J. Software Engineering & Applications, **2010, 3: 27-33**
doi:10.4236/jsea.2010.31003 Published Online January 2010 (http://www.SciRP.org/journal/jsea)

Properties of Nash Equilibrium Retail Prices in Contract Model with a Supplier, Multiple Retailers and Price-Dependent Demand

Koichi NAKADE, Satoshi TSUBOUCHI, Ibtinen SEDIRI

Department of Industrial Engineering and Management, Nagoya Institute of Technology, Japan.
Email: nakade@nitech.ac.jp, chd18509@stn.nitech.ac.jp

Received September 3rd, 2009; revised October 9th, 2009; accepted October 19th, 2009.

ABSTRACT

Recently, price contract models between suppliers and retailers, with stochastic demand have been analyzed based on well-known newsvendor problems. In Bernstein and Federgruen [6], they have analyzed a contract model with single supplier and multiples retailers and price dependent demand, where retailers compete on retail prices. Each retailer decides a number of products he procures from the supplier and his retail price to maximize his own profit. This is achieved after giving the wholesale and buy-back prices, which are determined by the supplier as the supplier's profit is maximized. Bernstein and Federgruen have proved that the retail prices become a unique Nash equilibrium solution under weak conditions on the price dependent distribution of demand. The authors, however, have not mentioned the numerical values and proprieties on these retail prices, the number of products and their individual and overall profits. In this paper, we analyze the model numerically. We first indicate some numerical problems with respect to theorem of Nash equilibrium solutions, which Bernstein and Federgruen proved, and we show their modified results. Then, we compute numerically Nash equilibrium prices, optimal wholesale and buy-back prices for the supplier's and retailers' profits, and supply chain optimal retailers' prices. We also discuss properties on relation between these values and the demand distribution.

Keywords: *Supply Chain Management, Nash Equilibrium, Stochastic Demand, Competing Retailers*

1. Introduction

Recently, price contract models between suppliers and retailers with stochastic demand have been analyzed based on well-known newsvendor problems. Cachon [1] has reviewed models with one supplier and one retailer under several types of contracts. In a market, however, many retailers exist and they compete in order to attract the maximum number of consumers. In this context, Yano and Gilbert [2] have been interesting in contracting models in which the demand is stochastic and depends on price. Wang *et al*. [3] and Petruzzi [4] have studied decentralized price setting newsvendor problems under multiplicative retail demand functions. Song *et al.* [5] have analyzed theoretically the optimal prices and the fraction of a total profit under individual optimization to that under supply chain optimization.

In Bernstein and Federgruen [6], they have analyzed a contract model with single supplier and multiple retailers and price dependent demand, where retailers compete on retail prices. Each retailer decides a number of products he procures from the supplier and his retail price to maximize his own profit. This is achieved after giving the wholesale and buy-back prices, which are determined by the supplier as the supplier's profit is maximized. They have proved that the retail prices become a unique Nash equilibrium solution under weak conditions on the price dependent distribution of demand. They, however, have not mentioned the numerical values and properties on these retail prices, the number of products and their individual and overall profits.

In this paper, we analyze the model numerically. We first indicate some numerical problems with respect to the theorem of Nash equilibrium solutions, which Bernstein and Federgruen [6] proved, and we show their modified results. Then we present Nash equilibrium prices, optimal wholesale and buy-back prices for the supplier's and retailers' profits, and optimal retail prices under supply chain optimization, analytically and numerically. We also discuss the properties on a relationship between these values and the demand distribution.

In the next section, we present the competing retailers model introduced in [6], and we discuss the sufficient conditions on the existence and the uniqueness of the Nash solution. In Section 3 we investigate the model with exponential and uniform distribution functions and with linear and Logit demand functions. In Section 4, we present numerical results and discuss the behavior of Nash equilibrium solutions and properties of the profits and prices.

2. Competing Retailers' Model

The model of competing retailers for one supplier S and N retailer $R_i, 1 \le i \le N$ introduced in [6], is shown in Figure 1.

This model is set under wholesale and buyback payment scheme. The supplier S incurs retailer $R_i, 1 \le i \le N$ a wholesale price w_i for each product, combined with an agreement to buyback unsold inventory at b_i. We assume that the supplier has ample capacity to satisfy any retailer demand and produce products at a constant production cost rate c_i, which includes the transportation cost to retailer R_i. When w_i and b_i are given, each retailer R_i orders his quantity y_i and chooses his retail price p_i. A salvage rate $-\infty < v_i < +\infty$ is adopted in the supply chain. To avoid trivial setting, the model parameters are chosen as $v_i < b_i < w_i$ and $v_i < c_i$ for $1 \le i \le N$.

