Network Structure-Based Critical Bus Identification for Power System Considering Line Voltage Stability Margin

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Abstract

Voltage security assessment of power system is an important and all-inclusive aspect of power system operation and preventive control actions. Fast and accurate detection of critical components of the power system is one essential approach for preventing the occurrence of voltage collapse phenomenon. Over the years, several approaches for voltage collapse point identification and prevention have been widely studied using the continuous power flow approach, minimum singular value of eigenvalues, Jacobian matrices, and power transfer concept. In this work, critical node (bus) identification based on power system network structure is proposed. In this approach, the power system is treated as a multidimensional graph with several nodes (buses) linked together by the transmission lines. An improved line voltage stability margin estimator which is based on active and reactive power changes in a power system is used as the weight of each transmission line and an adaptation of the degree of centrality approach is used to determine the criticality of the system buses. A comparative analysis with other bus voltage stability indices is presented to test the suitability of the proposed approach using the IEEE 14, 30, 57 and 118 bus test systems.

Keywords

1. Introduction

Voltage stability studies and formulation of efficient stability indices have been

the subject of research over the past few decades. Voltage stability indices play a very important role in the planning and operation of modern power system, when considering stability constraint. Due to the continuous changing nature of power system, lack of continuous monitoring of critical portion of the power system can lead the system to a worse condition of voltage instability. Hence, In very recent time, derivation of voltage stability indices with better performance especially in terms of computational requirements has been a major area of research focus especially for applications such as voltage stability prediction, voltage collapse margin prediction, estimation of transmission line power capability and power outage prediction [1] [2] [3] [4] [5]. Power system stability is usually classified into rotor angle, voltage and frequency stability, but these events are usually not independent or isolated from one another [6] [7] [8]. A voltage collapse at a bus can lead to large excursions in rotor angle and frequency. Similarly, large frequency deviations can lead to large changes in voltage magnitude. Hence, dealing with voltage stability issues and its associate problems in power system operation affect the whole condition of the power system [9] [10].

Online and offline voltage stability analysis of power system is attracting increasing attention due to continuous rise in electricity demand and deregulation policy of the electricity markets which has forced power systems to operate close to their stability limits. Approaches for monitoring the voltage stability condition of power systems are classified into either node or line stability indices [11] [12]. This classification is based on the system component of interest; whether we are trying to identify the buses or lines that position the power system closer to the point of voltage collapse. Some of the node voltage stability indices are P-Q-V curves, L-Index, P-index, V/V0 index and Modal analysis [5] [13] [14] [15]. Some of the line voltage stability are Lmn, FVSI, VCPI, NLSI, LQP [16]-[21]. The continuation power flow (CPF) has been extensively employed for estimating the voltage stability margin of a power system [22] [23] [24] [25] and a new approach for line voltage stability margin estimation which can double as voltage stability index was developed in our previous work, this tool is known as the critical boundary index, CBI [26].

Several other approaches for voltage collapse point identification and prevention has been widely studied in available literature [27]-[34]. The network component sensitivity model for identifying critical components of a large network is recently being applied for static voltage stability assessment. Critical node identification is an important tool for understanding network vulnerability; a node is considered to be critical if its role in maintaining credible system performances is vital such that when it is removed the system deteriorates greatly [35] [36]. This approach has been widely used in several area of studies such as communication network, transportation network, smart grid network, financial risk-management, chemical and physical structural analysis etc. [37]-[45]. However, the use of this approach for voltage stability assessment of power system is limited and it is just gaining attention in recent time.

In [46], a voltage stability index based on the sensitivity of local network and neighbouring network to the local voltage phasors was presented. This index is known as the equivalent node voltage collapse index (ENVCI) and it is derived based on equivalent system model (ESM). Voltage stability assessment and important nodes identification in transmission network using graph theory and network restructuring is been considered in recent research studies. [47] introduced a network response structural characteristics theory participation factor (NRSCTPF) sensitivity approach for critical nodes determination based on network response structural characteristics indices (NRSCIs) and eigenvalue decomposition. This approach was found to be less computationally intensive.

Most of the existing indices are based on approximation and these approximations are premised mainly on the relationship between the power system's reactive power limit and bus voltage magnitude. More so, Most of the existing indices are based on minimum singular value of eigen values and Jacobian matrices [48]. However, the relationship between power system loading (real and reactive demand) and voltage stability can not be overlooked, especially by considering it from the structure of the power system network; and this can be inferred from the relationship between the bus voltage profile and the power transfer capacity of the connecting lines. Hence, the impact of active and reactive power variation on the point at which voltage collapse occurs is not well-elaborated in most of the existing reports. Also, the structural correlation between the buses and the connecting lines from neighbouring buses are not clearly emphasized. In this work, we have considered the importance of active and reactive power changes on the voltage stability of power system using the relationship between nodes (buses) and optimal stability margin of the connecting links (lines). The critical boundary index, *CBI* is a measure of the effect of active and reactive load variation on the stability margin of the transmission lines. The effectiveness of the proposed approach is tested on IEEE 14, 30, 57 and 118 bus test systems. The results are compared with other approaches in existing literature in order to validate the proposed method in terms of accuracy and computational requirement. The rest of the paper is organized as follows: concept of voltage stability margin estimation using CBI is discussed in Section 2. The concept of bus criticality determination using graph theory was discussed in Section 3. Simulation results are discussed in Section 4 and the study was concluded in Section 5.

