

# A Liability Division Method for Harmonic Pollution Based on Line-Transferred Power Components

Jianchun Peng\*, Jun Zhou, Hui Jiang

College of Mechatronics and Control Engineering, Shenzhen University, Shenzhen, China  
Email: [jcpeng@szu.edu.cn](mailto:jcpeng@szu.edu.cn)

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## Abstract

The existing liability division methods for harmonic pollution are either inexplicit or incomplete in physical meaning. To compensate these defects, two new methods are proposed based on line-transferred power components in this paper. At first, all harmonic sources are represented by ideal equivalent current source, line current components and bus voltage components of a source are determined by stimulation of this source with all other sources disabled. Then, the line-transferred power component owing to a source under all sources action together is determined by the theory of line-transferred power components, and called source's line-transferred power component. At last, the liability of a source for line-end harmonic pollution is divided by two methods: the ration of the source's line-transferred active power component to the total line-transferred power, and the ration of projection of the source's line-transferred complex power component to absolute value of the total line-transferred complex power. These two methods are taken into account not only harmonic voltage but also harmonic current in the liability division. Simulation results show that the proposed liability division method based on active power component is the most effective and ideal one.

## Keywords

Harmonic Pollution, Liability Division, Line-Transferred Power, Power Component, Grid

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## 1. Introduction

With the development of modern industry and the use of a large number of power electronic devices, the harmonic sources have been increasing in power network. They seriously influence the normal work of sensitive loads and precision devices in the grid [1]-[4]. For the power quality pollution caused by harmonic waves, a management scheme of rewards and punishments is suggested in [5]. Therefore, a comprehensive, reasonable and quantitative liability division method of a source for harmonic pollution is crucial under the coexistence of several harmonic sources.

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\*Corresponding author.

Considering the voltage sensitivity of electrical equipment, many researchers proposed a liability division method for harmonic pollution based on harmonic voltage. A non-invasive liability division method was proposed in [6] for harmonic pollution based on the fast components of harmonic current injected by harmonic sources and the statistical features of independent random vectors. A liability division method was proposed in [7] for assessing harmonic pollution based on the improvement of traditional least squares method and M-estimation robust regression. It overcomes the disadvantages of the traditional least squares method, such as the low accuracy, sensitivity to singular data. A liability division method was proposed in [8] for quantify the harmonic pollution, which is based on the estimated harmonic impedance and background harmonic voltage using complex least squares method. It overcomes the shortage of the linear regression method by using the real and imaginary components respectively to estimate the harmonic impedance and quantify the harmonic liability.

There are some other researchers proposing a liability division method for harmonic pollution based on harmonic currents. A liability division method was proposed in [9] for harmonic pollution from the perspective of harmonic currents based on QR-RLS (Recursive Least-Squares Method based on QR decomposition). A liability division method was proposed in [10] for harmonic pollution, which apportions the voltage and current harmonic step by step considering the filter, initial phase angles of harmonic voltage and current as well as the number of harmonic sources.

In addition, from the viewpoint of harmonic power, a qualitative liability division method for harmonic pollution was proposed in [11]. The method evaluated the liability of each harmonic source qualitatively according to the power response produced by the action alone of this source. Compared to the harmonic voltage or harmonic current based method in which the V and I characteristics are not involved completely, this qualitative liability division method compensates this drawback. However, it is a challenging problem that how to quantitatively divide the harmonic pollution liability among sources based on harmonic power, because the power does not meet the superposition theorem.

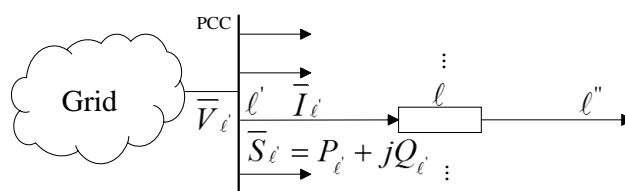
The electrical characteristics of harmonics are reflected by not only the voltage (or electric field) but also the current (or magnetic field). It is the power (there is no any other quantity) that involves both of them (voltage and current). So only the quantitative liability division method for harmonic pollution based on harmonic power is the most complete and explicit one. In this paper, employing our achievements, two quantitative liability division methods for harmonic pollution based on the harmonic power are proposed. Simulation and analysis show that the quantitative liability division method for harmonic pollution based on line-transferred active power component is the most desirable one.

