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Einstein's Elevator in Cosmology

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Abstract

The metrics of gravitational and cosmological models are brought into canonical form in comoving coordinates. The FWR curvature parameter k is read from this and it is shown that $k=0$ does not correlate to a flat model, but for a spatially positively curved geometry in which reference systems which are in free fall exist. This also corresponds to Einstein's elevator principle. Moreover, we will show that our subluminal cosmos is associated with the $R_h = ct$ model of Melia, assuming that $k=0$ is related to a free-falling system in the sense described above.

Keywords

Cosmology, Einstein's Elevator, Canonical Form of the Metric

1. Introduction

One of the main features of general relativity is the identification of the gravitational forces with the effect of the curvature of space on observers, but also the possibility of "transforming away" the gravitational effect, that is to keep an observer without force by an appropriate choice of a reference system. This does not mean that one can eliminate the curvature of space by an observer transformation, but only that one can annul the effect of the curvature of space for certain observers. This is the case for a freely falling observer in the Schwarzschild field of a stellar object. Such an effect can also be expected for cosmological models which expand in free fall.

The effect has become known in the literature as Einstein's elevator. We will recall this effect for the case of the Schwarzschild field, but we will also show that it is useful to consider it for cosmological models with position-independent spatial curvature and obeying the cosmological principle. We will discuss the elevator principle when referring to the de Sitter cosmos and to a subluminal model. We will resort to earlier results [1] [2] and we will discuss the problems from the perspective of Einstein's elevator principle outlined in detail therein. In modern research, Einstein's elevator is denoted as "Week Einstein Equivalence Principle" (WEEP). Kopeikin [3] [4] [5] has recently treated the problem of WEEP in connection with cosmological FRW metrics. He has shown that the cosmological expansion could be detected in local gravitational experiments.

The introduction of the elevator principle will be crucial for the structure of the universe. It determines whether a metric with the curvature parameter $k=0$ is flat or positively curved and whether the universe is expanding in free fall.

2. Einstein's Elevator

The line element of the Schwarzschild field in the standard form is

$$\mathrm{d}s^2=\frac{1}{1-\frac{2M}{r}}\mathrm{d}r^2+r^2\,\mathrm{d}\Omega^2-\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 \tag{2.1}$$

with r as the radial coordinate and Ω as the solid angle. We want to bring this line element into the *canonical form*, in which the FRW[1]-curvature parameter k occurs. It is said that the curvature properties of the model are attributed to k. This method was first used in cosmology. Later, it was introduced by McVittie [6] for collapsing gravity models. Based on the Schwarzschild model we will examine what the FRW method is able to accomplish and we will use this knowledge for cosmology.

To bring the Schwarzschild metric into the canonical form, we should remember that the radius of curvature ρ of the Schwarzschild parabola is twice as long as its extension $\mathcal{R}=\mathcal{R}(r)$ to the directrix of the Schwarzschild parabola

$$\rho=2\mathcal{R}=\sqrt{\frac{2r^3}{M}},\quad \mathcal{R}=r\sqrt{\frac{r}{2M}}\,. \tag{2.2}$$

Thus we have

$$\sqrt{\frac{2M}{r}}=\frac{r}{\mathcal{R}} \tag{2.3}$$

and the Schwarzschild metric in canonical form

$$\mathrm{d}s^2=\frac{1}{1-\frac{r^2}{\mathcal{R}^2}}\mathrm{d}r^2+r^2\,\mathrm{d}\Omega^2-\left(1-\frac{r^2}{\mathcal{R}^2}\right)\mathrm{d}t^2\,. \tag{2.4}$$

It is now similar to the de Sitter metric which we will discuss in the next Section. By comparison with the FRW standard form

$$\mathrm{d}s^2=\frac{1}{1-k\frac{r^2}{\mathcal{R}^2}}\mathrm{d}r^2+r^2\,\mathrm{d}\Omega^2-\mathrm{d}T^2 \tag{2.5}$$

we find $k=1$. Flamm's paraboloid appears positively curved in the canonical representation of the metric (2.4). But in contrast to the FWR definition Flamm's paraboloid is open and infinite. Since already deviations from the usual interpretation of the quan-

[1]Friedman-Robertson-Walker.

tity k occur in the Schwarzschild model, we will from now on denote k as the *form parameter* of the model.

