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Einstein's Gravitational Field Approach to Dark Matter and Dark Energy

—Geometric Particle Decay into the Vacuum Energy Generating Higgs Boson and Heavy Quark Mass

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Abstract

During an interview at the Niels Bohr Institute David Bohm stated, "according to Einstein, particles should eventually emerge ... as singularities, or very strong regions of stable pulses of (the gravitational) field" [1]. Starting from this premise, we show spacetime, indeed, manifests stable pulses (*n-valued gravitons*) that decay into the vacuum energy to generate all three boson masses (*including Higgs*), as well as heavy-quark mass; and all in precise agreement with the 2010 CODATA report on fundamental constants. Furthermore, our relativized quantum physics approach (RQP) answers to the mystery surrounding dark energy, dark matter, accelerated spacetime, and why ordinary matter dominates over antimatter.

Keywords

Dark Energy, Dark Matter, Einstein, Higgs Particle, Geometric Particles, Fundamental Quanta, General Relativity, Bosons, Quarks, Mass Hierarchy Problem, Accelerated Spacetime, Standard Model of Particle Physics, Relativized Quantum Physics, RQP, Bohm, Consistency Condition

1. Einstein's Relativized Quantum Physics (RQP)

Though not well-known, Einstein endeavored much of his life to general-relativize quantum physics (RQP), rather than quantizing gravity [2]. Albeit he did not succeed, his legacy lives on. In this paper we continue on with Einstein's original idea that quanta manifests from the gravitational fields. To show this, we begin by re-imagining local flat spacetime geometry in an analogous way to the how Max Planck solved the blackbody radiation

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problem by means of light quanta absorption and emission. However, instead of a perfect absorber and emitter of light, we propose a background of fluctuating vacuum energy intermixed with gravitons. By the principle of general relativity, as vacuum energy fluctuates, it will induce graviton oscillation. Taken together, this is enough information to construct a modified flat-spacetime metric out of normal coordinates [3]. After a straightforward general relativistic calculation on the modified metric, a pair of covariant and contravariant energy momentum tensors emerge as n-valued raising and lowering operators. By assuming these RQP operators can be detached from flat spacetime, and then transported to curved spacetime (*without loss of their basic spin-like structure*); we show that these energy operators generate all three boson masses (*including the Higgs*) [4], as well as heavy-quark mass, and do so in precise agreement with 2010 CODATA [5]. RQP thus solves part of the mass hierarchal problem, which is unable to be done by the Standard Model of particle physics. Moreover, our RQP proposal provides a general relativistic explanation for the origin of dark matter, dark energy, as well as accelerated spacetime, and why ordinary matter dominates over antimatter.

2. Mathematical and Physical Development of RQP Spacetime Metric

Our strategy is to consider flat spacetime at the microscopic level, where vacuum energy fluctuations induce graviton oscillations. Under such a general relativistic scenario, no longer can flat spacetime be described by the Minkowski metric:

$$g_{\mu\nu} \neq \eta_{\mu\nu} \tag{1}$$

Instead, we construct a spacetime metric based on graviton oscillations induced by vacuum energy fluctuations. Our mathematical starting point is to apply the classical Lagrangian representing a vibrating system of particles about a point of equilibrium:

$$L = \frac{1}{2}\left(T_{ij}\dot{\eta}_i\dot{\eta}_j - V_{ij}\eta_i\eta_j\right) \tag{2}$$

The η_i's represent small deviations from the generalized coordinates q_{0i}, and they are expressed by the equation: $q_i = q_{0i} + \eta_i$. The η's subsequently become the generalized coordinates for the equations of motion, wherein the kinetic energy has only diagonal components:

$$T_i\ddot{\eta}_i - V_{ij}\eta_j = 0 \quad (\text{no sum over } i) \tag{3}$$

The solution to the previous equation has the form of normal coordinates [6] given by:

$$\eta_i = C_\kappa \mathrm{e}^{-i\omega_\kappa t} \tag{4}$$

Assuming these coordinates quasi-describe graviton oscillations, by the principle of equivalence, let the general relativistic coordinate system e_μ experience accelerations expressed by preceding equation. To simplify matters, and for purposes of clarification, let the coefficients C_κ be set equal to one, also let negative one-half be introduced into the natural exponent. These small changes allow for the physics—the motion and energy of gravitons undergoing oscillations—to become mathematically and physically apparent. Of notable importance, should the resulting covariant and contravariant energy momentum tensors (*calculated from the modified flat spacetime metric*), be in accordance with the vibrational motion of the gravitons we first imagined flat spacetime undergoing, it will further substantiate the RQP approach. After a straightforward, but lengthy, general relativistic calculation, this turns out to be the case, as we show in subsequent sections.

Continuing on, with construction of the modified spacetime metric, we apply Rayleigh's principle [7] [8] to the coordinate frequencies ω_κ to reduce them to their fundamental mode of oscillation ω having the greatest intensity. This implies the average kinetic energy $\langle T\rangle$ becomes equal to the average potential energy $\langle U\rangle$. Joining all of these normal coordinate ideas together, allows us to construct a basis for the general relativistic coordinate system; moreover it allows computation of the modified flat-spacetime-metric (*representing graviton oscillation*):

$$g_{\mu\nu} \equiv e_\mu \cdot e_\nu = \mathrm{e}^{i\omega t}\delta^\mu{}_\nu = \mathrm{e}^{i\omega t}\eta_{\mu\nu} \tag{5}$$

However, for the final version of the modified flat spacetime metric, we introduce the $\sqrt{n}$ into the natural exponent:

$$g_{\mu\nu} = e^{\sqrt{n}(i\omega)t}\eta_{\mu\nu} \quad n = 0,1,2,3,\cdots \tag{6}$$

At first we did not understand the importance of introducing the $\sqrt{n}$ into the spacetime metric; that occurred later when we observed the contravariant energy momentum tensor was undergoing cyclic complex phasing from real to imaginary energies, as a function of n. [It is noteworthy to mention that when n = 0, or when $\sqrt{n}(\omega t) = m\pi$ (*where m and n are natural numbers*), the modified flat-spacetime metric reduces to Minkowski, implying flat spacetime-nodes dynamically appear throughout the cosmos. RQP is thus understood to be a massless spacetime theory, joined discretely to a massive one with certain de Broglie wave-like characteristics.]

3. RQP Operators

After a straightforward general relativistic calculation, the following covariant and contravariant n-valued energy momentum tensor-operators were calculated directly from the modified flat spacetime metric [9]:

$$T_{\mu\nu} = \frac{nc^4}{16\pi G}\begin{pmatrix} -\frac{3}{2}\omega^2 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\omega^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\omega^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\omega^2 \end{pmatrix} \tag{7}$$

$$T^{\mu\nu} = \frac{n^2c^4}{16\pi G}\begin{pmatrix} -\frac{3}{2}\omega^2 & 0 & 0 & 0 \\ 0 & \frac{1}{2}\omega^2 & 0 & 0 \\ 0 & 0 & \frac{1}{2}\omega^2 & 0 \\ 0 & 0 & 0 & \frac{1}{2}\omega^2 \end{pmatrix} \tag{8}$$

The pair of energy-operators is related via the raising and lowering properties of the RQP spacetime metric:

$$T^{\mu\nu} = g^{\mu\alpha}g^{\nu\beta}T_{\alpha\beta} = e^{-2i\omega t}\eta^{\mu\alpha}\eta^{\nu\beta}T_{\alpha\beta} \tag{9}$$

Whereas the covariant energy momentum tensor generates only real energy, we see the contravariant energy-momentum-tensor is necessarily complex; furthermore it expresses cyclically phasing between real and imaginary energies. Since one tensor is real, while the other complex (*except at certain n values*), and that the covariant and contravariant energy momentum tensors are paired to act on the same spacetime point, in effect this causes spacetime to undergo constructive and destructive spacetime interference. Destructive interference results in antimatter canceling out with ordinary matter, except under very narrow constraints, e.g. antimatter tunneling. This is why ordinary matter dominates at the cosmic scales. It is also the reason why the contravariant tensor was made a function of n^2, instead of n.

We conclude this section by pointing out that the RQP operators obey the conservation of energy principle, via a divergence on the energy-operators:

$$T^{\mu\nu}{}_{;\nu} = T_{\mu\nu;\nu} = 0 \tag{10}$$

Historically, James Clerk Maxwell applied such a divergent consistency formulation to the four electromagnetic equations. Discovering the four equations were inconsistent, Maxwell altered Ampere's law so they would be consistent with each other. From the consistent electromagnetic equations, Maxwell answered one of the oldest questions posed by humankind—the nature of light. Today we understand conservation principles to be a powerful tool in legitimizing (*or negating*) proposed physical theories (as the gravitational version has for RQP); also such principles help in ascertaining new physical law and their properties [for further details on Maxwell's Consistency Condition, see **Appendix I**].

However equation (10) involves gravity, and thus required a modified divergence formulation, called the Maxwell-Fang Consistency Formulation (MFCC) [refer to **Appendix II** for details on the Maxwell-Fang Consistency Condition (MFCC) applied to our RQP theory].

