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Incomparability through Superposition of Many States under LOCC

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Abstract

In this paper we investigate the global property of the states constructed through superposition of many states by using the concept of incomparability under LOCC as the inherent property of the states. In our work we are able to form a bridge between comparable and incomparable classes of states through linear superposition of a states and the concept of strong incomparability criterian under LOCC. PACS number(s): 03.67.Hk, 03.65.Ud, 03.65.Ta, 03.67.–a, 89.70+c.

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1. Introduction

Quantum information, quantum entanglement and quantum computation become the most attracting and useful branches of quantum mechanics. Specially quantum entanglement, has been regarded as the fundamental resource was firstly introduced by Einstein, Podolsky and Rosen (EPR) [1] and by *Schödinger* [2]. In [1] EPR raised a question wheather quantum mechanics is local and complete theory or not. The most significant progress towards this resolution of EPR problem was made by Bell [3] through famous Bell's inequality and the latter feature of quantum mechanics called usually a non-local theory. Quantum entanglement mainly reveals some non-local behaviors of quantum mechanics and performs many informative tasks like Teleportation, Dense Coding, Cloning and many others [4-6]. Such type of informative tasks are possible if and only if quantum entanglement exists positively. So characterization and quantification of entanglement [7,8] are both not only necessary important but also a serious task to the scientists in the field of quantum information and computation.

For a few decades many researchers tried to observe the underline physics of quantum entanglement [9,10] and suggested many algorithms and concepts to prove some new results for both characterizing and quantification of quantum entanglement [11,12]. Recently, Lindu, Popescu and Smolin [13] have raised the following problem: given a bipartite quantum state $|\Gamma\rangle$ and a certain decomposition of it as a superposition of two other states.

In $|\Gamma\rangle = \alpha|\psi\rangle + \beta|\varphi\rangle$ what is the relation between the entanglement of $|\Gamma\rangle$ and those of the two states in the superposition? They also considered the following examples to illustrate the above problem.

$$|\gamma\rangle = \frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle \quad \text{and}$$

$$|\gamma'\rangle = \frac{1}{\sqrt{2}}|\varphi^+\rangle + \frac{1}{\sqrt{2}}|\varphi^-\rangle$$

where $|\varphi^\pm\rangle = \frac{1}{\sqrt{2}}|00\rangle \pm \frac{1}{\sqrt{2}}|11\rangle$. The first example very clearly explains that $|\gamma\rangle$ is a maximally entangled state but each state is fully separable [14,15]. That is, superposition of fully separable states form a maximally entangled state ,and the second example provides the most opposite information of the first *i.e.* $|\gamma'\rangle$ is separable but each state is maximally entangled. So just observing the above facts it is quite understood that the superposition of states will give some new physics and concepts in the area of detection and characterization of quantum entanglement.

The aim of this paper is to use the notion of incomparability [16-18] under LOCC which is a branch of memorization criteria to discuss some interesting situations in which the state property incomparability under LOCC is remained same or not to the state generated through the superposition of many states.

This paper is organized as follows: first, in section II, we give the idea of incomparability and some fundamental results and concepts of this topic. Section III is devoted to discuss the main results. The paper is ended in section IV with a brief conclusion of our new achievement.

2. Notion of Incomparability

For achieving our goal of our work it is now quite essential to define the condition for a pair of states to be incomparable with each other and the notion of incomparability of a pair of bipartite pure states is a consequence of Nielsen's [5,19] famous and fundamental majorization criterion. To illustrate the notion suppose we consider the conversion of the pure bipartite state $|\Psi\rangle$ to $|\Phi\rangle$ shared between two parties, say, Alice and Bob by deterministic LOCC with the consideration that the pair $\left(|\Psi\rangle,|\Phi\rangle\right)$ is in their Schmidt bases $\left\{|i_A\rangle,|i_B\rangle\right\}$ with decreasing order of Schmidt coefficients: $|\Psi\rangle=\sum_{i=1}^{d}\sqrt{\alpha_i}\,|i_Ai_B\rangle$, $|\Phi\rangle=\sum_{i=1}^{d}\sqrt{\beta_i}\,|i_Ai_B\rangle$, where $\alpha_i\geq\alpha_{i+1}\geq 0$ and $\beta_i\geq\beta_{i+1}\geq 0$, for $i=1,2,\cdots,d-1$, and $\sum_{i=1}^{d}\alpha_i=1=\sum_{i=1}^{d}\beta_i$. The Schmidt vectors corresponding to the states $|\Psi\rangle$ and $|\Phi\rangle$ are $\lambda_\Psi\equiv(\alpha_1,\alpha_2,\cdots,\alpha_d)$ and $\lambda_\Phi\equiv(\beta_1,\beta_2,\cdots,\beta_d)$. Now the Nielsen's criterion $|\Psi\rangle\rightarrow|\Phi\rangle$ is possible with certainty under LOCC if and only if λ_Ψ is majorized by λ_Φ, denoted by $\lambda_\Psi\prec\lambda_\Phi$ and described as,

$$\sum_{i=1}^{k}\alpha_i\leq\sum_{i=1}^{k}\beta_i\quad\forall\ k=1,2,\cdots,d \tag{1}$$