From (2.1) or (2.4) we calculate the components of the Ricci-rotation coefficients[2]. The radial and the two lateral components are

$$U_m = \left\{-\alpha v \frac{1}{\rho}, 0, 0, 0\right\}, \quad B_m = \left\{\frac{a}{r}, 0, 0, 0\right\}, \quad C_m = \left\{\frac{a}{r}, \frac{1}{r}\cot\vartheta, 0, 0\right\}. \tag{2.6}$$
$$v = -\sqrt{2M/r} = -r/\Re, \quad a = 1/\alpha = \sqrt{1 - 2M/r} = \sqrt{1 - r^2/\Re^2}$$

The geometric quantity U is the negative of the physical quantity, the force of gravity.

Lemaître has found a coordinate transformation associated with a freely falling observer. The metric in these coordinates is

$$\mathrm{d}s^2 = \mathrm{K}^2\left[\mathrm{d}r'^2 + \Re^2\,\mathrm{d}\Omega^2\right] - \mathrm{d}t'^2, \quad \mathrm{K} = \frac{r}{\Re}. \tag{2.7}$$

Herein K is referred to as scale factor as it is done in the cosmological models. The line element is of type $k = 0$. According to the FRW classification the model would be referred to as flat. If one calculates from this metric the Ricci-rotation coefficients one has

$$'U_{m'} = \left\{0, 0, 0, -\frac{i}{\rho}\right\}, \quad B_{m'} = \left\{\frac{1}{r}, 0, 0, \frac{i}{\Re}\right\}, \quad C_{m'} = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0, \frac{i}{\Re}\right\}. \tag{2.8}$$

It is noteworthy that the space-like components of the lateral field quantities

$$B_{\alpha'} = \left\{\frac{1}{r}, 0, 0\right\}, \quad C_{\alpha'} = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0\right\}, \quad \alpha' = 1', 2', 3' \tag{2.9}$$

are precisely those that one would expect for a flat geometry in polar coordinates. But it would be premature to call the geometry flat. The basic geometric structure of a model cannot be modified by a coordinate transformation. We want to get to the bottom of the matter.

From the coordinate transformation $x^i \to x^{i'}$ of Lemaître the matrix of the coordinate transformation can be determined with $\Lambda_i^{i'} = x^{i'}{}_{|i}$. Since for the two systems, the tetrads can be read from (2.1) and (2.7) and one can calculate the associated Lorentz transformation of this coordinate transformation with $L_m^{m'} = \overset{m'}{e}_{i'}\Lambda_i^{i'}\underset{m}{e}^{i}$:

$$L_m^{m'} = \begin{pmatrix} \alpha & & & i\alpha v \\ & 1 & & \\ & & 1 & \\ -i\alpha v & & & \alpha \end{pmatrix}, \quad \alpha = \sqrt{1 - \frac{2M}{r}}, \quad v = -\sqrt{\frac{2M}{r}}. \tag{2.10}$$

For the lateral field quantities one obtains with

$$B_{m'} = L_{m'}^m B_m, \quad C_{m'} = L_{m'}^m C_m \tag{2.11}$$

[2]Details for the calculation with the tetrad method can be found in papers published about 1900 by Ricci, Bianchi, Levi-Cività, furthermore by Treder [7], Liebscher and Treder [8], and also in our paper [9]. Wanas [10] [11] has treated new cosmological models with the help of tetrads.

from (2.6) first the components

$$B_{m'} = \left\{\alpha a \frac{1}{r}, 0, 0, \frac{i}{\mathfrak{R}}\right\}, \quad C_{m'} = \left\{\alpha a \frac{1}{r}, \frac{1}{r}\cot\vartheta, 0, \frac{i}{\mathfrak{R}}\right\}, \tag{2.12}$$

which a free falling observer would measure. However, since the velocity of a freely falling object in the Schwarzschild field is coupled to the angle of ascent η of the Schwarzschild parabola via

$$r = \mathfrak{R}\sin\eta, \quad v = -\frac{r}{\mathfrak{R}} = -\sin\eta$$

one has for the Lorentz factor and the metric factor according to (2.6)

$$a\alpha = 1. \tag{2.13}$$

This means that the geometric quantity a is supplemented with the kinematic quantity α to 1. Thus, the parameter a for the curvature of space in (2.8) is just *hidden*.

