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Reply to "A Simple Derivation of the Lorentz Transformation"

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Abstract

The theory of Relativity is consistent with the Lorentz transformation. Thus Pr. Lévy proposed a simple derivation of it, based on the Relativity postulates. A reply is provided: Some related results (five ones) are found and developed step by step which would invalid it. So Lorentz transformation would not be simply derived by this way. Finally an alternative demonstration of Lorentz transformation is reminded, consistent with Quantum Mechanics.

Keywords

Lorentz Transformation Derivation, Relativity Theory

1. Introduction

Today, the Relativity Theory does almost the unanimity among the scholarly community. However the problem is that the Relativity Theory and the Quantum Mechanics do not describe the same world. "They imagine from a radically different point of view the basic notions of physics that are force, space, time or matter. One considers force as an exchange of quantum, whereas the other conceives it as a deformation of space time; one [the Q.M.] sees space-time flat and static, the other [the R.T.], bumpy and dynamic; one confers a preponderant role at random, the other, non-existent …" [1]. That means that one of these two previous theories should be invalidated. Because the Relativity Theory [2] is consistent with the Lorentz transformation, this demonstration is its keystone. Thus ten years ago another derivation [3] of the Lorentz transformation was proposed by Pr. Jean-Michel Lévy based on the two postulates of the Theory of Relativity. It is a simple one, based on a thought experiment. If the mathematical equations look correct, the physical arguments to transit from an equation to another one can be discussed. It is the topic of this paper.

2. Vocabulary and Notations

a) A thought experiment

"A thought experiment is a device with which one performs an intentional, structured process of intellectual deliberation in order to speculate, within a specifiable problem domain, about potential consequents (or antecedents) for a designated antecedent (or consequent)" (*Yeates,* 2004*, p.* 150). That means it is not a true experiment with real measures coming from instruments. To sum up, a thought experiment is closer to a demonstration that to an experiment; if results are paradoxical it could come either from a mistake in the demonstration or from an erroneous hypothesis.

b) Coordinate and length, moment and duration:

In this paper, we will distinguish the coordinate *x* from the length *L*, and the moment *t* from the duration *T*. Length and duration will be noted in capital letters, coordinate and moment in small letters. Relation between them is:

$$\begin{cases} L_{AB} = x_B - x_A \\ T_{AB} = t_B - t_A \end{cases} \quad (1.a\text{-}b)$$

c) Referential frames

To distinguish them,

- the frame linked to the object (the mirrors) will be noted without nothing: (K)
- the first moving frame (from the mirror in reference) and its elements will be noted with an apostrophe: (K')
- the second moving frame and its elements will be noted with a double apostrophe: (K")

d) Galilean referential frames:

According to the Relativity theory and to the Newtonian mechanics too, physical laws are the same in Galilean referential frames, *i.e.* in rectilinear and uniform translation. We get the same result:

- either the frame (K') is moving from the mirrors, or the mirrors are moving within the motionless frame (K').
- the same, either the frame (K") is moving from the mirrors, or the mirrors are moving within the motionless frame (K").

But an apparent trajectory is not a physical law and so it can be different in two different frames.

e) Regularity and comparison between clocks:

A clock is an instrument or a device used to measure time, and the results are comparable if this machine remains regular. For example, the well-known pendulum clock remains regular if we do not change neither the length of the pendulum nor the gravity of the place. If this clock is moving at a constant speed on a horizontal plan without change of gravity, there is no effect, it can be compared to a motionless one. But if this clock is moving on a vertical direction with a change of apparent gravity, its timing will change, it cannot be compared to the previous ones. In his article, the device with the mirrors is qualified of "clock", which suggests that the dribble of the photons is regular and comparable whatever the situation is.

3. The Arguments

Lorentz transformation is derivate in a decade of equations by Pr. Jean-Michel Lévy, as retraced in **Appendix A**. If these equations are mathematically understandable, we will check here how these equations are physically justified, or not.

a) No clear distinction between (K') and (K"):

T'_{ABC} is the duration the photon takes to cover the path length L'_{ABC} in (K')

The author says *the clock moving parallel to its mirrors has period* $[T'_{ABC}]$ *in the lab* [K']; *therefore we can be sure that the clock moving perpendicularly to its mirrors also has period* $[T''_{ABC}]$ *in the lab frame* [K"][1]. The lab (K') is not necessarily equals to the lab (K"). Equality of T duration is a reasonable hypothesis; but it is not demonstrated at this step.

b) Confusion between T' and T":

Let us call for the light path: A the starting point of the first mirror, B the reflection point of the second mirror and C the arrival point on the first mirror.