We briefly mention here some advantages MFCC [10] has over numerous other approaches to particle physics. First, MFCC can be applied to both flat and curved spacetime. Moreover MFCC is able to generate the following (*and with less assumptions*):

a) Einstein's wave equation without the need for the principle of equivalence;

b) The Scherk and Schwarz dual string model, and show the torsional part developed in their theory, is not an inherent aspect of string theory as claimed);

c) It correctly reproduces the Callan, Coleman and Jackiw model;

d) It reproduces Yang Mills theory;

e) From MFCC several fundamental identities, essential to solving the aforementioned equations, were calculated;

f) The algorithmic structure of MFCC is conducive to the language of computer programming. Once MFCC is evolved into computational form, many tedious calculations can be performed to produce a quick, accurate result for any graviton-particle interaction. This would be extremely handy in proving or disproving numerous claims about spacetime and particle manifestations. For example, MFCC could explain why the predicted microscopic black holes particles have not been detected at those premiere particle accelerators;

g) Finally, MFCC has validated RQP; and so MFCC has helped to extend general relativity to include the Standard Model of Particle Physics.

4. RQP Operators Arranged into Vibrational Form

In this section we link the vibrational motion of flat spacetime gravitons, described by the modified flat-spacetime metric, to the energy modes of the energy momentum tensors calculated from this modified metric. If the said metric is in physical harmony with the RQP operators, it will further validate our RQP approach. Stated another way, by constructing a spacetime metric based on oscillating gravitons, and then operating on it with the general relativistic wave equation, we show below, the resulting energy momentum tensor describes vibrational energies; hence both the metric and operators describe the same physical phenomena in their own respective, harmonic way. To show this, we begin by defining the moment of inertia I, and angular frequency ω as follows:

$$I \equiv \frac{c^4}{8\pi G}\begin{bmatrix} \frac{-3}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & \frac{1}{2} \end{bmatrix}; \quad \tilde{\omega}^2 \equiv \begin{bmatrix} \omega^2 & 0 & 0 & 0 \\ 0 & \omega^2 & 0 & 0 \\ 0 & 0 & \omega^2 & 0 \\ 0 & 0 & 0 & \omega^2 \end{bmatrix} \tag{11}$$

Inserting these relations by into the RQP operators, they take on the following n-valued vibrational energy form given by:

$$T_{\mu\nu} \equiv \frac{n}{2} I \tilde{\omega}^2 \tag{12}$$

Likewise, the contravariant energy momentum operator becomes:

$$T^{\mu\nu} \equiv \frac{n^2}{2} I \tilde{\omega}^2 \tag{13}$$

These last two rotational kinetic energy equations support our initial assumption that spacetime is indeed comprised of oscillating gravitons.

5. Boson Mass Calculation

Now that the raising and lowering operators have been calculated, our continued strategy is one of application—to evolve the RQP energy-density tensors into mass generators for the purposes of calculating boson and heavy-quark mass. We start by detaching the RQP operators from their general relativistic wave equations, and transporting them to regions of very energetic spacetime, to a region where enough physical parameters are known to facilitate conversion of operator energy-densities into particle densities, and finally into graviton and heavy quark mass.

The most suitable spacetime candidate to aide in this conversion is inside the core of our Milky Way galaxy, where an abundance of n-valued gravitons will certainly have been excited into higher energy states—as represented by the time component of the RQP operators:

$$T_{00} = -\frac{3}{2}\frac{n\omega^2 c^4}{16\pi G}, \text{ and } T^{00} = -\frac{3}{2}\frac{n^2\omega^2 c^4}{16\pi G} \tag{14}$$

Rather than speculating on various graviton particle outcomes occurring within a galactic singularity (*multiverses*, *wormholes*, *entropy…etc.*), we simply assume the n-valued gravitons flip sign from negative to positive [11]. In effect, this causes the n-valued gravitons to change from attractive to repulsive, whereupon they immediately flux from the galactic core (*somewhat like Hawking radiation*) [12]-[16]. Assuming these graviton energy-states are unstable, then as the gravitons stream out into the vast reaches of the galaxy, at some point they will decay and release gravitational-quanta [17]. Should this quanta be the correct n-valued amount (*analogous to the photoelectric effect*), it will be absorbed into the vacuum energy to generate boson and heavy quark mass (*as-well-as the other Standard Model particles*). If the n-valued quanta are not the correct amount, then these quanta will flow on like dark-light, accumulate, and cause accelerated spacetimes.

The remainder of this section completes the dark matter model envisioned by RQP for the Milky Way galaxy. We then calculate ground-state graviton particle density number N_D, which we apply to the energy operators to form graviton mass generators. In the next section we then calculate heavy quark mass.

a) As evidenced by galactic rotation curves [18]-[22], dark matter is prevalent throughout all known spiral galaxies;

b) The fact that the dark halos are responsible for the observed galactic rotation curves [23]-[28], and that dark halos are spherically symmetric around spiral galaxies (*including our own Milk Way*), suggests the halo was formed from particles fluxing radially from the galactic core. We assume this is to be the case, and that these halo-forming particles are energized gravitons fluxing from the galactic core;

c) Moreover, because the dark halo has spherical boundaries, and that is assumed to be formed by gravitons fluxing from the core (*no other particle could escape from the singularity*), and that these flipped gravitons are traveling at speeds near that of light, if said gravitons are to become part of the dark halo, then some process other than gravity must be present to slow down these fast gravitons—or else they would sail beyond the limits of the Milky Way galaxy (*as some statistically do*). The simplest explanation for graviton bremsstrahlung, is by conservation of energy: as gravitational energy is released during graviton decay, gravitons must slow, or violate conservation of energy;

d) During this vast continual graviton decay, all-the-while energetic gravitons continue to stream from the galactic core. Though the details need to be worked out, nevertheless the outcome is the same; stasis is achieved between the flipped and the ground state gravitons. This is fortunate, for it allows us to compute the number density of ground-state gravitons—which we then apply to the energy operators, to convert them to boson and heavy quark mass generators;

e) To determine the needed number density N_D, we simply divide the observed dark matter energy density [29] [30], by the Goldhaber mass limit for a graviton [*note that a small complication arises*, *in that*, *both the dark matter density and graviton mass*, *have acceptable range limits*]. Depending on the method provided by Goldhaber and Nieto [31]-[34], graviton mass ranges from:

$$10^{-55}\,\text{kg} \rightarrow 10^{-69}\,\text{kg} \tag{15}$$

We take the average of the two extremes for the graviton mass: 1.0×10^{-58} kg/particle, and a ground-state graviton density of: 1.0×10^{-27} kg/m^3 (*Bovy and Tremaine and other report a larger average halo density of*: 5.36×10^{-21} kg/m^3 [35]. *However*, *our RQP halo density represents only the ground state gravitons*; *the ener-*

getic graviton contribution were "*sifted out*"). From this we get a reasonable graviton number density:

$$N_D = \frac{1.0\times10^{-27}\ \text{kg/m}^3}{1.0\times10^{-58}\ \text{kg/particle}} = 1.0\times10^{31}\ \text{part/m}^3 \tag{16}$$

Note that N_D could be precisely determined by working backwards from what follows; however, we continue forward with the N_D value to determine the energy per particle from the operators, then finally both the boson and heavy-quark mass per particle, as follows:

$$\frac{T_{00}}{N_D} = n\frac{\left(\frac{3}{2}\omega_g^{\ 2}\right)\left(\frac{c^4}{16\pi G}\right)}{1.00\times10^{31}} = nE = n\left(1.425801587\times10^{-11}\right)\text{J/particle}, \tag{17}$$

$$\frac{T^{00}}{N_D} = n^2\frac{\left(\frac{3}{2}\omega_g^{\ 2}\right)\left(\frac{c^4}{16\pi G}\right)}{1.00\times10^{31}} = n^2E = n\left(1.425801587\times10^{-11}\right)\text{J/particle} \tag{18}$$

where ω_g is the graviton angular frequency calculated in our precursor paper [36]:

$$\omega_g = 2\pi\nu_g = 2\pi\left(1.000000000\times10^{-12}\right)\text{s}^{-1}, \tag{19}$$

with $c = 299,792,458\ \text{m/s}$ and $G = 6.67428(67)\times10^{-11}$. Finally we convert the energy operators into mass generators:

$$m_B = n\frac{E}{c^2} = n\left(1.586418216\times10^{-28}\ \text{kg}\right) \tag{20}$$

$$m_B = n^2\frac{E}{c^2} = n^2\left(1.586418216\times10^{-28}\ \text{kg}\right) \tag{21}$$

f) Applying superposition to the two mass generators, we are able to generate the n-valued boson masses. We calculate the W-boson mass first, by setting $n = 7$, for the covariant mass, and $n = 30$ for the contravariant mass. This leads to the following bosonic results:

$$m_{B1} = n\frac{E}{c^2} = 7\left(1.586418216\times10^{-28}\ \text{kg}\right) = 1.110492751\times10^{-27}\ \text{kg} \tag{22}$$

The contravariant mass is calculated to be:

$$m_{B2} = n^2\frac{E}{c^2} = 30^2\left(1.586418216\times10^{-28}\ \text{kg}\right) = 1.427776395\times10^{-25}\ \text{kg} \tag{23}$$

Adding the generated masses together yields the theoretical W-boson mass value of:

$$\begin{aligned} M_W &= m_{W1} + m_{W2} \\ &= 1.110492751\times10^{-27}\ \text{kg} + 1.427776395\times10^{-25}\ \text{kg} = 1.438881318\times10^{-25}\ \text{kg} \end{aligned} \tag{24}$$

Comparing the theoretical W-boson mass to the empirically measured mass of 80.403(29) GeV/c^2, as reported in CODATA [37], (*converted to SI units via*: $1\ \text{GeV/c}^2 = 1.782661845(39)\times10^{-27}\ \text{kg}$), leads to a CODATA W-boson mass of: $1.433(32)\times10^{-25}\ \text{kg}$. Indeed, the theoretical and empirical W-boson mass value are in precise agreement.

