Scientific Research

Perishable Inventory Management in Healthcare

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ABSTRACT

This study addresses a problem encountered in a nation-wide, large-scale healthcare supply chain that comprises several hundred medical organizations (hospitals, clinics, pharmacies, etc.) and provides highly advanced medical care to several million people. The medical products in the system are perishable, meaning that they become unusable beyond a certain expiry date. It is necessary to track the ages of units in stock and to plan and control the inventory accordingly. The models developed herein represent a multi-echelon, multi-supplier inventory system and unite together aspects of perishability and outsourcing under deterministic demand for medical products, which include both perishable and deteriorating goods. The objective of the study is to determine the optimal number of products to be purchased from regular and outsource suppliers so as to meet the required demand at the minimum operating cost. The solution is a network-flow model that can be used to determine the trade-off between the quantities of items to be ordered from the two types of suppliers in each time period. In addition, the study analyzes different distribution policies to account for the perishable nature of the products. Further insights are obtained by applying the model to a case study of a real-life healthcare supply chain from which interesting results are drawn.

KEYWORDS

Supply Chain; Inventory Management; Perishable Inventory; Outsourcing

1. Introduction

Many classic inventory models rely on the assumption that the lifetime of inventory items is indefinitely long. Real-life inventory systems, however, can consist of *perishable* products, *i.e.*, products with a finite lifetime, which become unusable after a given *expiry date*. Some perishable products also *deteriorate*, *i.e.*, their quality diminishes gradually over time (although it may take some time until these products expire completely). Typical examples of deteriorating products are fruits, vegetables, meats, drugs and medical products [1]. Clearly, an inventory management approach that ignores product perishability and deterioration is likely to yield suboptimal outcomes.

Inventory management of medical products raises particular challenges, as medical inventory is highly sensitive to storage conditions (temperature, humidity, etc.)

and to quality of service. Early perishable medical inventory models were formulated to address problems related to blood bank inventory control [2]. Since then, a variety of models have been developed to address medical inventory management problems of substantial social and economic importance [3].

In the present paper we develop a model that considers perishability and deterioration of medical goods under deterministic demand. The model further incorporates aspects of outsourcing. Specifically, we propose a single-product capacitated lot sizing model of perishable medical inventory (e.g., a drug or other pharmaceutical product), over a finite planning horizon. The expiry dates of the products in the inventory are known, and the "ages" of the products are tracked as time passes. The supply chain we model consists of three echelons: multiple suppliers (a regular supplier and an outsource supplier), a centralized distribution center (DC) where medical

products are processed and stored, and the end user (several hospitals). The demand of the hospitals in each period is deterministic but time-varying and is fulfilled by the inventory stock in the DC. The regular supplier and the outsource supplier are assumed to charge different prices for the product and to supply products with different expiry dates. In general, outsource suppliers usually charge more per product compared with regular suppliers, but outsourcing can lower overall costs, as the supplier bears the holding costs [4-7]. Perishability adds an extra dimension to the problem: Inventory managers do not want to have excessive quantities of perishable products in stock, so they are likely to order fewer products than they would in the case of non-perishable products. In addition to provide efficient turn-over of the inventory stored at the DC, inventory managers prefer to send products with a short expiry date from the DC to the hospitals so that these products will not perished while stocked. Also, the price of a product may depend on the expiry date of the product provided by suppliers. The problem consists of determining, in each period, the quantities of items to be purchased from the regular supplier and from the outsourced supplier such that all demand is satisfied and the total cost of purchasing, shipping and inventory holding is minimized.