2. Evaluation of Lines Voltage Stability Margin (CBI)

Relationship between voltage stability and power flow along each branch of a power system is analyzed using a simple two bus system shown in **Figure 1**.

$$P_k - jQ_k = \left(V_k \angle \delta_k\right)^* \frac{V_l \angle \delta_l - V_k \angle \delta_k}{r_{lk} + jx_{lk}} \tag{1}$$

The transmitting end terminal voltage is V_l, receiving terminal voltage is V_k, apparent power sent to the receiving bus is $S_k = P_k + jQ_k$, and the line
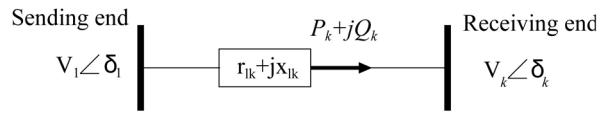
Figure 1. Transmission line model.

impedance is $\hat{z}_{lk} = r_{lk} + jx_{lk}$. * represent the complex conjugate.

$$P_k r_{lk} + x_{lk} Q_k + j(P_k x_{lk} - r_{lk} Q_k) = V_l V_k \cos(\delta_l - \delta_k) - jV_l V_k \sin(\delta_l - \delta_k) - V_k^2 \quad (2)$$

separating the real and imaginary parts of the above equation, the following equations are obtained;

$$P_k r_{lk} + x_{lk} Q_k + V_k^2 = V_l V_k \cos(\delta_l - \delta_k) \quad (3)$$

$$P_k x_{lk} - r_{lk} Q_k = -V_l V_k \sin(\delta_l - \delta_k) \quad (4)$$

Solving for V_k^2 from Equations (3) and (4) with $\left(\sin^2 \delta + \cos^2 \delta = 1\right)$ yields the following equation.

$$V_k^2 = -\left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right) \pm \sqrt{\left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right)^2 - \left(r_{lk}^2 + x_{lk}^2\right)\left(P_k^2 + Q_k^2\right)} \quad (5)$$

With the condition that $V_k > 0$, the receiving end voltage V_k is obtained as

$$V_k = \sqrt{-\left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right) \pm A} \quad (6)$$

where

$$A = \sqrt{\left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right)^2 - \left(r_{lk}^2 + x_{lk}^2\right)\left(P_k^2 + Q_k^2\right)}$$

There is a point at which there is a limit to the amount of power that can be transmitted along the transmission line prior to system collapse. These stability condition is obtained by setting A to zero as shown in the following expression:

$$\sqrt{\left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right)^2 - \left(r_{lk}^2 + x_{lk}^2\right)\left(P_k^2 + Q_k^2\right)} = 0 \quad (7)$$

Hence, the locus of a point that corresponds to the voltage stability limit is obtained as:

$$C(X,Y) = \left(r_{lk} P_k + x_{lk} Q_k - \frac{V_l^2}{2}\right)^2 - \left(r_{lk}^2 + x_{lk}^2\right)\left(P_k^2 + Q_k^2\right) \quad (8)$$

(P,Q)-V characteristic which shows the combination of P-Q characteristics for different V is shown on **Figure 2**. The P-Q characteristic depicts the voltage stability limit as obtained from Equation (5).

From **Figure 3**, we obtain the voltage stability limit point $C(X,Y)$ with respect to the current operating point $B(P_o, Q_o)$ using Lagrangian undetermined multiplier method as shown.

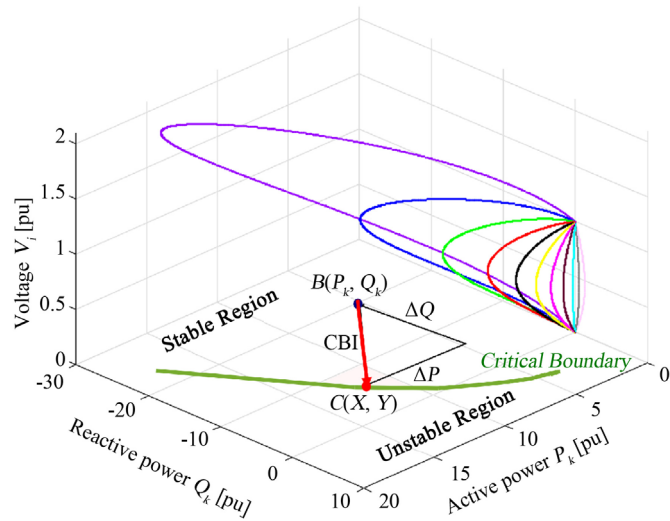The shortest distance between current operating point B and voltage