## 2. Concept of Harmonic Pollution Liability

In a power grid, there often coexist several harmonic sources. **Figure 1** shows a point of common coupling (PCC) connected to several lines. The harmonic voltage and current as well as complex power overline end- $\ell'$ , denoted by  $\bar{V}_{\ell'}$ ,  $\bar{I}_{\ell'}$  and  $\bar{S}_{\ell'}$ , are responses under all harmonic sources action together. The level of harmonic pollution liability of a source should be decided by the ration of the source's component contribution to the sum of component contributions over all sources.

For a harmonic wave over line end- $\ell'$  (or the PCC), the liability of a source for harmonic pollution is defined as the ration of the source's line-transferred component contribution to the sum of component contributions over all sources, and briefly called this source's harmonic pollution liability.

Electrical quantities representing harmonics are harmonic current, voltage and power. Currently, there are only two quantitative liability division methods for harmonic pollution, one is based on harmonic current and the other is based on harmonic voltage. Assume that there are  $n$  harmonic sources in a grid. For line end- $\ell'$ , a source's harmonic pollution liability based on harmonic voltage is calculated by



**Figure 1.** A PCC connected to several lines.

$$L_{\ell k}^v = V'_{\ell k} / |\bar{V}_{\ell'}| \times 100\% . \quad (1)$$

$L_{\ell k}^v$  is the harmonic pollution liability of source- $k$  ( $k = 1, 2, 3, \dots, n$ ) over line end- $\ell'$ .  $V'_{\ell k}$  is the projection of phasor  $\bar{V}_{\ell k}$  (response of harmonic voltage component over line end- $\ell'$  under source- $k$  action alone) on phasor  $\bar{V}_{\ell'}$  (total response of harmonic voltage over line end- $\ell'$ ).

A source's harmonic pollution liability based on harmonic current is calculated by

$$L_{\ell k}^i = I'_{\ell k} / |\bar{I}_{\ell'}| \times 100\% . \quad (2)$$

$L_{\ell k}^i$  is the harmonic pollution liability of source- $k$  over line end- $\ell'$ .  $I'_{\ell k}$  is the projection of phasor  $\bar{I}_{\ell k}$  (response of harmonic current component over line end- $\ell'$  under source- $k$  action alone) on phasor  $\bar{I}_{\ell'}$  (total response of harmonic current over line end- $\ell'$ ).

As the voltage is an important power quality index, the research of voltage-based liability division method for harmonic pollution is relatively deeper than that of current-based one. The level of harmonic voltage (current) will increase sharply when series (parallel) harmonic impedances match in a grid. So the harmonic voltage- or current-based method can't reflect completely the electrical characteristics of harmonics. In addition, the calculation of voltage- or current-based harmonic pollution liability needs the projection of component on the total, which makes not only the physical meaning inexplicit but also the divergence in the level of harmonic pollution liabilities big [12]. Thus, it would be more complete and reasonable to take into account both harmonic voltage and current, or take into account harmonic power, in determining harmonic pollution liability. However, there is no power-based quantitative liability division method for harmonic pollution till now. This is because "power does not meet the superposition theorem". In this paper, employing our achievements, two liability division methods for harmonic pollution based on harmonic power are proposed.

### 3. Liability Division Method for Harmonic Pollution Based on Power

#### 3.1. Theory of Line-Transferred Power Component

By circuit theory, power does not meet the superposition theorem. In order to quantitatively divide the harmonic pollution liability among sources based on power, source- $k$ -driven line-transferred complex power component under coexistence of all harmonic sources must be determined at first. A formula for determining source- $k$ -driven line-transferred complex power component is derived and proved in [13] [14] based on the additive, effectiveness and symmetry, which complies with the circuit laws without any assumption.

$$\bar{S}_{\ell k} = 0.5\bar{V}_{\ell k} \sum_{k=1}^n \bar{I}_{\ell k}^* + 0.5\bar{I}_{\ell k}^* \sum_{k=1}^n \bar{V}_{\ell k} . \quad (3)$$

