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Time Dependent Entropy and Decoherence in a Modified Quantum Damped Harmonic Oscillator

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Abstract

The time dependence of probability and Shannon entropy of a modified damped harmonic oscillator is studied by using single and double Gaussian wave functions through the Feynman path method. We establish that the damped coefficient as well as the system frequency and the distance separating two consecutive waves of the initial double Gaussian function influences the coherence of the system and can be used to control its decoherence.

Keywords

1. Introduction

The study of dissipative systems and their quantization is of a great theoretical and practical value in view of many different situations in which dissipative phenomena with a quantum origin manifest themselves [1]. Within these dissipative systems, the damped harmonic oscillator (DHO) is the simplest quantum systems displaying the dissipation of energy. It is of great physical importance and has found many applications especially in quantum optics [2] [3]. For example, it plays a central role in the quantum theory of lasers and masers [4]. Moreover, damped harmonic oscillators are used to investigate the quantum decoherence (QD) phenomenon whose role

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became relevant in many interesting physical problems such as quantum computation and quantum information processing [5], material science [6]-[8], heavy ion collisions [9], quantum gravity and cosmology [10]-[22], and condensed matter physics [23]-[25]. In many cases, physicists are interested in understanding the causes of QD in order to prevent decoherence from damaging quantum states and protect the information stored in these states. Thus, decoherence is responsible for washing out the quantum interference effects which are desirable to be seen as signals in some experiments. However, QD has negative effects in many areas such as quantum computation and quantum control of atomic and molecular processes. The physics of information and computation is a domain where decoherence is an obvious major obstacle in the implementation of information-processing hardware. It takes advantage on the superposition principle [26]. QD is a condition that has to be satisfied in order that a system could be considered as classical. This condition requires that the system should be in one of relatively permanent states (called by Zurek "*preferred states*") and the interference between different states should be negligible [27] [28]. The loss of coherence can be achieved by introducing an interaction between the system and environment [23] [29] [30].

Nowadays, a great deal of research is dedicated to understanding decoherence in harmonic oscillator [31] [32]. Isar *et al.* [33] determine the degree of quantum decoherence of a harmonic oscillator interacting with a thermal bath using Lindblad theory [34] [35]. Other authors [36] use a semi-classical approach to examine decoherence in a harmonic oscillator coupled to a thermal harmonic bath. Darius *et al.* [37] exploit the Feynman path integral to study the memory in a non-locally damped oscillator. Moreover, Ozgur *et al.* [32] determine the time dependence of Leipnik's entropy in the damped harmonic oscillator via path integral techniques. Another strategy to describe dissipative quantum systems is based on the idea of Bateman [38].

In this paper, we investigate the coherence of the damped harmonic oscillator using the Caldirola-Kanai model [39] but based on the idea of Bateman [38]. This model is known as a popular model used to describe dissipative systems coupled to a harmonic bath. It has many applications like reproducing classical effects or giving a good Hamiltonian necessary to exhibit the phenomenon of decoherence [38]. This paper is organized as follows. In Section 2, we present the mathematical tools based on the path integral formalism. We also discuss the case of the damped harmonic oscillator and build the associated propagator. In Section 3, we derive the general expressions of the thermodynamic parameters of the system, and analyze the effects of the damping constant on the distribution probability and the Shannon entropy for a single Gaussian wave function. The influence of the system's frequency has also been measured on these parameters. Hence, we consider a double Gaussian wave function and make numerical investigations appreciate the impact of the damping constant, the distance separating the two wave functions and the system's frequency on the thermodynamic characteristics of the system. Discussion and concluding remarks are given in the last section.

2. Fundamental Definitions

We start by presenting the model which consists of a particle of mass m, labeled by the position variable q and the momentum p. Then follows the description of the used mathematical tools which is the path integral formalism introduced by Feynman [40]. These tools suggest that the transformation function called propagator is analogue to $\exp\left(\frac{i}{\hbar}S_{cl}\right)$ in which S_{cl} stands for the action, solution of Hamilton-Jacobi equation. On the other way, the transition amplitude of the particle (of mass m) from the position q_a at time t_a to the position q_b at time t_b, known as the propagator, represents the solution of the Schrodinger equation. Nowadays, several problems of physics are solved via these techniques [32] [41].

Next, we consider the Bateman Hamiltonian [38] defined as:

$$H = \bar{p}p - \gamma\left[x\bar{p} - \bar{x}p\right] + \Omega^2\bar{x}x \tag{1}$$

where $\bar{p}$ and $\bar{x}$ are the mirror variables corresponding to the coordinate x and the momentum p. The quantities γ and Ω are respectively the damped coefficient and the system frequency. The associated lagrangian is given by

$$L = \dot{\bar{x}}\dot{x} - x\bar{x} + \gamma\left(x\dot{\bar{x}} - \dot{x}\bar{x}\right) \tag{2}$$

Using Euler-Lagrange equation, we derive the following two motion equations [42]:

$$\begin{cases} \ddot{x} + \gamma\,\dot{x} + \Omega^2 x = 0 \\ \ddot{\overline{x}} + \gamma\,\dot{\overline{x}} + \Omega^2 \overline{x} = 0 \end{cases} \tag{3}$$