Figure 2. (P,Q)-V characteristics showing the voltage stability limit [26].
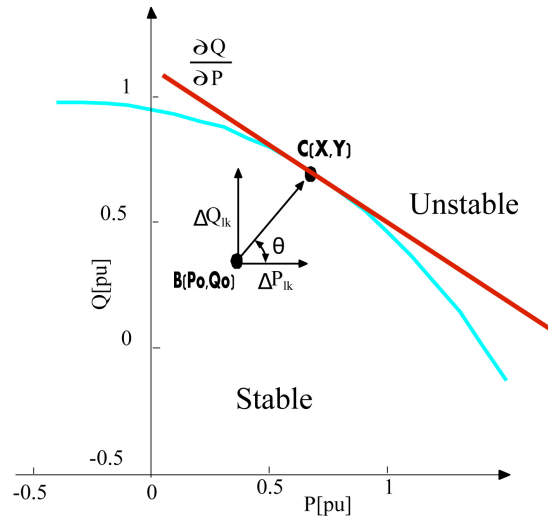
Figure 3. (P,Q) characteristics showing the voltage stability limit.

instability point C is $d(X,Y)$:

$$d(X,Y) = \left((X - P_o)^2 + (Y - Q_o)^2 \right)^{\frac{1}{2}} \tag{9}$$

The minimum point on C that corresponds to the shortest distance from point B is thus obtained:

$$F(X,Y,\lambda) = d(X,Y) - \lambda C(X,Y) \tag{10}$$

$$F(X,Y,\lambda) = \left\{ (X - P_o)^2 + (Y - Q_o)^2 \right\}^{\frac{1}{2}} \\ - \lambda \left\{ \left(r_{lk}X + x_{lk}Y - \frac{V_l^2}{2} \right)^2 - \left(r_{lk}^2 + x_{lk}^2 \right) \left(X^2 + Y^2 \right) \right\} \tag{11}$$

Partial differentiation of the above equation with respect to X, Y, λ yields the following equation.

$$
\begin{aligned}
&(X - P_o)\left\{(X - P_o)^2 + (Y - Q_o)^2\right\}^{-\frac{1}{2}} \\
&- 2\lambda\left\{\left(r_{lk}X + x_{lk}Y - \frac{V_l^2}{2}\right)r_{lk} + \left(r_{lk}^2 + x_{lk}^2\right)X\right\} = 0
\end{aligned}
\tag{12}
$$

$$
\begin{aligned}
&(Y - Q_o)\left\{(X - P_o)^2 + (Y - Q_o)^2\right\}^{-\frac{1}{2}} \\
&- 2\lambda\left\{\left(r_{lk}X + x_{lk}Y - \frac{V_l^2}{2}\right)x_{lk} + \left(r_{lk}^2 + x_{lk}^2\right)Y\right\} = 0
\end{aligned}
\tag{13}
$$

$$
\left(r_{lk}X + x_{lk}Y - \frac{V_l^2}{2}\right)^2 + \left(r_{lk}^2 + x_{lk}^2\right)\left(X^2 + Y^2\right) = 0
\tag{14}
$$

We obtain the values of X, Y, λ by simultaneously solving Equations (12) to (14) using *fsolve* function in matlab. The optimal voltage stability margin known as critical boundary index, CBI of a line is evaluated as [26]:

$$
\Delta P_{lk} = X - P_o
\tag{15}
$$

$$
\Delta Q_{lk} = Y - Q_o
\tag{16}
$$

$$
CBI_{lk} = \sqrt{\Delta P_{lk}^2 + \Delta Q_{lk}^2}
\tag{17}
$$

The CBI is the measure of the available apparent power transfer limit of the transmission line with regards to voltage stability and as it approaches zero for any transmission line, the voltage stability level of the line deteriorates.

3. Estimation of Bus Criticality Index

The detail analysis of power systems as complex networks using graph theory is presented in [49]. As shown in **Figure 4**, each link (line) between any pair of nodes (buses) is described by two parameters; the electrical distance between
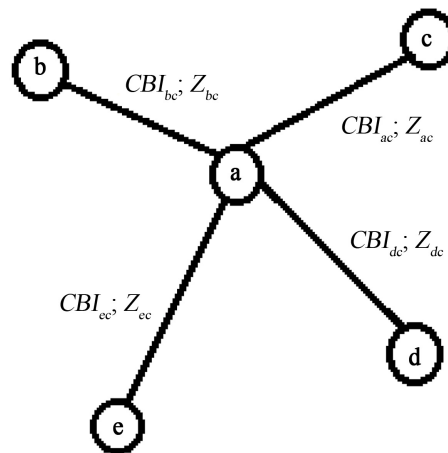

Figure 4. A 5 node system showing the link parameters under consideration.

nodes (impedance value) and the optimal voltage stability margin (CBI value). In this study, the voltage stability criticality index of a bus is determined based on the approach of weighted degree of node centrality [35] [37] [45] using the line optimal voltage stability margin as the line weight:

3.1. Electrical Degree of Bus Centrality

Degree of a node (bus) is the most basic indicator of the importance of the node to the performance of the network. Freeman in [41] describes the degree of a reference node of a network as the number of other nodes connected to that node. In order to measure this index with respect to a power system network, the electrical distance of connected links are used as the weight of each link and this is described as shown in the given equation:

$$C_d(l) = \frac{\sum_k \|\hat{y}_{lk}\|}{n-1}; \quad (l \neq k) \tag{18}$$

where $C_d(l)$ is the weighted degree of node centrality for node l, $\hat{y}_{lk}$ is the impedance value of each line connected to the reference node l and n is the total number of system nodes. The most critical node will have the highest C_d value. The traditional electrical degree of centrality has the same value as $C_d(l)$. The traditional electrical degree of centrality is described below [50]:

$$C_d^Y(l) = \frac{\sum_k \|Y_{l,l}\|}{n-1} \tag{19}$$

where $Y_{l,l}$ is the admittance matrix value at the diagonal corresponding to the node l. The higher the value of $C_d^Y(l)$, the more critical is node l.

