta = 0$ versus $H_1 : \delta \neq 0$. The null hypothesis implies that the conditional variance σ_t^2 does not affect the returns y_t. We first examine a simple semi-parametric GARCH model of the form

$$\sigma_t^2 = m\left(u_{t-1}\right) + \gamma \sigma_{t-1}^2, \qquad (14)$$

where the functional form of $m(\cdot)$ is not necessary specified parametrically. In case $m\left(u_{t-1}\right) = a + \beta u_{t-1}^2$ the model is reduced to the standard GARCH(1,1) specification. Under the null $H_0 : \delta = 0$ using (13), we obtain $u_{t-1} = y_{t-1} - a_0 - a_1 y_{t-2}$. Then, we can generalize (14) as follows

$$\mathrm{var}\left(y_t \mid I_{t-1}\right) = \sigma_t^2 = g\left(y_{t-1}, y_{t-2}\right) + \gamma \sigma_{t-1}^2, \qquad (15)$$

where

$$g\left(y_{t-1}, y_{t-2}\right) = m\left(y_{t-1} - a_0 - a_1 y_{t-2}\right) = m\left(u_{t-1}\right)$$

Expression (15) allows the conditional variance to depend on lagged values of y_t. Denoting $z_{t-1} = \left(y_{t-1}, y_{t-2}\right)$ and substituting (15) recursively yields

$$\sigma_t^2 = g\left(z_{t-1}\right) + \gamma g\left(z_{t-2}\right) + \gamma^2 g\left(z_{t-3}\right) \\ + \cdots + \gamma^{d-1} g\left(z_{t-d}\right) + \cdots \qquad (16)$$

Given that $0 < \gamma < 1$, we may approximate (16) by a finite lag model of length d:

$$\sigma_t^2 = g\left(z_{t-1}\right) + \gamma g\left(z_{t-2}\right) + \gamma^2 g\left(z_{t-3}\right) \\ + \cdots + \gamma^{d-1} g\left(z_{t-d}\right) \qquad (17)$$

Equation (17) is a restricted additive model with the restriction that the different additive functions $g(\cdot)$ are proportional to each other. The model allows lagged y_{t-s} being included at the right-hand side of (17). [26], in a similar framework, suggests a kernel-based method to estimate the model. Although Equation (17) is only a two-dimensional non-parametric model, it can be difficult to estimate by the popular kernel method, especially when d is large. Moreover, when d is large, the kernel method can give quite unreliable estimates due to its failure to impose the additive model structure. In this work we opt to estimate (17) by the non-parametric series method. The advantage of using a series method is that the additive proportional model structure is imposed directly and the estimation is performed in one step. To

this end, let $\left\{\phi_i\left(y\right)\right\}_i^\infty = 0$ denote a series-based function that can be used to approximate any univariate function $m\left(y\right)$. We can use a linear combination of the product base function to approximate $g\left(y_{t-1}, y_{t-2}\right)$,

i.e. $\displaystyle\sum_{i=0}^{q}\sum_{i'=0}^{q} a_{ii'}\phi_i\left(y_{t-s}\right)\phi_{i'}\left(y_{t-s-1}\right)$

$$= a_{00}\phi_0\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots + a_{0q}\phi_0\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right) \\ + a_{10}\phi_1\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots + a_{0q}\phi_1\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right) \\ + \cdots + a_{q0}\phi_q\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots \\ + a_{qq}\phi_q\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right), \qquad s = 1, \cdots, d.$$

After re-arranging terms, the approximating function is of the form:

$$\sigma_t^2 \cong a_{00}\sum_{s=1}^{d}\gamma^{s-1}\phi_0\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots \\ + a_{0q}\sum_{s=1}^{d}\gamma^{s-1}\phi_0\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right) \\ + a_{10}\sum_{s=1}^{d}\gamma^{s-1}\phi_1\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots \\ + a_{1q}\sum_{s=1}^{d}\gamma^{s-1}\phi_1\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right) \\ + a_{q0}\sum_{s=1}^{d}\gamma^{s-1}\phi_q\left(y_{t-s}\right)\phi_0\left(y_{t-s-1}\right) + \cdots \\ + a_{qq}\sum_{s=1}^{d}\gamma^{s-1}\phi_q\left(y_{t-s}\right)\phi_q\left(y_{t-s-1}\right). \qquad (18)$$