$\bar{S}_{\ell k} = P_{\ell k} + jQ_{\ell k}$  is the source- $k$ -driven line-end- $\ell'$ -transferred complex power component.  $P_{\ell k}$  and  $Q_{\ell k}$  are the real and imaginary parts of  $\bar{S}_{\ell k}$ .  $\bar{I}_{\ell k}^*$  is the conjugation of  $\bar{I}_{\ell k}$ . Obviously, the sum of line-end- $\ell'$ -transferred complex power components over all sources is equal to the total complex power over line-end- $\ell'$ .

$$\bar{S}_{\ell'} = \sum_{k=1}^n \bar{S}_{\ell k} . \quad (4)$$

$\bar{S}_{\ell'} = P_{\ell'} + jQ_{\ell'}$  is the total complex power over line-end- $\ell'$  under all sources action together.  $P_{\ell'}$  and  $Q_{\ell'}$  are the real and imaginary parts of  $\bar{S}_{\ell'}$ .

With formula (3), the harmonic pollution liability can be quantitatively divided among harmonic sources based on power.

#### 3.2. Liability Division Method for Harmonic Pollution Based on Line-Transferred Active Power Component

The active power is the physical quantity really reflecting the capability of power transmission in a grid, while the reactive power is just the amplitude of the power travelling back and forth in the grid. A source's harmonic pollution liability based on line-transferred active power component can be determined by the following formula.

$$L_{\ell k}^p = P_{\ell'k} / P_{\ell'} \times 100\% . \quad (5)$$

$L_{\ell k}^p$  is the harmonic pollution liability of source- $k$  over line end- $\ell'$  based on line-transferred active power component.

### 3.3. Liability Division Method for Harmonic Pollution Based on Line-Transferred Complex Power Component

Similar to the quantitative liability division methods for harmonic pollution based on voltage or current, the projection of phasor  $\bar{S}_{\ell'k}$  on phasor  $\bar{S}_{\ell'}$  can be used to quantitatively divide the harmonic pollution liability of source- $k$  over line end- $\ell'$ . As a result, a source's harmonic pollution liability based on line-transferred complex power component can be determined by

$$L_{\ell k}^s = S'_{\ell'k} / |\bar{S}_{\ell'}| \times 100\% . \quad (6)$$

$L_{\ell k}^s$  is the harmonic pollution liability of source- $k$  over line end- $\ell'$  based on line-transferred complex power component.  $S'_{\ell'k}$  is the projection of phasor  $\bar{S}_{\ell'k}$  on phasor  $\bar{S}_{\ell'}$ .

## 4. Case Study

The two newly proposed methods for quantitative division of harmonic pollution liability, represented by (5) and (6), are based on line-transferred active-and complex-power components, and called *P-method* and *S-method*, respectively. A case study is performed to show the effectiveness of the two methods. And the simulation results by the voltage- and current-based methods (respectively called *V-method* and *I-method*) are also given for comparison.

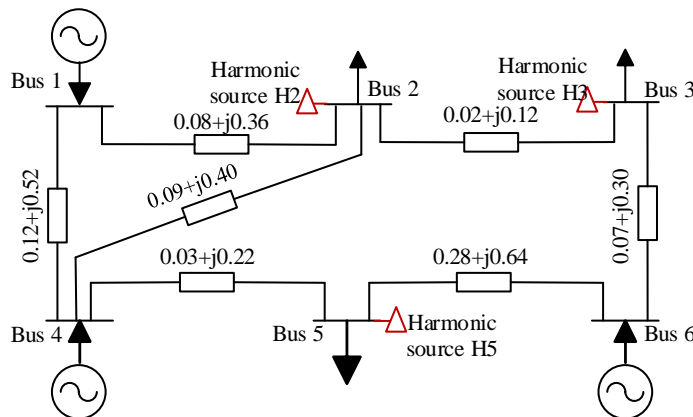
The IEEE 6-bus system shown in **Figure 2** is used for the test. The system contains three generators, six bus, and seven transmission lines. Line impedances (in p.u.) at rated frequency are also shown in the figure.

Assume that there are three harmonic sources (their frequency is 5 times of the rated) located at buses 2 and 3 as well as 5, and denoted by H2 and H3 as well as H5, respectively. In the test, all harmonic sources are represented by ideal equivalent current source, all loads in the harmonic domain are represented by impedance model II as in [15]. The harmonic currents of the three harmonic sources are  $0.3780 + j0.2823$ ,  $0.8193 + j0.6608$  and  $0.9450 + j1.0318$  p.u., respectively.