By symmetry in (K'):

$$T'_{AB} = T'_{BC} \tag{2}$$

Then the author adds for the perpendicular direction (K") *it will reach the front mirror after a time* $[T''_{AB}]$ *and will need a further time lapse* $[T''_{BC}]$[2] *for the return leg*, which means:

$$T''_{AB} \neq T''_{BC} \tag{3}$$

It is paradoxical to get in the same time an equality and an inequality of duration for the same phenomenon!

c) Durations of photon paths would be the same along different inter-mirrors distances:

Now, let us demonstrate what happens in (K') and (K"), details in **Appendix B**.

Using rectilinear uniform translations, mirrors are studied within three different referential frames: in the first one, mirrors are motionless; in the second one, mirrors are moving in parallel to their plan; and in the third one, mirrors are moving perpendicularly to their plan. This mirrors device appears to have different light paths either this device is motionless or in motion (please check **Appendix B**). The same, this mirrors device appears to have a different light path either this device is moving in parallel to the mirrors or perpendicularly to them. And in (K"), we can add that one way path and its duration are not symmetric with the return path and its duration.

Then the light path L'_{ABC} in (K') is *a priori* different from the light path L''_{ABC} in (K"), and their duration too:

[1]Terms in square brackets [] have been added by myself.
[2]Terms in square brackets [] have been added by myself.

$$\begin{cases} L'_{ABC} \neq L''_{ABC} \\ T'_{ABC} \neq T''_{ABC} \end{cases} \tag{4.a-b}$$

Mirrors device can produce different durations in a referential frame or in another, as it is the case for the pendulum clock. Then (see **Appendix B**) we get

$$T''_{ABC} = \gamma^2 \cdot \frac{2L''_0}{c} \tag{5}$$

So

$$\begin{cases} \text{I) if } \left(L''_0 = L'_0\right) \text{ then } \left(T''_{ABC} = \gamma^2 \cdot T_0\right) \\ \text{II) if } \left(T''_{ABC} = T'_{ABC}\right) \text{ then } \left(L''_0 = \frac{1}{\gamma} \cdot L_0\right) \\ \text{III) if } \left(L''_0 \neq L'_0 \text{ and } T''_{ABC} \neq T'_{ABC}\right) \text{ then } \left(T''_{ABC} = \gamma^2 \cdot \frac{2L''_0}{c}\right) \end{cases} \tag{6.a-b-c}$$

i) If the inter-mirrors distance remains the same in (K') and (K"), $L''_0 = L'_0$, so using Equations ((B3), (B11) and (B17)) of **Appendix B**, the paths of light and their duration change ($T''_{ABC} \neq T'_{ABC}$).

ii) If we consider period remained the same in (K') and (K"), $T''_{ABC} = T'_{ABC}$, so using again Equations ((B3), (B11) and (B17)), the inter-mirrors distance (not the path length but the real inter-mirrors distance) would have changed ($L''_0 \neq L'_0$), which would be quite surprising! If you are not convinced, please consider that the observer is moving and that the mirrors are motionless.

If not (same clock device, and so same period and same inter-mirrors distance), we would get:

$$\gamma^2 \cdot \frac{1}{c} = \gamma \cdot \frac{1}{c} \tag{7}$$

which is impossible! Then we would have to suggest that the main hypothesis in the demonstration is not valid, *i.e.* that the postulate to get the same celerity c of the photon in (K), (K') and (K") is not valid. (see **Figure 1**).

iii) If inter-mirrors distance were different in (K') and (K") and two-ways duration too, then duration in (K") would be by hypothesis different from duration in (K')

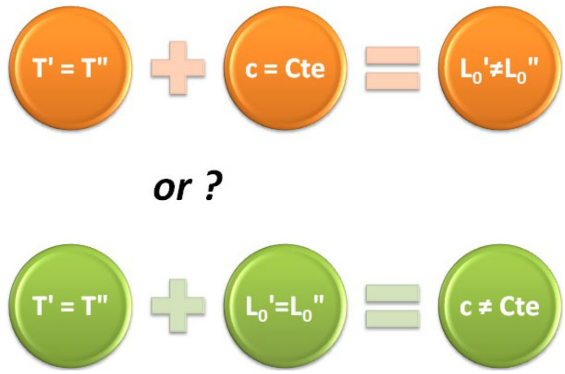

Figure 1. Two competitive explanations.