Special attention should be paid to the conversion $U_m \to {}'U_{m'}$. The radial field quantities are also components of the Ricci-rotation coefficients, but they transform inhomogeneously. Since the free fall takes place in the [1] [4]—slice of the space the inhomogeneous transformation law is limited to the radial field quantities [1] and the transformation formula is reduced to

$${}'U_{m'} = L^{m}_{m'}U_m + {}'L_{m'}, \quad {}'L_{1'} = i\alpha^2 v_{|4'}, \quad {}'L_{4'} = -i\alpha^2 v_{|1'}. \tag{2.14}$$

The ${}'L$ -terms are calculated using $v = -\sqrt{2M/r}$. Finally, one obtains in accordance with (2.8)

$$U_m = \left\{-\alpha v \frac{1}{\rho}, 0, 0, 0\right\} \quad \to \quad {}'U_{m'} = \left\{0, 0, 0, -\frac{i}{\rho}\right\}. \tag{2.15}$$

We recognize that in the freely falling system the spatial components of the U -field are all zero. In detail, one has

$${}'U_{1'} = U_{1'} + {}'L_{1'} = 0. \tag{2.16}$$

This means that the gravitational force $U_{1'}$ is canceled by the dynamic term ${}'L_{1'}$, *i.e.* by the counter force. This is the principle of *Einstein*'*s elevator*. Observers who are in a free-falling elevator are not subjected to gravitational forces, they hover. Since these observers do not experience any gravitational forces, they might think that space is flat. Thus,

$${}'U_{m'} = L^{m}_{m'}U_m + {}'L_{m'}$$

is *Einstein*'*s elevator equation* and

$${}'U^{s'}{}_{|s'} + {}'U^{s'}\,{}'U_{s'} = -\frac{1}{\mathfrak{R}^2} \tag{2.17}$$

the *Friedman equation* for free fall in the Schwarzschild field in tetrad form.

Although the metric which relates to the free fall is of the type $k = 0$ and the space-like part of this metric appears flat and the field quantities appear flat as well, the 3-dimensional space is nevertheless curved and is represented by Flamm's paraboloid. The apparent flatness of the space is due to the effect of Einstein's elevator. The quantity k, common in cosmology, cannot be related to the curvature of space, it describes the form of the metric. $k=0$ indicates that the metric relates to a freely falling system.

We have dealt with the problem so minutely because in cosmological models the problem is the same and the formal treatment does not differ much from what has just been put forward.

3. Einstein's Elevator and the Cosmos of de Sitter

By de Sitter [12] [13] [14] [15] was proposed a cosmological model with the line element

$$\mathrm{d}s^2 = \frac{1}{\cos^2\eta}\mathrm{d}r^2 + r^2\,\mathrm{d}\vartheta^2 + r^2\sin^2\vartheta\,\mathrm{d}\varphi^2 - \cos^2\eta\,\mathrm{d}t^2 . \tag{3.1}$$

It is the line element on a pseudo-hyper sphere with the time-independent radius $\mathcal{R}_0$. The space is positively curved and closed. With

$$r = \mathcal{R}_0 \sin\eta \tag{3.2}$$

it can be brought into the canonical form

$$\mathrm{d}s^2 = \frac{1}{1-\frac{r^2}{\mathcal{R}_0^2}}\mathrm{d}r^2 + r^2\,\mathrm{d}\vartheta^2 + r^2\sin^2\vartheta\,\mathrm{d}\varphi^2 - \left(1-\frac{r^2}{\mathcal{R}_0^2}\right)\mathrm{d}t^2 . \tag{3.3}$$

We read from this metric $k=1$. We calculate the Ricci-rotation coefficients from (3.3) and we find the following field quantities

$$U_m = \left\{-\alpha v\frac{1}{\mathcal{R}_0},0,0,0\right\},\quad B_m = \left\{\frac{a}{r},0,0,0\right\},\quad C_m = \left\{\frac{a}{r},\frac{1}{r}\cot\vartheta,0,0\right\} \tag{3.4}$$

with the definitions

$$a = \cos\eta = \sqrt{1-r^2/\mathcal{R}_0^2} = 1/\alpha, \quad v = r/\mathcal{R}_0 = \sin\eta . \tag{3.5}$$

B and C are the typical lateral field quantities for a spherical geometry. The quantity U_1 is a force acting at any point and wants to pull apart neighboring points.