The vital physical phenomenon that the non-increasement of entanglement by LOCC is observed as a consequence of the notion: If $|\Psi\rangle\rightarrow|\Phi\rangle$ is possible under LOCC with certainty, then $E\left(|\Psi\rangle\right)\geq E\left(|\Phi\rangle\right)$ [where $E(\cdot)$ denote the von-Neumann entropy of the reduced density operator of any subsystem and known as the entropy of entanglement]. Now in case of failure of the above criterion (1), it is usually denoted by $|\Psi\rangle\not\rightarrow|\Phi\rangle$. Also it may happen that $|\Phi\rangle\rightarrow|\Psi\rangle$ under LOCC. And if it happens that both $|\Psi\rangle\not\rightarrow|\Phi\rangle$ and $|\Phi\rangle\not\rightarrow|\Psi\rangle$ then we denote it as $|\Psi\rangle\not\leftrightarrow|\Phi\rangle$ and describe $\left(|\Psi\rangle\rightarrow|\Phi\rangle\right)$ as a pair of incomparable states. One of the peculiar features of such incomparable pairs is that we are unable to say which state has a greater amount of entanglement content than that of the other. For 2×2 systems there are no pair of incomparable pure entangled states as described above. Now we want to mention explicitly the criterion of incomparability for a pair of pure entangled states $|\Psi\rangle,|\Phi\rangle$ of $m\times n$ system where $\min\{m,n\}=3$. Suppose the Schmidt vectors corresponding to the two states are (a_1,a_2,a_3) and (b_1,b_2,b_3) respectively, where $a_1>a_2>a_3>0$, $b_1>b_2>b_3>0$, $a_1+a_2+a_3=1=b_1+b_2+b_3$. Then it follows from Nielsen's criterion that $|\Psi\rangle,|\Phi\rangle$ are incomparable if and only if, either of the pair of relations.

$$\begin{aligned}&a_1>b_1\ \&\ a_3>b_3\\&b_1>a_1\ \&\ b_3>a_3\end{aligned} \tag{2}$$

will hold.

A most powerful usefulness of incomparability is that if a pair of states is incomparable then we can not compare the amount of entanglement of the pair.

3. Main Results

Now we arrive at that stage where we can discuss some newly interesting basic consequences of superposition through the concept of incomparability under LOCC. Now for our discussion we take two parties say A and B. Now

$$|\Gamma\rangle=\alpha|\psi\rangle+\beta|\varphi\rangle \tag{3}$$

where $\alpha^2+\beta^2=1$ and

$$|\Gamma'\rangle=\alpha'|\psi'\rangle+\beta'|\varphi'\rangle \tag{4}$$

where $\alpha'^2+\beta'^2=1$ be their explicit forms. Now we write the explicit forms of $|\psi\rangle$, $|\psi'\rangle$, $|\phi\rangle$ and $|\phi'\rangle$ in the following

$$\begin{aligned}|\psi\rangle&=\sum_{i=0}^{2}\sqrt{a_i}\,|ii\rangle,\\|\varphi\rangle&=\sum_{j=0}^{2}\sqrt{b_j}\,|jj\rangle,\\|\psi'\rangle&=\sum_{i=0}^{2}\sqrt{\alpha_i}\,|ii\rangle\ and\\|\varphi'\rangle&=\sum_{i=0}^{2}\sqrt{\beta_j}\,|jj\rangle\end{aligned} \tag{5}$$

Now we discuss the problem that we have already raised in this paper in the following cases through imposing restrictions on $\alpha,\beta,\alpha',\beta'$, and also a_i,b_i,α_i,β_i for all $i=0,1,2$.