Consequently, his Equation (A4) in (K”) is not justified.

d) Round trip with Lorentz transformation

Let us now consider Equation (A4) would be valid for all directions. When we go from the frame (K) to (K”), length would contract:

$$L_0'' = \frac{1}{\gamma} L_0 \tag{8}$$

which is nothing less that:

$$L_0 = \gamma \cdot L_0'' \tag{8.bis}$$

But according to the author when we go back from (K”) to (K), length would contract as

$$L_0 = \frac{1}{\gamma} L_0'' \tag{9}$$

and so

$$L_0 = \frac{1}{\gamma^2} L_0 \tag{10}$$

which would be paradoxical!

It is this paradox the author use to demonstrate Equation (A6).

Consequently, his Equation (A6) is not demonstrated

e) Length is not coordinate

Let us now consider that when we go from K to K’, length would contract: $L' = \frac{1}{\gamma} L$. And when we go back from (K’) to (K), length would contact too: $L = \frac{1}{\gamma} L'$ according to the author hypothesis. The trouble in author’s demonstration comes from confusion between the length and the coordinate (check **Appendix C**). In fact:

$$\begin{cases} \text{in } (\text{K}): X_{O'M} = x_M - x_{O'} \\ \text{in } (\text{K}'): X'_{OM} = x'_M - x'_O \end{cases} \tag{11.a-b}$$

But

$$\begin{cases} \text{at } t \neq 0: x_{O'} \neq 0 \\ \text{at } t' \neq 0: x'_O \neq 0 \end{cases} \tag{12.a-b}$$

Then

$$\begin{cases} X_{O'M} \neq \dfrac{x'_M}{\gamma} \\ X'_{OM} \neq \dfrac{x_M}{\gamma} \end{cases} \tag{13.a-b}$$

Correct equations are given in Appendix, Equation (C19.a-b).

Consequently, his Equations ((A6) & (A7)) are not demonstrated.

Remark: More globally, trouble comes from the thought experiment which is not an indisputable true experiment. To a thought experiment can often be opposed another thought experiment (see **Appendix D**); it can also be proposed another demonstration of Lorentz derivation, compatible with the Quantum Mechanics: the Neo-Newtonian Mechanics, developed in a previous article [4].

4. Conclusions

The derivation of Lorentz transformation is the keystone of the Relativity Theory. This derivation is not as simple as the title of Pr. Lévy's article suggests it. A thought experiment is not a true experiment with real measures; it is closer to a demonstration. If we found a single logical mistake in the reasoning, it would be enough to invalid this derivation. And we would have found five ones:

- a mixture between the two referential frames in motion,
- a mixture between the duration of the second and the third referential frame,
- using a constant light velocity, a same light path duration along a variable inter-mirrors distance,
- an incorrect use of the Lorentz transformation,
- and a confusion between length and coordinate.

So, this relativistic derivation does not appear to be a valid one. Then it could open up a field to the Neo-Newtonian Mechanics, consistent with the Quantum Mechanics.

Acknowledegments

I would like to thank the referee for his valuable suggestion to introduce the subject.

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Appendix

Appendix A) Equations extracted from "A simple derivation of the Lorentz transformation (...)" of Professor Jean-Michel LEVY

$$T_0 = \frac{2 \cdot L_0}{c} \tag{A1}$$

$$(c \cdot T/2)^2 = L_0^2 + (v \cdot T/2)^2 \tag{A2}$$

$$T = \frac{T_0}{\sqrt{1-(v/c)^2}} = \gamma \cdot T_0 \tag{A3}$$

$$\begin{cases} c \cdot t = L + v \cdot t \\ v \cdot t' = L - c \cdot t' \end{cases} \tag{*.a-b}$$

$$T = L/c \cdot \left(1 - v^2/c^2\right) \tag{**}$$

$$L = L_0 \sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{L_0}{\gamma} \tag{A4}$$

$$OM = OO' + O'M \tag{A5}$$

$$x = v \cdot t + \frac{x'}{\gamma} \tag{A6}$$

$$\frac{x}{\gamma} = v \cdot t' + x' \tag{A7}$$

Appendix B) Lengths and their durations in three different referential frames

a) First referential frame, linked to the mirrors:

L_0 is the distance between the two mirrors (in this referential frame), and L_{AB} is the length covered by light from A to B (see **Figure A1**)

$$L_{AB} = L_0 \tag{B1}$$

In this referential frame, mirrors are motionless.