3.2. Proposed Bus Voltage Stability Criticality Indices

The approach for identifying critical buses is based on the voltage stability limit of transmission lines connected to the buses as defined below:

$$BVSI(l) = \frac{\sum_k \|CBI_{lk}\|}{n-1}; \quad (l \neq k) \tag{20}$$

n is the total number of buses and the criticality of a bus l increases as the BVCI value reduces. Hence, the bus with the lowest BVSI value is most susceptible to voltage collapse phenomenon in the power system network.

4. Simulation Results

The derived voltage stability-based critical bus identification approaches were tested on IEEE 14, 30, 57 and 118 buses power systems. A comparative assessment with some other approaches found in literature is presented. To verify the accuracy of the proposed approaches, voltage collapse point was identified by monitoring the voltage magnitude at identified critical buses with heavy loading at all buses at constant power factor, accordingly:

$$P'_{loadi} = P_{loadi} \times \sigma \tag{21}$$

$$Q'_{loadi} = Q_{loadi} \times \sigma \qquad (22)$$

where, i is the bus number and σ is the loading factor. Three simulation conditions are considered as given on **Table 1** and the point at which the power flow solution diverges is the system collapse point under each case.

For the IEEE 14 bus system, buses 14, 12, 11, 8 and 3 are identified as the weakest buses as shown in **Table 2**. The electrical degree of centrality, C_d identified bus 8 as the weakest; BVSI identified bus 14 as the weakest bus. **Figure 5** shows that bus 14 is most sensitive to continuous load increase as its voltage at the point of system collapse was reduced to 0.6858 pu at $\sigma = 3.71$ under heavy real loading, 0.6877 pu at $\sigma = 5.99$ under heavy reactive loading and 0.5774 pu at $\sigma = 3.46$ under simultaneous real and reactive power loading. **Table 3** shows the values and ranking of the 5 most critical buses at the identified system collapse point under heavy real, reactive and both real and reactive loading

Table 1. Simulation conditions.

Case	Simulation condition
1	Increase only Active power (Equation (21))
2	Increase only Reactive power (Equation (22))
3	Increase Active and Reactive power (Equations (21), (22))

Table 2. Critical bus analysis of IEEE 14 bus at base load.

	C_d		BVSI	
Rank	Bus	Value	Bus	Value
1	8	0.4367	14	0.0991
2	14	0.4559	8	0.1028
3	12	0.5292	12	0.1239
4	11	0.7173	11	0.1640
5	3	0.7968	3	0.1815
6	13	0.9831	13	0.2230
7	10	1.2203	10	0.2818
8	6	1.4740	6	0.3359
9	7	1.5120	7	0.3497
10	1	1.5706	1	0.3546

Table 3. Critical bus analysis of IEEE 14 bus at heavy load.

Case 1		Case 2		Case 3	
Bus	Value	Bus	Value	Bus	Value
8	0.0028	14	0.0421	14	0.0258
14	0.0429	8	0.0553	8	0.0301
11	0.0457	11	0.0735	11	0.0597
10	0.0603	12	0.0865	12	0.0780
7	0.0896	13	0.1281	1	0.0906
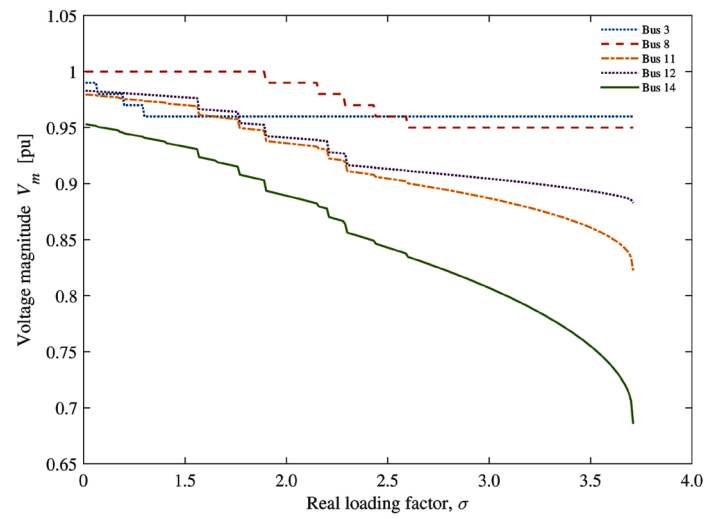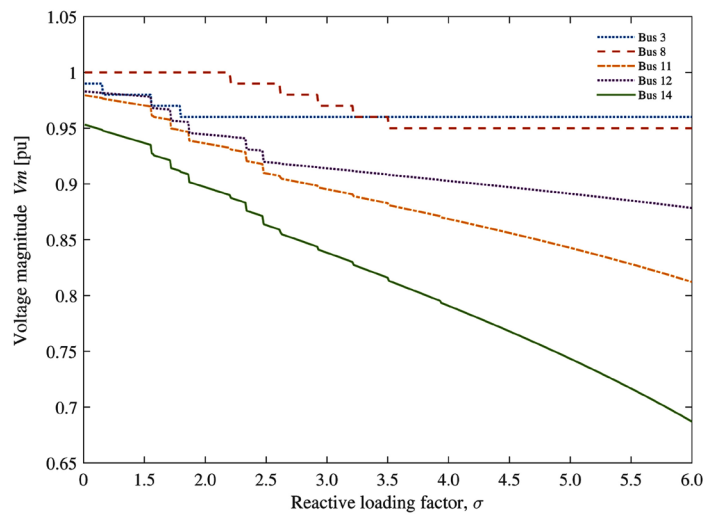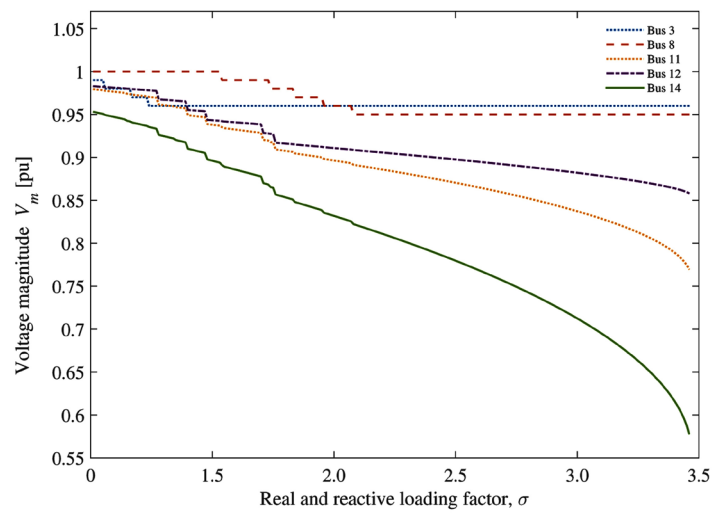
Figure 5. Voltage collapse point on IEEE 14 system. (a) Only active power increase; (b) Only reactive power increase; (c) Active and reactive power increase.