Bateman's dual Hamiltonian describes classical mechanics correctly, but this model faces some difficulties. It violates Heisenberg's principle for $\gamma \neq 0$. Therefore, to solve quantum mechanical problem, Caldirola-Kanai [39] build a theory based on the idea of Bateman dissipative system by considering the standard Hamiltonian of harmonic oscillator with time dependent mass given by [39] $m(t) = m_0 \exp(2\gamma t)$. Hence, the Hamiltonian and the Lagrangian of the system become respectively:

$$H = \frac{p^2}{2m(t)} + \frac{1}{2}m(t)\omega^2 x^2,$$

$$L = p\dot{x} - H = \exp(2\gamma t)\left[\frac{1}{2}m\dot{x}^2 - \frac{1}{2}m\omega^2 x^2\right]. \tag{4}$$

Here, ω is the system frequency. From the Lagrangian theory and exploiting quantities (4), the equation of motion takes the form:

$$\ddot{x} + 2\gamma\,\dot{x} + \Omega^2 x = 0 \tag{5}$$

The classical solution of (5) is given by

$$x(t) = C_1 \exp(a_1 t) + C_2 \exp(a_2 t) \tag{6}$$

where in a_1 and a_2 are complex quantities defined as: $a_1 = -\gamma - i\Omega$, $a_2 = -\gamma + i\Omega$ with $\Omega = \sqrt{\omega^2 - \gamma^2}$. The integration constants C_1 and C_2 are evaluated when the particle moves from the position x_a at the time t_a to the position x_b at time t_b. The determination of the propagator is convenient for founding quantum mechanical solution for this Hamiltonian. Therefore, the classical action S_{cl} is defined as:

$$S_{cl} = \int L(x,\dot{x},t)\mathrm{d}t = \int_{t_a}^{t_b} \frac{m}{2}\left(\dot{x}^2 - \omega^2 x^2\right)\mathrm{d}t = \frac{m_0}{2}\int_{t_a}^{t_b}\left(\dot{x}^2 - \omega^2 x^2\right)\mathrm{d}t$$

whose computation for the current study case leads to:

$$S_{cl} = \frac{m_0\Omega\mu_1}{\exp\{-2\gamma(t_a + t_b)\}\sin\{\Omega(t_a - t_b)\}} + \frac{m_0\gamma\mu_2}{\exp\{-2\gamma(t_a + t_b)\}} \tag{7}$$

in which we set:

$$\mu_1 = \left[x_b^2\exp(-2\gamma t_a) + x_a^2\exp(-2\gamma t_b)\right] - 2x_a x_b\exp\left[-\gamma(t_a + t_b)\right] + \cos\{\Omega(t_a - t_b)\},$$
$$\mu_2 = x_b^2\exp(-2\gamma t_a) - x_a^2\exp(-2\gamma t_b).$$

From the classical action, the expression of the corresponding propagator is defined below.

$$\aleph(x,x',t) = \frac{m\Omega}{2\pi i\hbar\sin(\Omega t)}\exp\left(\frac{i}{\hbar}S_{cl}\right) \tag{8}$$

Substituting (7) into (8), we obtain the following expression for the quantum propagator of damped harmonic oscillator:

$$\aleph(x_a,x_b,t) = \left[\frac{m_0\Omega}{2\pi i\hbar\exp\{2\gamma(t_a + t_b)\}\sin\{\Omega(t_b - t_a)\}}\right]^{1/2}\exp\left(\frac{i}{\hbar}S_{cl}\right) \tag{9}$$

This result is identical to the one establish in [41] using the propagator method developed by Um *et al.* [43]. It also appears from (9) that the propagator $\aleph(x_a,x_b,t)$ depends on the damped coefficient γ that links the system

with the environment in which it evolves.

Hereafter, we intend to use the propagator (9) and derive some characteristic parameters (such as the distribution probability and the Shannon entropy) of the system subjected respectively to single and double Gaussian wave functions. These investigations aim to measure the impact of the environment on the behavior of the system when the latter progresses.

3. Calculations and Results

3.1. System Properties under a Single Gaussian Wave Function

In this section, we exploit the single Gaussian wave function to examine the impact of the environment on the distribution probability and the Shannon entropy for a specific damped harmonic oscillator and therefore, to measure its coherence.

3.1.1. Distribution Probability

Making use of [31], we determine the distribution probability for a single Gaussian wave packet to find the particle at coordinate x at the time t. Nowadays, several problems of physics are solved via path integral techniques. It gives analytical solution for various coupling problems. This probability can be written in the Feynman-Hibbs form as [44]:

$$P(x,t) = |\varphi(x,t)|^2 = \int dx_b' \int dx_b \aleph^*(x_a, x_b', t) \aleph(x_a, x_b, t) \varphi(x_b', 0) \varphi(x_b, 0) \tag{10}$$

which presents the link between the distribution probability $P(x,t)$ and the propagator (9) for a given wave function φ. In expression (10), $\varphi(x_b, 0)$ designates the initial Gaussian wave packet centered at $x_0 = 0$ with [31]:

$$\varphi(x_b, 0) = \left[2\pi\sigma^2\right]^{-1/4} \exp\left(-\frac{x^2}{4\sigma^2}\right) \tag{11}$$