$$L_{AB} = L_{BA} \tag{B2}$$

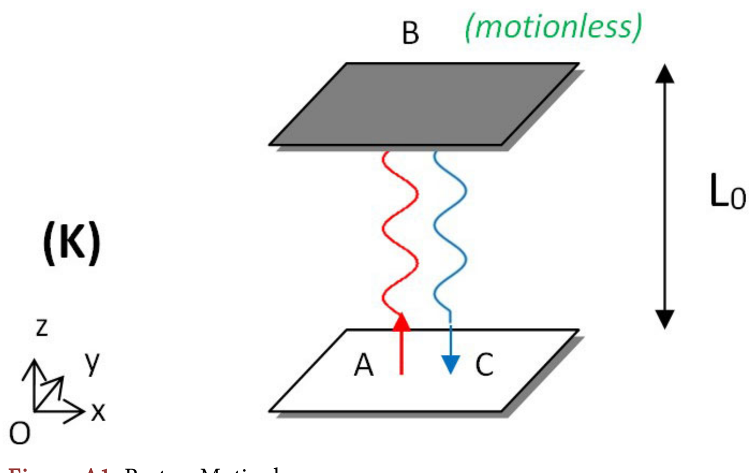

Figure A1. Rest or Motionless.

$$L_0 = c \cdot \frac{T_0}{2} \tag{B3}$$

$$L_{AB} = c \cdot \frac{T_{AB}}{2} \tag{B4}$$

Then, the total duration T_{ABC} for light for moving from A to C is:

$$T_{ABC} = T_0 \tag{B5}$$

b) Second referential frame, motion parallel to the mirrors:

It is equivalent having the observer moving on a parallel way of the mirrors, or seen from the observer having the mirrors moving on a parallel way.

L'_0 is the distance between the two mirrors (in the frame (K')), and L'_{AB} is the length covered by light from A to B (see Figure A2)

$$X'_{AB} = v_x \cdot T'_{AB} \tag{B6}$$

According to the second postulate of the Relativity Theory, light celerity is constant and equal in the two reference frames:

$$L'_{AB} = c \cdot T'_{AB} \tag{B7}$$

$$L'_{AB} = L'_{BC} \tag{B8}$$

$$Z'_{AB} = L'_0 \tag{B9}$$

$$\left(L'_{AB}\right)^2 = \left(X'_{AB}\right)^2 + \left(Z'_{AB}\right)^2 \tag{B10}$$

Because the motion is parallel to the mirrors, it is sure the inter-mirrors distance does not change:

$$L'_0 = L_0 \tag{B11}$$

So

$$\left(c \cdot T'_{AB}\right)^2 = \left(v_x \cdot T'_{AB}\right)^2 + \left(c \cdot \frac{T_0}{2}\right)^2 \tag{B12}$$

$$\left(T'_{AB}\right)^2 = \frac{c^2}{c^2 - v_x^2}\left(\frac{T_0}{2}\right)^2 \tag{B13}$$

$$\left(T'_{AB}\right)^2 = \frac{1}{1 - v_x^2/c^2}\left(\frac{T_0}{2}\right)^2 \tag{B14}$$

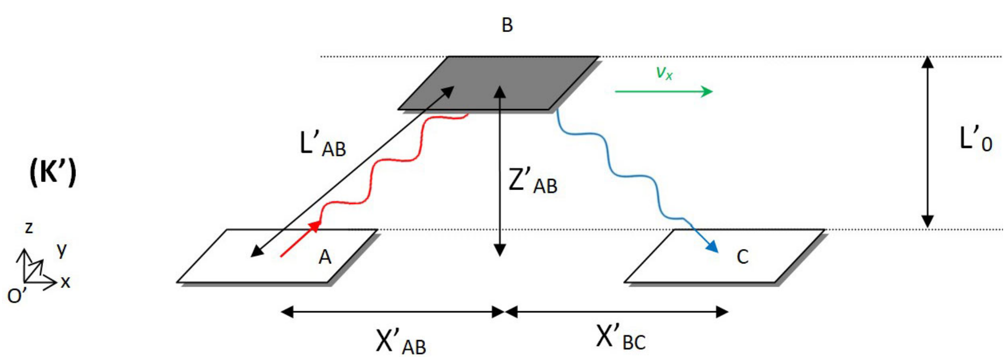

Figure A2. Motion according to V_x (or V_y).

$$T'_{AB} = \gamma\left(\frac{T_0}{2}\right) \tag{B15}$$

Idem for T'_{BC}

$$T'_{BC} = \gamma\left(\frac{T_0}{2}\right) \tag{B16}$$

Then

$$T'_{ABC} = \gamma \cdot T_0 \tag{B17}$$

In this referential frame we find the same result that this in the simple derivation article.