By Lemaître [16] [17] a coordinate transformation has been found which transforms the metric (3.1) into the expanding metric

$$ds^2 = e^{2\psi'}\left[\mathrm{d}r'^2 + r'^2\mathrm{d}\vartheta^2 + r'^2\sin^2\vartheta\,\mathrm{d}\varphi^2\right] - \mathrm{d}t'^2 . \tag{3.6}$$

Therein $e^{\psi'} = \mathcal{K}$ is the position-independent but time-dependent scale factor and $\{r',\vartheta,\varphi,t'\}$ the coordinates comoving with the expansion. The following relations apply:

$$r = \mathcal{K}r',\quad t' = \mathcal{R}_0\psi',\quad \frac{1}{\mathcal{K}}\mathcal{K}_{|m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}_0}\right\}. \tag{3.7}$$

From the metric (3.6) we derive the field quantities

$$'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}_0}\right\},\quad B_{m'} = \left\{\frac{1}{r},0,0,-\frac{i}{\mathcal{R}_0}\right\},\quad C_{m'} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0,-\frac{i}{\mathcal{R}_0}\right\} \tag{3.8}$$

which differ from those of the freely falling Schwarzschild system only in the time-like components, *i.e.* the tidal forces. The Schwarzschild geometry is parabolic, the dS geometry spherical. The Schwarzschild reference system contracts, the dS reference system expands. The geometry only *seems to be flat*. The same arguments as in the Schwarzschild geometry speak against the flatness of space, if the metric is written in the expanding form (3.6).

The field quantities in (3.8) could also have been derived with the Lorentz transformation

$$L^{m}_{m'} = \begin{pmatrix} \alpha & & & -i\alpha v \\ & 1 & & \\ & & 1 & \\ i\alpha v & & & \alpha \end{pmatrix}, \tag{3.9}$$

whereby for the variables U the inhomogeneous transformation law

$$'U_{m'} = L^{m}_{m'} U_m + 'L_{m'}, \quad 'U_{1'} = U_{1'} + 'L_{1'} = 0 \tag{3.10}$$

applies. We also find just like in (2.15)

$$U_m = \left\{ -\alpha v \frac{1}{\mathcal{R}_0}, 0, 0, 0 \right\} \quad \rightarrow \quad 'U_{m'} = \left\{ 0, 0, 0, -\frac{i}{\mathcal{R}_0} \right\}. \tag{3.11}$$

(3.10) is the Einstein elevator equation for the dS-Universe. The reference system established with the Lorentz transformation (3.9) expands in freefall. The form parameter read from the metric (3.6) is $k = 0$. Here k again stands for the form of a metric which is determined by a freely falling coordinate system and does not imply any statements about the curvature of space.

4. Einstein's Elevator and the $R_h = ct$ Model

In a previous paper [2] we have presented a cosmological model which is an exact solution of Einstein's field equations and in which the stress-energy-momentum tensor includes pressure and density of matter

$$\kappa p = -\frac{1}{\mathcal{R}^2}, \quad \kappa \mu_0 = \frac{3}{\mathcal{R}^2}. \tag{4.1}$$

The model is based on the de Sitter universe. The geometry is a pseudo-hyper sphere and its radius a function of time. Thus, the model expands and is positively curved and closed. The equator of the pseudo-hyper sphere is the cosmic horizon. For galaxies drifting apart due to expansion, the highest attainable velocity is the velocity on the horizon, namely the velocity of light. The model is subluminal; there are no superluminal values for the recession velocities of galaxies.

Since this subluminal model is an extension of the dS cosmos, we will start from the dS metric as seed metric, but the radius of curvature of the pseudo-hyper sphere will be time-dependent. Thus, a new quantity enters into the theory

$$\mathcal{F}_{m'} = \frac{1}{\mathcal{R}} \mathcal{R}_{|m'} = \left\{ 0, 0, 0, \frac{1}{\mathcal{R}} \mathcal{R}_{|4'} \right\}, \quad \frac{1}{\mathcal{R}} \mathcal{R}_{|4'} = -\frac{i}{\mathcal{R}}. \tag{4.2}$$

Again, the system $\{r',t'\}$ is comoving with the expansion of the cosmos. The field quantities for this system are analogous to (3.8)

$$'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}}\right\}, \quad B_{m'} = \left\{\frac{1}{r},0,0,-\frac{i}{\mathcal{R}}\right\}, \quad C_{m'} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0,-\frac{i}{\mathcal{R}}\right\}. \tag{4.3}$$

All three field quantities seem to be flat. Since the subluminal cosmos expands in freefall, the supposed flatness of space is due to the elevator effect.