Next we compute the Z^0 boson mass by setting $n = 1$ for the covariant operator and $n = 32$ for the contravariant operator.

$$m_{Z1} = n\frac{E}{c^2} = (1)\left(1.586418216\times10^{-28}\ \text{kg}\right) = 1.586418216\times10^{-28}\ \text{kg} \tag{25}$$

$$m_{Z2} = n^2\frac{E}{c^2} = (32)^2\left(1.586418216\times10^{-28}\ \text{kg}\right) = 1.626078671\times10^{-25}\ \text{kg} \tag{26}$$

Combining the n-valued masses, we have:

$$M_{Z_0} = m_{Z1} + m_{Z2} = 1.586418216\times10^{-28}\,\text{kg} + 1.6244992253^{-25}\,\text{kg} = 1.62607867\times10^{-25}\,\text{kg} \quad (27)$$

This mass value is in precise agreement with the CODATA (2013) report on the Z^0 boson mass of: 91.1876(21) GeV/c^2 (1.6255(66) × 10^{-25} kg). Again theoretical and empirical particle mass values are in strong agreement.

Finally, we compute the Higgs mass by selecting n = 32 for the covariant contribution, and n = 37 for the contravariant mass contribution:

$$m_{H1} = n\frac{E}{c^2} = (32)\left(1.586418216\times10^{-28}\,\text{kg}\right) = 5.076538291\times10^{-27}\,\text{kg} \quad (28)$$

The contravariant mass contribution is given by:

$$m_{H2} = n^2\frac{E}{c^2} = (37)^2\left(1.586418216\times10^{-28}\,\text{kg}\right) = 2.171806538\times10^{-25}\,\text{kg} \quad (29)$$

Combining these two masses via superposition of the two energy operators, we determine the theoretical mass value for the Higgs-boson:

$$M_{Z_0} = m_{Z1} + m_{Z2} = 1.586418216\times10^{-28}\,\text{kg} + 1.6244992253^{-25}\,\text{kg} = 1.62607867\times10^{-25}\,\text{kg} \quad (30)$$

We see the RQP mass result for the Higgs particle, is in precise agreement that reported by CERN of 126.0 GeV or 2.246153925 × 10^{-25} kg.

What we have just shown, is that from a modified flat spacetime metric, from which we calculated the energy momentum tensors turned operators, we are able to generate all three boson masses in precise agreement with experiment. Overall, this offers confirmation of the General Relativized Quantum Physics approach first envisioned by Einstein.

In the next section we extend the RQP approach to show it precisely generates the three heavy quark masses.

6. Heavy Quark Mass Calculation

By adhering to the same format in the preceding section, we now generate heavy quark masses from the covariant and contravariant particle mass generators:

$$m_{Q1} = n\frac{E}{c^2} = n\left(1.586418216\times10^{-28}\,\text{kg}\right) \quad (31)$$

$$m_{Q2} = n^2\frac{E}{c^2} = n^2\left(1.586418216\times10^{-28}\,\text{kg}\right) \quad (32)$$

The Top Quark mass is calculated by setting n = 9 for the covariant mass, and n = 44 for the contravariant mass:

$$m_{T1} = n\frac{E}{c^2} = 9\left(1.586418216\times10^{-28}\,\text{kg}\right) = 1.427776394\times10^{-27}\,\text{kg} \quad (33)$$

The contravariant mass is calculated to be:

$$m_{T2} = n^2\frac{E}{c^2} = 44^2\left(1.586418216\times10^{-28}\,\text{kg}\right) = 3.071305666\times10^{-25}\,\text{kg} \quad (34)$$

Superposition yields the theoretical Top Quark mass of:

$$M_{TQ} = m_{T1} + m_{T2} = 1.427776394\times10^{-27}\,\text{kg} + 3.071305666\times10^{-25}\,\text{kg} = 3.08558343\times10^{-25}\,\text{kg} \quad (35)$$

Converting the CODATA Top Quark mass value 173.07(29) GeV/c^2 to kilograms, leads to a Top Quark mass of: $3.085252855 \times 10^{-25}\,\text{kg}$. We immediately recognize the RQP theoretically calculated Top Quark mass is in precise agreement to the empirically determined Top Quark mass.

We next calculate Charm Quark mass with $n = 5$ for the covariant operator and $n = 32$ for the contravariant operator:

$$m_{C1} = n\frac{E}{c^2} = (5)\left(1.586418216 \times 10^{-28}\,\text{kg}\right) = 7.93209108 \times 10^{-28}\,\text{kg} \tag{36}$$

Setting $n = 32$ for the contravariant operator, we have:

$$m_{C2} = n^2\frac{E}{c^2} = (3)^2\left(1.586418216 \times 10^{-28}\,\text{kg}\right) = 1.427776394 \times 10^{-27}\,\text{kg} \tag{37}$$

Combining the two masses via superposition of the energy operator mass, we have:

$$\begin{aligned} M_{CQ} &= m_{C1} + m_{C2} \\ &= 0.793209108 \times 10^{-27}\,\text{kg} + 1.427776394 \times 10^{-27}\,\text{kg} = 2.2209855502 \times 10^{-27}\,\text{kg} \end{aligned} \tag{38}$$

Whereas, CODATA (2013) reports a Charm Quark mass of 1.275 GeV/c^2, or $M_{CQ} = 2.272893741(44) \times 10^{-27}$ kg. The empirically determined mass for the Charm Quark is recognized immediately to be in strong agreement with the theoretical result determined by RQP.

The Bottom Quark mass is determined by setting $n = 32$ for the covariant contribution and $n = 37$ for the contravariant mass contribution:

$$m_{B1} = n\frac{E}{c^2} = (11)\left(1.586418216 \times 10^{-28}\,\text{kg}\right) = 1.745060038 \times 10^{-27}\,\text{kg} \tag{39}$$

Setting $n = 6$ for the contravariant operator, we have:

$$m_{B2} = n^2\frac{E}{c^2} = (6)^2\left(1.586418216 \times 10^{-28}\,\text{kg}\right) = 5.711105578 \times 10^{-27}\,\text{kg} \tag{40}$$

Combining the two masses via superposition of the energy operator mass, we have:

$$\begin{aligned} M_{BQ} &= m_{B1} + m_{B2} \\ &= 1.745060038 \times 10^{-27}\,\text{kg} + 5.711105578 \times 10^{-27}\,\text{kg} = 7.456165616 \times 10^{-27}\,\text{kg} \end{aligned} \tag{41}$$

Bottom Quark has a mass of 4.18 GeV or 1 GeV/c^2 = 1.782661845(39) × 10^{-27} kg, the mass of a Bottom Quark is 7.451526148 × 10^{-27} kg, which shows Bottom Quark mass generated by RQP, is in precise agreement with the experimental determined values.

7. Conclusions

We have shown that our general relativized quantum physics approach RQP precisely generates all three boson masses, and three heavy-quark masses. By adjusting the approximated dark halo particle density, in the future we will be able to generate particle masses to even greater precision.

In closing, we hope to soon calculate the remaining Standard Model particle masses. Upon doing so we will have completely solved the mass hierarchal problem, which is unable to be done by the Standard Model of particle physics. This implies that quantum physics is a subset of general relativity, just as Einstein had imagined.

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References

[1] Interview with David Bohm at the Niels Bohr Institute in Copenhagen (1989)

[2] Lehmkuhl, D. Talk at MCMP. "Einstein's Approach to Quantum Mechanics". http://www.youtube.com/watch?v=zbsbc0MfdlE

[3] Christensen, W.J. (2007) *GERG*, **39**, 105-110. http://dx.doi.org/10.1007/s10714-006-0360-8

[4] Christensen, W.J. (2015) *Journal of Modern Physics*, **6**, 194-200. http://dx.doi.org/10.4236/jmp.2015.63025

[5] Mohr, P.J., Taylor, B.N. and Newell, D.B. (2012) *Reviews of Modern Physics*, **84**, 1527-1605. http://dx.doi.org/10.1103/RevModPhys.84.1527

[6] Goldstein, H., Poole, C. and Safko, J. (2002) Classical Mechanics. Addison Wesley, Boston, 250.