In order to obviously show the features of the four harmonic pollution liability division methods (*P-method*, *S-method*, *V-method* and *I-method*), only the simulation results of the six lines, 2-1, 2-3, 3-6, 4-2, 5-6, 5-4, are selected and listed in **Table 1-3**.

For intuitiveness, the bar graphs of one source's harmonic pollution liabilities over individual lines by different methods are shown in **Figure 3-5**.

Look at the H3's harmonic pollution liabilities shown in **Figure 3** and **Table 1**: By *I-method*, the range of the harmonic pollution liabilities is  $[-58.681\%, 176.646\%]$ , the biggest of all. The standard deviation of them



**Figure 2.** The IEEE 6-bus system.

**Table 1.** Liabilities of H3 for line-end harmonic pollution by different methods.

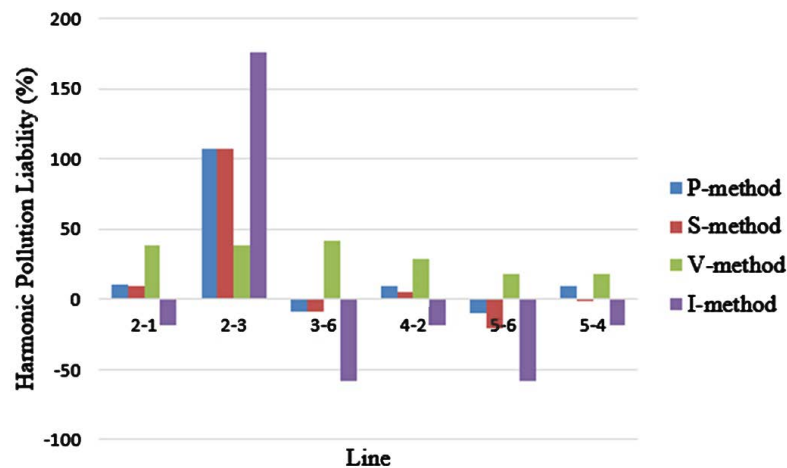
Line	P-method (%)	S-method (%)	V-method (%)	I-method (%)
2-1	10.158	9.672	38.150	-18.806
2-3	106.932	107.398	38.150	176.646
3-6	-8.321	-8.321	42.037	-58.681
4-2	9.744	4.805	28.415	-18.806
5-6	-10.057	-20.036	18.608	-58.681
5-4	9.545	-0.099	18.608	-18.806

**Table 2.** Liabilities of H2 for line-end harmonic pollution by different methods.

Line	P-method (%)	S-method (%)	V-method (%)	I-method (%)
2-1	0.734	0.754	18.054	-16.545
2-3	-18.810	-18.730	18.054	-55.514
3-6	4.609	4.030	16.674	-8.615
4-2	0.486	-1.377	13.791	-16.545
5-6	4.153	-0.026	8.563	-8.615
5-4	0.734	0.754	18.054	-16.545

**Table 3.** Liabilities of H5 for line-end harmonic pollution by different methods.

Line	P-method (%)	S-method (%)	V-method (%)	I-method (%)
2-1	89.107	89.573	43.796	135.351
2-3	11.878	11.332	43.796	-21.132
3-6	103.712	104.292	41.288	167.296
4-2	89.770	96.572	57.794	135.351
5-6	105.904	120.063	72.829	167.296
5-4	90.088	104.090	72.829	135.351