The demand $D_i(p)$ is random and depends on the price vector $p = (p_1, p_2, \cdots, p_N)$, with a cumulative distribution function $\tilde{G}_i(x \mid p_1,...,p_N)$. It is restrained to a multiplicative form $D_i(p) = d_i(p)\varepsilon_i$, where ε_i is a random variable with a cumulative distribution function

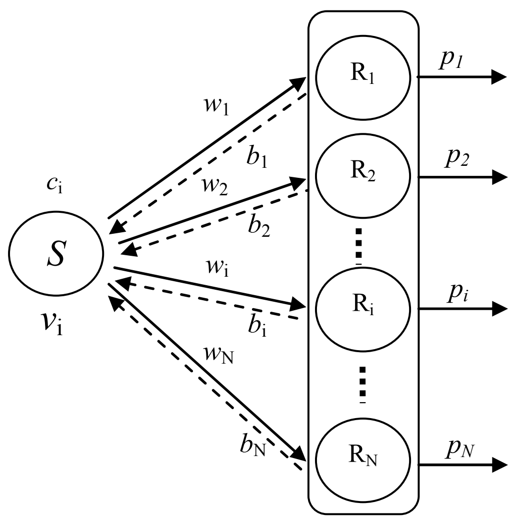

Figure 1. Competing retailers' model

$G_i(.)$ and a probability density function $g_i(.)$. In addition ε_i is assumed to be positive only on $x \in [x^i_{\min}, x^i_{\max}]$ and independent of the price vector p. This implies that $\tilde{G}_i(x|p) = G_i\left(\frac{x}{d_i(p)}\right)$.

The demand function $d_i(p)$ depends on the whole price vector. It is supposed that $d_i(p)$ decreases in p_i and increases in p_j for all $j \neq i$, $1 \le I \le N$, that is $\frac{\partial d_i(p)}{\partial p_i} < 0$ and $\frac{\partial d_i(p)}{\partial p_j} \ge 0$.

Let $y = (y_1, y_2, \cdots, y_N)$ denotes the order vector of the model. The expected profit function for the retailer R_i is given by

$$\pi_i(p,y) = p_i E\left[\min\{y_i, D_i(p)\}\right] + b_i E\left[y_i - D_i(p)\right]^+ - w_i y_i,$$

where $[a]^+ = \max(0,a)$. It can be rewritten as

$$\pi_i(p,y) = (p_i - w_i)y_i - (p_i - b_i)E\left[y_i - D_i(p)\right]^+ \quad (1)$$

From (1), the retail prices p impact on the profits of all retailers and his order quantity, however, affects only his own profit. In addition the retailer wants to maximize his order quantity. Then, the derivation of the retailer i's profit function given by Equation (1) on y_i is equal to zero

$$\frac{\partial \pi_i(p,y)}{\partial y_i} = 0 \quad (2)$$

Therefore, the retailer i's optimal corresponding order can be obtained from (1) and (2) by

$$y_i(p) = d_i(p) G_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) \quad (3)$$

This result reduces the no-cooperative game in the *(p, y)-space* to a *p-space* game. In this space the retailers compete only on prices (*reduced retailer game*). Then, considering the Equations (1) and (3), we get the retailers profits as a function of p only, as

$$\tilde{\pi}_i(p) = d_i(p)\left[(p_i - w_i)G_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) - (p_i - b_i)E\left[G_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) - \varepsilon_i\right]^+\right]$$
$$= \pi_i^{\det}(p \mid w_i) L_i(f_i(p_i)) \quad (4)$$

where $\pi_i^{\det}(p \mid w_i) = (p_i - w_i)d_i(p)$ is the profit function with a deterministic demand $y_i = d_i(p)$, $f_i(p_i) = \frac{(p_i - w_i)}{(p_i - b_i)}$

is the critical fractile, and

$$L_i(f) = G_i^{-1}(f_i) - f_i^{-1}E\left[G_i^{-1}(f_i) - \varepsilon_i\right]^+ = \left(\int_{-\infty}^{G_i^{-1}(f_i(p_i))} ug_i(u)du\right) / f_i$$