[7] Marion, J.B. and Thornton, S.T. (1988) Classical Dynamics of Particles and Systems. 3rd Edition, 458.

[8] Quick, R.M. and Miller, H.G. (1985) *Physical Review D*, **31**, 2682. http://dx.doi.org/10.1103/PhysRevD.31.2682

[9] Christensen, W.J. (2015) *Journal of Modern Physics*, **6**, 194-200. http://www.scirp.org/journal/jmp

[10] Fang, J., Christensen, W.J. and Nakashima, M.M. (1996) *Letters in Mathematical Physics*, **38**, 213-216. http://dx.doi.org/10.1007/BF00398322

[11] Grib, A.A. and Pavlov, Y.V. (2011) *International Journal of Modern Physics D*, **20**, 675-684. http://dx.doi.org/10.1142/S0218271811019013

[12] Williams, A.J. (2011) *Physical Review D*, **83**, Article ID: 123004. http://dx.doi.org/10.1103/PhysRevD.83.123004

[13] Harko, T. and Lobo, F.S.N. (2011) *Physical Review D*, **83**, Article ID: 124051.

Harko, T. and Lobo, F.S.N. (2013) Cosmological Anisotropy from Non-Comoving Dark Matter and Dark Energy. http://arxiv.org/abs/1304.0757

[14] Peebles, P.J.E. (2000) *The Astrophysical Journal*, **534**, L127.

[15] Corbel, S., Coriat, M., Brocksopp, C., Tzioumis, A.K., Fender, R.P., Tomsick, J.A., *et al.* (2013) *Monthly Notices of the Royal Astronomical Society*, **428**, 2500-2515. http://dx.doi.org/10.1093/mnras/sts215

[16] Hawking, S. (1974) *Nature*, **248**, 30-31. http://dx.doi.org/10.1038/248030a0

[17] Bertone, G., Fornasa, M., Taoso, M. and Zentner, A.R. (2009) *New Journal of Physics*, **11**, Article ID: 105016. http://dx.doi.org/10.1088/1367-2630/11/10/105016

[18] Bird, C., Kowalewski, R. and Pospelov, M. (2006) *Modern Physics Letters A*, **21**, 457-478. http://dx.doi.org/10.1142/S0217732306019852

[19] Falkowski, A. and No, J.M. (2013) *Journal of High Energy Physics*, **2013**, 34.

[20] Dreiner, H.K., Huck, M., Krämer, M., Schmeier, D. and Tattersall, J. (2013) *Physical Review D*, **87**, Article ID: 075015. http://dx.doi.org/10.1103/PhysRevD.87.075015

[21] Peter, A.H.G. (2010) *Physical Review D*, **81**, Article ID: 087301. http://dx.doi.org/10.1103/PhysRevD.81.087301

[22] Rix, H.-W. and Bovy, J. (2013) *The Astronomy and Astrophysics Review*, **21**, 61.

[23] Salucci, P., Lapi, A., Tonini, C., Gentile, G., Yegorova, I. and Klein, U. (2007) *Monthly Notices of the Royal Astronomical Society*, **378**, 41-47. http://dx.doi.org/10.1111/j.1365-2966.2007.11696.x

[24] Boylan-Kolchin, M., Springel, V., White, S.D.M. and Jenkins, A. (2010) *Monthly Notices of the Royal Astronomical Society*, **406**, 896.

[25] Einasto, J. (2010) Dark Matter. http://arxiv.org/abs/0901.0632

[26] Nestia, F. and Salucci, P. (2013) The Dark Matter Halo of the Milky Way, AD 2013. http://arxiv.org/abs/1304.5127

[27] Gnedin, O.Y., Brown, W.R., Geller, M.J. and Kenyon, S.J. (2010) *The Astrophysical Journal*, **720**, L108-L112. http://dx.doi.org/10.1088/2041-8205/720/1/L108

[28] Anderhalden, D., Schneider, A., Maccio, A.V., Diemand, J. and Bertone, G. (2013) *Journal of Cosmology and Astroparticle Physics*, **2013**, Article ID: 014.

[29] van der Marel, R.P., Fardal, M., Besla, G., Beaton, R.L., Sohn, S.T., Anderson, J., *et al.* (2012) *The Astrophysical Journal*, **753**, 8. http://dx.doi.org/10.1088/0004-637X/753/1/8

[30] Robotham, A.S.G., Baldry, I.K., Bland-Hawthorn, J., Driver, S.P., Loveday, J., Norberg, P., *et al.* (2012) *Monthly Notices of the Royal Astronomical Society*, **424**, 1448-1453. http://dx.doi.org/10.1111/j.1365-2966.2012.21332.x

[31] Goldhaber, A.S. and Nieto, M.M. (1974) *Physical Review D*, **9**, 1119-1121. http://dx.doi.org/10.1103/PhysRevD.9.1119

[32] Goldhaber, A.S. and Nieto, M.M. (2010) *Reviews of Modern Physics*, **82**, 939-979. http://dx.doi.org/10.1103/RevModPhys.82.939

[33] Baskaran, D., Polnarev, A.G., Pshirkov, M.S. and Postnov, K.A. (2008) *Physical Review D*, **78**, Article ID: 044018. http://dx.doi.org/10.1103/physrevd.78.044018

[34] Finn, L.S. and Sutton, P.J. (2002) *Physical Review D*, **65**, Article ID: 044022. http://dx.doi.org/10.1103/PhysRevD.65.044022

[35] Bovy, J. and Tremaine, S. (2012) *The Astrophysical Journal*, **756**, 89. http://dx.doi.org/10.1088/0004-637X/756/1/89

[36] Christensen, W.J. (2007) *General Relativity and Gravitation*, **39**, 105-110. http://dx.doi.org/10.1007/s10714-006-0360-8

[37] Mohr, P.J., Taylor, B.N. and Newell, D.B. (2012) Table 1.1. Reviewed 2013 by P.J. Mohr (NIST). Mainly from the CODATA Recommended Values of the Fundamental Physical Constants: 2010 by P. J. Mohr, B. N. Taylor, and D.B. Newell in Rev. Mod. Phys. 84, 1527 (2012). http://dx.doi.org/10.1103/RevModPhys.84.1527

[38] Maxwell, J.C. (1865) *Philosophical Transactions of the Royal Society of London*, **155**, 459-512. http://dx.doi.org/10.1098/rstl.1865.0008

[39] Huggins, E.R. (1962) Quantum Mechanics of the Interaction of Gravity with Electrons: Theory of Spin-Two Field Coupled to Energy. Ph.D. Dissertation, California Institute of Technology, Pasadena.

[40] Feynman, R. (1962-63) Lectures on Gravitation. California Institute of Technology, Pasadena.

[41] Fierz, M. and Pauli, W. (1939) *Proceedings of the Royal Society A*, **173**, 211-232. http://dx.doi.org/10.1098/rspa.1939.0140

[42] Rosen, N. (1940) *Physical Review*, **57**, 150-153. http://dx.doi.org/10.1103/PhysRev.57.150

[43] Gupta, S.N. (1952) *Proceedings of the Royal Society A*, **65**, 608.

[44] Gupta, S.N. (1954) *Physical Review*, **96**, 1683-1685. http://dx.doi.org/10.1103/PhysRev.96.1683

[45] Fang, J. and Fronsdal, C. (1979) *Journal of Mathematical Physics*, **20**, 2264-2271. http://dx.doi.org/10.1063/1.524007

[46] Pitts, J.B. (2006) *Journal of Physics*: *Conference Series*, **33**, 270-284. http://dx.doi.org/10.1088/1742-6596/33/1/031

[47] Padmanabhan, T. (2008) *International Journal of Modern Physics D*, **17**, 367-398. http://dx.doi.org/10.1142/S0218271808012085

[48] Bizdedea, C., Cioroianu, E.M., Danehkar, A., Iordache, M., Saliu, S.O. and Săraru, S.C. (2009) *The European Physical Journal C*, **63**, 491-519. http://dx.doi.org/10.1140/epjc/s10052-009-1105-0

[49] Butcher, L.M., Hobson, M. and Lasenby, A. (2009) *Physical Review D*, **80**, Article ID: 084014. http://dx.doi.org/10.1103/PhysRevD.80.084014

[50] Takook, M.V., Tanhayi, M.R. and Fatemi, S. (2010) *Journal of Mathematical Physics*, **51**, Article ID: 032503. http://dx.doi.org/10.1063/1.3321581

[51] Fronsdal, C. (1978) *Physical Review D*, **18**, 3624-3629. http://dx.doi.org/10.1103/PhysRevD.18.3624

[52] Ward, J.C. (1950) *Physical Review*, **78**, 182. http://dx.doi.org/10.1103/PhysRev.78.182

[53] Takahashi, Y. (1957) *Nuovo Cimento*, **6**, 370.