The quantity $\varphi(x_b', 0)\varphi^*(x_b, 0)$ defines the pure electronic density matrix and $\sigma^2 \hbar/2m\Omega$. Based on (9), we evaluate the propagator

$$\aleph^*(x_a, x_b', t)\aleph(x_a, x_b, t) = \left[\frac{m_0\Omega\exp(2\gamma t_b)}{2\pi\hbar\sin\{\Omega(t_b - t_a)\}}\right]\exp\left(\frac{i}{\hbar}S_{cl}\right)$$

which takes the following form after substitution of the classical action S_{cl}:

$$\aleph^*(x_a, x_b', t)\aleph(x_a, x_b, t) = \left[\frac{m_0\Omega\exp(2\gamma t_b)}{2\pi\hbar\sin\{\Omega(t_b - t_a)\}}\right]\exp\left[a\left(x_b^2 - x_b'^2\right) + b\left(x_b - x_b'\right)\right] \tag{12}$$

With

$$a = \frac{i}{\hbar}\left\{m_0\exp(2\gamma t_b)\left[\Omega\cot\Omega(t_b - t_a) + \gamma\right]\right\}, \quad b = -\frac{2ix_a m_0\Omega\exp\left[\gamma(t_b + t_a)\right]}{\hbar\sin\left[\Omega(t_b - t_a)\right]} \tag{13}$$

Therefore, the distribution probability (10) yields the following expression:

$$P = \frac{2\sigma m_0\Omega\exp(2\gamma\, t_b)}{\hbar\sin(\Omega t)}\left(\frac{1}{2\pi}\right)^{1/2}\left\{\frac{1}{\sqrt{1 - 16a^2\sigma^4}}\exp\left[\frac{2b^2\sigma^2}{1 - 16a^2\sigma^4}\right]\right\} \tag{14}$$

This relation shows that the distribution probability is an explicit function of space and time. **Figure 1(a)** presents its evolution when the damping coefficient γ is null (pure state). This plot shows that the probability decreases with space and increases with time. We observe that the probability behaves like in the case of free particle as shown in [32]. For non-zero values of γ **Figure 1(b)** gives the evolution of the same probability.
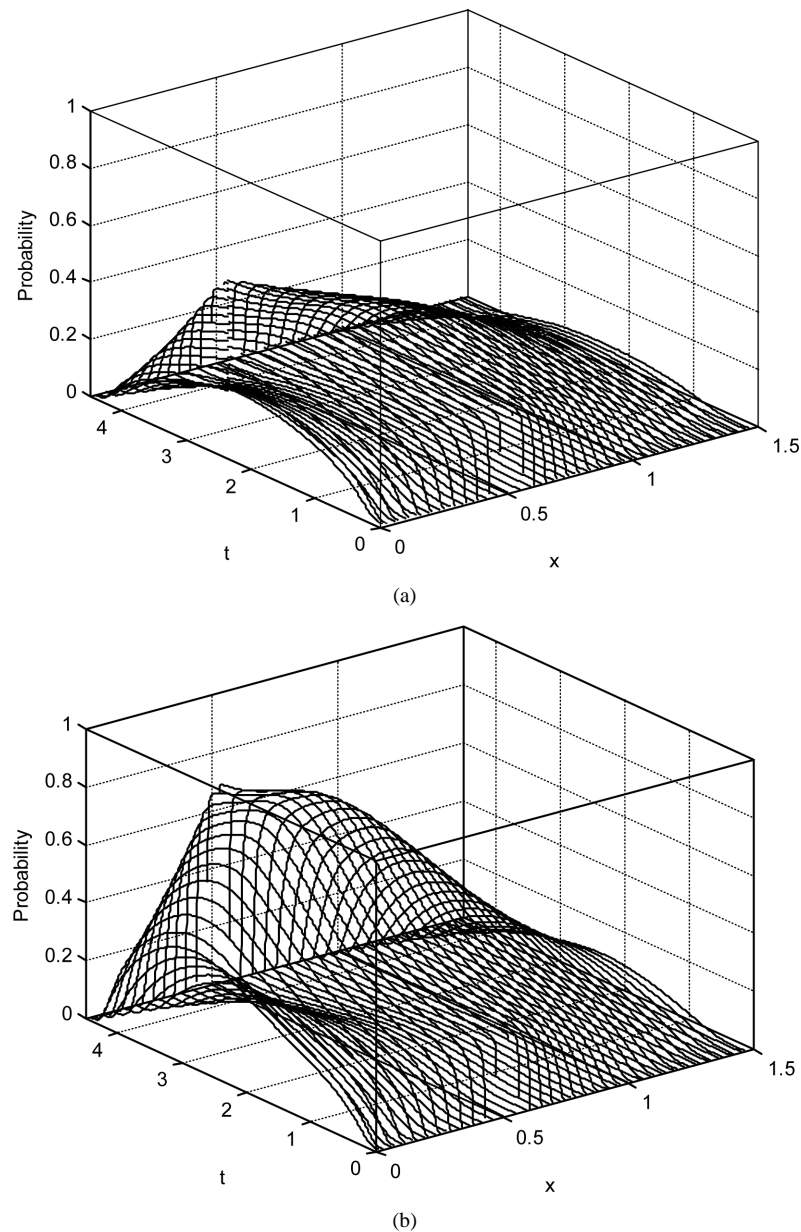
(a)

(b)

Figure 1. Distribution probability for a single Gaussian wave function with the parameters $\Omega = 0.7$, $m_0 = 1$, $\hbar = 1$ and two values of γ (a): $\gamma = 0$ (pure state) and (b): $\gamma = 0.3$ (damped state).