This work was motivated by the supply chain management for a nation-wide, large-scale healthcare supply chain called CLALIT Health Services (Israel), the largest healthcare organization in Israel. CLALIT has an annual budget of $3.3 billion (USD). It employs more than 30,000 employees and provides highly advanced medical care to 55% of the total population. CLALIT's healthcare providers issue 5 million prescriptions per year, so that approximately 5000 types of items from the organization's DC are sent annually to its 14 hospitals, 1380 primary and specialized clinics and 400 pharmacies. Annual operating costs amount to $17.2 million, while transportation costs stand at $2.3 million. The logistics operations make up about 50% - 60% of the total operating costs. Today the average monthly orders are around $3.5 million. The goal of CLALIT's managers is to improve the general healthcare service, which includes two main aspects: 1) providing hospitals and other medical organizations with their required products in a timely fashion; and 2) decreasing the total cost of all operations in the supply chain by using a centralized DC. In the final part of the work, we examine a case study on inventory management in a CLALIT Health Services DC.

2. Literature Review

An extensive overview of perishable-inventory management models can be found in [8]. [9] provide an up-to-date review. Many perishable-inventory management models in the literature (e.g., more than 80% of the

models reviewed in [9]) are deterministic, *i.e.*, they assume that products' shelf-life characteristics or the characteristics of demand follow deterministic patterns. This assumption, although limiting from a real-life point of view, yields comparatively simple models, and is adopted in this paper.

The inventory management problems considered in this work are based on a classical modeling approach called *dynamic economic lot sizing* (ELS). This approach has been extensively studied in the literature on inventory control, production planning and capacity planning. Wagner and Whitin [10] solved the single-product problem (called the *single-product ELS*) efficiently using dynamic programming. Researchers continue to devote attention to variations of this problem and have developed numerous practical extensions to Wagner and Whitin's model, including finite production capacity models [11] and multi-echelon models [12-15]. The general purpose of these extensions is to represent and analyze real problems faced by manufacturing companies. Numerous studies have developed applications of ELS to perishable goods [16-22].

To the best of our knowledge, there are no published studies of perishable inventory with outsourcing, in spite of the fact that outsourcing strategies are among the most effective ways to decrease losses in perishable medical inventory. Outsourcing problems are complex and entail considerable implications for firms' strategies. [4] were the first to consider the trade-offs associated with outsourcing, production and inventory decisions. They assumed that a given deterministic demand can be fulfilled in a given period either by production or by subcontracting (outsourcing) in that period. They provided polynomial time algorithms for a large variety of cost functions. [23] dealt with a production/inventory system under uncertain capacity levels and the existence of outsourcing opportunities. In their model, outsourcing was associated with a fixed cost, whereas production was not. The authors characterized the optimal outsourcing policy under certain conditions and elaborated on the optimal policy of production with internal resources. [5,6] considered outsourcing in inventory problems of non-perishable products. They considered a lot-size model for outsourcing with time-varying costs under a finite planning horizon. The model minimizes the total costs of production, inventory holding and outsourcing. These authors assumed that unsatisfied demand cannot be backlogged and that all demand is satisfied through outsourcing, with the outsourcing cost being time-varying. They focused on a special case of concave costs in which conservation and stock-out periods exist. Since such a situation may lead to excessive inventory holding costs, outsourcing may constitute a cheaper alternative that increases profit. The authors presented and proved several properties of

optimal solutions, and developed a strongly polynomial algorithm with complexity of $O(T^4 \log T)$. [7] also considered non-perishable products but allowed for backlogging. They assumed that holding, backlogging and outsourcing cost are linear, and developed a polynomial algorithm with complexity of $O(T^4 \log T)$.

Our work is closely related to the network flow models in [19,24]. [19] takes into account perishability, converts the problem to a minimum network problem and constructs a network flow with flow loss. We extend their approach, developing a mathematical model that can incorporate multiple customers and multiple suppliers. [24] consider a problem with a single supplier, a single non-perishable product, and multiple customers, where the customer demand is deterministic. Their model is a generalization of the traditional Wagner-Whitin model. The authors solve the problem as a network flow model on a specially constructed network with a single source node and multiple destination nodes, each corresponding to the demand of customer i in period j. We extend this work by adding deterioration rate and expiry date and by allowing for outsourcing. In addition, we examine different methods for inventory management in the DC.