We define $\hat{L}_i(p_i) \equiv \int_{-\infty}^{G_i^{-1}(f_i(p_i))} ug_i(u)du$ and we apply the logarithm to (4), we get for $1 \le i \le N$

$$\log\tilde{\pi}_i(p) = \log(p_i - b_i) + \log d_i(p) + \log\hat{L}_i(p_i) \quad (5)$$

The supplier profit function is given by $\Pi_M = \sum_{i=1}^{N}\left((w_i - c_i)y_i - (b_i - v_i)E\left[y_i - D_i(p)\right]^+\right)$. Using Equation (3), it can be expressed as

$$\Pi_M = \sum_{i=1}^{N} d_i(p)\left((w_i - c_i)G_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) + (b_i - v_i)E\left[G_i^{-1}\left(\frac{p_i - w_i}{p_i - b_i}\right) - \varepsilon_i\right]^+\right). \quad (6)$$

Differentiating (5) on p_i for $1 \le i \le N$

$\frac{\partial \log\tilde{\pi}_i(p)}{\partial p_i} = \frac{1}{d_i(p)}\frac{\partial d_i(p)}{\partial p_i} + U_i(p_i),$ with

$$U_i(p_i) = \frac{1}{p_i - b_i} + \frac{(w_i - b_i)\cdot G_i^{-1}(f_i(p_i))}{(p_i - b_i)^2\hat{L}_i(p_i)} \quad (7)$$

Bernstein and Federgruen [6] have proved that the existence of a Nash solution p^* for the reduced retailer game is assured by the following condition (A).

(A): For each $i \in \{1,...,N\}$*, the function* $\log d_i(p)$ *is increasing in* (p_i, p_j) *for all* $i \ne j$.

It is assumed in the same reference [6] that each retailer R_i chooses his price p_i from a closed interval $\left[p_i^{\min}, p_i^{\max}\right]$. The authors proved the uniqueness of the Nash solution in the price space $\prod_i\left[\max\left(p_i^{\min}, 2w-b\right)_i, p_i^{\max}\right]$. This has provided the following conditions (D) and (S) to hold:

$$\text{(D)}: -\frac{\partial^2 \log\pi_i^{\det}(p \mid w_i = b_i)}{\partial p_i^2} \ge \sum_{j\ne i}\frac{\partial^2 \log\pi_i^{\det}(p \mid w_i = b_i)}{\partial p_i \partial p_j},$$

$$\text{(S)}: \psi_i(x) \equiv \left[-2x + \frac{\overline{G}_i(x)}{g_i(x)}\right]\int_{-\infty}^{x} ug_i(u)du - \overline{G}_i(x)x^2 \le 0,$$

for all $i \in \{1,...,N\}$, where $x \ge m_i$ (m_i is the median of the distribution G_i). However, the solution under the above conditions may exist on the boundary of the area $\prod_i\left[\max\left(p_i^{\min}, 2w-b\right)_i, p_i^{\max}\right]$. In this case, it does not satisfy $\frac{\partial \log\tilde{\pi}_i(p)}{\partial p_i} = 0$. the condition (S) is modified to (S'), as

$$\text{(S')}: \Psi_i(x) \equiv \left[-2x + \frac{\overline{G}_i(x)}{g_i(x)}\right]\int_{-\infty}^{x} ug_i(u)du - \overline{G}_i(x)x^2 \le 0$$

for all $x \in [x_{\min}^i, x_{\max}^i]$. Then, the following theorem can be obtained.

Theorem : If conditions (A), (D) and (S') hold, then there is a unique set of Nash equilibrium prices on $\prod_i[w_i, \infty)$ *which satisfy* $\frac{\partial \log\tilde{\pi}_i(p)}{\partial p_i} = 0$ *for all* $i \in \{1,...,N\}$.

Proof: In the same way as in Bernstein and Federgruen [6], there is a unique Nash solution p^* in $\prod_i[w_i, \infty)$ which satisfies $p_i^* > w_i$, because for each $i \in \{1,...,N\}$, $\pi_i(p) = 0$ when $p_i = w_i$ whereas $\pi_i(p) > 0$ when $p_i > w_i$. This implies that $\frac{\partial \log\tilde{\pi}_i(p)}{\partial p_i} = 0$ when $p = p^*$ for all $i \in \{1,...,N\}$.