There are $\left(q+1\right)^2 + 1$ parameters, namely γ and $a_{ij}\left(i, j = 0, \cdots, q\right)$. Note that the number of parameters in model (18) does not depend on the number of lags included in the model. For example, if q is fixed, then the number of parameters is also fixed. Therefore, we can let $d \to \infty$ as $T \to \infty$ (with $d/T \to 0$). Asymptotically, it allows an infinite lag structure without the curse of dimensionality problem (since q is independent of d). The estimation procedure is as follows: Under $H_0 : \delta = 0$ we obtain $y_t = a_0 + a_1 y_{t-1} + u_t$. Define $\widetilde{y}_t = y_t - a_0 - a_1 y_{t-1}$, then $E\left(\widetilde{y}_t^2 \mid \Omega_{t-1}\right) = \sigma_t^2$ and

$$\widetilde{y_t} = \sigma_t^2 + v_t, \qquad (19)$$

where $E\left(\widetilde{y}_t^2 \mid \Omega_{t-1}\right) = 0$. Using the series approximation of σ_t^2 in (18) we substitute out σ_t^2 in (19), and replacing a_0 and a_1 by the least squares estimators of a_0 and a_1, we can estimate the parameters γ and a_{ij}'s using nonlinear least squares methods. Alternatively, one can estimate a_{ij}, $i, j = 0, 1, \cdots, q$, by least squares, regressing y_t^2 $\left(\widetilde{y_t^2}\right)$ on the series approximating base functions, and searching over the grid $\gamma \in \left[0,1\right]$ for optimizing values. In order to ensure that the above pro-

cedure leads to a consistent estimate of the $g(\cdot)$ function, we need to let $q \to \infty$ and $q/T \to 0$ as $T \to \infty$. The condition $q \to \infty$ ensures that the asymptotic bias goes to zero. For example, if one uses power series as the base function, then it is well known that the approximation error of using a q-th order polynomial goes to zero as $q \to \infty$. The condition $q/T \to 0$ ensures that the estimation variance goes to zero as sample increases. See [27,28] for more details on the rate of convergence of series estimation.

Let $\widehat{\sigma}_t^2$ denote the resulting nonparametric series estimator of σ_t^2. Replacing σ_t^2 by $\widehat{\sigma}_t^2$ in Equation (13), we get

$$y_t = a_0 + a_1 y_{t-1} + \delta \widehat{\sigma}_t^2 + \varepsilon_t, \qquad (20)$$

where $\varepsilon_t = u_t + \delta \left(\sigma_t^2 - \widehat{\sigma}_t^2 \right)$. We then estimate the parameters vector $a = (a_0, a_1, \delta)'$ by least squares. Equation (20) contains a non-parametrically generated regressor $\widehat{\sigma}_t^2$. Let $\hat{a} = (\hat{a}_0, \hat{a}_1, \hat{\delta})'$ denote the resulting estimator. If one estimate the conditional variance ignoring the additive structure of σ_t^2, then can use the results of [30] and obtain the asymptotic distribution of $\hat{a}$ from:

$$\sqrt{n} (\hat{a} - a) \to N(0, \Sigma) \qquad (21)$$

where Σ is the asymptotic covariance matrix. Based on the least squares estimation of (18), the non-parametric estimator $\widehat{\sigma}_t^2$, is consistent under the null hypothesis of $H_0 : \delta = 0$.

Next, we discuss the selection of the number of lags d and the order of series approximation q, in finite sample applications. For a fixed value of q, the number of parameters to be estimated is fixed and does not depend on d. In particular, choosing a large value of d does not lead to over fitting because the number of parameters that need to be estimated does not vary as d increases. Therefore, it makes sense to select the value of q that minimizes the AKAIKE information criteria.