CASE: I

For two states $|\Gamma\rangle$ and $|\Gamma'\rangle$ of two parties A and B we have imposed the following restriction that the states $|\psi\rangle$, $|\psi'\rangle$ and $|\phi\rangle$ and $|\phi'\rangle$ are mutually incomparable to each other under LOCC. So the fundamental question is wheather this incomparability under LOCC of states becomes the global phenomenon of $|\Gamma\rangle$ and $|\Gamma'\rangle$ *i.e.* clearly to say that $|\Gamma\rangle$ and $|\Gamma'\rangle$ are incomparable under LOCC to each other or not and we illustrate all the results through the following tabular form:

CASE: II

Two states $|\Gamma\rangle$ and $|\Gamma'\rangle$ of two parties A and B with

the consideration that the states $(|\psi\rangle,|\psi'\rangle)$ are comparable pair and $(|\phi\rangle,|\phi'\rangle)$ are mutually incomparable to each other under LOCC. So natural question is about the status of $|\Gamma\rangle$ and $|\Gamma'\rangle$ *i.e.* clearly wheather the states $|\Gamma\rangle$ and $|\Gamma'\rangle$ are incomparable or comparable pair under LOCC to each other or not and we illustrate all the results through the following tabular form:

CASE: III

Here we consider two states $|\Gamma\rangle$ and $|\Gamma''\rangle$ of two parties A and B, where $|\Gamma''\rangle = \alpha'|\psi'\rangle + \beta'|\varphi\rangle$ with the assumption that that the states $(|\psi\rangle,|\psi'\rangle)$ are mutually incomparable to each other under LOCC. So the

Table 1. Relationship between $|\Gamma\rangle$ and $|\Gamma'\rangle$ as provided in case: I.

Restriction on α,β	Some Other Considerations	Nature of the Pair (Γ *and* Γ')
$\alpha \geq \alpha' and \beta \geq \beta'$		Incomparable
$\alpha < \alpha' and \beta < \beta'$	$(\alpha^2 a_0 > \alpha'^2 \alpha_0)$ and $(\beta^2 b_0 > \beta'^2 \beta_0)$; $(\alpha^2 a_2 > \alpha'^2 \alpha_2)$ and $(\beta^2 b_2 > \beta'^2 \beta_2)$	Incomparable
$\alpha < \alpha' and \beta < \beta'$	$(\alpha^2 a_0 > \alpha'^2 \alpha_0)$ *and* $(\beta^2 b_0 > \beta'^2 \beta_0)$; $(\alpha^2 a_2 < \alpha'^2 \alpha_2)$ *and* $(\beta^2 b_2 < \beta'^2 \beta_2)$	Comparable

Table 2. Relationship between $|\Gamma\rangle$ and $|\Gamma'\rangle$ as provided in case: II.

Restriction on α,β	Some Other Considerations	Nature of the Pair $(\Gamma and \Gamma')$
$\alpha > \alpha' and \beta \geq \beta'$	$(\alpha^2 a_2 > \alpha'^2 \alpha_2)$	Incomparable
$\alpha < \alpha' and \beta < \beta'$	$(\alpha^2 a_0 < \alpha'^2 \alpha_0)$ and $(\beta^2 b_0 < \beta'^2 \beta_0)$ $(\beta^2 b_2 > \beta'^2 \beta_2)$	Incomparable
$\alpha < \alpha' and \beta = \beta'$	$(\alpha^2 a_0 > \alpha'^2 \alpha_0)$ *and* $(\alpha^2 a_2 > \alpha'^2 \alpha_2)$	Incomparable
$\alpha = \alpha' and \beta = \beta'$	$(\alpha^2 a_2 > \beta^2 \beta_2)$ *and* $(\beta^2 b_2 > \alpha^2 \alpha_2)$	Incomparable
$\alpha = \alpha' and \beta > \beta'$	$(\alpha^2 a_2 > \beta'^2 \beta_2)$ *and* $(\beta^2 b_2 > \alpha^2 \alpha_2)$	Incomparable
$\alpha < \alpha' and \beta > \beta'$	$(\alpha^2 a_0 > \beta'^2 \beta_0); (\beta^2 b_0 > \alpha'^2 \alpha_0)$ and $(\alpha^2 a_2 > \beta'^2 \beta_2); (\beta^2 b_2 > \alpha'^2 \alpha_2)$ or $(\alpha^2 a_0 < \beta'^2 \beta_0); (\beta^2 b_0 < \alpha'^2 \alpha_0)$ and $(\alpha^2 a_2 < \beta'^2 \beta_2); (\beta^2 b_2 < \alpha'^2 \alpha_2)$	Incomparable
$\alpha = \alpha' and \beta = \beta'$	$(\alpha^2 a_2 < \beta^2 \beta_2)$ *and* $(\beta^2 b_2 < \alpha^2 \alpha_2)$	Comparable
$\alpha = \alpha' and \beta > \beta'$	$(\alpha^2 a_2 < \beta'^2 \beta_2)$ *and* $(\beta^2 b_2 < \alpha^2 \alpha_2)$	Comparable
$\alpha < \alpha' and \beta > \beta'$	$(\alpha^2 a_0 > \beta'^2 \beta_0); (\beta^2 b_0 > \alpha'^2 \alpha_0)$ and $(\alpha^2 a_2 < \beta'^2 \beta_2); (\beta^2 b_2 < \alpha'^2 \alpha_2)$ or $(\alpha^2 a_0 < \beta'^2 \beta_0); (\beta^2 b_0 < \alpha'^2 \alpha_0)$ and $(\alpha^2 a_2 > \beta'^2 \beta_2); (\beta^2 b_2 > \alpha'^2 \alpha_2)$	Comparable
$\alpha < \alpha' and \beta < \beta'$	$(\alpha^2 a_0 > \alpha'^2 \alpha_0)$ and $(\beta^2 b_0 > \beta'^2 \beta_0)$ $(\beta^2 b_2 > \beta'^2 \beta_2)$	Comparable
$\alpha = \alpha' and \beta < \beta'$	$(\beta^2 b_0 > \beta'^2 \beta_0)$ and $(\beta^2 b_2 < \beta'^2 \beta_2)$	Comparable