In the following, the retailers sell products at these equilibrium prices, whereas the supplier knows the behavior of the retailers and determines the wholesale and buyback prices to maximize his own profit. This system is called "*individual optimization"*. On the other hand, the problem of determining retail prices and quantities of products to maximize the entire profits of supply chain is called "*supply chain optimization*"

3. Determination of the Nash Equilibrium

As shown in Figure 2, we study a two competing retailers' model. Each retailer $i \in \{1, 2\}$ faces a random demand $D_i(p)$, where $p = (p_1, p_2)$. We assume two types of cumulative distribution functions of demand. We consider first the exponential case and then the uniform one.

3.1 Exponential Case

The cumulative distribution function in the exponential

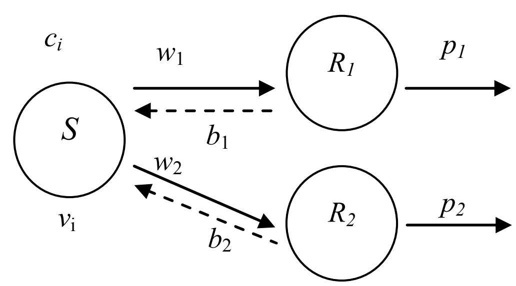

Figure 2. Two competing retailers' model

case is given by $G_i(x)=1-e^{-x}$ for all $x\geq 0$, where $E\varepsilon_i$ is set as one without loss of generality. The inverse function of $G_i(x)$ is given by $G_i^{-1}(y)=-\log(1-y)$ for all $0\leq y<1$. With $f_i(p_i)=\dfrac{(p_i-w_i)}{(p_i-b_i)}$, we get

$$\widehat{L}_i(p_i)=\frac{1}{p_i-b_i}\left(p_i-w_i+(w_i-b_i)\log(\frac{w_i-b_i}{p_i-b_i})\right)$$

Then, using (7), we obtain

$$U_i(p_i)=\frac{(p_i-w_i)}{(p_i-b_i)\left(p_i-w_i+\log\left(\frac{w_i-b_i}{p_i-b_i}\right)(w_i-b_i)\right)}$$

1) Linear demand function

The linear demand is given for $j\neq i, i, j\in\{1,2\}$ by

$$d_i(p)=\alpha_i-\beta_i p_i+\sum_{j\neq i}\beta_{ij}p_j \text{ with } \alpha_i>0,\beta_i,\beta_{ij}\geq 0 \quad (8)$$

With this demand function, we obtain the system of equations

$$\begin{cases}\dfrac{\partial\log\tilde{\pi}_1(p)}{\partial p_1}=\dfrac{-\beta_1}{\alpha_1-\beta_1p_1+\beta_{12}p_2}+U_1(p_1)=0,\\ \dfrac{\partial\log\tilde{\pi}_2(p)}{\partial p_2}=\dfrac{-\beta_2}{\alpha_2-\beta_2p_2+\beta_{21}p_1}+U_2(p_2)=0.\end{cases}$$

It can be rewritten as

$$\begin{cases}p_1=\dfrac{\beta_2-\alpha_2U_2(p_2)+\beta_2p_2U_2(p_2)}{\beta_{21}U_2(p_2)},\\ p_2=\dfrac{\beta_1-\alpha_1U_1(p_1)+\beta_1p_1U_1(p_1)}{\beta_{12}U_1(p_1)}.\end{cases} \quad (9)$$

The optimal order quantities y_1 and y_2 can be evaluated to

$$\begin{cases}y_1(p)=(\alpha_1-\beta_1p_1+\beta_{12}p_2)\log\left(\dfrac{p_1-b_1}{w_1-b_1}\right),\\ y_2(p)=(\alpha_2-\beta_2p_2+\beta_{21}p_1)\log\left(\dfrac{p_2-b_2}{w_2-b_2}\right).\end{cases}$$

Since

$$E\left[G_i^{-1}\left(\frac{p_i-w_i}{p_i-b_i}\right)-\varepsilon_i\right]^+=\log\left(\frac{p_i-b_i}{w_i-b_i}\right)+\left(\frac{w_i-p_i}{p_i-b_i}\right),$$

from (6), the supplier profit function can be expressed as

$$\Pi_M=\sum_{i=1}^{2}d_i(p_i)\left((w_i-c_i-b_i)\log\left(\frac{p_i-b_i}{w_i-b_i}\right)+b_i\frac{p_i-w_i}{p_i-b_i}\right).$$