[54] Yoneya, T. (1974) *Progress of Theoretical Physics*, **51**, 1907-1920. http://dx.doi.org/10.1143/PTP.51.1907

[55] Callan, C.G., Coleman, S. and Jackiw, R. (1970) *Annals of Physics*, **59**, 42-73. http://dx.doi.org/10.1016/0003-4916(70)90394-5

Appendix I: Maxwell's Consistency Approach

Rather than starting with the four electromagnetic field equations, as Maxwell did [38], we begin from the modern approach of the action S, where:

$$S = \int L \mathrm{d}x^4 \tag{42}$$

Here, L represents the Lagrangian density for any system under consideration. Specifically, the Lagrangian for electromagnetic fields interacting with matter, and is given by:

$$L = L_{em} - j_\mu A_\mu \tag{43}$$

Identical indices are understood to be summed. The free part of the Lagrangian density (self-interaction) is given by:

$$L_{em} = -\frac{1}{16\pi} F_{\mu\nu} F_{\mu\nu} = -\frac{1}{8\pi} \left(A_{\mu,\nu} A_{\mu,\nu} - A_{\mu,\nu} A_{\nu,\mu} \right) \tag{44}$$

The interaction Lagrangian density term is given by:

$$L_{\text{int}} = -j_\mu A_\mu \tag{45}$$

[Note: Only partial derivatives are being used here. Also keep in mind the structure of the Lagrangian is a scalar product of the electromagnetic field A_μ with the electric current density j_μ. In a like manner our interaction Lagrangian for gravity under CC considerations will have the gravitational field coupled to its source.]

The Euler-Lagrange equation of motion, for an arbitrary field component ϕ, is given by[1]:

$$\frac{\delta L}{\delta \phi} = \frac{\partial L}{\partial \phi} - \left(\frac{\partial L}{\partial \phi_{,\mu}} \right)_{,\mu} = 0 \tag{46}$$

Substituting L_{em} into the Euler-Lagrange equation of motion yields the four well-tested Maxwell's electromagnetic equations:

$$j_{\mu,\mu} = \left(A_{\mu,\nu,\nu} - A_{\nu,\mu,\nu} \right)_{,\mu} = 0 \tag{47}$$

The above result is key to our understanding of how to build a consistent field theory for gravity. By examining the left hand side of this divergent equation, we see it is simply the equation of continuity and must physically, as well as mathematically, equal zero (*unless conservation of charge fails*). The right hand side of the equation is designed so that the terms A_μ, in the equation of motion, have zero divergence identically (this requirement will also be fundamental to the CC gravitational approach, and though a gauge approach may yield a similar result, it does not do so as viscerally and so we are not led to the correct theory for gravity and dark matter). Hence, we have shown the electromagnetic field equations are consistent.

Appendix II: Maxwell-Fang Consistency Approach

If we are to build any viable theory in physics, its foundation must be based on those successful traditions from both physics and mathematics. For example, the ideas for special relativity were established when young Einstein considered the consequences of James Clerk Maxwell's electromagnetic equations from various inertial coordinate frames. Likewise, to build a consistent theory for gravity, we again turn to the seminal work of James Clerk Maxwell, in which he demanded the divergence of the source for electromagnetism equal zero. That is: $j_{\mu,\mu} = 0$; which led to the discovery of the composition of light. By this successful example, we too will require the divergence of the source for gravity-the energy momentum tensor ($T_{\mu\nu}$) equals zero:

$$T_{\mu\nu,\nu} = 0 \tag{48}$$

However, a consistent theory for gravity is not as straightforward as it was for the electromagnetic equations. For one thing, gravity interacts with itself. Not only this, but the source for gravity includes a variety of other sources beyond current density, such as mass-energy, momentum and radiation (just to name a few). In short, Maxwell's consistent condition points us in the right direction, but needs modifications if we are to produce an

[1]This mathematics leading to a consistent approach for gravity was developed by E. Huggins in his dissertation see pages 8-10.

analogous theory for gravity. So what then is the next step? Since we are seeking a gravitational approach, our theory has to be constrained by Einstein's requirement that the gravitational field ultimately must have a geometrical interpretation. Fortunately, Feynman [39] showed the gravitational equations of motion, obtained from flat spacetime, from an action principle on the Lagrangian, is equivalent to Einstein's geometric requirement[2].

Developing a theory for pure gravity is one thing, but what do we do when gravity interacts with matter? In theory, one could build infinitely many Lagrangians, but which of them would represent real nature? Especially when there are so many particles that were not even known during the development of general relativity? The answer is to continue on with the modern approach to gravity through linearization of the gravitational field [40]; and to treat gravity as a spin-2 particle $h_{\mu\nu}$ (graviton). And that is just what occurred historically, until R. Feynman imposed that the interaction Lagrangian represent a coupling to gravity in the following manner:

$$L_{\text{int}} = -\frac{K}{2} h_{\mu\nu} T_{\mu\nu} \tag{49}$$

From the form of the coupling above, the gravitational field, $h_{\mu\nu}$ must be symmetric on $\mu\nu$. The scalar product of the anti-symmetric part of $h_{\mu\nu}$ with symmetric tensor $T_{\mu\nu}$ is then zero, implying that the anti-symmetric part of $h_{\mu\nu}$ would not couple to matter and therefore never be seen. The full Lagrangian for gravity proposed by Feynman is composed of the free part L_g, as well as the interaction Lagrangian. That is:

$$L = L_g - \frac{K}{2} h_{\mu\nu} T^m_{\mu\nu} \tag{50}$$

The Euler-Lagrange equations of motion for gravity are then given by:

$$\frac{\delta L}{\delta h_{\mu\nu}} = \frac{\delta L_g}{\delta h_{\mu\nu}} - \frac{K}{2} T^m_{\mu\nu} = 0 \tag{51}$$

To have the same type of consistency for the gravitational field equations, as with the electromagnetic field equations, Feynman demanded that the divergence of the free part of the Lagrangian equals zero; that is:

$$\partial_\nu \cdot \frac{\partial L_g}{\partial h_{\mu\nu}} = 0 \tag{52}$$

This automatically forces the divergence of interaction part of the Lagrangian to be zero; otherwise the total divergence of Einstein's field equations would not equal zero, implying we would lose any sense of conservation of the source. This leads us to the founding principle for gravity:

$$T^m_{\mu\nu,\nu} = 0 \tag{53}$$

In terms of the consistency formulation, the gravitational field equations must satisfy the condition:

$$\left\{ G_{\mu\nu} + T_{\mu\nu} \right\}_{;\nu} = 0 \tag{54}$$

Conceptually this gravitational condition on the field equations, is no different to the consistent formulation imposed on the electromagnetic equations by James Clerk Maxwell.

From the proceeding we develop a general consistent approach for gravity.

a. Here we provide additional information about the consistency formulation for particle fields interacting with gravity first developed by M. Fierz and W. Pauli in the 1930s [41]. Based on their seminal work, subsequent researchers (*all luminary*) contributed to the gravitational consistency formulation [42]-[50]. Then after R. Feynman work in this field in the 1960s, the consistency formulation was put into its final and complete form by J. Fang. In this paper we refer to this gravitational consistency formulation interacting with all particle fields, as the Maxwell-Fang consistency condition (MFCC). The main condition of Fang's consistency condition, is that the divergence of the wave equation must equal zero; not identically, nor by particular solutions, but rather by collecting various divergent terms into numerous forms of the wave equations, and then setting them all to zero. Only in this way can the correct Lagrangian (leading to a set of consistent set of field equations for gravity) be determined. Schematically, the consistency condition is written as a variation on the Lagrangian followed by a

[2]The Lagrangian approach to gravity first had its origin in curved spacetime and was introduced by the eminent mathematician David Hilbert, a few days after Einstein completed his wave equations for the general theory of relativity.

divergence on the field equations:

$$\frac{\delta L}{\delta \phi} = 0 \Rightarrow \partial \cdot \frac{\delta L}{\delta \phi} = 0 \tag{55}$$

To understand the basic idea behind the consistency condition formulation for linearized gravity, let us consider a system of coupled fields, Φ, under variation given by:

$$\frac{\delta L}{\delta \Phi} = 0 \tag{56}$$

Let the subset Φ' be that of the massless gauge fields. Such field equations can always be decomposed into linear (L) and nonlinear (NL) parts, in which variation of the fields, yields:

$$\left[\frac{\delta L}{\delta \Phi'}\right]^{L} + \left[\frac{\delta L}{\delta \Phi'}\right]^{NL} = 0 \tag{57}$$

For reasons previously given, the divergence of linear part of the wave equation must equal zero:

$$\partial \cdot \left[\frac{\delta L}{\delta \Phi'}\right]^{L} = 0 \tag{58}$$

The above condition, together with gauge invariance, imposes a strict requirement on the nonlinear parts, so that the divergence must also equal zero (*otherwise the field equations would be inconsistent mathematically*):

$$\partial \cdot \left[\frac{\delta L}{\delta \Phi'}\right]^{NL} = 0 \tag{59}$$

It was J. Fang, who first understood that the nonlinear divergence can go to zero in three distinct ways:

1) The divergences on the nonlinear parts of the massless gauge field equations are zero identically (*that is they vanish as mathematical identities--without referring to the field equations*). However, such an approach provides no physical content;

2) The divergences of the nonlinear parts of the massless gauge field equations are zero neither identically, but only analytically for some particular solutions (not general enough), which happen to satisfy the consistency condition:

$$\partial \cdot \left[\frac{\delta L}{\delta \Phi'}\right]^{NL} = 0 \tag{60}$$

3) The divergences of the nonlinear parts of the massless gauge field equations are equal to the sum of various terms, where each of these is proportional to the original field equations or various contracted field equations. Therefore, the divergence of the nonlinear parts of the massless gauge field equation vanish by field equations, and is expressed mathematically by:

$$\partial \cdot \left[\frac{\delta L}{\delta \Phi'}\right]^{NL} = A\frac{\delta L}{\delta \Phi} + B\left[\frac{\delta L}{\delta \Phi}\right] + C\left(\partial \cdot \frac{\delta L}{\delta \Phi}\right) + \cdots = 0 \tag{61}$$

where *A*, *B*, *C* are some structure functions for the field equations, or for various contracted field equations (trace, differentiation, etc.), which could depend on the field Φ itself, and derivatives of the fields.