Table 3. Relationship between $|\Gamma\rangle$ and $|\Gamma''\rangle$ as provided in case: III.

Restriction on α,β	Some Other Considerations	Nature of the Pair $(\Gamma and \Gamma')$
$\alpha \geq \alpha' and \beta \geq \beta'$		Incomparable
$\alpha < \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 < \alpha'^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 < \alpha'^2 \alpha_2\right)$	Incomparable
$\alpha < \alpha' and \beta = \beta'$	$\left(\alpha^2 a_0\right) > or < \left(\alpha'^2 \alpha_0\right) and \left(\alpha^2 a_2\right) > or < \left(\alpha'^2 \alpha_2\right)$	Incomparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 > \beta'^2 b_0\right); \left(\beta^2 b_0 > \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 > \beta'^2 b_2\right); \left(\beta^2 b_2 > \alpha^2 \alpha_2\right)$ or $\left(\alpha^2 a_0 < \beta'^2 b_0\right); \left(\beta^2 b_0 < \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 < \beta'^2 b_2\right); \left(\beta^2 b_2 < \alpha^2 \alpha_2\right)$	Incomparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 > \beta'^2 b_0\right); \left(\beta^2 b_0 > \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 < \beta'^2 b_2\right); \left(\beta^2 b_2 < \alpha^2 \alpha_2\right)$ or $\left(\alpha^2 a_0 < \beta'^2 b_0\right); \left(\beta^2 b_0 < \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 > \beta'^2 b_2\right); \left(\beta^2 b_2 > \alpha^2 \alpha_2\right)$	Comparable
$\alpha < \alpha' and \beta = \beta'$	$\left(\alpha^2 a_0\right) < or > \left(\alpha'^2 \alpha_0\right) and \left(\alpha^2 a_2\right) > or < \left(\alpha'^2 \alpha_2\right)$	Comparable

Table 4. Relationship between $|\Gamma\rangle$ and $|\Gamma''\rangle$ as provided in case: IV.

Restriction on α,β	Some Other Considerations	Nature of The Pair $(\Gamma and \Gamma')$
$\alpha > \alpha' and \beta \geq \beta'$	$\left(\alpha^2 a_2 > \alpha'^2 \alpha_2\right)$	Incomparable
$\alpha < \alpha' and \beta \leq \beta'$	$\left(\alpha^2 a_0 < \alpha'^2 \alpha_0\right)$	Incomparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 > \beta'^2 b_0\right); \left(\beta^2 b_0 > \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 > \beta'^2 b_2\right); \left(\beta^2 b_2 > \alpha^2 \alpha_2\right)$ or $\left(\alpha^2 a_2 > \beta'^2 b_2\right); \left(\beta^2 b_2 > \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_0 < \beta'^2 b_0\right); \left(\beta^2 b_0 < \alpha^2 \alpha_2\right)$	Incomparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\alpha^2 a_2 > \beta'^2 b_2\right) and \beta^2 b_2 > \alpha^2 \alpha_2$	Incomparable
$\alpha = \alpha' and \beta > \beta'$	$\left(\alpha^2 a_2 > \beta'^2 b_2\right) and \beta^2 b_2 > \alpha^2 \alpha_2$	Comparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 > \beta'^2 b_0\right); \left(\beta^2 b_0 > \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 < \beta'^2 b_2\right); \left(\beta^2 b_2 < \alpha^2 \alpha_2\right)$ or $\left(\alpha^2 a_0 < \beta'^2 b_0\right); \left(\beta^2 b_0 < \alpha^2 \alpha_0\right)$ and $\left(\alpha^2 a_2 > \beta'^2 b_2\right); \left(\beta^2 b_2 > \alpha^2 \alpha_2\right)$	Comparable
$\alpha = \alpha' and \beta = \beta'$		Comparable
$\alpha > \alpha' and \beta = \beta'$	$\left(\alpha^2 a_2 < \alpha'^2 \alpha_2\right)$	Comparable
$\alpha < \alpha' and \beta \leq \beta'$	$\left(\alpha^2 a_0 > \alpha'^2 \alpha_0\right)$	Comparable