Remark: v_x can be changed to v_y or to any velocity parallel to the mirrors.

<u>c) Third referential frame, motion perpendicular to the mirrors:</u>

It is equivalent having the observer moving on perpendicular way of the mirrors, or seen from the observer having the mirrors moving on a perpendicular way.

L''_0 is the distance between the two mirrors (in the frame (K”)), and L''_{AB} is the length covered by light from A to B (see Figure A3)

$$Z'''_{AB} = v_z \cdot T''_{AB} \tag{B18}$$

According to the second postulate of the Relativity Theory, light celerity is constant and equal in the two reference frames:

$$L''_{AB} = c \cdot T''_{AB} \tag{B19}$$

$$L''_{AB} = L''_0 + Z''_{AB} \tag{B20}$$

$$Z''_{AC} = v_z \cdot \left(T''_{AB} + T''_{BC}\right) \tag{B21}$$

$$L''_{BC} = c \cdot T''_{BC} \tag{B22}$$

$$L''_{AB} = L''_{BC} + Z''_{AC} \tag{B23}$$

$$T''_{ABC} = T''_{AB} + T''_{BC} \tag{B24}$$

Then using the Equations (B18)-B(20) for the first one, and Equations (B21)-

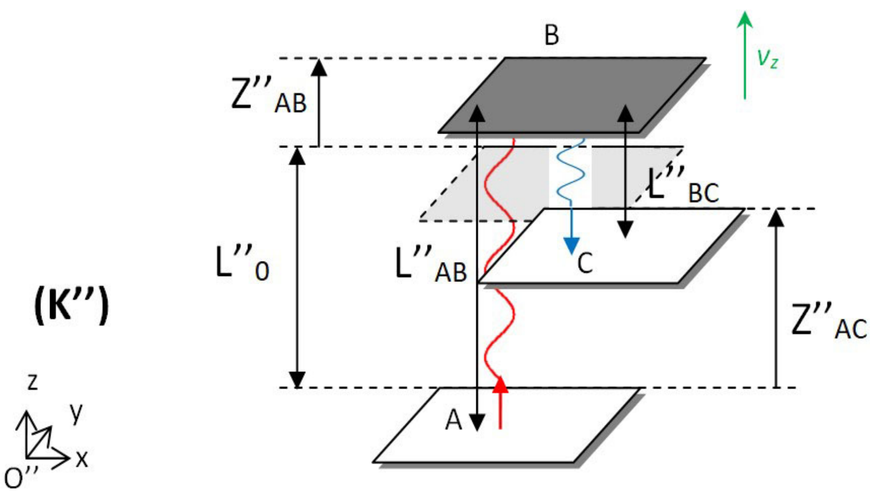

Figure A3. Motion according V_z.

(B23) for the second one:

$$\begin{cases} c \cdot T''_{AB} = L''_0 + v_z \cdot T''_{AB} \\ v_z \cdot \left(T''_{AB} + T''_{BC}\right) = L''_{AB} - c \cdot T''_{BC} \end{cases} \quad \text{(B25.a-b)}$$

And using (B19) again:

$$\begin{cases} c \cdot T''_{AB} = L''_0 + v_z \cdot T''_{AB} \\ c \cdot T''_{AB} = c \cdot T''_{BC} + v_z \cdot \left(T''_{AB} + T''_{BC}\right) \end{cases} \quad \text{(B26.a-b)}$$

$$\begin{cases} T''_{AB} = \dfrac{L''_0}{c - v_z} \\ T''_{BC} = \dfrac{L''_0}{c + v_z} \end{cases} \quad \text{(B27.a-b)}$$

$$T''_{ABC} = \frac{2c}{c^2 - \left(v_z\right)^2} L''_0 \quad \text{(B28)}$$

$$T''_{ABC} = \gamma^2 \cdot \frac{2L''_0}{c} \quad \text{(B29)}$$

Appendix C) Length and coordinate

The author describes the same relation in (K) and (K'):

$$\|\boldsymbol{OM}\| = \|\boldsymbol{OO'}\| + \|\boldsymbol{O'M}\| \quad \text{(C1)}$$