Let us reexamine the three cases above more carefully: Case 1) is simply too strong a condition and must rejected because it would exclude many well established theories such as electrodynamics; SU(2) Yang-Mills gauge theory and Einstein's theory of gravitation. Case 2) is not general enough to provide criteria for constructing the Lagrangian describing gravity coupled to particles of various spin; and so case 2) is also rejected. Case 3) has just the right condition to allow one to build up the Lagrangian order-by-order in flat spacetime representing the gravitational system. This same criteria may also be applied to curved spacetime to determine if the chosen Lagrangian produces inconsistent field equations. Furthermore, case 3) has been shown to be compatible with other established theories, such as general relativity; Scherk-Schwarz dual model for string theory and Yang-Mills theory. Finally, it allows the massless gauge fields Φ' to maintain the freedom of the deformed gauge transformation:

$$\Phi' \to \Phi' + (\partial\chi)\Gamma \tag{62}$$

where χ stands for a set of gauge parameters, which are a function of x_μ. And Γ is some structure function. Thus the gauge is modified, but not lost.

Instead of using Case 3 in the given form and postulating the structure functions A, B, C, ⋯, it is more feasible, and equivalent, to use the consistency condition (CC) in the form:

$$\frac{\delta L}{\delta\Phi} = 0 \Rightarrow \partial\cdot\frac{\delta L}{\delta\Phi'} = 0 \tag{63}$$

where $\delta L/\delta\Phi' = 0$ represents the set of all field equations and $\delta L/\delta\Phi' = 0$ refers only to the subset of field equations for Φ' that are massless gauge fields. In this approach, the nonlinear part of the Lagrangian of the system $L(\Phi)$ is the yet-unknown Lagrangian to be determined by CC. Whereas the lowest order term in L(Φ) is the usual Fierz-Pauli type free-field Lagrangian constructed from the standard group theoretical approach [51].

Although other attempts at a consistency formulation are valid, without Fang's subtle understanding and completion of the consistency formulation, all previous attempts are incomplete and so lead to Lagrangians that, in general, inaccurately represent real nature.

Appendix III: MFCC Example

In this section we provide an example of how the Maxwell-Fang Consistency Condition (MFCC) can generate String Theory (Scherk and Schwarz), both on flat and curved spacetime.

How exactly then does a flat spacetime approach work to determine the Lagrangian for some unknown system of spin particles interacting with gravity? In particular, for the spin-0 (2-form) particles denoted by "a", interacting with gravitational spin-2 particles, denoted by "h", described by dual-string model Lagrangian. We begin by forming all possible combinations of the two fields up to third expanded order. Symbolically this is written as:

$$L(\Phi) = L^2(a) + L^2(h) + L(a^3) + L^3(a^2\cdot h) + L^3(h^3) \tag{64}$$

The metric is expanded as:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{65}$$

Here $\eta_{\mu\nu}$ is recognized as flat spacetime or Minkowski metric, and $h_{\mu\nu}$ represents the graviton field. Note, that any derivatives on the expanded metric $g_{\mu\nu}$ leaves only derivatives of $h_{\mu\nu}$. Variation on the linearized Lagrangian up to third order, with respect the spin-0 field "a" (two-form) and spin-2 fields "*h*", is given by:

$$\frac{\delta L}{\delta a_{\mu\nu}} = \frac{\delta L^2(a)}{\delta a_{\mu\nu}} + \frac{\delta L^3(a^2\cdot h)}{\delta a_{\mu\nu}} = 0 \tag{66}$$

$$\frac{\delta L}{\delta h_{\mu\nu}} = \frac{\delta L^2(h)}{\delta h_{\mu\nu}} + \frac{\delta L^3(a^2\cdot h)}{\delta h_{\mu\nu}} + \frac{\delta L^3(h^3)}{\delta h_{\mu\nu}} = 0 \tag{67}$$

The variation produces the linearized wave equation with both correct and incorrect terms. To get rid of the unwanted terms, or to show the Lagrangian is already correct, the next step of the consistency condition is to take the divergence of the wave equation. Then terms are grouped together in such a way as to form structures of the wave equation. By the consistency condition these grouped wave equations are set to zero. The remaining terms must cancel algebraically. If this occurs, then the order-by-order Lagrangian correctly represents gravity interacting with particles. If not, terms must be added to make this main condition of CC hold. This is exactly what was done with a general form of a Lagrangian representing spin-0 particles interacting with gravity. Note this is not yet the Scherk-Schwarz dual model Lagrangian. However, without prejudice, we discovered (*after many pages of calculations*), the dual-string Lagrangian was sifted out via CC. To show this, we choose for the sake of the reader to develop a schemata that indicates what occurs to various Lagrangian terms. Note that we use unspecified coefficients to produce the most general Lagrangian. Part of the consistency condition is to solve for these coefficients up to a single coefficient, which can only be determined experimentally. Put another way, mathematics alone cannot yield the values to fundamental constants.

The CC applied to spin-0 particles interacting with spin-2 graviton particles, is given in schemata form, where each interaction and free Lagrangian term $L(\Phi)=L^2(a)+L^2(h)+L(a^3)+L^3(a^2\cdot h)+L^3(a\cdot h^2)+L^3(h^3)+\cdots$ is shown under the CC term to either go to zero or not. If it goes to zero it is not a correct expanded Lagrangian term:

$$L(a^3)=a_1\left(a_{\mu\nu}a_{\lambda\nu,\mu}a_{\lambda\sigma,\sigma}\right)\xrightarrow{CC}0 \tag{68}$$

$$\begin{aligned}L(a^2\cdot h)=&\,a_2\left(ha_{\mu\nu,\nu}a_{\mu\lambda,\lambda}\right)+a_3\left(ha_{\mu\nu,\lambda}a_{\mu\nu,\lambda}\right)+a_4\left(ha_{\mu\nu,\lambda}a_{\lambda\nu,\mu}\right)+a_5\left(h_{\mu\nu}a_{\lambda\mu,\mu}a_{\lambda\sigma,\sigma}\right)\\&+a_6\left(h_{\mu\nu}a_{\nu\lambda,\lambda}a_{\mu\sigma,\sigma}\right)+a_7\left(h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\lambda,\sigma}\right)+a_8\left(h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\sigma,\lambda}\right)+a_9\left(h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\sigma\lambda,\mu}\right)\\&+a_{10}\left(h_{\mu\nu}a_{\lambda\sigma,\nu}a_{\lambda\sigma,\mu}\right)+a_{11}\left(h_{\lambda\sigma,\sigma}a_{\mu\nu}a_{\mu\nu,\lambda}\right)+a_{12}\left(h_{,\lambda}a_{\mu\nu}a_{\mu\nu,\lambda}\right)+a_{13}\left(h_{\lambda\sigma,\sigma}a_{\mu\nu}a_{\lambda\nu,\mu}\right)\\&+a_{14}\left(h_{,\lambda}a_{\mu\nu}a_{\lambda\nu,\mu}\right)+a_{15}\left(h_{\lambda\nu,\mu}a_{\mu\nu}a_{\lambda\sigma,\sigma}\right)+a_{16}\left(h_{\mu\lambda,\lambda}a_{\mu\nu}a_{\nu\sigma,\sigma}\right)+a_{17}\left(h_{\mu\lambda,\sigma}a_{\mu\nu}a_{\nu\lambda,\sigma}\right)\xrightarrow{CC}\end{aligned} \tag{69}$$