Table 5. Relationship between $|\Gamma\rangle$ and $|\Gamma'\rangle$ as provided in case: V.

Restriction on α,β	Some Other Considerations	Nature of the Pair $(\Gamma and \Gamma')$
$\alpha > \alpha' and \beta > \beta'$	$\left(\alpha^2 a_2 > \alpha'^2 \alpha_2\right)$ and $\left(\beta^2 b_2 > \beta'^2 \beta_2\right)$	Incomparable
$\alpha < \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 < \alpha'^2 \alpha_0\right)$ and $\left(\beta^2 b_0 < \beta'^2 \beta_0\right)$	Incomparable
$\alpha = \alpha' and \beta = \beta'$		Comparable
$\alpha > \alpha' and \beta > \beta'$	$\left(\alpha^2 a_2 < \alpha'^2 \alpha_2\right)$ and $\left(\beta^2 b_2 < \beta'^2 \beta_2\right)$	Comparable
$\alpha = \alpha' and \beta > \beta'$	$\left(\beta^2 b_2 < \beta'^2 \beta_2\right)$	Comparable
$\alpha > \alpha' and \beta = \beta'$	$\left(\alpha^2 a_2 < \alpha'^2 \alpha_2\right)$	Comparable
$\alpha < \alpha' and \beta < \beta'$	$\left(\alpha^2 a_0 > \alpha'^2 \alpha_0\right)$ and $\left(\beta^2 b_0 > \beta'^2 \beta_0\right)$	Comparable
$\alpha = \alpha' and \beta < \beta'$	$\left(\beta^2 b_0 > \beta'^2 \beta_0\right)$	Comparable
$\alpha < \alpha' and \beta = \beta'$	$\left(\alpha^2 a_0 > \alpha'^2 \alpha_0\right)$	Comparable

fundamental question is weather this incomparability under LOCC of states becomes the global phenomenon of $|\Gamma\rangle$ and $|\Gamma''\rangle$ *i.e.* clearly to say that wheather $|\Gamma\rangle$ and $|\Gamma''\rangle$ are incomparable under LOCC to each other or not and we illustrate all the results through the following tabular form:

CASE: IV

For two states $|\Gamma\rangle$ and $|\Gamma''\rangle$ of two parties A and B with the restriction that the states $(|\psi\rangle,|\psi'\rangle)$ are comparable to each other under LOCC and our investigation of the status of the pair $|\Gamma\rangle$ and $|\Gamma''\rangle$ leads some interesting consequences that have been provided through the following tabular form:

CASE: V

Here the two states $|\Gamma\rangle$ and $|\Gamma'\rangle$ of two parties A and B with the basic considerations that the states $|\psi\rangle,|\psi'\rangle$ and $|\phi\rangle$ and $|\phi'\rangle$ are comparable to each other under LOCC. So the fundamental question is wheather the notion of incomparability under LOCC of states can play the crucial role to change the status of $|\Gamma\rangle$ and $|\Gamma'\rangle$.

Through observations some peculiar facts have been revealed and we illustrate all the results through the following tabular form:

4. Conclusion

Through the long path of observation and investigation the most interesting physical fact that superposition of many states which forms a bridge between the two classes of states *i.e.* comparable and incomparable has been vividly revealed and some simple algebra and intuitive ideas become helpful to come to arrive the serious conclusion that the extension of dimension of $(|\psi\rangle,|\psi'\rangle)$ and $(|\phi\rangle,|\phi'\rangle)$ will give exactly the same results if we are thinking of strong incomparability instead of general sense of incomparability of states.The result, that constructing of locally comparable states through the superposition of locally incomparable states may be helpful for revealing some new physics of detection of entanglement. The concept, that converting incomparability of states to comparability under LOCC, directly may considered as an information processing task and detecting entanglement of the states which are used for constructing a new state.

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