3. Description of the Data

To estimate the models we use daily US-dollar denominated returns[1] on stock indices of seventeen countries: the twelve stock markets of the European Monetary Union, Italy (ITA), Greece (GRE), Germany (GER), France (FRA), Finland (FIN), Belgium (BEL), Austria (AUS), Ireland (IRL), Netherlands (NETH), Luxemburg (LUX), Spain (SPN), Portugal (POR), as well as the United States of America (USA), the United Kingdom (UK),

Japan (JAP), Sweden (SWD) and Russia (RUS). We also include two world stock-indices: the international index of DataStream's stock-exchange markets (D-W.I.) (this includes stocks from the most developed markets worldwide and it remains through many years one of the most recognized indices in the world) and the M.S.C.I (Morgan Stanley Capital International) world index, which is a stock index of the international market. The last index includes stocks from 23 developed markets. It is computed since 1969 and has been a common evaluation index of the international stock-markets.

The data set starts from 4th December of 1992 and end up at 5th December of 2007, comprising 3.914 observations. The source is the DataStream database. Daily returns of each country are calculated using the first logarithmic differences of general indices that include dividends.

4. Forecasting Power of the Models

In this section we empirically investigate the four models, stated in Section 2, in terms of their forecasting ability. The test statistic is the mean absolute error (MAE) which is a measurement of the co-cumulative forecasting error of the model. In our analysis the MAE is calculated from the monthly sub-samples of observations. Therefore, we are able to obtain a MAE figure for each month. The results are presented on **Table 1**. We notice that the best forecasting model is the semi-parametric. In almost all markets the average MAE of the semi-parametric model is smaller of that of the parametric models. The second best forecasting model is EGARCH except of Japan and Italy where the GARCH-M and AGARCH- M models provide a smaller MAE. Moreover, the models GARCH-M and AGARCH-M have a similar forecasting ability with AGARCH-M being slightly ahead. Based on the results reported in **Table 1** we conclude that forecasting ability of the semi-parametric model is broadly superior.

5. Empirical Results

Notice that the conditional variance of the returns for each market is calculated using the stock returns instead of the excess stock returns. The excess return of a stock is defined as the difference between return and the risk-free rate that is dominant in the market. [6,18,29,30] agree that using returns is broadly equivalent with excess returns. In the present study for the estimation of the conditional variance we use returns computed by log price differences. For the three models: GARCH-M, EGARCH-M, and AGARCH-M, we decide the appropriate distribution and number of lags p and q based on the AKAIKE criterion. So, for the models GARCH-M

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[1]As suggested by [31], calculating the returns in U.S. dollars eliminates the local inflation.

Table 1. Results of the mean of the mean absolute forecasting errors of the four models.

MARKET	AUS	BEL	FIN	GER	GRE	IRL	ITA	JAP	LUX
garch-m	0.01035	0.009	0.01266	0.01	0.01237	0.01121	0.0072	0.01240	0.005394
egarch-m	0.008160	0.008	0.03916	0.0077	0.01101	0.01045	0.0096	0.13893	0.00393
agarch-m	0.010169	0.009	0.00877	0.0095	0.01242	0.01118	0.0068	0.01225	0.005452
semi-par	2.52E-07	8.9E-6	0.01264	0.007	0.00843	0.01047	0.0067	0.00926	0.00317
FRA	**NETH**	**POR**	**RUS**	**SPN**	**SWD**	**UK**	**USA**	**D-W.I.**	**M.S.C.I.**
0.00994	0.009523	0.0085	0.0165	0.0100	0.01372	0.0086	0.007	0.00702	0.0067297
0.00926	0.008817	0.0075	0.0115	0.0080	0.01228	0.00808	0.0073	0.00622	0.006395
0.00979	0.009236	0.0084	0.0142	0.0099	0.01341	0.0082	0.0075	0.00687	0.006534
0.0074891	0.007609	0.0060	0.0110	0.0074	0.00991	0.0075	0.0055	0.00508	0.005276

Table 2. Parameter estimates for all four models.