3. Model

3.1. Problem Description

We consider a single-product multi-echelon medical supply chain comprising the following components: 1) several hospitals with different shipping charges; 2) a DC with a storage facility; 3) two suppliers: one regular supplier and one outsourcing supplier. The flow of products in the medical distribution system is depicted in **Figure 1**.

The assumptions of the problem are as follows:

1) Products are perishable and deteriorating.

2) The demands at the hospitals are deterministic but time-varying over a finite horizon.

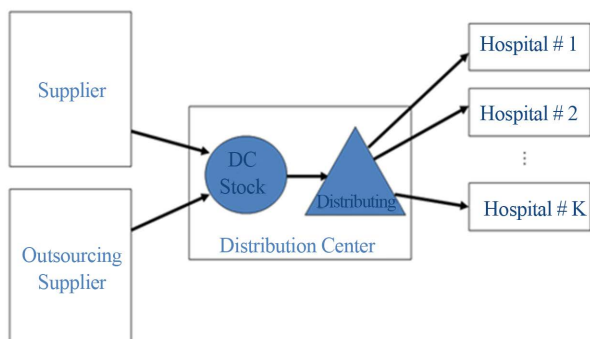3) Unsatisfied demand cannot be backlogged and is satisfied through outsourcing.

Figure 1. Flow of medical products in the healthcare supply chain.

4) The cost of purchasing products from a supplier (regular or outsource) consists of a fixed cost and a unit cost.

5) The outsource supplier charges a higher price per unit than the regular supplier does.

6) Products from the outsource supplier have a sufficiently long expiry date (*i.e.*, they will not expire during the planning horizon).

7) Products from the regular supplier may have a long expiry date or a short expiry date, and their unit cost depends on the expiry date of the product.

8) Shipping costs from the DC to the hospitals vary across hospitals, reflecting different distances from the DC to the hospitals, and are proportional to the number of products delivered.

Our goal is to develop a model for determining the optimal quantities to be purchased from the regular supplier and from the outsource supplier in each period, such that all demand is satisfied, and the total cost of purchasing, shipping and inventory holding is minimized.

3.2. Notations and Variables

The following parameters are assumed to be given input data:

K The number of hospitals, indexed as $k = 1, \dots, K$;

t The period (week) when a product arrives at the DC; for the sake of simplicity, t is also the product's index, $t = 1, 2, \dots, T$;

s A current index indicating the week of delivery to hospitals, $s = 1, 2, \dots, T$;

τ A current index indicating how many weeks a product has been stored in the DC, $\tau = 0, 1, \dots, T$; $\tau = s - t$;

α The proportion that is lost due to deterioration during one week, $0 \leq \alpha \leq 1$;

e_t Expiry date of product t measured by the number of remaining weeks until expiry;

d_{sK} Demand of hospital k in week s;

Q_s The set of all products t available on week s, satisfying: $t \leq s$ and $e_t > s$. This set is called a set of valid products;

$\overline{X}_t$ Given maximum capacity of the regular supplier on week t;

h Unit holding cost;

p^O Unit price from an outsource supplier;

f_t^O Fixed purchasing setup cost from an outsource-supplier on week t;

p Unit price from the regular supplier. This value depends on the expiry date e_t, such that:

$$p = \begin{cases} p^L & e_t > e \\ p^S & \text{otherwise} \end{cases}$$

where e is a given threshold, and

$$p^O > p^L > p^S$$

f_t Fixed purchasing setup cost from a regular supplier on week t;

c_{SK} Unit cost to ship a product in week s from the DC to hospital k.

We introduce the following variables:

$0 \leq X_{t\tau}$: The number of units of product t purchased from the regular supplier that have been stored at the DC for τ weeks;

$0 \leq Y_{t\tau}$: The number of units of product t purchased from the outsourcing supplier that have been stored at the DC for τ weeks;

$0 \leq z_{t\tau}$: The number of units of product t supplied by the regular supplier that are distributed from the DC after τ weeks of storage;

$0 \leq u_{t\tau k}$: The number of units of product t supplied by the outsource supplier that are distributed from the DC after τ weeks of storage;

$0 \leq Z_{sk}$: The number of products shipped from the DC on week s to hospital k.