From the obtained curve, one observes that the probability increases with the growth of the damping coefficient. This result traduces the fact that the interaction between the system and the environment induces a modification of the former. It also establishes the existence of the critical value γ_c of γ over which the probability will exceed unit. The determination of γ_c is useless here since we are not concerned with the consequences of its existence in the present work. Furthermore, we intend to deeply examine another aspect of the coherence of the system by investigating the influence of the environment on the Shannon entropy.

3.1.2. Shannon Entropy

It is well known that the major way to appreciate the purity of a system is to study the evolution of its entropy. When this quantity tends to zero, we obtain a pure state. Decoherence stands for the loose of information in the

system. This occurs when the exchange between the environment and the system affects the evolution of the concerned system. Mathematically, the entropy S_1 is defined by Boltzmann-Shannon as:

$$S_1 = -K_B \int_{-\infty}^{+\infty} P \ln P \, \mathrm{d}x \tag{15}$$

in which K_B represents the Boltzmann constant and P is the distribution probability defined by (14) for a single Gaussian function. Therefore, it appears that the Shannon entropy S_1 is an explicit function of time. But it also deals with the system frequency. Indeed, **Figure 2** gives the evolution of this entropy in terms of time and system frequency for pure system. From this graph, we note that the entropy oscillates with time. Therefore, the information is periodically transferred between the environment and the system.

However, when $\gamma \neq 0$, **Figure 3** presents the behavior of temporal evolution of the entropy versus the system
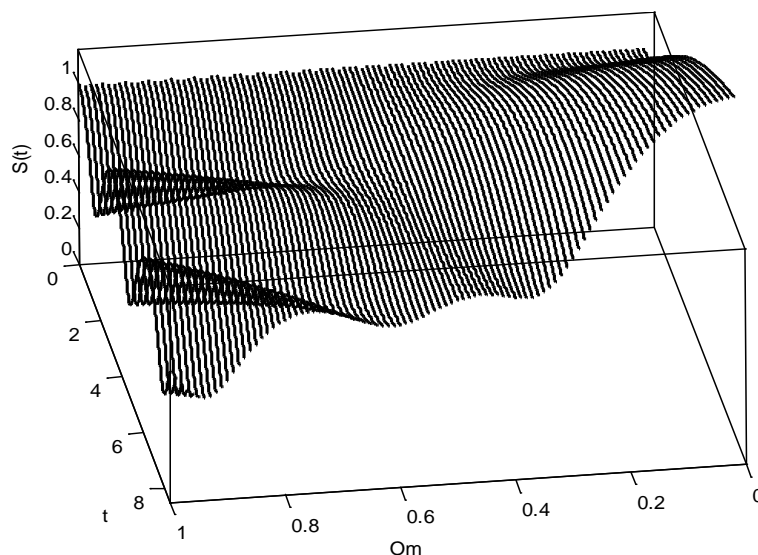

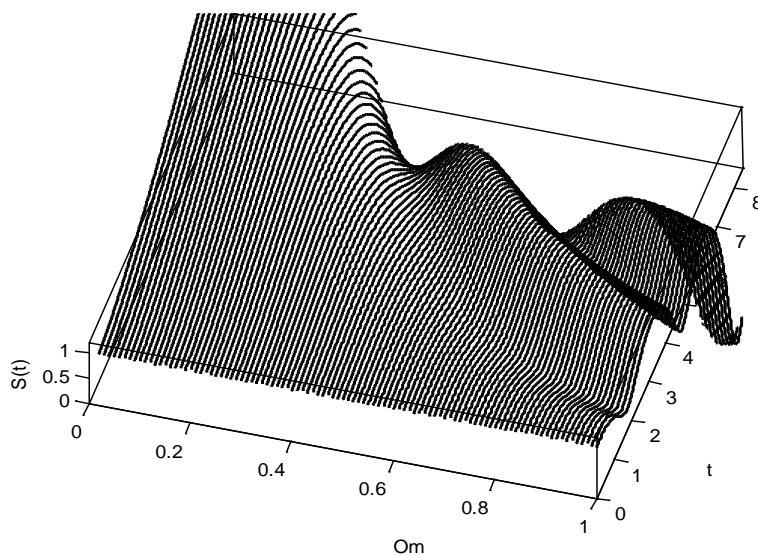**Figure 2.** The 3D entropy for a simple harmonic oscillator with the parameters of **Figure 1(a)**.