MARKET	AUS	BEL	FIN	GER	GRE	IRL	ITA	JAP	LUX
garch-m									
δ	0.0094***	0.01	−0.041***	0.00492	0.06	−0.008***	−0.0341	0.055***	0.039
(t-stat)	(14.5)	(0.003)	(−25.6)	(0.0021)	(0.036)	(−23.2)	(−0.016)	(39.6)	(0.016)
$\alpha_i + \beta_i$	0.8764	0.9921	0.9921	0.964	0.9928	0.9341	0.9808	0.9612	0.9944
egarch-m									
δ	−0.204	0.297	−0.0128	−0.0065	0.15	0.26	−2.061	2.788	−0.585
(t-stat)	(−0.03)	(0.087)	(−0.011)	(−0.002)	(0.088)	(0.121)	(−1.27)	(0.91)	(−0.162)
agarch-m									
δ	0.0083***	0.03	−0.0144	0.01***	0.072	0.00057	0.0403	0.18***	0.029
(t-stat)	(11.0)	(0.01)	(−0.013)	(30.2)	(0.042)	(0.531)	(0.0186)	(−175.00)	(0.0119)
$\alpha_i + \beta_i$	0.8401	0.9849	0.989	0.939	0.986	0.9003	0.959	0.954	0.9941
κ_2	0.0037***	0.062***	0.0029***	0.0031***	0.0032***	0.0037***	0.003***	0.0049***	0.00048
(t-stat)	(2.18)	(3.15)	(1.56)	(4.26)	(3.63)	(4.47)	(4.39)	(4.67)	(0.621)
semi-par									
δ	0.0002***	0.06***	−6.57***	−16***	−9.6***	−13.27***	−13***	−3.62**	6.66***
(t-stat)	(8.26)	(16.0)	(−7.97)	(−10.8)	(−9.5)	(−6.7)	(−9.247)	(−2.32)	(2.91)
FRA	**NETH**	**POR**	**RUS**	**SPN**	**SWD**	**UK**	**USA**	**D-W.I.**	**M.S.C.I.**
0.0115	0.018	0.031	0.019	0.008	0.0003	0.0085	0.05	0.008	0.03
(0.0042)	(0.0077)	(0.01)	(0.0067)	(0.0026)	(0.00016)	(0.0025)	(0.022)	(0.0023)	(0.0092)
0.9603	0.9906	0.9964	0.9536	0.939	0.993	0.9466	0.998	0.955	0.9944
1.059	0.79	0.248	−0.367	0.059	0.778	−0.33	0.533	3.919	−0.146
(0.361)	(0.298)	(0.046)	(−0.366)	(0.0019)	(0.44)	(−0.054)	(0.18)	(1.13)	(0.0361)
1.054	0.067	0.029	0.05	0.01***	0.0791	0.06	4.58	0.054	0.081
(0.018)	(0.024)	(0.0097)	(0.0172)	(−196.00)	(0.035)	(0.018)	(0.018)	(0.014)	(0.021)
0.936	0.987	0.9954	0.9485	0.9272	0.9876	0.9274	0.9826	0.9257	0.9839
−0.0041***	0.0043***	0.001***	0.00244	0.003***	0.0063***	0.0043***	0.004***	0.0032***	0.0035***
(5.11)	(−5.26)	(1.53)	(1.30)	(2.90)	(4.69)	(14.6)	(8.62)	(8.24)	(8.39)
−9.83***	−12.86***	−15.0***	−8.16***	−22.0***	−8.96***	−15.87***	−16.0***	−25.4***	−15.94***
(−6.36)	(−11.49)	(−8.5)	(−8.07)	(−9.85)	(−9.04)	(−9.75)	(−12.21)	(−10.77)	(−7.85)

*denotes statistical significance at 10% level, * *denotes statistical significance at 5% level, * **denotes statistical significance at 1% level.