3.3. Constraints and Objective Function

We define the problem as a network flow problem with the following constraints:

Constraints 1 and 2. Balance of supply flow at the DC: The quantity $X_{t\tau}$ of product t stored at the DC for τ weeks is equal to the sum of the quantity $z_{t\tau}$ that is distributed from the stock and quantity $X_{t,\tau+1}$ (reduced by the corresponding deterioration coefficient $1 - \alpha$) that is stored for the next period $\tau + 1$.

$$X_{t\tau} = X_{t\tau+1}/(1-\alpha) + z_{t\tau} \qquad (1)$$

Correspondingly, for the outsource supplier:

$$Y_{t\tau} = Y_{t\tau+1}/(1-\alpha) + u_{t\tau} \qquad (2)$$

Constraint 3. Shipping of products to the hospitals: The number of products Z_{sk} shipped from the DC to hospital k at period s should satisfy the demand of hospital k at period s.

$$Z_{sk} \geq d_{sk} \quad s = 1, 2, \cdots, T; \ k = 1, 2, \cdots, K \qquad (3)$$

Constraint 4. Distribution of products from the DC: All products distributed from the DC at period s must be valid (as defined by the set Q_s), and the number of distributed products must be equal to the total number of products shipped to all hospitals.

$$\sum_{t \in Q_s} z_{t\tau} + \sum_{t \in Q_s} u_{t\tau} = \sum_{k=1}^{K} Z_{sk} \quad s = 1, 2, \cdots, T \qquad (4)$$

Constraint 5. Expired products are not valid. This means that when product t expires, that is, $t + \tau > e_t$, the product becomes useless and its value is lost:

$$u_{t\tau} = 0 \quad z_{t\tau} = 0 \ \text{if} \ t + \tau > e_t \qquad (5)$$

Constraint 6. Product flow from the regular supplier to the DC does not exceed the given capacity:

$$X_{t\tau} \leq \overline{X_t} \quad t = 1, 2, \cdots, T; \ \tau = 0 \qquad (6)$$

Constraint 7. Natural constraints on variables:

$$X_{t\tau}; Y_{t\tau}; Z_{sk}; z_{t\tau}; u_{t\tau} \ \text{are integers} \qquad (7)$$

Defining

$$\delta(x) = \begin{cases} 1 & x > 0 \\ 0 & \text{otherwise} \end{cases}$$

The objective is to minimize the total cost of purchasing, inventory holding and shipping:

$$\min \sum_{t:e_t > e} p^L X_{t0} + \sum_{t:e_t \leq e} p^S X_{t0} + \sum_{t=1}^{T} p^O Y_{t0} +$$

$$\sum_{t=1}^{T} \left[f_t \delta(X_{t0}) + f_t^O \delta(X_{t0}) + h \sum_{\tau=1}^{T} (X_{t\tau} + Y_{t\tau}) \right] + \sum_{s=1}^{T} \sum_{k=1}^{K} c_{sk} Z_{sk}$$

4. Solution Methodology

The solution methods for the above problem will be described using graph theory. We use a graph to present the process of supplying medical products to a DC, storing them there and subsequently distributing them. In this graph each node corresponds to the position of the product in the warehouse every week, and arcs depict how the product flows in time and space. Notice that the graphs presented in **Figure 2** generalizes a simpler graph model suggested in [14,15] for the inventory of non-perishable products.

Adding a single source node to this graph, our minimization problem formulated in Section 3 is equivalent to a minimum cost network flow. (See [19,24] for discussionson theconstruction of the graph).