The retailers' profit functions are given by

$$\begin{cases}\tilde{\pi}_1(p)=d_1(p)\left((b_1-w_1)\log\left(\dfrac{p_1-b_1}{w_1-b_1}\right)+(p_1-w_1)\right),\\ \tilde{\pi}_2(p)=d_2(p)\left((b_2-w_2)\log\left(\dfrac{p_2-b_2}{w_2-b_2}\right)+(p_2-w_2)\right)\end{cases}$$

2) Logit demand function

Now, the problem is studied with a logistic demand function, expressed by

$$d_i(p)=\frac{k_ie^{-\lambda p_i}}{C_i+\sum_{j=1}^{2}k_je^{-\lambda p_j}} \text{ for } \lambda, C_i, \text{and } k_i>0. \quad (10)$$

With this demand function we obtain the system of equations

$$\begin{cases}\dfrac{\partial\log\tilde{\pi}_1(p)}{\partial p_1}=\dfrac{-\lambda(C_1+k_2e^{-\lambda p_2})}{C_1+k_1e^{-\lambda p_1}+k_2e^{-\lambda p_2}}+U_1(p_1)=0,\\ \dfrac{\partial\log\tilde{\pi}_2(p)}{\partial p_2}=\dfrac{-\lambda(C_2+k_1e^{-\lambda p_1})}{C_2+k_1e^{-\lambda p_1}+k_2e^{-\lambda p_2}}+U_2(p_2)=0.\end{cases}$$

Then, we have

$$\begin{cases}p_1=-\dfrac{1}{\lambda}\log\dfrac{\lambda C_2-C_2U_2(p_2)-k_2e^{-\lambda p_2}U_2(p_2)}{k_1(-\lambda+U_2(p_2))},\\ p_2=-\dfrac{1}{\lambda}\log\dfrac{\lambda C_1-C_1U_1(p_1)-k_1e^{-\lambda p_1}U_1(p_1)}{k_2(-\lambda+U_1(p_1))}.\end{cases}$$

The order quantities are given by

$$\begin{cases}y_1(p)=\left(\dfrac{k_1e^{-\lambda p_1}}{C_1+k_1e^{-\lambda p_1}+k_2e^{-\lambda p_2}}\right)\log\left(\dfrac{p_1-b_1}{w_1-b_1}\right),\\ y_2(p)=\left(\dfrac{k_2e^{-\lambda p_2}}{C_2+k_1e^{-\lambda p_1}+k_2e^{-\lambda p_2}}\right)\log\left(\dfrac{p_2-b_2}{w_2-b_2}\right).\end{cases}$$

The supplier profit function and retailers' profit functions are obtained in the same way as for the linear demand function.

3.2 Uniform Case

The cumulative distribution function in the uniform case is given by

$$\begin{cases}G_i(x)=\dfrac{x-(1-a_i)}{2a_i},\\ 1-a_i\leq x\leq 1+a_i, 0\leq a_i\leq 1, i=1,2,\end{cases}$$

where $E\varepsilon_i=1$. The inverse function of $G_i(x)$ is given by $G_i^{-1}(y)=1-a_i+2a_iy$ for $0\leq y\leq 1$. With $f_i(p_i)=(p_i-w_i)/(p_i-b_i)$, we get

$$\widehat{L}_i(p_i)=\left(\frac{p_i-w_i}{p_i-b_i}\right)\left(1-a_i+a_i\left(\frac{p_i-w_i}{p_i-b_i}\right)\right)$$

Then by (7) for $i \in \{1,2\}$

$$U_i(p_i) = \frac{1}{p_i - b_i}\left(1 + \left(\frac{w_i - b_i}{p_i - w_i}\right)\frac{1 - a_i + 2a_i\left(\dfrac{p_i - w_i}{p_i - b_i}\right)}{1 - a_i + a_i\left(\dfrac{p_i - w_i}{p_i - b_i}\right)}\right)$$

1) Linear demand function

With the linear demand given by (8) and $U_i(p_i)$, we obtain

$$\begin{cases} p_1 = \dfrac{\beta_2 - \alpha_2 U_2(p_2) + \beta_2 p_2 U_2(p_2)}{\beta_{21} U_2(p_2)}, \\ p_2 = \dfrac{\beta_1 - \alpha_1 U_1(p_1) + \beta_1 p_1 U_1(p_1)}{\beta_{12} U_1(p_1)}. \end{cases}$$