**Figure 3.** Bar graphs of H3's harmonic pollution liabilities versus lines.

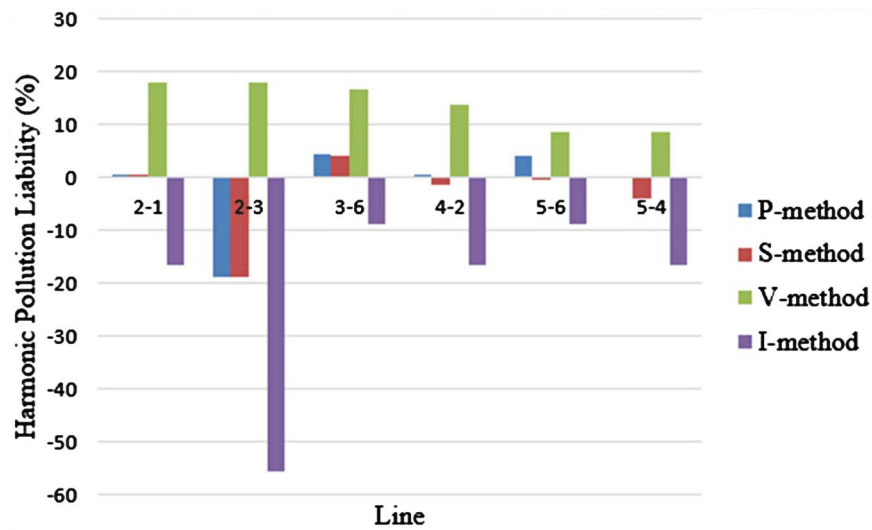


Figure 4. Bar graphs of H2's harmonic pollution liabilities versus lines.

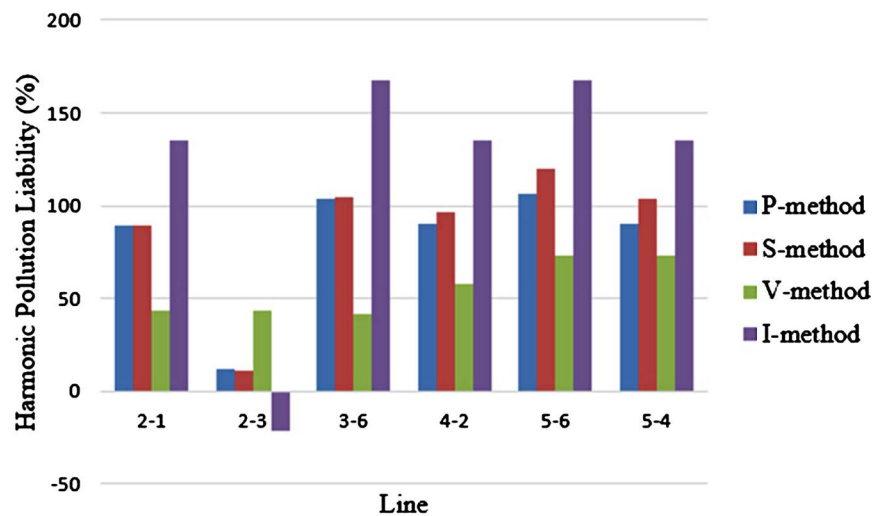


Figure 5. Bar graphs of H5's harmonic pollution liabilities versus lines.

is 0.885, also the biggest of all. It indicates that the *I-method* is the most unreasonable and extreme method. By *V-method*, the range of the harmonic pollution liabilities is [18.608%, 42.037%], the smallest of all. The standard deviation of them is 0.104, also the smallest of all. Thus *V-method* goes to another extreme opposite to the *I-method*. By *P-method* and *S-method*, the range of the harmonic pollution liabilities are respectively [-10.057%, 106.932%] and [-20.036%, 107.398%], the medium among all. The standard deviations of them are respectively 0.437 and 0.462, also the medium among all. Both *P-method* and *S-method* are reasonable viewing from the ranges of their harmonic pollution liabilities.

The real part of a complex power means the average power delivered by the grid, while the imaginary part of a complex power is the amplitude of power travelling back and forth in the grid. Their physical meanings are quite different. In addition, the *S-method* needs projection of complex power component on total complex power. These make the *S-method* inexplicit in physical meaning.

In conclusion, the *P-method* (liability division method for harmonic pollution based on line-transferred active power component) is the most ideal method, which is not only explicit in physical meaning but also complete and reasonable.

For harmonic source H2 or H5, the same conclusion can be made from **Figure 4** and **Table 2**, or **Figure 5** and **Table 3**.

## 5. Conclusions

The range and standard deviation of liabilities for harmonic pollution by the current-based method are the biggest of all. Those by the voltage-based method are the smallest of all. The two methods go to opposite extremes and are unreasonable. The newly proposed two methods, which respectively based on line-transferred active power component and line-transferred complex power component, take all factors into account (complete) and give reasonable levels of liabilities for harmonic pollution.

However, the method based on line-transferred complex power component needs projection of the complex power component, thus inexplicit in physical meaning. As a result, the liability division method for harmonic pollution based on line-transferred active power components is not only complete and explicit but also reasonable, and it is worth of recommendation.

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