Figure 3. The 3D entropy as function of time and the system frequency for the parameters of **Figure 1** with $\gamma = 0.1$. The transfer of information between the system and the environment has no periodicity.

frequency Ω. This plot shows that the Shannon entropy loses its periodicity and decays with the system frequency. Furthermore, this entropy S_1 grows with time and the damped factor as shown on **Figure 4**. These results indicate that the damped factor enhances the transfer of information between the system and the environment. Also the fact that the envelope of the curve of Shannon entropy increase implies that this information is losing in time. This traduces the decoherence of the system.

3.2. System Properties for a Double Gaussian Case

In this section, we focus our attention on the study of the effects of the double Gaussian approximation function on
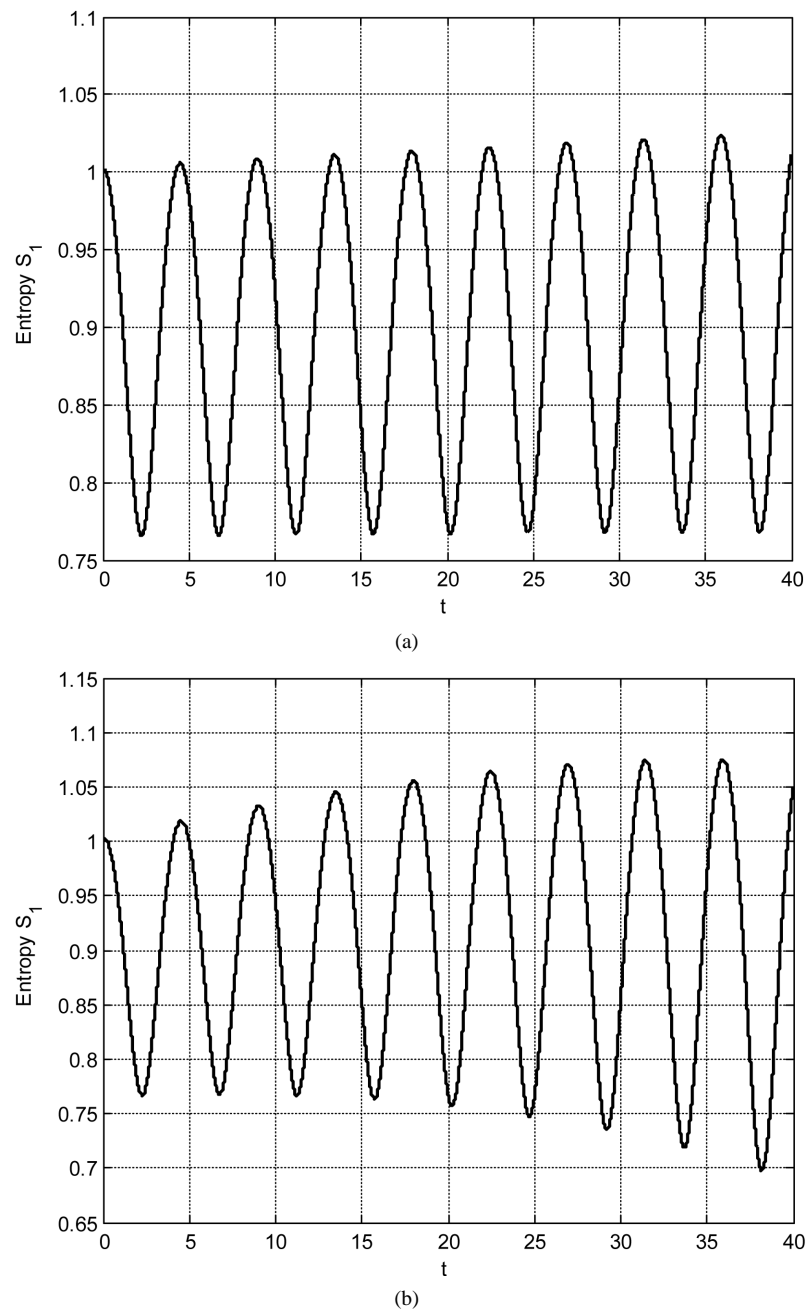

(a)

(b)

Figure 4. Behavior of the Shannon entropy $S_1(t)$ for several values of the damped factor (a): $\gamma = 0.001$ and (b): $\gamma = 0.006$.

the interaction between our damped harmonic oscillator and its environment. For this purpose, the initial state for the double Gaussian wave function is given by [31]

$$\varphi'(x_b,0) = \left(8\pi\sigma^2\right)^{-1/4}\left[1+\exp\left(\frac{-d^2}{8\sigma^2}\right)\right]\left\{\exp\left[\frac{-1}{4\sigma^2}\left(x-\frac{d}{2}\right)^2\right]+\exp\left[\frac{-1}{4\sigma^2}\left(x+\frac{d}{2}\right)^2\right]\right\} \tag{16}$$

in which d defines the distance between the top of the two successive waves in the double Gaussian state. To appreciate the impact of this new wave packet on the thermodynamic parameters of the system, we seek separately its distribution probability and Shannon entropy.