respectively. Contrary to the result of the continuous power flow that confirms bus 14 as the most critical under the three loading condition based on the voltage magnitude, the proposed index indicates bus 8 as the most critical under case 1 while bus 14 was identified as the most critical under both case 2 and case 3. The proposed index failure to accurately identify the critical bus under case 1 shows the importance of reactive power to voltage stability analysis.

For the IEEE 30 bus system, buses 26, 29 and 30 are the clear candidates for the most critical bus as presented in Table 4. C_d, BVSI identified bus 26 as the most critical closely followed by bus 30. From the result for IEEE 57 as shown in Table 5, buses 18, 19 and 31 are identified as the critical buses susceptible to voltage collapse. In Table 6, buses 87, 117 and 43 are clearly indicated as the critical buses from the result obtained from IEEE 118.

In Table 7, the results obtained from the proposed algorithms are compared with more established indices from the literature. They are mostly derived,

Table 4. Critical bus analysis of IEEE 30 bus at base load.

	C_d		BVSI	
Rank	Bus	Value	Bus	Value
1	26	0.0754	26	0.0179
2	30	0.1178	30	0.0275
3	29	0.1406	29	0.0336
4	11	0.1658	11	0.0416
5	14	0.2372	14	0.0621
6	13	0.2463	13	0.0663
7	23	0.2677	23	0.0674
8	25	0.3127	25	0.0779
9	16	0.3215	16	0.0829
10	27	0.3602	27	0.0882

Table 5. Critical bus analysis of IEEE 57 bus at base load.

	C_d		BVSI	
Rank	Bus	Value	Bus	Value
1	31	0.0497	18	0.0136
2	19	0.0561	19	0.0168
3	20	0.0565	31	0.0192
4	57	0.0705	20	0.0196
5	42	0.0870	57	0.0250
6	18	0.0973	42	0.0283
7	25	0.1031	25	0.0313
8	30	0.1035	54	0.0368
9	54	0.1225	41	0.0381
10	41	0.1350	30	0.0387

Table 6. Critical bus analysis of IEEE 118 bus at base load.

Rank	C_d		BVSI	
	Bus	Value	Bus	Value
1	87	0.0408	87	0.0094
2	117	0.0594	117	0.0133
3	43	0.0832	43	0.0192
4	72	0.0884	72	0.0192
5	107	0.0897	107	0.0196
6	86	0.1077	111	0.0252
7	111	0.1087	86	0.0256
8	53	0.1192	53	0.0261
9	33	0.1235	33	0.0280
10	98	0.1239	112	0.0292

Table 7. Comparison of critical bus identification approaches at base load.