5. Case Study

5.1. Motivation

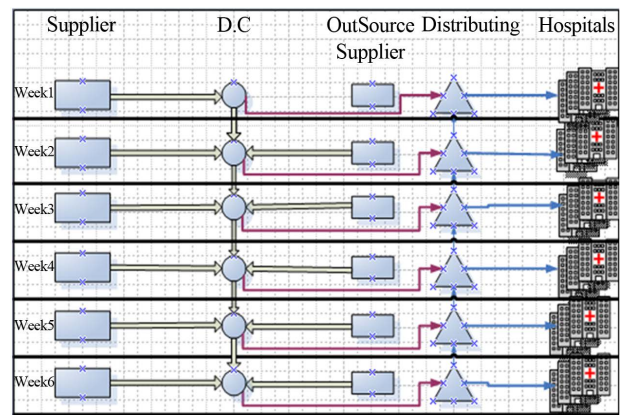This section addresses an inventory management prob-

Figure 2. The distribution model graph.

lem encountered in a central warehouse of the company CLALIT Health Services, the largest healthcare organization in Israel. CLALIT comprises several hundred medical organizations (hospitals, clinics, pharmacies, etc.) and provides highly advanced medical care to several million people. The medical products in the system, such as drugs and other pharmaceuticals, are perishable. It is necessary to keep track of the ages of units in stock and to plan and control the inventory accordingly.

The planning and distributing process of inventory management in the company is organized as follows:

- The demand planning department orders the needed quantity of medical items based on known demands from hospitals.
- Then the supplier sends the order quantity to the DC.
- The quantity and quality of the perishable items received at the DC are checked, and then these items are moved to a *put-away* inventory storage (if there is no inventory in the DC the items will be moved to *short-term storage* without being sent to the put-away storage see **Figure 3**).
- When the need for a product arises, the workers pick the needed product from its storage location; this stage is called *picking*. This is a human-driven stage in which many types of human errors can occur. For instance, workers may not always pay attention to products' expiry dates and may pick the items that arrived first instead of the products with the closest expiry dates.
- In order to avoid this type of human error, the DC has special check-points at which the expiry date and quantity of the products are examined. The check-points are depicted by red flags in **Figure 3**.

In order to provide efficient turn-over of the inventory stored at the DC, the DC manager seeks to prioritize products with the shortest expiry dates for delivery to hospitals in the first turn (*i.e.*, these products are distributed earlier than products with longer expiry dates).

5.2. Example

This section provides a numerical example corresponding to a 5-period, two hospital problem, in a setting similar to that of CLALIT Health Services. We build on an example in [24], which we extend to incorporate outsourcing and perishability.

We use the following input data, borrowed from [24]: hospitals 1 and 2 have demands of 20 and 50 units in each period, respectively. The purchasing costs are $p^O = 3$, $p^L = 2$, $p^S = 1$. Delivery costs for the 5 periods, respectively, are (1,4,1,4,1) for hospital 1 and (4,2,4,2,4) for hospital 2. The holding cost is $h = 1$. Since our model allows for outsourcing, we modify the input data as follows: The maximum capacity of the regular supplier is 150 for each period. Fixed purchasing setup costs for both suppliers, in line with [24], are (1,10,10,4,1) for each of 5 periods.

Additional input data, which are specific for our model, include the following: the coefficient of deterioration is $\alpha = 0.1$. The expiry threshold is $e = 3$ periods (so that if a product's expiry date is less than or equal to 3, the product is classified as a short-expiry product).

The expiry dates of products that are purchased from the regular supplier are given in **Table 1**. Products purchased from the outsource supplier are assumed to have sufficiently long expiry dates, such that they will not expire during the planning horizon.

The optimal solution to this example is presented in **Table 2**. Taking into account product deterioration, the DC should purchase 148 products from the regular supplier and 87 products from the outsourcing supplier in period 1 in order to satisfy the demand of hospital 1 and hospital 2 in periods 1, 2 and 3. To satisfy the demand levels in periods 4 and 5, it is optimal to purchase the demanded quantities in each period from the regular supplier. The total cost is 2364.