$$\begin{aligned}&\frac{1}{2}(a_{15}+a_{17})\left(ha_{\mu\nu,\nu}a_{\mu\lambda,\lambda}\right)-\frac{1}{2}(a_{10}+a_{14}+a_{15}+a_{17})\left(ha_{\mu\nu,\lambda}a_{\mu\nu,\lambda}\right)\\&+\frac{1}{2}(2a_{10}+2a_{14}+a_{15}+a_{17})\left(ha_{\mu\nu,\lambda}a_{\lambda\nu,\mu}\right)-a_{15}\left(h_{\mu\nu}a_{\lambda\mu,\mu}a_{\lambda\sigma,\sigma}+h_{\mu\nu}a_{\nu\lambda,\lambda}a_{\mu\sigma,\sigma}\right)\\&+(2a_{10}+a_{15})\left(h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\lambda,\sigma}\right)+a_{10}\left(-2h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\sigma,\lambda}-4h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\sigma\lambda,\mu}+h_{\mu\nu}a_{\lambda\sigma,\nu}a_{\lambda\sigma.\mu}\right)\\&-\frac{1}{2}(2a_{12}+a_{14}+a_{15}+a_{17})\left(h_{\lambda\sigma,\sigma}a_{\mu\nu}a_{\mu\nu,\lambda}\right)+a_{12}\left(h_{,\lambda}a_{\mu\nu}a_{\mu\nu,\lambda}\right)+(a_{15}+a_{17})\left(h_{\lambda\sigma,\sigma}a_{\mu\nu}a_{\lambda\nu,\mu}\right)\\&+a_{14}\left(h_{,\lambda}a_{\mu\nu}a_{\lambda\nu,\mu}\right)+a_{15}\left(h_{\lambda\nu,\mu}a_{\mu\nu}a_{\lambda\sigma,\sigma}\right)-a_{17}\left(h_{\mu\lambda,\lambda}a_{\mu\nu}a_{\nu\sigma,\sigma}-h_{\mu\lambda,\sigma}a_{\mu\nu}a_{\nu\lambda,\sigma}\right)\xrightarrow{F.R}\\&\cong 1-\text{parameter Lagrangian }(a_{10}\neq 0;a_{12}=a_{14}=a_{15}=a_{17}=0)\end{aligned}$$

Hence $L(a^2\cdot h)$ reduces to:

$$\begin{aligned}L(a^2\cdot h)=&-\frac{1}{2}(a_{10})\left(ha_{\mu\nu,\lambda}a_{\mu\nu,\lambda}\right)+(a_{10})\left(ha_{\mu\nu,\lambda}a_{\lambda\nu,\mu}\right)+(2a_{10})\left(h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\lambda,\sigma}\right)\\&+a_{10}\left(-2h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\mu\sigma,\lambda}-4h_{\mu\nu}a_{\nu\lambda,\sigma}a_{\sigma\lambda,\mu}+h_{\mu\nu}a_{\lambda\sigma,\nu}a_{\lambda\sigma.\mu}\right)\end{aligned} \tag{70}$$

Please note this third order Lagrangian term has been reduced to a single coefficient a_{10}, and is thus a valid Lagrangian term. The remaining third order interaction Lagrangian is examined under CC and field redefinition FR. The result shows this interaction Lagrangian is not allowed. Under CC schemata this is shown as follows:

$$\begin{aligned}L(a^2\cdot h)=&\,a_{18}\left(h_{,\mu}h_{\mu\nu,\lambda}a_{\lambda\nu}\right)+a_{19}\left(h_{\mu\nu,\nu}h_{\mu\lambda,\sigma}a_{\sigma\lambda}\right)+a_{20}\left(h_{\mu\lambda,\nu}h_{\mu\nu,\sigma}a_{\sigma\lambda}\right)+a_{21}\left(h_{\mu\lambda,\mu}h_{,\sigma}a_{\sigma\lambda}\right)\\&+a_{22}\left(h_{\mu\nu,\nu}h_{\mu\lambda}a_{\lambda\sigma,\sigma}\right)+a_{23}\left(h_{,\mu}h_{\mu\lambda}a_{\lambda\sigma,\sigma}\right)+a_{24}\left(h_{\mu\lambda,\mu}ha_{\lambda\sigma,\sigma}\right)+a_{25}\left(h_{\mu\lambda,\nu}h_{\mu\nu}a_{\lambda\sigma,\sigma}\right)\\&+a_{26}\left(h_{\mu\lambda,\mu}h_{\nu\sigma}a_{\sigma\lambda,\nu}\right)+a_{27}\left(h_{\mu\lambda,\nu}h_{\mu\sigma}a_{\sigma\lambda,\nu}\right)\xrightarrow{CC}\\&a_{18}\left(h_{,\mu}h_{\mu\nu,\lambda}a_{\lambda\nu}-h_{\mu\nu,\nu}h_{\mu\lambda,\sigma}a_{\sigma\lambda}+h_{\mu\nu,\nu}h_{\mu\lambda}a_{\lambda\sigma,\sigma}-h_{,\mu}h_{\mu\lambda}a_{\lambda\sigma,\sigma}+h_{\mu\lambda,\mu}h_{\nu\sigma}a_{\sigma\lambda,\nu}-h_{\mu\lambda,\nu}h_{\mu\sigma}a_{\sigma\lambda,\nu}\right)\xrightarrow{FR}0\end{aligned} \tag{71}$$

The consistency condition with field redefinition has revealed that $L(a\cdot h^2)$ does not partake in gravity interacting with spin-0 particles.

Finally, to clarify what is meant by field-redefinition (FR) in the schemata immediately above, is to say that two infinite series polynomial type Lagrangians: $L(\{a\}\cdot\Phi)$ and $L(\{b\}\cdot\Phi)$, having the same linear parts but each containing a different set of free parameters $\{a\}$ and $\{b\}$ specifying their non-linear parts, are regarded as completely equivalent if $L(\{a\}\cdot\Phi)$ transforms into $L(\{b\}\cdot\Phi)$ under field redefinition FR:

$$\Phi\to\Phi+\{c\cdot\Phi\cdot\Phi\}+\{\text{higher terms}\} \tag{72}$$

Here the polynomial $\{c\cdot\Phi\cdot\Phi\}$ contains terms from all possible combinations contracted out of any two fields in Φ, without derivatives. The most general form of FR for the case above is as follows:

$$h_{\mu\nu}\to h_{\mu\nu}+(c'\cdot h\cdot h)+c_1\left(a_{\mu\lambda}a_{\nu\lambda}\right)+c_2\left(\delta_{\mu\nu}a_{\lambda\sigma}a_{\lambda\sigma}\right)+c_3\left(h_{\mu\lambda}a_{\nu\lambda}+h_{\nu\lambda}a_{\mu\lambda}\right) \tag{73}$$

$$a_{\mu\nu} \to a_{\mu\nu} h + c_4 \left(a_{\mu\nu} h \right) + c_5 \left(a_{\mu\lambda} h_{\nu\lambda} - a_{\nu\lambda} h_{\mu\lambda} \right) \tag{74}$$

Dual tensors, like $\varepsilon^{\mu\nu\lambda\sigma} a_{\lambda\sigma} h$ and $\varepsilon^{\mu\nu\lambda\sigma}$ ($a_{\lambda\kappa} a_{\sigma\kappa} - a_{\sigma\kappa} h_{\lambda\kappa}$) are allowed for, but not included here because of their odd parity under space inversion.

Appendix IV: String Theory in Curved Spacetime MFCC Approach

In this section we show the Scherk-Schwarz Lagrangian satisfies the consistency condition in curved spacetime. This is a two-step procedure: 1) Variation of the Lagrangian with respect to each of the fields $\left\{ g_{\mu\nu}, a_{\mu\nu}, \phi \right\}$, thereby generating three distinct wave equations; 2) Apply the divergence to each of these wave equations, and show they equal zero; if so we have correct gravitational theory, just as we have shown with RQP theory.

Step 1) Rather making many tedious variational calculations with full covariant derivatives, we simply show the final result of the dual-string wave equations:

$$\frac{\delta L}{\delta a} = \left(a^{\beta\gamma;\alpha} + a^{\gamma\alpha;\beta} - a^{\beta\alpha;\gamma} \right) f_{;\alpha} + \left(a^{\beta\gamma;\alpha}{}_{;\alpha} + a^{\gamma\alpha;\beta}{}_{;\alpha} - a^{\beta\alpha;\gamma}{}_{;\alpha} \right) f = 0 \tag{75}$$

$$\frac{\delta L}{\delta \phi} = D_\nu \left(\partial^\nu \phi \right) - R \frac{\mathrm{d} f(\phi)}{\mathrm{d}\phi} - a F_{\alpha\beta\gamma} F^{\alpha\beta\gamma} \frac{\mathrm{d} f(\phi)}{\mathrm{d}\phi} = 0 \tag{76}$$

$$\begin{aligned} \frac{\delta L}{dg} = & \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right] + \frac{1}{2 f(\phi)} \left[\left(\partial^\mu \phi \right) \left(\partial^\nu \phi \right) - \frac{1}{2} g^{\mu\nu} \left(\partial_\lambda \phi \right) \left(\partial^\lambda \phi \right) - 2 g^{\mu\nu} f^{\lambda}_{;\lambda} - 2 g^{\mu\lambda} f^{\nu}_{;\lambda} \right] \\ & + a \left[-\frac{1}{2} g^{\mu\nu} F^2 + \frac{1}{2} a_{\beta\gamma}{}^{;\nu} a^{\beta\gamma;\mu} + \frac{1}{2} a^{\mu}_{\gamma;\alpha} a^{\nu\gamma;\alpha} + \frac{1}{2} a^{\nu}_{\beta;\alpha} a^{\beta\mu;\alpha} - a^{\beta\gamma;\nu} a^{\mu}_{\beta;\gamma} - a^{\beta\mu;\alpha} a_{\beta\alpha}{}^{;\nu} - a^{\nu\gamma;\alpha} a^{\mu}_{\alpha;\gamma} \right] = 0 \end{aligned} \tag{77}$$