The optimal order quantities are given by

$$\begin{cases} y_1(p) = (\alpha_1 - \beta_1 p_1 + \beta_{12} p_2)\left(1 - a_1 + 2a_1\left(\dfrac{p_1 - w_1}{p_1 - b_1}\right)\right), \\ y_2(p) = (\alpha_2 - \beta_2 p_2 + \beta_{21} p_1)\left(1 - a_2 + 2a_2\left(\dfrac{p_2 - w_2}{p_2 - b_2}\right)\right). \end{cases}$$

The supplier profit function is equal to

$$\Pi_M = \sum_{i=1}^{2}\left[d_i(p_i)\left((w_i - c_i)\left(1 - a_i + 2a_i\left(\frac{p_i - w_i}{p_i - b_i}\right)\right) - a_i b_i\left(\frac{p_i - w_i}{p_i - b_i}\right)^2\right)\right].$$

The retailers' profit functions are given by

$$\begin{cases} \tilde{\pi}_1(p) = d_1(p)\left((p_1 - w_1)\left(1 - a_1 + 2a_1\left(\dfrac{p_1 - w_1}{p_1 - b_1}\right)\right) - a_1(p_1 - b_1)\left(\dfrac{p_1 - w_1}{p_1 - b_1}\right)^2\right), \\ \tilde{\pi}_2(p) = d_2(p)\left((p_2 - w_2)\left(1 - a_2 + 2a_2\left(\dfrac{p_2 - w_2}{p_2 - b_2}\right)\right) - a_2(p_2 - b_2)\left(\dfrac{p_2 - w_2}{p_2 - b_2}\right)^2\right). \end{cases}$$

2) Logit demand function

With the Logit function given by (10), we obtain p_1 and p_2 as

$$\begin{cases} p_1 = -\dfrac{1}{\lambda}\log\dfrac{\lambda C_2 - C_2 U_2(p_2) - k_2 e^{-\lambda p_2} U_2(p_2)}{k_1(-\lambda + U_2(p_2))}, \\ p_2 = -\dfrac{1}{\lambda}\log\dfrac{\lambda C_1 - C_1 U_1(p_1) - k_1 e^{-\lambda p_1} U_1(p_1)}{k_2(-\lambda + U_1(p_1))}. \end{cases}$$

The optimal order quantities are given by

$$\begin{cases} y_1(p) = \dfrac{k_1 e^{-\lambda p_1}}{C_1 + k_1 e^{-\lambda p_1} + k_2 e^{-\lambda p_2}}\left(1 - a_1 + 2a_1\left(\dfrac{p_1 - w_1}{p_1 - b_1}\right)\right), \\ y_2(p) = \dfrac{k_2 e^{-\lambda p_2}}{C_2 + k_1 e^{-\lambda p_1} + k_2 e^{-\lambda p_2}}\left(1 - a_2 + 2a_2\left(\dfrac{p_2 - w_2}{p_2 - b_2}\right)\right). \end{cases}$$

The supplier profit function and retailers' profit functions are obtained in the same way as for the linear demand function.

3.3 Supply Chain Optimization

When the supplier and the retailers determine the prices and the order quantities to maximize the overall profit of the supply chain, the wholesale and buyback prices are meaningless because they are payments between the supplier and the retailers. As the whole of the supply chain is equivalent to a single retailer with wholesale price (c_1, c_2) and buyback (ν_1, ν_2), and by using (3), the optimal order quantity (the amount of products) is given by $y_i^I(p) = d_i(p) G_i^{-1}\left(\frac{p_i - c_i}{p_i - \nu_i}\right)$. By using (4), the overall expected profit of the supply chain is

$$\tilde{\pi}^I(p) = \sum_{i=1}^{2}(p_i - c_i) d_i(p) L_i\left(\frac{p_i - c_i}{p_i - \nu_i}\right), \tag{11}$$

where, the retail prices (p_1, p_2) are given. The optimal retail prices (p_1^I, p_2^I) in the integrated supply chain maximize the profit function given by (11).