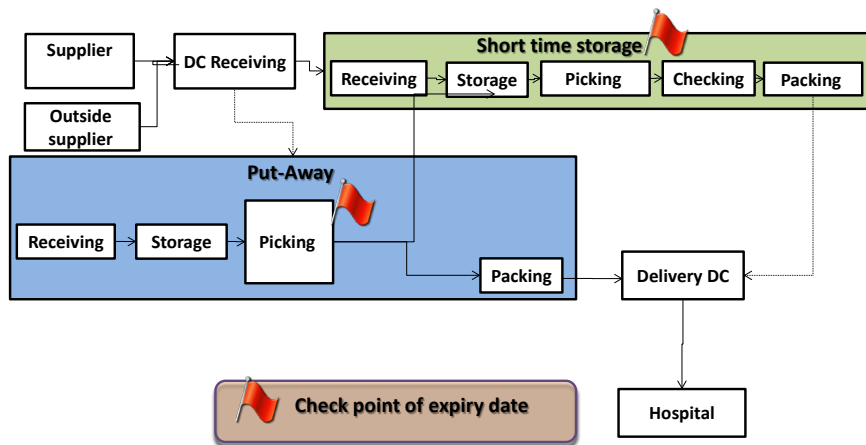This optimal solution is obtained under the "best-case

Figure 3. Flow chart of inventory flows in CLALIT DC.

Table 1. Expiry dates of products.

Period	Expiry Date
1	2
2	3
3	2
4	5
5	5

Table 2. Optimal ordering quantities.

	Order	
Period	Regular supplier	Outsourcing
1	148	87
2	0	0
3	0	0
4	70	0
5	70	0

Figure 4. Flow of products under the best-case scenario.

Figure 5. Flow of products under the worst-case scenario.

scenario", which is distributing first to the hospitals the shorter expiry dates purchased from the regular supplier. Specifically, the supply during the first week is 235 products (148 short-expiry products purchased from the regular supplier and 87 long-expiry products purchased from the outsource supplier). Under the "best-case scenario", at the end of week 1, the DC delivers 70 short-expiry products (20 to hospital 1 and 50 to hospital 2), and the remaining 165 products are stored at the DC. After deterioration during one week of storage, the remaining stock at the beginning of period 2 is 13 long-expiry products and 78 short-expiry products. Next, under the "best-case scenario" the DC fulfills the demand of week 2 by delivering the 70 short-expiry date products and the remaining long-expiry products are stored at the DC. Finally, at the beginning of period 3, only 70 long-expiry products remain in the DC (after deterioration) and are delivered to satisfy the demand (see the flow of products in **Figure 4**).

Note that for this example, under the "worst-case scenario" if long-expiry products are delivered before short-expiry products, the DC will need to place an additional order (from the regular supplier) in the third period. This extra purchase will be necessary because at the end of period 2 all the short-expiry products will have expired (see **Figure 5** for a depiction of the product flow in this case). Therefore under the "worst-case scenario" the total cost of the example is 3204. This example demonstrates the importance of the correct inventory management and the distribution of orders according to the products expiry date.
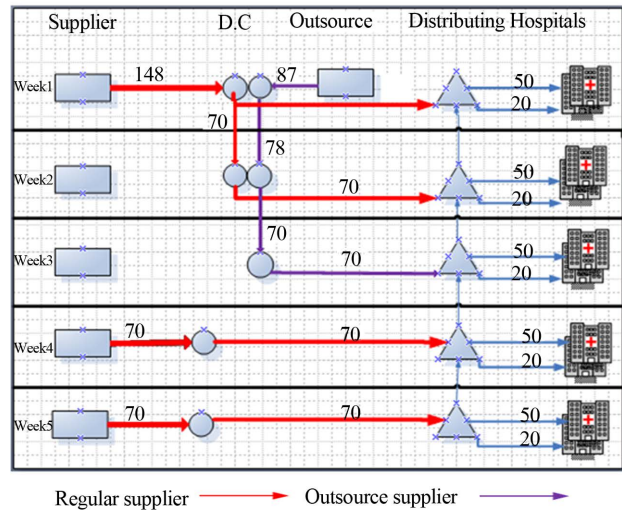
6. Conclusion

