

# Study of Configural Reasoning and Written Discourse in Geometric Exercises of Proving

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## Abstract

**This article presents a study of configural reasoning and written discourse developed by students of the National Polytechnic School of Ecuador when performing geometrical exercises of proving.**

## Keywords

**Proving, Configural Reasoning, Truncation, Visualization, Apprehension**

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## 1. Introduction

The study of geometry at all levels of education is very important because it “can be seen as a reflective tool that allows human beings to solve various problems and understand a world that offers a wide range of various geometric forms, each of the scenarios that comprise it, whether they be natural or artificial” [1].

According to McClure [2] the teaching of Euclidean Geometry prepares students for a more rigorous mathematical training because it deals with familiar objects that can be designed both visually and verbally; the statements made about these objects are easily understood and often blunt; logical methods involved tend to be less subtle than other introductory parts of mathematics, involving fewer quantifiers; it is possible to make a serious mathematical learning in this area without a perfect knowledge of axiomatic systems and the rules to work with them.

Jones *et al.* [3], is convinced that Euclidean geometry, in pre-university education, is a good time in which to learn about mathematical proofs and demonstrations because the demonstrations are usually short, require only a few concepts, are supported by visual properties and are quite formal in structure.

## 2. The Mathematical Proof

A traditional approach to defining mathematical proof states that: “a mathematical proof is a formal, logical line of reasoning that begins with a set of axioms and progresses through logical steps towards a conclusion,” Griffiths [4]. When a statement is postulated, you have to judge whether that statement is a logical consequence of the foregoing statements, either by identification of a theorem or logical rule which the statement could be derived from, or by building a subtest that formally indicates how the new statement can be deduced from prior statements. From this perspective, informal reasoning processes like drawing diagrams or inspecting specific examples play a minimal role in this process and are therefore not sufficient to determine if any aspect of a proof

is correct. This vision of demonstrations has been put into doubt by many mathematicians and philosophers today [5].

Harel and Sowder [6] introduced the framework of *testing schemes*, which are defined as ways in which university students gain self-assurance and persuade others of the accuracy of mathematical statements. These researchers found that students primarily built three types of arguments: 1) those that rely on features that are external to the students (the structure of the argument, an authority figure, or meaningless symbolic manipulation) 2) empirical arguments using different types of examples (visual-spatial imagery, numerical substitutions, measurements), and 3) deductive arguments ranging from those expressed in terms of generic examples up to those in which students exhibit some understanding of the dependence of their argument on a given axiomatic system.

Alcolea [7] perceives mathematical proofs as a *substantive argument*. The role of the latter is to provide gradual support for an affirmation, and even if this support is not necessarily directed towards a *necessary logical conclusion* or on the other hand an arbitrary one, it does originate in the need for a credible presentation of contexts, relationships, explanations, justifications, etc. He considers that the mathematician does a demonstration by a convincing considered argument with the participation of other factors that are not usually included in the published evidence. Thus, in practice, *any mathematical proof is a convincing piece of argument, addressed to the competent expert*.

### 3. Visualization, Apprehensions and Coordination

From the psychological point of view, vision and perception are associated two functions: the first called epistemological and related function with direct access any physical object and the second synoptic function, interpreted as the simultaneous apprehension of several objects or specific complete field. Visualization is the vision manifested in synoptic function [8].

Visual perception (*epistemological function*) requires physical examination, it cannot grasp the object at once, as a whole. On the contrary, visualization can have a complete “snapshot” apprehension of any organization of relations.

When solving geometry problems we are constantly interacting with figures and for these to constitute a mathematical object they must be a combination of several related gestalts that determine what we are observing (configuration) and be linked to a (mathematical) proposition that sets some properties represented by the gestalt (hypotheses). Based on Duval two ways of apprehending a figure emerge from these conditions: *perceptual apprehension and discursive apprehension*.

#### 3.1. The Reasoning like Configural Process: Coordination of Operative and Discursive Apprehensions

Perceptual apprehension is characterized as simply identifying a configuration and discursive as cognitive action produces an association of mathematical statements identified with given configuration. In the process of solving problems we interact with geometric figures, making changes to the original settings. These changes are manifestations of operational apprehension, which can be of two types: when new geometric elements (operational figural apprehension of change) are added when the components and subassemblies are manipulated like pieces of a puzzle (operational apprehension of reconfiguration). After each change, new properties can be made visible and they can be associated with definitions, axioms, theorems (*discursive apprehension*); after this analysis further changes may be made to the settings previously obtained (*operational apprehension*) repeating the discursive/operational apprehension cycle in a coordinated manner until the solution is reached or the strategy being followed is abandoned.

The configural reasoning should be understood as the development of coordinated action of discursive/operational apprehension that the students made when solving a geometric problems; generating an interaction between the initial configuration and any changes in this with the appropriate mathematical statements [9].

#### 3.2. Outcomes of Configural Reasoning

Torregrosa and Quesada [9] study concludes that the configural reasoning can lead two different phenomena:

- The coordination provides solution to distinguishing two types of processes: a) *Truncation*: capturing the idea that allows deductive problem solving and b) *Conjecture without proof*: the problem is resolved assuming as valid certain assumptions that arise from the mere perception.