4. Numerical Examples

4.1 Geometric Analysis of the Nash Solution

The system of equations on (p_1, p_2) that solves the profit functions for the two retailers is obtained in Section 3. In the case of exponential demand and linear functions, we denote the right hand sides of two equations in (9) by $f_2(p_2)$ and $f_1(p_1)$, respectively. Then the equations (9) become $p_1 = f_2(p_2)$ and $p_2 = f_1(p_1)$. Note that in other cases the equations satisfied by (p_1, p_2) form $p_1 = f_2(p_2)$ and $p_2 = f_1(p_1)$ similarly. Geometrically, to analyze the behavior of the system around the Nash solution, we plot the functions $f_i(p_i)$ for p_1 and p_2 in Figure 3. There are multiple solutions for the equations, but there is a unique Nash solution (p_1, p_2) with $p_i > w_i (i = 1, 2)$, which has been proved in the theorem of Section 2.

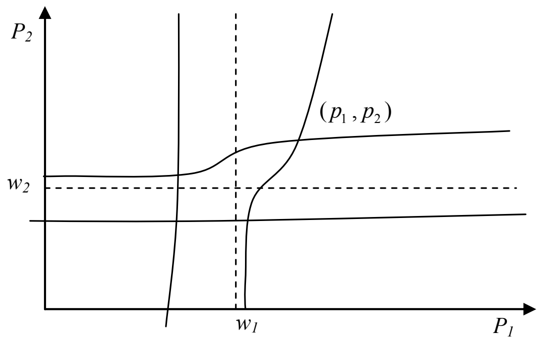

Figure 3. Nash solution and system of equations

Given wholesale and buyback prices, we derive these Nash retail prices, and profits of the supplier and two retailers. We compute them for all combinations of wholesale and buyback prices, which are integers and satisfy $c_i \le w_i \le w_i^U$ and $v_i \le b_i \le w_i$, where w_i^U is set as the upper bound for the optimal wholesale price for the supplier, and derive optimal wholesale and buyback prices for the supplier. We also compute the overall profits and retail prices under the supply chain optimization, and compare them with the ones under individual optimization.

4.2 Numerical Results

In numerical examples we set parameters as shown in the following:

$(v_1, v_2) = (0,0)$,

$(\alpha_1, \alpha_2) = (100,100)$, $(\beta_1, \beta_2) = (1,1)$,

$(\beta_{12}, \beta_{21}) = (0.3, 0.3)$ (linear function),

$\lambda = 0.03$, $(C_1, C_2) = (0.005, 0.005)$,

$(k_1, k_2) = (1,1)$, (Logit function).

The program is coded by C and the computations are done by using Fujitsu C compiler on PC. In Table 1, we assume exponential demand and Logit functions, whereas in Table 2 the linear function is assumed. In these tables two cost parameter settings are considered: $(c_1, c_2) = (30,30)$ (symmetric) and $(c_1, c_2) = (30,20)$ (anti-symmetiric).

The values in tables are the optimal profit for supplier, the profit for each retailer; entire expected profit (sum of supplier's and retailers' profits), optimal whole-sale and buyback prices for the supplier, Nash equilibrium retail prices and order quantities. The values in paranthesis () are the total profit, optimal retail prices and order quantities for retailers under the supply chain optimization.

In the cases of Tables 1 and 2, optimal whole sale prices and buybacks determined by the supplier give more profits to the supplier than retailers. In the symmetric cost cases, the optimal retail prices of two retailers become the same. Compared to supply chain optimization, the retail prices are higher and the quantities of orders are smaller in the individual optimal case. It is because under the chain optimization more amounts of demand are satisfied by decreasing retail prices and increasing order quantities, whereas in the individual optimal case the supplier wants to obtain its own profit, which leads to higher wholesale prices and as a result retail prices become higher. In the anti-symmetric cost case, the optimal wholesale price to the retailer with the smaller production cost is smaller than that to another retailer, which leads to more profits for the former retailer. The reason is that the retailer with small wholesale price sets the less retail price and more quantities of order, which implies that more amounts of demand occur in total and the supplier can sell more products to customers. In particular, with Logit demand function the demand depends on the retail prices more intensively, and the wholesale prices, retail prices and the order quantities change more.

