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Stress Intensity Factors in Two Bonded Elastic Layers Containing Crack Perpendicular on the Interface with Different Elastic Properties

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Abstract

Thin bonded films have many applications (*i.e.* in information storage and processing systems, and etc.). In many cases, thin bonded films are in a state of residual tension, which can lead to film cracking and crack extension in one layer often accompanies failure in whole systems. In this paper, we analyze a channel crack advanced throughout thickness of an elastic thin film bonded to a dissimilar semi-infinite substrate material via finite element method (FEM). In order to simplify modeling, the problem is idealized as plane strain and a two-dimensional model of a film bonded to an elastic substrate is proposed for simulating channel crack in thin elastic film. Film is modeled by common 4-node and substrate by infinite 4-node meshes. The stress intensity factor (SIF) for cracked thin film has been obtained as a function of elastic mismatch between the substrate and the film. The results indicate that in elastic mismatch state, SIF is more than match state. On the other hand, mismatch state is more sensitive to crack than match state. And SIF has also increased by increasing Young's modulus and Poisson ratio of film.

Keywords

Thin Film, Channeling Crack, Infinite Element, SIF

1. Introduction

Many modern materials and material systems are layered. The potential applications of fracture mechanics of

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layered materials range over a broad spectrum of problem areas; included are: protective coating, multilayer capacitors, thin film/substrate systems for electronic packages, layered structural composites of many varieties, reaction product layers, and adhesive joints [1]. Many applications in microelectronics (e.g., interconnects and electronic packaging) often involve integrated structures with dissimilar materials. Stresses are introduced during the processes of fabrication, reliability testing, and operation. The stress field concentrates at the junctions of dissimilar materials, at the corners, or, if there exists a crack, at the crack tip [2]. In all of these applications, the films are very thin, with thicknesses measured in nanometers or micrometers, and they are bonded to comparatively thick substrates, with thicknesses typically measured in millimeters or centimeters.

Many cracking patterns in film-substrate systems have been observed and analyzed [1] [3]. A crack nucleates from a flaw either in the film or at the edge, and propagates both towards to interface and laterally through the film. Depending on the material, the crack may stop at the interface (**Figure 1(a)**), penetrate into the substrate (**Figure 1(b)**), or bifurcate onto the interface (**Figure 1(c)**) [4].

Irwin [5] claims that the stress field in the vicinity of a crack tip can adequately be defined by a single parameter proportional to the SIF. When the intensity of the local tensile stresses at the crack tip attains a critical value, a previously stationary or slow-moving crack propagates rapidly. This critical value defines the "fracture toughness" and it is a constant for a particular material. If the size of the hugest flaw in a particular structure is known, minimum toughness standards can be established for the materials in this structure. In the application of most of the current fracture criteria, the SIF and the crack opening displacement are the mostly used quantities [6].

The objective of this study is investigating sensitivity of two bonded elastic layers to a single crack perpendicular to the interface between film and substrate. This paper also has considered different elastic ratio of film and substrate. Because there are similar works in the literature (The problem of a crack perpendicular to the interfaces may be found in [7]-[11]), the main emphasis here is on using infinite meshes to simulating substrate (to be close to real problems) and also presenting results in different form, *i.e*. plotting SIF versus elastic properties instead of Dundurs' parameters [12], to have better understanding.

2. Background of Analytical Methods

In this section, we first give an overview of the fracture mechanics modes and then previous analytical works of SIF on both homogeneous and layered systems.

Three linearly independent cracking modes are used in fracture mechanics. These load types are categorized as Mode I, II, or III as shown in the **Figure 2**. Mode I, shown to the left, is an opening (tensile) mode where the crack surfaces move directly apart. Mode II is a sliding (in-plane shear) mode where the crack surfaces slide over one another in a direction perpendicular to the leading edge of the crack. Mode III is a tearing (anti-plane shear) mode where the crack surfaces move relative to one another and parallel to the leading edge of the crack. Mode I is the most common and important load type encountered in engineering design [13].

2.1. SIF in Single Edge Notched Tension Specimen

The SIF equation for a single edge notch and homogeneous properties in an infinite specimen is [14],