3.2.1. Distribution Probability

The corresponding distribution probability P' is obtained by substituting expression (16) into (10). The computation yields the upcoming quantity:

$$P' = \frac{16\pi A\sigma^2}{\sqrt{1-16a^2\sigma^4}}\left[e^{u_1}+e^{u_2}\right]\cdot\left[e^{v_1}+e^{v_2}\right] \tag{17}$$

wherein $u_1 = \dfrac{\left(d+4b\sigma^2\right)^2}{16\sigma^2\left(1-4a\sigma^2\right)}$; $u_2 = \dfrac{\left(d-4b\sigma^2\right)^2}{16\sigma^2\left(1-4a\sigma^2\right)}$; $v_1 = \dfrac{\left(d-4b\sigma^2\right)^2}{16\sigma^2\left(1+4a\sigma^2\right)}$; $v_2 = \dfrac{\left(d+4b\sigma^2\right)^2}{16\sigma^2\left(1+4a\sigma^2\right)}$ and

$$A = \frac{\left(8\pi\sigma^2\right)^{-1/2}m_0\Omega\exp\left(2\gamma t_b - \dfrac{d^2}{8\sigma^2}\right)}{2\pi\hbar\sin\left(\Omega t\right)\left[1+\exp\left(-\dfrac{d^2}{8\sigma^2}\right)\right]}.$$

One could note that the distribution probability depends not only on time, position and system frequency, but also on the distance separating the two successive peaks of the double Gaussian function. In the limit case $d=0$, we recover the probability (14) that deals with a single Gaussian wave function.

The Spatiotemporal evolution of the probability (17) is plotted on **Figure 5**. These figures confirm the fact that, the probability grows with the increment of the damped factor γ (showing that the information is losing with the increasing of the damping coefficient). Analysis of **Figure 5(a)** and **Figure 1(a)** shows that the distance d improves the probability meanwhile the double Gaussian wave function is welcome for the study of the probability of this particle.

3.2.2. Shannon Entropy

In this subsection, we investigate the Shannon entropy relates to the double Gaussian wave function for a specific damped harmonic oscillator. Owing to the definition, this entropy is:

$$S_2(t) = -K_B\int_{-\infty}^{+\infty}P'\ln P'\mathrm{d}x \tag{18}$$

in which the probability P' is defined by (17). Hereafter, we explore the influence of each system characteristics on the evolution of this entropy.

First, **Figure 6** presents the effects of the distance d on the behavior of the entropy $S_2(t)$ for the pure state. From these curves, one observes that the entropy amplitude has an unchanged time behavior for given values of d. It appears from these curves that the factor d can be used to control the transfer of information between system and its environment.

Next, we examine the influence of the damped factor $\gamma\neq 0$ on the entropy (18) as shown on **Figure 7**. These plots confirm the fact that the presence of γ induces the decoherence of the system. Comparison of **Figure 6(b)** and **Figure 7(b)** shows that the growth of γ affects the coherence of the system. At the end, we compare the plot of **Figure 7(a)** and **Figure 7(b)**, and appreciate the cumulative effects of d and γ on the coherence of the system. These graphs let appear that d and γ contribute to the decoherence of the system: one (γ) increases the magnitude of the entropy while the other (d) reduces the periodicity of the information transferred.
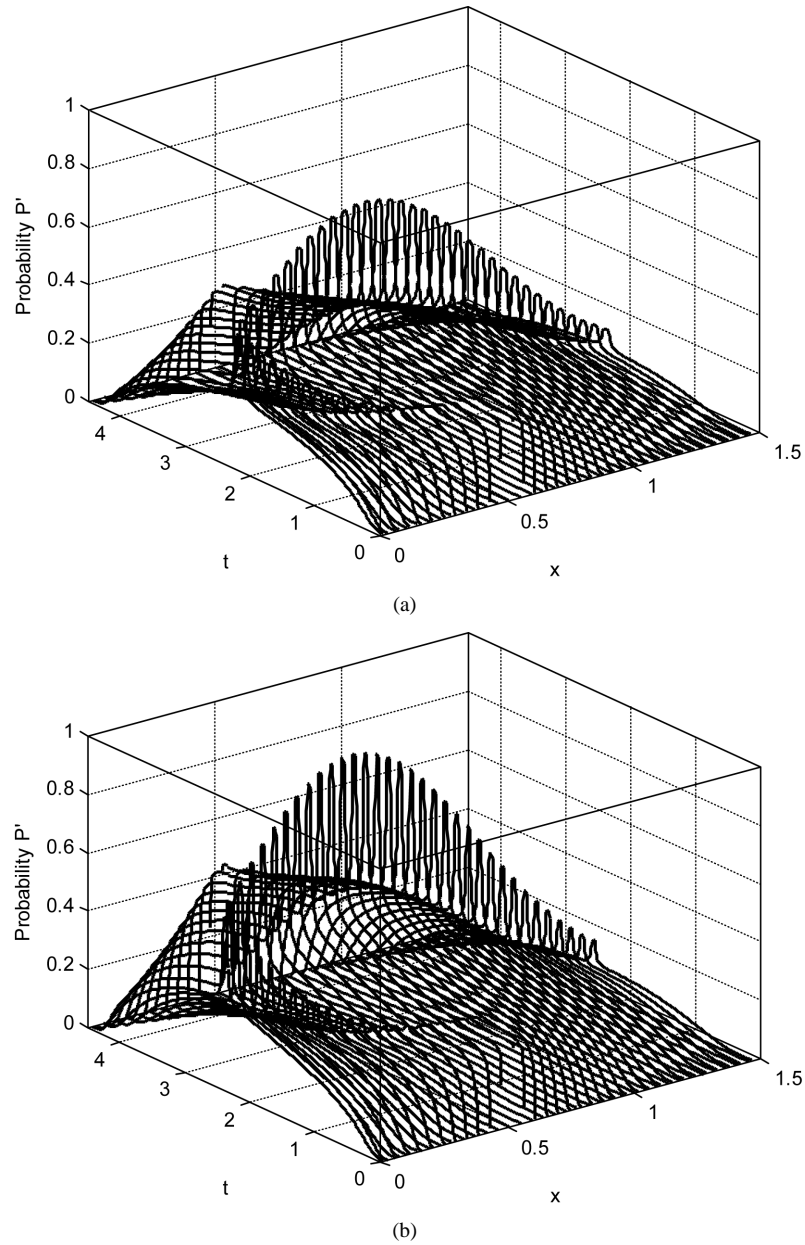
(a)