Indices	IEEE 14	IEEE 30	IEEE 57	IEEE 118
BVSI	14, 8, 12, 11	26, 30, 29, 11	18, 19, 31, 20, 57	87, 117, 43, 72
C_d [46]	8, 14, 12, 11	26, 30, 29, 11	31, 19, 20, 57, 42	87, 117, 43, 72
ENVSI [50]	14, 4, 9, 13	30, 29, 27	33, 32	44, 23, 43
L-index [14] [51]	14, 9, 10, 8, 11	-	31, 33, 32, 30	44, 45, 43
P-Index [14]	14, 9, 10, 5, 4	-	31, 33, 32, 30	44, 22, 45
CMAT [52]	-	30, 27, 29, 28	31, 33, 32, 30	-
IMAT [52]	-	27, 30, 29, 26	31, 33, 32, 30	-
VCPI [52]	-	27, 30, 29, 23	31, 33, 32, 30	-
PVDBI [47]	-	27, 30, 29, 23	31, 33, 32, 30	-
NRSCTPF [47]	14, 10, 9, 12, 13	-	33, 31, 32, 30	-

approximately, based on the relationship between the power system's reactive power limit and bus voltage magnitude. Also, except for C_d and NRSCTPF, all the other indices are not exclusively based on the structural relationship of the power system components; hence the reasons for the difference in the results as the system size increases. For IEEE 14, the proposed algorithm agree with the existing algorithms as bus 14 is shown to be the most critical. For IEEE 30 bus system, bus 26, 30, 29, 27 are clear candidates for the bus responsible for voltage collapse under continuous loading. Although, different set of buses are identified by the proposed indices and the existing indices as the bus number increases, yet we can infer that bus 31 and bus 43 for IEEE 57 and IEEE 118 bus system, respectively are within the range of critical buses, as indicated by the aggregation of all the considered indices.

5. Conclusion

A formulation of power system voltage stability analysis model based on

network structure is presented. The proposed index can perform the dual function of line voltage stability assessment and weakest bus determination. The proposed indices have shown the ability to yield accurate information relating to the closeness of a power system to the voltage collapse point. The computational approach is quite simple and straightforward and the computational requirement is less. The accuracy of the proposed index is tested by comparison with other existing approach and the proposed index agrees with existing indices, especially with the electrical degree of centrality.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

References

[1] Echavarren, F., Lobato, E., Rouco, L. and G´omez, T. (2011) Formulation, Computation and Improvement of Steady State Security Margins in Power Systems. Part I: Theoretical Framework. *International Journal of Electrical Power* & *Energy Systems*, **33**, 340-346. https://doi.org/10.1016/j.ijepes.2010.08.031

[2] Echavarren, F., Lobato, E., Rouco, L. and G´omez, T. (2011) Formulation, Computation and Improvement of Steady State Security Margins in Power Systems. Part II: Results. *International Journal of Electrical Power* & *Energy Systems*, **33**, 347-358. https://doi.org/10.1016/j.ijepes.2010.08.026

[3] Dobson, I. and Chiang, H.-D. (1989) Towards a Theory of Voltage Collapse in Electric Power Systems. *Systems* & *Control Letters*, **13**, 253-262. https://doi.org/10.1016/0167-6911(89)90072-8

[4] Chiang, H.-D., Dobson, I., Thomas, R.J., Thorp, J.S. and Fekih-Ahmed, L. (1990) On Voltage Collapse in Electric Power Systems. *IEEE Transactions on Power Systems*, **5**, 601-611. https://doi.org/10.1109/59.54571

[5] Dobson, I. (1992) Observations on the Geometry of Saddle Node Bifurcation and Voltage Collapse in Electrical Power Systems. *IEEE Transactions on Circuits and Systems I*: *Fundamental Theory and Applications*, **39**, 240-243. https://doi.org/10.1109/81.128018

[6] Taylor, C.W. (1994) Power System Voltage Stability. McGraw-Hill, New York City.

[7] Makarov, Y.V., Dong, Z.Y. and Hill, D.J. (1998) A General Method for Small Signal Stability Analysis. *IEEE Transactions on Power Systems*, **13**, 979-985. https://doi.org/10.1109/59.709086

[8] Makarov, Y.V., Dong, Z.Y. and Hill, D.J. (2008) On Convexity of Power Flow Feasibility Boundary. *IEEE Transactions on Power Systems*, **23**, 811-813. https://doi.org/10.1109/TPWRS.2008.919307

[9] Ajjarapu, V. (1998) Bibliography on Voltage Stability. *IEEE Transactions on Power Systems*, **13**, 115-125. https://doi.org/10.1109/59.651622

[10] Cutsem, T.V. and Vournas, C. (2003) Voltage Stability of Electric Power Systems. 3rd Edition, Kluwer, Massachusetts.

[11] Chitare, P.B., Balijepalli, V.M. and Khaparde, S. (2013) Online Assessment of Voltage Stability in Power Systems with Pmus. *International Journal of Emerging Electric Power Systems*, **14**, 115-122. https://doi.org/10.1515/ijeeps-2013-0031

[12] Danish, M.S.S., Yona, A. and Senjyu, T. (2015) A Review of Voltage Stability Assessment Techniques with an Improved Voltage Stability Indicator. *International Journal of Emerging Electric Power Systems*, **16**, 107-115.