In this work, we studied an inventory problem with multiple suppliers, namely, a regular supplier and an outsource supplier. We considered deterministic demand for perishable medical items. Over the planning horizon, the demand can vary over time due to changes in the consumption rate of the products in the hospitals. Our network flow model makes it possible to find a trade-off between the quantities of items that the DC should order from different suppliers at each time period. To illustrate the application of our model, we provide a numerical example. This example confirms that managerial effort is required to ensure that short-expiry units are distributed to hospitals and do not expire in the DC. The suggested mathematical model provides a tool for optimizing such managerial efforts.

REFERENCES

[1] Y. Ge and J. Zhang, "Dynamic Pricing of Perishable Products under Consumer Factor," *Journal of Service Science and Management*, Vol. 4, No. 4, 2011, pp. 440-444. http://dx.doi.org/10.4236/jssm.2011.44049

[2] S. Nahmias, "Perishable Inventory Theory: A Review," *Operations Research*, Vol. 30, No. 4, 1982, pp. 680-708. http://dx.doi.org/10.1287/opre.30.4.680

[3] Z. Shen, M. Dessouky and F. Ordonez, "Perishable Inventory Management System with a Minimum Volume Constraint," *Journal of the Operational Research Society*, Vol. 62, No. 12, 2011, pp. 2063-2082. http://dx.doi.org/10.1057/jors.2010.181

[4] A. Atamturk and D. S. Hochbaum, "Capacity Acquisition, Subcontracting and Lot Sizing," *Management Science*, Vol. 47, No. 8, 2001, pp. 1081-1100. http://dx.doi.org/10.1287/mnsc.47.8.1081.10232

[5] X. Liu, C. Wang, X. Luo and D. Wang, "A Model and Algorithm for Outsourcing Planning," *Proceedings of the 2005 IEEE International Conference on E-Business Engineering*, *ICEBE*'05, Beijing, 2005, pp. 1-4.

[6] X. Liu and J. Zhang, "A Capacitated Production Planning with Outsourcing: A General Model and Its Algorithm," *Lecture Notes in Computer Science*, Vol. 4113, No. 2006, pp. 997-1002. http://dx.doi.org/10.1007/11816157_121

[7] C. Chu, F. Chu, J. Zhong and S. Yang, "A Polynomial Algorithm for a Lot-Sizing Problem with Backlogging, Outsourcing and Limited Inventory," *Computers & Industrial Engineering*, Vol. 52, No. 1, 2013, pp. 200-210. http://dx.doi.org/10.1007/11816157_121

[8] S. K. Goyal and B. C. Giri, "Recent Trends in Modeling of Deteriorating Inventory," *European Journal of Operational Research*, Vol. 134, No. 1, 2001, pp. 1-16. http://dx.doi.org/10.1016/S0377-2217(00)00248-4

[9] M. Bakker, J. Riezebos and R. H. Teunter, "Review of Inventory Systems with Deterioration since 2001," *European Journal of Operational Research*, Vol. 221, No. 2, 2012, pp. 275-284. http://dx.doi.org/10.1016/j.ejor.2012.03.004

[10] H. M. Wagner and T. M. Whitin, "A Dynamic Version of the Economic Lot Size Model," *Management Science*, Vol. 5, No. 1, 1958, pp. 89-96. http://dx.doi.org/10.1287/mnsc.5.1.89

[11] M. Florian and M. Klein, "Deterministic Production Planning with Concave Costs and Capacity Constraints," *Management Science*, Vol. 18, No. 1, 1971, pp. 12-20. http://dx.doi.org/10.1287/mnsc.18.1.12