In both cases the entire expected profits in the individual optimal cases is about 80 to 85 % of that under supply chain optimization. When the chain consists of

Table 1. Exponential demand and logit function

i	1	2	1	2
C_i	30	30	30	20
$\Pi_M(p)$	32.195		35.792	
$\pi_i(p_i, y_i)$	10.227	10.227	8.917	13.843
Entire expected profits	52.649 (62.430)		58.552 (70.153)	
w_i	98	98	100	88
b_i	47	47	47	47
p_i	175.420 (172.428)	175.420 (172.428)	175.376 (182.095)	168.444 (161.07)
y_i	0.311 (0.606)	0.311 (0.606)	0.276 (0.444)	0.418 (0.965)

Table 2. Exponential demand and linear function

a_i	0.1	0.3	0.5	0.7
$\Pi_M(p)$	2531.42	2352.36	2176.38	2003.38
$\pi_i(p_i, y_i)$	513.03	481.51	450.56	414.12
Entire expected profits	3557.49 (4303.71)	3315.38 (3999.12)	3077.51 (3700.00)	2832.62 (3407.00)
$w_1(=w_2)$	87	87	87	87
$b_1(=b_2)$	75	75	75	74
$p_1(=p_2)$	110.31 (87.08)	110.97 (88.46)	111.69 (89.96)	112.55 (91.56)
$y_1(=y_2)$	23.51 (40.26)	24.55 (41.75)	25.59 (43.20)	26.05 (44.57)

Table 3. Uniform demand and linear function

i	1	2	1	2
C_i	30	30	30	20
$\Pi_M(p)$	1200.548		1473.307	
$\pi_i(p_i, y_i)$	242.306	242.306	228.119	380.888
Entire expected profits	1685.160 (2041.22)		2082.314 (2515.01)	
w_i	89	89	89	82
b_i	77	77	77	73
p_i	116.154 (96.902)	116.154 (96.902)	115.532 (97.788)	112.445 (90.259)
y_i	22.105 (37.717)	22.105 (37.717)	21.233 (34.608)	32.826 (58.887)

one supplier and one retailer, it is shown in Song *et al.* (2008) that the fraction is 3/4(in linear case) or $2/e = 0.736$ (in Logit case). The competition among retailers makes retail prices lower, which makes the fraction higher.

In Table 3, the uniform distribution of demand is assumed with the symmetric production costs ($(c_1, c_2) = (30, 30)$), and the a_i, which corresponds to the width of the uniform distribution, is changed from 0.1 to 0.7. It implies that large a_i means the high variance of demand. As the variance increases, retail prices are higher, and profits of the supplier and retailers decrease. This is because when the variance increases, the quantity of order must be increased to apply the fluctuation of demand, whereas the retail price must be also increased to obtain profits of retailers.

When a_i changes the optimal wholesale prices and buyback prices for the supplier are almost the same. Note that even if it is compared with results in the exponential case shown in Figure 2, which has more variance than these uniform distributions, the difference on these prices is very small. It means that the optimal wholesale and buyback prices for the supplier are robust in the variance of the demand distribution.

5. Concluding Remarks

In this paper, we first show the sufficient condition that unique Nash equilibrium retail prices exist and they are greater than wholesale prices. We then give the equations whose solutions are those retail prices. In numerical examples we compute these equilibrium prices, optimal wholesale and buy-back prices for the supplier and supply chain optimal retailers' prices, and discuss properties on these values. In future research, a two-supplier problem and other types of problems will be modeled and the properties will be discussed analytically and numerically.

6. Acknowledgments

This work was supported by Grant-in-Aid for Scientific Research (C) 19510145.

REFERENCES

[1] G. P. Cachon, "Supply chain coordination with contracts, in supply chain management, design, coordination and operation," Elsevier, Vol. 11, Amsterdam, pp. 229–339.

[2] C. A. Yano and S. M. Gilbert, "Coordinated pricing and production/procurement decisions: A review," A. Chakravarty, J. Eliashberg, eds. "Managing business interfaces: marketing, engineering and manufacturing perspectives," Kluwer Academic Publishers, Boston, MA, 2003.

[3] Y. Wang, J. Li, and Z. Shen., "Channel performance under consignment contract with revenue sharing," Management Sci. Vol. 50, No. 1, pp. 34–47, January 2004.

[4] N. C. Petruzzi, "Newsvendor pricing, purchasing and consignment: Supply chain modeling implications and insights," Working Paper, College of Business, University of Illinois, Urbana-Champaign, IL, 2004.

[5] Y. Song, S. Ray, and S. Li, "Structural properties of buyback contracts for price-setting newsvendors," MSOM, Vol. 10, pp. 1–18, January 2007.

[6] F. Bernstein and A. Federgruen, "Decentralized supply chains with competing retailers under demand uncertainty," Management Science, Vol. 51, pp. 18–29, January 2005.