[13] Dobson, I. and Lu, L. (1993) New Methods for Computing a Closest Saddle Node Bifurcation and Worst Case Load Power Margin for Voltage Collapse. *IEEE Transactions on Power Systems*, **8**, 905-913. https://doi.org/10.1109/59.260912

[14] Kamel, M., Karrar, A.A. and Eltom, A.H. (2017) Development and Application of a New Voltage Stability Index for On-Line Monitoring and Shedding. *IEEE Transctions on Power Systems*, **33**, 1231-1241. https://doi.org/10.1109/TPWRS.2017.2722984

[15] Gao, B., Morison, G.K. and Kundur, P. (1992) Voltage Stability Evaluation Using Modal Analysis. *IEEE Transctions on Power Systems*, **7**, 1529-1542. https://doi.org/10.1109/59.207377

[16] Moghavvemi, M. and Omar, F.M. (1998) Technique for Contingency Monitoring and Voltage Collapse Prediction. *IEEE Proceedings—Generation, Transmission and Distribution*, **145**, 634-640. https://doi.org/10.1049/ip-gtd:19982355

[17] Ismail, M. and Rahman, T.K. (2005) Estimation of Maximum Loadability in Power Systems by Using Fast Voltage Stability Index (Fvsi). *Journal of Power and Engineering Systems*, **25**, 181-189.

[18] Mohamed, A. and Jasmon, G.B. (1995) A New Clustering Technique for Power System Voltage Stability Analysis. *Electric Power Components and Systems*, **23**, 389-403. https://doi.org/10.1080/07313569508955632

[19] Moghavvemi, M. and Faruque, M.O. (2001) Technique for Assessment of Voltage Stability in Ill-Conditioned Radial Distribution Network. *IEEE Power Engineering Review*, **21**, 58-60. https://doi.org/10.1109/39.893345

[20] Yazdanpanah-Goharrizi, A. and Asghari, R. (2007) A Novel Line Stability Index (nlsi) for Voltage Stability Assessment of Power Systems. *Proceedings 7th WSEAS International Conference on Power Systems*, Beijing, 15-17 September 2007, 164-167.

[21] Lim, Z.J., Mustafa, M.W. and bt Muda, Z. (2012) Evaluation of the Effectiveness of Voltage Stability Indices on Different Loadings. *IEEE International Power Engineering and Optimization Conference*, Melaka, 6-7 June 2012, 543-547. https://doi.org/10.1109/PEOCO.2012.6230925

[22] Tachibana, M., Palmer, M.D., Senjyu, T. and Funabashi, T. (2014) Voltage Stability Analysis and (p,q)-v Characteristics of Multi-Bus System. *CIGRE AORC Technical Meeting*, Tokyo, 27-29 May 2014, 164-167.

[23] Ajjarapu, V. and Christy, C. (1991) The Continuation Power Flow: A Tool for Steady State Voltage Stability Analysis. *Power Industry Computer Application Conference*, Baltimore, 7-10 May 1991, 304-311.

[24] Corsi, S. and Taranto, G.N. (2007) Voltage Instability—The Different Shapes of the "Nose". *Bulk Power System Dynamics and Control VII Revitalizing Operational Reliability*, Charleston, 19-24 August 2007, 1-16.

[25] Sharma, P. and Kumar, A. (2016) Thevenin's Equivalent Based p-q-v Voltage Stability Region Visualization and Enhancement with Facts and Hvdc. *International Journal of Electrical Power & Energy Systems*, **80**, 119-127. https://doi.org/10.1016/j.ijepes.2016.01.026

[26] Furukakoi, M., Adewuyi, O.B., Danish, M.S.S., Howlader, A.M., Senjyu, T. and Funabashi, T. (2018) Critical Boundary Index (CBI) Based on Active and Reactive

Power Deviations. *International Journal of Electrical Power and Energy Systems*, **100**, 50-57. https://doi.org/10.1016/j.ijepes.2018.02.010

[27] Canizares, C.A., De Souza, A.C. and Quintana, V.H. (1996) Comparison of Performance Indices for Detection of Proximity to Voltage Collapse. *IEEE Trans-Actions on Power Systems*, **11**, 1441-1450. https://doi.org/10.1109/59.535685

[28] Amroune, M., Bourzami, A. and Bouktir, T. (2014) Weakest Buses Identification and Ranking in Large Power Transmission Network by Optimal Location of Reactive Power Supports. *Indonesian Journal of Electrical Engineering*, **12**, 7123-7130.

[29] Venkatesh, B., Rost, A. and Chang, L. (2007) Dynamic Voltage Collapse Index-Wind Generator Application. *IEEE Transactions on Power Delivery*, **22**, 90-94. https://doi.org/10.1109/TPWRD.2006.887097

[30] Musirin, I. and Abdul Rahman, T.K. (2002) Novel Fast Voltage Stability Index (FVSI) for Voltage Stability Analysis in Power Transmission System. *Student Conference on Research and Development*, Shah Alam, 17 July 2002, 265-268. https://doi.org/10.1109/SCORED.2002.1033108

[31] Mohamed, A., Jasmon, G.B. and Yusoff, S. (1989) A Static Voltage Collapse Indicator Using Line Stability Factors. *Journal of Industrial Technology*, **7**, 73-85.