[12] W. I. Zangwill, "Minimum Concave Cost Flows in Certain Networks," *Management Science*, Vol. 14, No. 7, 1968, pp. 429-450. http://dx.doi.org/10.1287/mnsc.14.7.429

[13] W. I. Zangwill, "A Backlogging Model and a Multi-Echelon Model of a Dynamic Economic Lot Size Production System—A Network Approach," *Management Science*, Vol. 15, No. 9, 1969, pp. 506-527. http://dx.doi.org/10.1287/mnsc.15.9.506

[14] Y. Perlman and I. Levner, "Modeling Multi-Echelon Multi-Supplier Repairable Inventory Systems with Backorders," *Journal of Service Science & Management*, Vol. 3, No. 4, 2010, pp. 440-448. http://dx.doi.org/10.4236/jssm.2010.34050

[15] E. Levner, Y. Perlman, T. C. E. Cheng and I. Levner, "A Network Approach to Modeling the Multi-Echelon Spare Part Inventory System with Backorders and Interval-Valued Demand," *International Journal of Production Economic*s, Vol. 132, No. 1, 2011, pp. 43-51. http://dx.doi.org/10.1016/j.ijpe.2011.03.004

[16] P. Nandakumar and T. E. Morton, "Near Myopic Heuristic for the Fixed Life Perishability Problem," *Management Science*, Vol. 18, 1990, pp. 1490-1498.

[17] L. R. Weatherford and S. E. Bodily, "A Taxonomy and Research Overview of Perishable-Asset Revenue Management: Yield Management Overbooking and Pricing," *Operations Research*, Vol. 40, No. 5, 1992, pp. 831-844. http://dx.doi.org/10.1287/opre.40.5.831

[18] A. Aggarwal and J. K. Park, "Improved Algorithms for Economic Lot Size Problems," *Operations Research*, Vol. 41, No. 3, 1993, pp. 49-571. http://dx.doi.org/10.1287/opre.41.3.549

[19] V. N. Hsu, "Dynamic Economic Lot Size Model with Perishable Inventory," *Management Science*, Vol. 46, No. 8, 2000, pp. 1159-1169. http://dx.doi.org/10.1287/mnsc.46.8.1159.12021

[20] L. Y. Chu, V. H. Hsu and Z. J. M. Shen, "An Economic Lot Sizing Problem with Perishable Inventory and Economic of Scale Costs: Approximation Solutions and Worst Case Analysis," *Naval Research Logistics*, Vol. 52, No. 6, 2005, pp. 536-548. http://dx.doi.org/10.1002/nav.20096

[21] N. Brahimi, S. Dauzere-Peres, N. Najid and A. Nordl, "Single Item Lot Sizing Problems," *European Journal of Operational Research*, Vol. 168, No. 1, 2006, pp. 1-16. http://dx.doi.org/10.1016/j.ejor.2004.01.054

[22] Q. G. Bai, Y. Z. Zhang and G. L. Dong, "A Note on Economic Lot Sizing Problem with Perishable Inventory And Economies of Scale Costs: Approximation Solutions and Worst-Case Analysis," *International Journal of Automation and Computing*, Vol. 7, No. 1, 2010, pp. 132-136. http://dx.doi.org/10.1007/s11633-010-0132-5

[23] J. Yang, X. Qi and Y. Xia, "A Production-Inventory System with Markovian Capacity and Outsourcing Option," *Operations Research*, Vol. 53, No. 2, 2005, pp. 328-349. http://dx.doi.org/10.1287/opre.1040.0165

[24] S. Chand, V. N. Hsu, S. Sethi and V. Deshpande, "A Dynamic Lot Size Problem with Multiple Customers: Customer-Specific Shipping and Backlogging Costs," *IIE Transactions*, Vo. 39, No. 11, 2007, pp. 1059-1069. http://dx.doi.org/10.1080/07408170601089497