(b)

Figure 5. Distribution probability versus space and time for a double Gaussian wave function with the parameters $\Omega = 0.7$, $m_0 = 1$, $\hbar = 1$, $d = 0.1$ and various values of γ: (a): $\gamma = 0$ and (b): $\gamma = 0.1$.

4. Conclusions

In this paper, we have examined the dynamics of the modified damped harmonic oscillator constructed from the Caldirola-Kanaï model and based on the idea of Bateman. For this purpose, the Feynman path integral method has been used to investigate the time dependent probability and the entanglement entropy exploiting the single and double Gaussian initial states. In these two initial states, we have shown that the Shannon entropy decreases with the system frequency and grows with the others parameters (such as time or damped factor). For both cases, we have established that the distribution probability possesses the same behavior. In the specific case of the double Gaussian approximation, we have obtained that the distribution probability and the Shannon's entropy have been improved by the distance between two consecutive peaks of the wave.
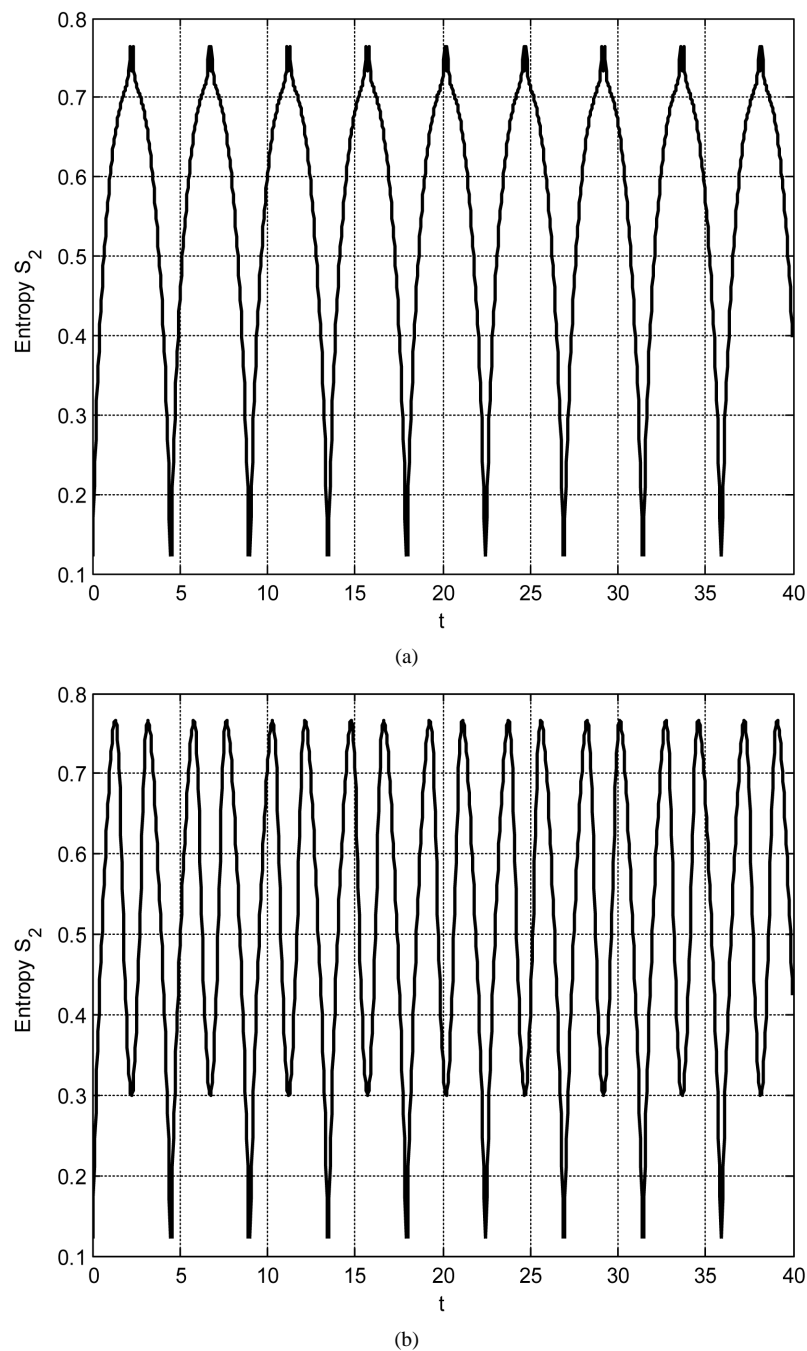
(a)

(b)

Figure 6. Shannon entropy $S_2(t)$ as function of time for the parameters of **Figure 5(a)** with several values of d: (a): $d = 0.1$ and (b): $d = 1.4$.