The constant "a" equals 1/6 and F^2 is given by:

$$F^2 \equiv F_{\alpha\beta\gamma} F^{\alpha\beta\gamma} \tag{78}$$

Step 2) Apply the divergence to these wave equations and show they equal zero:

$$\left(\frac{\delta L}{dg} \right)_{;\nu} = \left[G_{\mu\nu} + T_{\mu\nu} \right]_{;\nu} = 0 \tag{79}$$

Note the two terms are the free part of the gravitational field and the interactions terms, which is also the energy momentum tensor. Let us now expand the right hand side to make more visible the divergence.

$$\begin{aligned} & \left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right]_{;\nu} + \left\{ \frac{1}{2 f(\phi)} \left[\left(\partial^\mu \phi \right) \left(\partial^\nu \phi \right) - \frac{1}{2} g^{\mu\nu} \left(\partial_\lambda \phi \right) \left(\partial^\lambda \phi \right) - 2 g^{\mu\nu} f^{\lambda}_{;\lambda} - 2 g^{\mu\lambda} f^{\nu}_{;\lambda} \right] \right. \\ & \left. + a \left[-\frac{1}{2} g^{\mu\nu} F^2 + \frac{1}{2} a_{\beta\gamma}{}^{;\nu} a^{\beta\gamma;\mu} + \frac{1}{2} a^{\mu}_{\gamma;\alpha} a^{\nu\gamma;\alpha} + \frac{1}{2} a^{\nu}_{\beta;\alpha} a^{\beta\mu;\alpha} - a^{\beta\gamma;\nu} a^{\mu}_{\beta;\gamma} - a^{\beta\mu;\alpha} a_{\beta\alpha}{}^{;\nu} - a^{\nu\gamma;\alpha} a^{\mu}_{\alpha;\gamma} \right] \right\}_{;\nu} \end{aligned} \tag{80}$$

By analogy to the electromagnetic equation, the divergence of the free part of the wave equation must vanish, hence:

$$\left[R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R \right]_{;\nu} = 0 \tag{81}$$

What is then left is to show that the divergence of the interaction part of the wave equation, vanishes as well:

$$T^{\mu\nu}{}_{;\nu} = 0 \tag{82}$$

For the Scherk-Schwarz dual model, the divergence of the interaction terms (which are really the energy momentum tensor) must vanish. Hence:

$$\begin{aligned} \left(T^{\mu\nu} \right)_{;\nu} = & \left\{ \frac{1}{2 f(\phi)} \left[\left(\partial^\mu \phi \right) \left(\partial^\nu \phi \right) - \frac{1}{2} g^{\mu\nu} \left(\partial_\lambda \phi \right) \left(\partial^\lambda \phi \right) - 2 g^{\mu\nu} f^{\lambda}_{;\lambda} - 2 g^{\mu\lambda} f^{\nu}_{;\lambda} \right] \right. \\ & \left. + a \left[-\frac{1}{2} g^{\mu\nu} F^2 + \frac{1}{2} a_{\beta\gamma}{}^{;\nu} a^{\beta\gamma;\mu} + \frac{1}{2} a^{\mu}_{\gamma;\alpha} a^{\nu\gamma;\alpha} + \frac{1}{2} a^{\nu}_{\beta;\alpha} a^{\beta\mu;\alpha} - a^{\beta\gamma;\nu} a^{\mu}_{\beta;\gamma} - a^{\beta\mu;\alpha} a_{\beta\alpha}{}^{;\nu} - a^{\nu\gamma;\alpha} a^{\mu}_{\alpha;\gamma} \right] \right\}_{;\nu} = 0 \end{aligned} \tag{83}$$

After bringing in the covariant derivative with respect to "ν," and then cancelling hundreds of terms algebraically, as well as setting divergent structures of the wave equation to zero (main CC requirement to be given further discussion), we discover, indeed, the divergence of the interaction terms, that is, the divergence of the energy momentum tensor for the Scherk-Schwarz string theory, does indeed go to zero. Then, by the same assumption made by J. Maxwell, R. Feynman, and then J. Fang, the Scherk-Schwarz Lagrangian representing spin-0 particles interacting with gravity produces consistent field equations. This means the dual string model is a viable representation of nature; however, nature is the final judge.

Appendix V: Identities from the Consistency Formulation

During the development of QED, at first there were many infinities that could not be mathematically accounted for until Feynman absorbed some of these infinities into the mass and charge. Nevertheless it was not until the Ward-Takahashi identity [52] [53] was developed, did QED become a renormalizable theory. The Ward-Takahashi identity is a quantum version of the classical Noether's theorem. Recall previously in this paper, Yoneya [54] discovered that all the known string theories included a massless spin-two particle, which obeyed the correct Ward identities. Other examples exist, such as the identities associated QED and Einstein's gravity, respectively:

$$\left(\bar{\psi}\gamma_{\mu}\psi\right)_{,\mu} \equiv \bar{\psi}\left(\vec{\partial}+im+ieA\right)\psi+\bar{\psi}\left(\bar{\partial}-im-ieA\right)\psi \tag{84}$$

And

$$G_{\mu\nu,} \equiv \Gamma^{\sigma}_{\mu\nu}G_{\sigma\nu}\Gamma^{\sigma}_{\nu\nu}G_{\mu\sigma} \tag{85}$$

In our paper the consistency condition produces to new identities. One identity associated with the Callan-Coleman-Jackiw Lagrangian corresponding to the massless gauge field $h_{\mu\nu}$. And two identities associated with the Scherk-Schwartz Lagrangian corresponding to the two fields $h_{\mu\nu}$ and $a_{\mu\nu}$. For the Callan-Coleman-Jackiw Lagrangian we define the following symbolism in order to present the associated identity in compact notation:

$$L\phi \equiv g^{\mu\nu}\left(\phi_{,\nu}\right)_{;\mu} - 2aR\phi \tag{86}$$

The field equations generated from this Lagrangian are given as follows:

$$\left(G+T\right)^{\mu\nu} = 0 \tag{87}$$

$$L\phi = 0 \tag{88}$$

And the energy momentum tensor is given by:

$$T^{\mu\nu} = \frac{1}{2f\left(\phi\right)}\left[\left(\partial^{\mu}\phi\right)\left(\partial^{\nu}\phi\right)-\frac{1}{2}g^{\mu\nu}\left(\partial\phi\right)\cdot g\cdot\left(\partial\phi\right)+2g^{\mu\lambda}f^{\nu}_{;\lambda}-2g^{\mu\nu}f^{\lambda}_{;\lambda}\right] \tag{89}$$

$$f\left(\phi\right) = 1+a\phi^{2} \tag{90}$$

$$f^{\lambda} = \partial^{\lambda} f \tag{91}$$

After a lengthy calculation the generalized Fang identities for the Callan-Coleman-Jackiw Lagrangian [55] is given as:

$$\left(G+T\right)^{\mu\nu}_{\ \ ;\nu} \equiv \frac{1}{2\left(1+a\phi^{2}\right)}\left[\left(\partial^{\mu}\phi\right)\left(L\phi\right)-\left(4a\phi\right)\left(\partial_{\nu}\phi\right)\left(G+T\right)^{\mu\nu}\right] \tag{92}$$

Note that all the terms on the right-hand side of the identity involve the field equations.

For the Scherk-Schwarz Lagrangian again we define the following symbolism:

$$\left(La\right)^{\mu\nu} = a^{\mu\nu;\alpha}{}_{;\alpha} + a^{\nu\alpha;\mu}{}_{;\alpha} - a^{\mu\alpha;\nu}{}_{;\alpha} \tag{93}$$

The Scherk-Schwarz field equations are as follows:

$$(G+T)^{\mu\nu} = 0 \tag{94}$$

And

$$(La)^{\mu\nu} = 0 \tag{95}$$

The Fang current density for the Scherk-Schwarz Lagrangian is given by:

$$(La)^{\mu\nu}{}_{;\nu} \equiv 0 \tag{96}$$

The final identity is:

$$(G+T)^{\mu\nu}{}_{;\nu} \equiv 16\pi G\left(a_{\nu}{}^{\mu}{}_{;\sigma} - \frac{1}{2}a_{\nu\sigma}{}^{;\mu}\right)(La)^{\nu\sigma} \tag{97}$$