- Coordination fails to solve the problem and a phenomenon called *loop* occurs: it is up to a blocking situation where you cannot move towards the solution, causing a blockage of reasoning.

### 3.3. The Written Discourse of Students to Solve Problems of Geometry

The text produced by students when communicating the resolution of a problem may reflect their cognitive styles and the development of these types of processes, as some people reason better with words and others reason better with figures [10].

The interaction between configuration representations and discourse when students are solving geometric exercises of proving provide information on the geometric reasoning of students, as both written discourse and verbal expressions or gestures can be considered semiotic resources used by students when they are engaged in problem solving and in communicating those resolutions [11].

Clemente and Linares work [12] reveals three possible forms of written discourse in that account for student problem solving, they are: *graphics (G)*, *text (T)*, and *mixed graphics-text (G/T) format*.

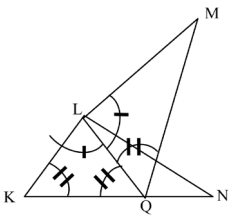
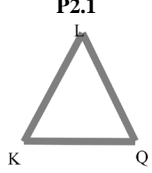
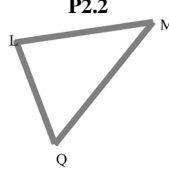
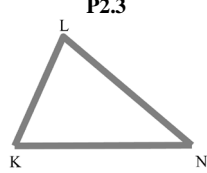
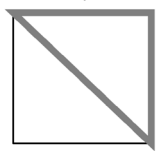
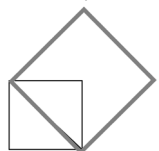
## 4. Case of Study

The study involved 46 students of the preparatory course of technologists who take a course of geometry. These students answered a questionnaire that included two proving problems (P2 and P4) as part of the course evaluation. For resolution the students should develop operative and discursive apprehensions identifying sub configurations to allow them to recognize some geometric objects to generate a proof, **Table 1**.

### 4.1. Results

In this section we show the numerical results of configural reasoning in relation to the identification of sub-assemblies and the association of knowledge. The style of the written discourse of students is also shown, **Table 2**.

**Table 1.** Problems.

	P2	P4
	Given the figure, prove that the QT segment is equal to KN.	
		Prove that the area of a square on the diagonal of another square has twice of his area.
	Sub configurations	
P2	<p><b>P2.1</b></p> 	<p><b>P2.2</b></p> 
	<p><b>P2.3</b></p> 	
P4	<p><b>P4.1</b></p> 	<p><b>P4.2</b></p> 

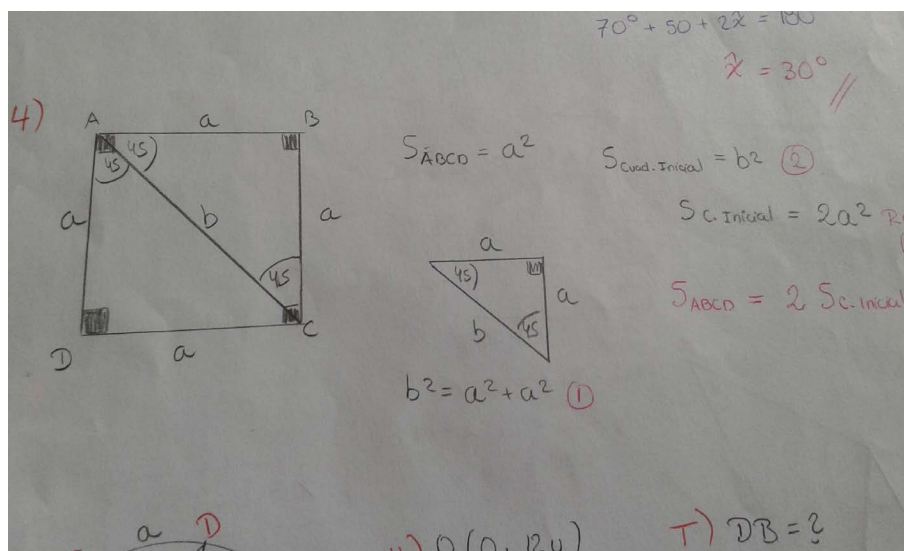


Figure 1. Written discourse.

Table 2. Numerical results.

		Sub configurations				Discourse style			Truncation	
		P1.1	P1.2		P1.3					
P2	Identified	Associated knowledge	Identified	Associated knowledge	Identified	Associated knowledge	G	T	G/T	
	25	25	30	8	33	25	2	1	38	29
		P2.1	P2.2		Others					
P4	Constructed	Associated knowledge	Constructed	Associated knowledge	Constructed	Associated knowledge	5	2	28	22
	22	19	19	11	23	23				

#### 4.2. “Text Style” (G/T) Example of Written Discourse (Figure 1)

Please see Figure 1.

### 5. Conclusions

The analysis of the table shows that the 29 (63.04%) students in P2 and 22 (47.82%) in P4 achieved truncation. In the P2 problem the sub configuration P2.2 is associated the less number of knowledge despite having been identified 30 (61.25%) times. In the P2 problem we note that a large number of new sub configurations 23 (50%) have been created with 100% of associated knowledge.

Written discourse analysis shows a strong tendency to mixed type G/T which perhaps is due to the nature of the proposed problems.

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