In the absence of the damped factor, we have recovered the results that link with the system in its pure state. Here, the information is exchanged periodically between the corresponding harmonic oscillator and the environment. For non-zero values of the damped factor, it has appeared that this transfer of information loses its periodicity traducing the loss of information in the system (decoherence phenomena). These results could be of great interest for engineering purposes since it becomes necessary to control the effects of the environment on the evolution of the system in order to reduce its decoherence. This phenomenon of controlling decoherence in an evolving system is essential in the construction of quantum computers that need the use of systems taken in their
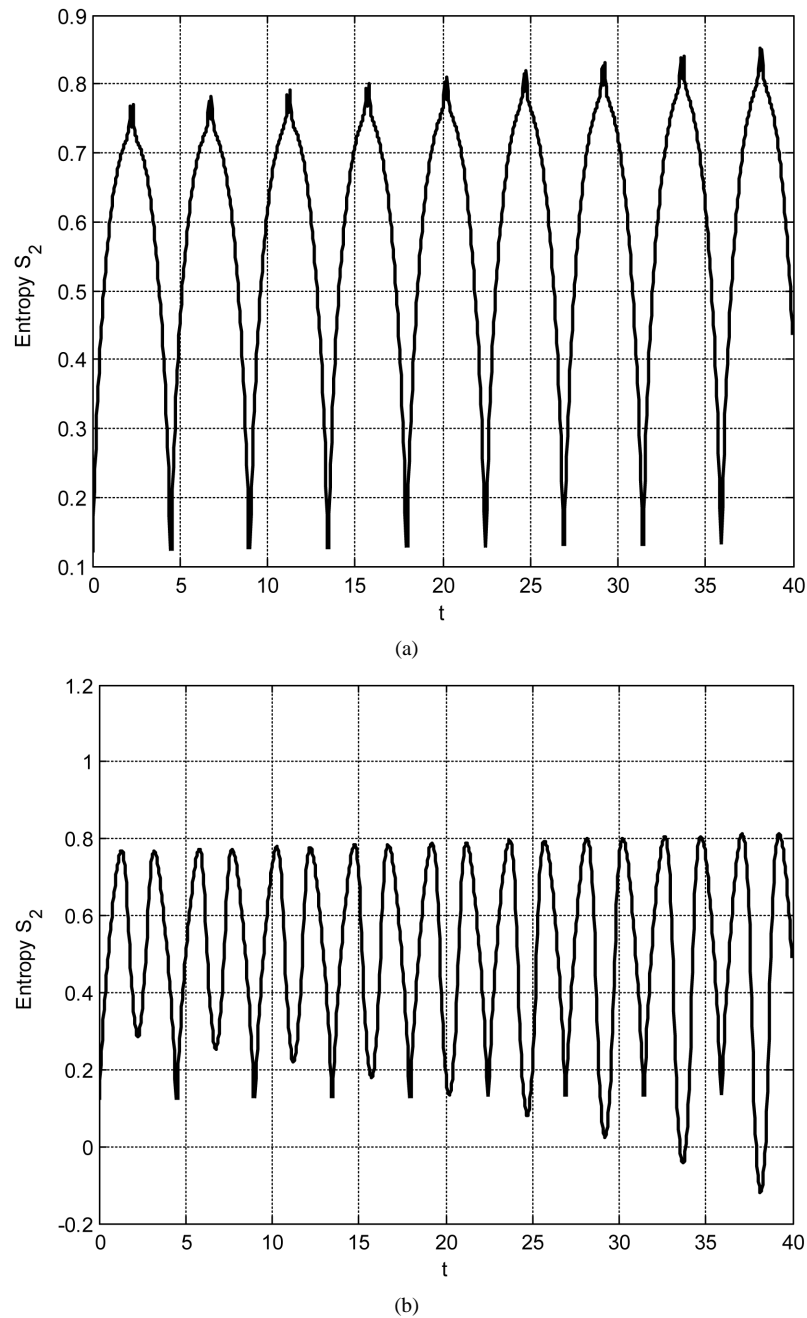
(a)

(b)

Figure 7. Temporal evolution of the Shannon entropy for the parameters of **Figure 6** with $\gamma = 0.003$ and various values of d: (a): $d = 0.1$ and (b): $d = 1.4$. The factor d can be used to control the decoherence of the system.

different superposition states. Our study has also shown that such aim could be achieved by acting on the damped coefficient, the frequency of oscillation and/or the type of state as control parameter.

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