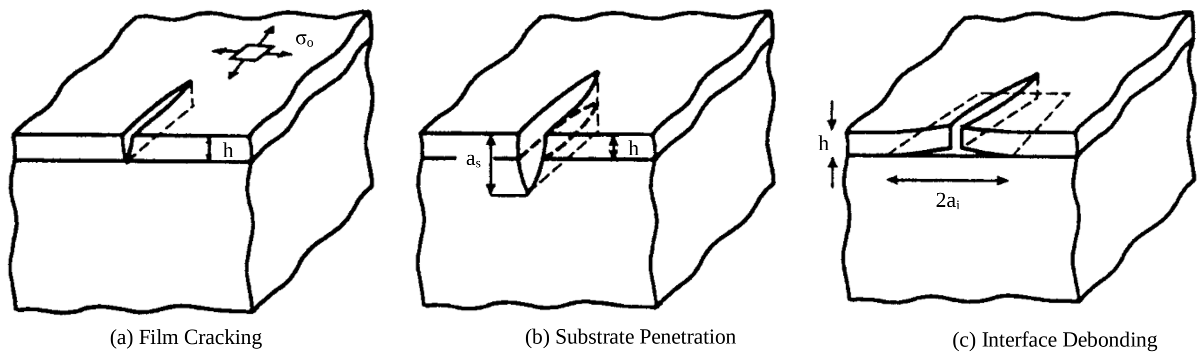

Figure 1. (a) A channeling crack within a thin film; (b) A channeling crack penetrating substrate; (c) A channeling crack with interface debonding [4].

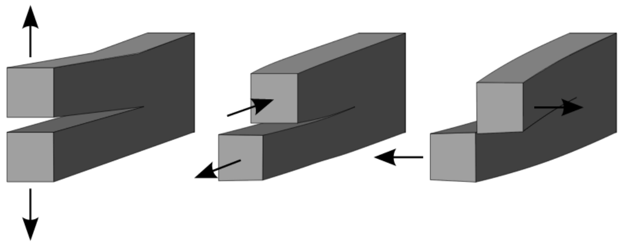

Figure 2. Mode I, Mode II, and Mode III crack loading [13].

$$K = \sigma ZY\sqrt{a} \tag{1}$$

where,

$$ZY = \frac{\sqrt{\pi}\left(1+2\left(\frac{a}{w}\right)\right)}{\left(1-\left(\frac{a}{w}\right)\right)^{\frac{1}{2}}} \times V \tag{2}$$

where,

$$\begin{aligned} V = {} & 1.12078 - 3.68220\left(\frac{a}{w}\right) + 11.95434\left(\frac{a}{w}\right)^2 - 25.85210\left(\frac{a}{w}\right)^3 \\ & + 33.09762\left(\frac{a}{w}\right)^4 - 22.4422\left(\frac{a}{w}\right)^5 + 6.17836\left(\frac{a}{w}\right)^6 \end{aligned} \tag{3}$$

Range of applicability of this equation: The defect depth, a, should be less than the specimen width, w, [14]. For different a/w ratio it has plotted in **Figure 3**.

2.2. SIF for Two Bonded Layers by Fully Cracked Film

Figure 4 shows a crack channeling through a pre-tensioned film on a semi-infinite substrate. The crack is confined by the film/substrate interface in the direction perpendicular to the interface.

For the fully cracked film problem, with its crack tip at the interface (**Figure 4**), the K_I is as the following form [15],

$$K_I = \sigma f_{(\alpha,\beta)}(\pi h)^s \tag{4}$$

where $f(\alpha,\beta)$ is a non-dimensionalized SIF and a function of Dundurs' (His work shows that for any problem of a composite body made of two isotropic, elastic materials with prescribed tractions, the material dependence of the problem is reduced from three dimensionless parameters to the two "Dundurs parameters" α and β) parameters.

For plane strain problems α and β are given by [12];

$$\alpha = \frac{\overline{E}_f - \overline{E}_s}{\overline{E}_f + \overline{E}_s} \tag{5}$$

$$\beta = \frac{1}{4}\frac{\overline{E}_f(1-\nu_f)(1-2\nu_s) - \overline{E}_s(1-\nu_s)(1-2\nu_f)}{\overline{E}_f(1-\nu_f)(1-2\nu_s) + \overline{E}_s(1-\nu_s)(1-2\nu_f)} \tag{6}$$

where $\overline{E} = E/1-\nu^2$, Furthermore, the compilation by Suga *et al*. [16] indicates that for most practical material

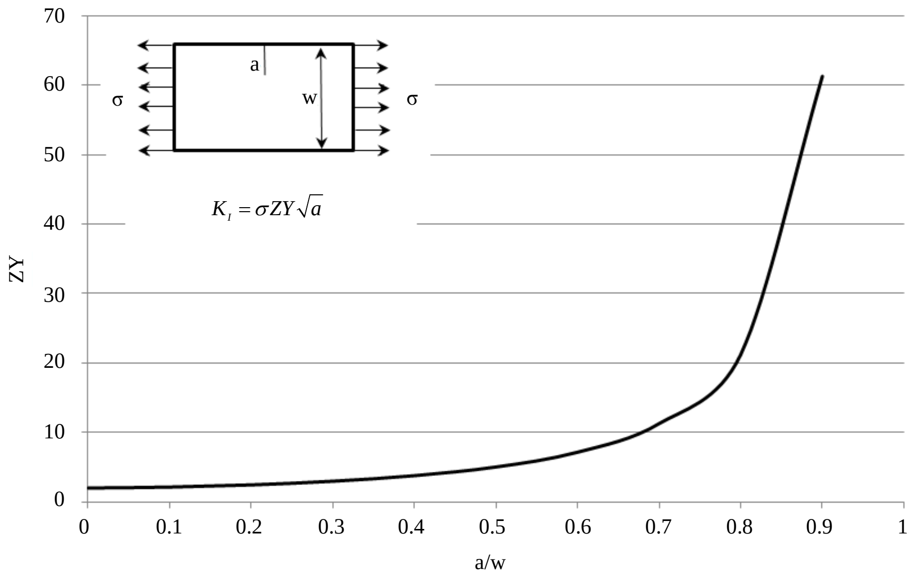

Figure 3. Plot of ZY (nondimensionalized SIF) vs the variation of a/w for plane strain condition the fully cracked film.