[32] Furukakoi, M., Howlader, H.O.R., Shigenobu, R. and Senjyu, T. (2017) Multi-Objective Optimal Operation for Transmission Systems Considering Voltage Stability. *International Journal of Emerging Electric Power Systems*, **18**. https://doi.org/10.1515/ijeeps-2016-0057

[33] Leonardi, B. and Ajjarapu, V. (2013) An Approach for Real Time Voltage Stability Margin Control via Reactive Power Reserve Sensitivities. *IEEE Transactions on Power Systems*, **28**, 615-625. https://doi.org/10.1109/TPWRS.2012.2212253

[34] Lin, L., Wang, J. and Gao, W. (2012) Effect of Load Power Factor on Voltage Stability of Distribution Substation. *IEEE Power and Energy Society General Meeting*, San Diego, 22-26 July 2012, 1-4. https://doi.org/10.1109/PESGM.2012.6345690

[35] Asif, W., Qureshi, H.K., Rajarajan, M. and Lestas, M. (2016) Combined Banzhaf and Diversity Index (CBDI) for Critical Node Detection. *Journal of Network and Computer Applications*, **64**, 76-88. https://doi.org/10.1016/j.jnca.2015.11.025

[36] Shen, Y., Nguyen, N.P., Xuan, Y. and Thai, M.T. (2013) On the Discovery of Critical Links and Nodes for Assessing Network Vulnerability. *IEEE/ACM Transactions on Networking*, **21**, 963-973. https://doi.org/10.1109/TNET.2012.2215882

[37] Yu, F., Xia, X., Li, W. and Tao, J. (2017) Critical Node Identification for Complex Network Based on a Novel Minimum Connected Dominating Set. *Soft Computing*, **21**, 5621-5629. https://doi.org/10.1007/s00500-016-2303-y

[38] Arulselvan, A., Commander, C.W., Shylo, O. and Pardalos, P.M. (2011) Cardinality Constrained Critical Node Detection Problem. In: Glpnar, N., Harrison, P. and Rstem, B., Eds., *Performance Models and Risk Management in Communications Systems*, Springer Optimization and Its Applications, Springer, Berlin, Vol. 46, 79-91.

[39] Kempe, D., Kleinberg, J. and Tardos, E. (2003) Maximizing the Spread of Influence through a Social Network. In: *Proceedings of the* 9*th ACM International Conference on Knowledge Discovery and Data Mining*, ACM, New York, 137-146. https://doi.org/10.1145/956750.956769

[40] Nguyen, D.T., Shen, Y. and Thai, M.T. (2013) Detecting Critical Nodes in Independent Power Networks for Vulnerability Assessment. *IEEE Transactions on Smart Grid*, **4**, 151-159. https://doi.org/10.1109/TSG.2012.2229398

[41] Freeman, L.C. (1977) A Set of Measures of Centrality Based on Betweenness. *Soci-*

ometry, **40**, 35-41. https://doi.org/10.2307/3033543

[42] Boginski, V. and Commander, C.W. (2008) Identifying Critical Nodes in Protein Interaction Networks.

[43] Joyce, K.E., Laurienti, P.J., Burdette, J. and Hayasaka, H.S. (2010) A New Measure of Centrality for Brain Networks. *PLoS ONE*, **5**, e12200. https://doi.org/10.1371/journal.pone.0012200

[44] Ventresca, M. and Aleman, D. (2015) Efficiently Identifying Critical Nodes in Large Complex Networks. *Computational Social Networks*, **2**, 6. https://doi.org/10.1186/s40649-015-0010-y

[45] Opsahl, T., Agneessens, F. and Skvoretz, J. (2010) Node Centrality in Weighted Networks: Generalizing Degree and Shortest Paths. *Social Networks*, **32**, 245-251. https://doi.org/10.1016/j.socnet.2010.03.006

[46] Wang, Y., Li, Y. and Lu, J. (2009) A New Node Voltage Stability Index Based on Local Voltage Phasors. *Electric Power Systems Research*, **79**, 265-271. https://doi.org/10.1016/j.epsr.2008.06.010

[47] Adebayo, I., Jimoh, A.A. and Yusuff, A. (2017) Voltage Stability Assessment and Identification of Important Nodes in Power Transmission Network through Network Response Structural Characteristics. *IET Generation, Transmission and Distribution*, **11**, 1398-1408. https://doi.org/10.1049/iet-gtd.2016.0745

[48] Karbalaei, F., Soleymani, H. and Afsharnia, S. (2010) A Comparison of Voltage Collapse Proximity Indicators. *Proceedings of* 2010 *IPEC Conference*, Singapore, 429-432. https://doi.org/10.1109/IPECON.2010.5697034

[49] Pagani, G.A. and Aiello, M. (2013) The Power Grid as a Complex Network: A Survey. *Physica A*, **392**, 2688-2700. https://doi.org/10.1016/j.physa.2013.01.023

[50] Wang, Z., Scaglione, A. and Thomas, R.J. (2010) Electrical Centrality Measures for Electric Power Grid Vulnerability Analysis. *Proceedings of the* 49*th IEEE Conference on Decision and Control*, Atlanta, 5792-5797. https://doi.org/10.1109/CDC.2010.5717964

[51] Yahia, E.Z., Elsherif, M.A. and Zaggout, M.N. (2015) Detection of Proximity to Voltage Collapse by Using L Index. *International Journal of Innovative Research in Science, Engineering and Technology*, **4**, 1755-1765.

[52] Adebayo, I. and Sun, Y. (2017) New Performance Indices for Voltage Stability Analysis in a Power System. *Energies*, **10**, 1-18.