One obtains the field quantities of the non-comoving system with a Lorentz transformation of the type (3.9). The relative velocity and the Lorentz factor are defined in the same way as in the dS cosmos

$$\alpha = 1\Big/\sqrt{1-r^2/\mathcal{R}^2}, \quad v = r/\mathcal{R}. \tag{4.4}$$

For the lateral field quantities B and C one obtains the expressions of the dS model, but for the radial field quantities the elevator equation is again responsible. In calculating the Lorentz term it has to be taken into account that v in (4.4) is now a function of time. With the elevator equation

$$U_m = 'U_m + L_m \tag{4.5}$$

and the Lorentz term

$$L_m = \{-i\alpha v,0,0,\alpha\}\, i\alpha^2 \frac{1}{\mathcal{R}} \tag{4.6}$$

one now gets

$$U_m = \hat{U}_m + f_m, \quad \hat{U}_m = \left\{-\alpha v \frac{1}{\mathcal{R}},0,0,0\right\}, \quad f_m = \left\{i\alpha^2 v\, \mathcal{R}^{\bullet}_4, 0,0,-i\alpha^2 v\, \mathcal{R}^{\bullet}_1\right\}. \tag{4.7}$$

Therein $\hat{U}$ is the dS expression for the radial forces, and f is an additional term which stems from the time dependence of the radius of curvature of the pseudo-hyper sphere. With $\mathcal{R} = \text{const}.$ one retrieves the dS model.

The Friedman equation for the subluminal model

$$'U^{s'}{}_{|s'} + 'U^{s'}\,'U_{s'} = 0 \tag{4.8}$$

is a subequation of Einstein's field equations. With $\partial_{4'} = \partial/i\partial T'$ and the proper time T' of the comoving observer one has

$$\frac{1}{\mathcal{R}}\mathcal{R}^{\bullet} - \frac{1}{\mathcal{R}} = 0, \quad \mathcal{R}^{\bullet} = 1, \quad \mathcal{R}^{\bullet\bullet} = 0. \tag{4.9}$$

The expansion in this model is constant. In a cosmos expanding in freefall comoving observers are not exposed to acceleration according to Einstein's elevator principle.

Let us consider this interesting result from a different perspective. If an observer does not perform an individual motion one has $\eta = \text{const}.$, η being the angle of ascent of the pseudo-hyper sphere. Differentiation of $r = \mathcal{R}\sin\eta$ leads to the Hubble equation

$$r^{\bullet} = \frac{1}{\mathcal{R}}\mathcal{R}^{\bullet} r = Hr. \tag{4.10}$$

At the equator $(r = \mathcal{R})$ of the pseudo-hyper sphere one has $r^{\bullet} = \mathcal{R}^{\bullet} = 1$ or in physical units

$$v = r^{\bullet} = c \,, \tag{4.11}$$

whereby the definition of the velocity $v = r/\mathcal{R} = \sin\eta$ has been taken into consideration. The expansion-related recession velocity of galaxies has the highest attainable value, the velocity of light at the equator. A galactic island formation is not possible in this model. This model has a horizon at

$$r_h = cT\,'. \tag{4.12}$$

No signal beyond the horizon can reach an observer at $\eta = 0$. Since all points on the hypersphere are equivalent, any observer at an arbitrary position in the universe has his individual horizon.

If one completely evaluates Einstein's field equations, one has for the pressure, the matter density, and the equation of state of the cosmos

$$\kappa p = -\frac{1}{\mathcal{R}^2}, \quad \kappa\mu_0 = \frac{3}{\mathcal{R}^2}, \quad p = -\frac{1}{3}\mu_0 \,. \tag{4.13}$$