a) In the frame (K):

$$OM = X_{OM} = x_M - x_O \quad \text{(C2)}$$

$$OO' = X_{OO'} = x_{O'} - x_O \quad \text{(C3)}$$

$$O'M = X_{O'M} = x_M - x_{O'} \quad \text{(C4)}$$

And because (O) is the origin of (K)

$$x_O = 0 \quad \text{(C5)}$$

And so Equations ((C2) and (C3)) become

$$OM = X_{OM} = x_M \quad \text{(C6)}$$

$$OO' = X_{OO'} = x_{O'} \quad \text{(C7)}$$

And then Equation (C1) in (K) is:

$$x_M = x_{O'} + x_M - x_{O'} \quad \text{(C8)}$$

which is coherent.

b) In the frame (K'):

$$OM = X'_{OM} = x'_M - x'_O \quad \text{(C9)}$$

$$OO' = X'_{OO'} = x'_{O'} - x'_O \quad \text{(C10)}$$

$$O'M = X'_{O'M} = x'_M - x'_{O'} \quad \text{(C11)}$$

And because (O') is the origin of (K')

$$x'_{O'} = 0 \quad \text{(C12)}$$

And so Equations ((C10) and (C11)) become

$$OO' = X'_{OO'} = -x'_O \tag{C13}$$

$$O'M = X'_{O'M} = x'_M \tag{C14}$$

And then Equation (C1) in (K') is:

$$x'_M - x'_O = -x'_O + x'_M \tag{C15}$$

which is coherent too.

c) The property on length:

Let us now use the property that length, not coordinate, would contract when we change of frame (cf Equation (A4)):

$$\begin{cases} L'_{OM} =^? \dfrac{1}{\gamma} L_{OM} \\ L_{O'M} =^? \dfrac{1}{\gamma} L'_{O'M} \end{cases} \tag{C16.a-b}$$

So in (K):

$$O'M = X_{O'M} =^? \frac{1}{\gamma} X'_{O'M} \tag{C17}$$

and in (K'):

$$OM = X'_{OM} =^? \frac{1}{\gamma} X_{OM} \tag{C18}$$

Consequently:

$$\begin{cases} O'M = x_M - x_{O'} =^? \dfrac{1}{\gamma} x'_M \\ OM = x'_M - x'_O =^? \dfrac{1}{\gamma} x_M \end{cases} \tag{C19.a-b}$$

$$\gamma\left(x_M - x_{O'}\right) - x'_O =^? \frac{1}{\gamma} x_M \tag{C20}$$

$$\left(\gamma^2 - 1\right) x_M =^? \gamma^2 \cdot x_{O'} + \gamma \cdot x'_O \tag{C21}$$

$$x_M =^? \frac{\gamma^2}{\gamma^2 - 1} \cdot x_{O'} + \frac{\gamma}{\gamma^2 - 1} \cdot x'_O \tag{C22}$$

That would means that whatever M is located, its value would be the same; it is impossible! So by *reduction ad absurdum*, Eq.(A4) as hypothesis is invalid.

Appendix D) Thought experiment: counter-proposal

a) Counter-proposal on duration

Let us imagine a third similar device called (D), but with its inter-mirrors distance half of the previous ones. Then photon will cover a two-ways in (D) when photon will cover a single way, or a return way, in the rest device (K), whatever the direction of the large clock is (parallel or perpendicular):

$$\begin{cases} L_{AD} = \dfrac{L_{AB}}{2} \\ T_{ADC} = T_{AB} = T_{BC} \end{cases} \tag{D1.a-b}$$

Then by simultaneity:

$$\begin{cases} T'_{ADC} = T'_{AB} = T'_{BC} \\ T''_{ADC} = T''_{AB} = T''_{BC} \\ T'_{ABC} = T''_{ABC} \end{cases} \quad \text{(D2.a-bc)}$$

So there is equality of duration between (K') and (K")

b) Counter-proposal on length

Let us imagine a bowl. Then:

$$D_{0x} = D_{0y} = D_{0z} \quad \text{(D3)}$$

D being the diameter of the bowl.

The perfect bowl will not change into a ellipsoid shape because it is running or because the observer is moving whatever his direction is, won't it? If not, how the bowl would roll?

Then

$$D'_{0x} = D''_{0z} \quad \text{(D4)}$$

So there is a equality of real distance (not of path length) between (K') and (K").

Now, please check again **Figure 1**.