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and AGARCH-M the best results are derived using the t-student distribution, while for EGARCH–M the normal distribution has prevailed.[2]

5.1. Interpretation of the Empirical Results

The parameter, δ, is estimated using the models of Section 2. In **Table 2** we notice that with the GARCH-M specification, parameter δ is statistically significant at 1% level in Austria, Finland, Ireland and Japan. Also we notice a negative relationship in Finland and Ireland and a positive relationship in the other two markets. On the other hand, with EGARCH-M models δ is not statistically significant at all levels. Next we estimate the asymmetric GARCH-M model. According to the results of **Table 2** it is obvious that only for Austria, Germany and Japan δ is statistical significant at 1% level. Also we observe a positive relationship between the returns and conditional variance for these three markets. Finally, we estimate the semi-parametric model using B-splines approximating base functions [32]. In all markets under examination δ is statistically significant at 1% level (with the exception of Japan where the estimate of the parameter δ is statistically significant at 5%). The negative relationship between return and conditional variance is dominant in almost all markets except Austria, Belgium and Luxemburg. Notice that the empirical results of the parametric models are in broad agreement with those of the semi-parametric model. However, the best forecasting ability and parameter estimate's statistical significance of the semi-parametric model renders it more reliable.

Estimates of asymmetry parameters are stated in **Table 3**. We notice that asymmetric response of the conditional variance is a dominant property of the countries under examination. Moreover, from **Table 2**, parameter estimates of κ_2 for all the markets, except France, exhibit positive signs at 1% level, confirming the existence of a negative asymmetry. Given the fact that the semi-parametric specification fits better the data this study tends to support the claim that volatility is negatively correlated with returns. Notice that the above results are consistent with the empirical findings of [10,31,33], but contradict empirical findings of an insignificant relationship reported in [29,30,34].

Table 3. Parameter estimates of the EGARCH-M model.

MARKET	AUS	BEL	FIN	GER	GRE	IRL	ITA	JAP	LUX
$\alpha_i\theta_1, \alpha_i = 1$	–0.065***	–0.06***	–0.068**	–0.05***	–0.0217	–0.0217	–0.03***	–0.046***	–0.00716
(t–stat)	(–2.87)	(–4.87)	(–2.31)	(–4.18)	(–1.28)	(–1.28)	(–2.79)	(–2.93)	(0.618)
$\alpha_2\theta_1$	–0.0350**		–0.04***	–0.09***	–0.0594			–0.067***	0.0097***
(t–stat)	(–2.10)		(–3.15)	(68.6)	(1.63)			(4.02)	(7.61)
$\alpha_3\theta_1$	–0.05646*		–0.0498*	–0.05***	–0.0014			–0.0092	–0.008559
(t–stat)	(–1,92)		(–8.79)	(–121)	(0.105)			(0.748)	(4.04)
$\alpha_4\theta_1$			–0.0199*		0.0313			0.0187	
(t–stat)			(–1.54)		(–1.44)			(–1.45)	
FRA	**NETH**	**POR**	**RUS**	**SPN**	**SWD**	**UK**	**USA**	**D–W.I.**	**M.S.C.I.**
–0.0525***	–0.058***	–0.05**	–0.0337	–0.0508	–0.08***	–0.08***	–0.1 ***	–0.11***	–0.06***
(–3.44)	(–5.31)	(–2.08)	(–0.62)	(–0.501)	(3.65)	(–3.48)	(–5.83)	(–5.24)	(–2.04)
		–0.03***	–0.061**	–0.0576	0.00968	–0.09***		–0.09***	
		(1.20)	(2.13)	(0.589)	(–0.497)	(72.5)		(–3.95)	
		–0.0786	–0.053**	–0.0392	0.00365			–0.0146	
		(0.621)	(1.94)	(0.309)	(–0.179)			(–0.643)	
		–0.0458	–0.027		0.0191				
		(–1.49)	(1.17)		(–1.28)				

*denotes statistical significance at 10% level, **denotes statistical significance at 5% level, ***denotes statistical significance at 1% level.

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[2]Full table of results are available upon request.

6. Conclusions

In this paper we empirically investigated the relationship between expected returns and conditional variance in twelve stock markets of the European Union as well as five large stock markets and two world indices. Most asset pricing models [1-4] predict a positive risk-return tradeoff. In order to investigate this, both parametric and semi-parametric estimation methods of the conditional variance have been applied to daily data from the above markets. Based on the semi-parametric specification, we find a statistically significant negative relationship of the risk-return tradeoff in most markets. There are only three exceptions: Austria, Belgium and Luxemburg. Furthermore, we find significant evidence of a negative asymmetry in almost all markets confirming the previous empirical findings.

7. Acknowledgements

8. References

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