Remarkably, these results are identical with those that Melia [18]-[25] derived from a model which he calls $R_h = ct$ model[3]. Melia derives his model in the comoving coordinate system, the underlying metric is of type $k = 0$ and is flat in accordance with the commonly accepted view. However, we are of the opinion that in the model of Melia also the elevator principle comes into effect. Due to $g_{4'4'} = 1$ it is apparent from the metric of Melia

$$\mathrm{d}s^2 = \mathcal{K}^2\left[\mathrm{d}r\,'^2 + r\,'^2\,\mathrm{d}\Omega^2\right] - \mathrm{d}t\,'^2 \tag{4.14}$$

that the universe described by this metric expands in freefall. Here, the scale factor is proportional to the cosmic time $t\,'$ according to the linear expansion of the universe. Therefore we put

$$\mathcal{K} = \frac{t\,'}{\mathcal{R}_0} \tag{4.15}$$

with $\mathcal{R}_0$ as a constant factor that can be put to 1 without loss of generality. From the above line element we read the 4-bein system

$$\overset{1'}{e}_{1'} = \mathcal{K}, \quad \overset{2'}{e}_{2'} = \mathcal{K}r\,' = r, \quad \overset{3'}{e}_{2'} = \mathcal{K}r\,'\sin\vartheta = r\sin\vartheta, \quad \overset{4'}{e}_{4'} = 1 \tag{4.16}$$

and we calculate from this the components of the Ricci-rotation coefficients

$${}'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}}\right\}, \quad B_{m'} = \left\{\frac{1}{r},0,0,-\frac{i}{\mathcal{R}}\right\}, \quad C_{m'} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0,-\frac{i}{\mathcal{R}}\right\}. \tag{4.17}$$

However, these quantities are identical with those of (4.3) which we have derived for our subluminal model. Thus, despite $k = 0$ it should be clear that Melia's model is not flat, but its basis is a pseudo-hyper sphere, *i.e.* the space-like part a positively curved closed space. This interpretation is convenient for the model because it avoids the need for explanation why an infinite homogeneous cosmos contains infinitely many stars and when and how an infinite number of stars have been created. Due to the identity of

[3]Melia's expression coincides with (4.12). Melia's coordinate time t corresponds to the proper time of the freely falling comoving system. This is referred to by us as $T\,'$.

both models our subluminal model undergoes a substantial support. Finally, Melia has shown [18]-[25] that the data found by observation are better matched to his model than to the standard model of cosmology. What is valid for the model of Melia, is also valid for our subluminal model. Five parameters must be adjusted for the standard model, so that the model fairly reproduces the observed data. This procedure, however, has certain arbitrariness. This relative freedom in the adjusting data is a consequence of the structure of the standard model: Starting from the pressure-free Friedman cosmos pressure is inserted by hand into the theory. The model thus obtained is not an exact solution of Einstein's field equations, leaving open the way for manipulation.

On the other hand the model of Melia is complemented by a full covariant field structure by our subluminal model. We have formulated the problem with field quantities which are closely related to the geometrical quantities and quite clearly reflect the prevailing conditions in the underlying pseudo-hyper sphere. We do not limit ourselves to discuss the Friedman equation, but we have presented the field equations and their subequations in a clear structure in both reference systems, the comoving and non-comoving systems. By this method one gains a deeper insight both into the physical and in the geometrical structures.

5. Conclusions

The standard model of cosmology allows galaxies to recede faster than light and make galactic island formation possible, *i.e.* without any causal connection and any exchange of information between galaxies. The standard model allows the fundamental laws of special and general relativity to be invalidated. The standard model is not an exact solution of Einstein's field equations: into the pressure-free Friedman cosmos pressure is inserted by hand. We have therefore raised the question as to whether it is possible to formulate a model that is an exact solution of Einstein's field equations, includes pressure, and respects the laws of special and general relativity. We have found the subluminal model, initially only with the intention to show the theoretical possibility of such a model without regard to conformity with astrophysical data.

Surprisingly, some of our results have been consistent with those of the $R_h = ct$ model of Melia. This model describes a flat and open cosmos with $k = 0$, while our subluminal model describes a positively curved and closed cosmos with $k = 1$. We have been able to show that the flatness of the Melia cosmos is only apparent, *i.e.* is caused by Einstein's elevator effect. Thus, both universes are identical and our subluminal model is supported by Melia's data, so ours is no longer a gedanken experiment, but a candidate for a model which can describe Nature sufficiently well. The introduction of Einstein's elevator principle into cosmology turns out to be important. Its legitimacy decides, whether we are living in an infinitely large universe with an infinite amount of stars or in a finite universe with a finite amount of stars.

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