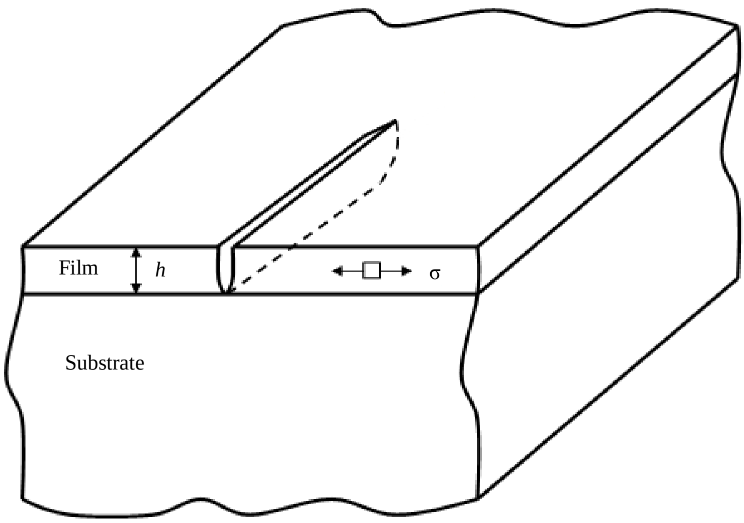

Figure 4. Steady-state crack channeling across the film for.

combinations, values of a typically lie between $\beta = 0$ and $\beta = \alpha/4$. The stress singularity exponent, s in Equation 7, is a function of α and β, too, and satisfies the following equation derived by Zak and Williams [17];

$$\cos(s\pi) - 2\frac{\alpha-\beta}{1-\beta}(1-s)^2 + \frac{\alpha-\beta^2}{1-\beta^2} = 0 \tag{7}$$

Values of s as a function of α for $\beta = 0$ and $\beta = a/4$ are plotted in **Figure 5**.

3. Finite Element Simulation

Consider a composite consisting of an infinite layer of width h and a half space (**Figure 6**). The half space can be assumed to approximate a semi-infinite substrate with average material constants as well as a homogeneous substrate. The layer is perfectly bonded to the half space (*i.e.* the bonding agent is neglected). There is a transverse crack in the layer. The film is subject to a uniform tensile stress σ and the substrate is stress-free (**Figure 6**).

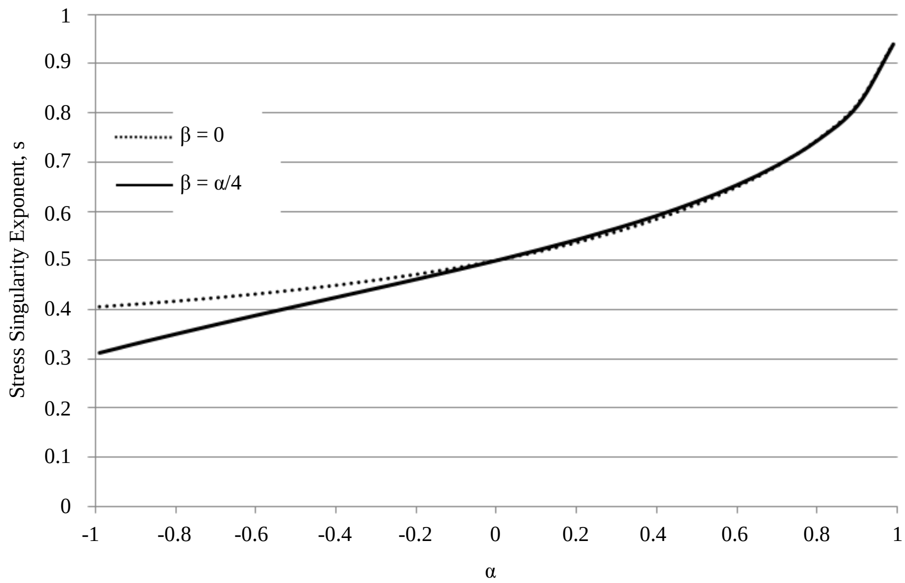

Figure 5. Plot of crack tip singularity exponent, s, vs α for $\beta = 0$ and $\beta = \alpha/4$.

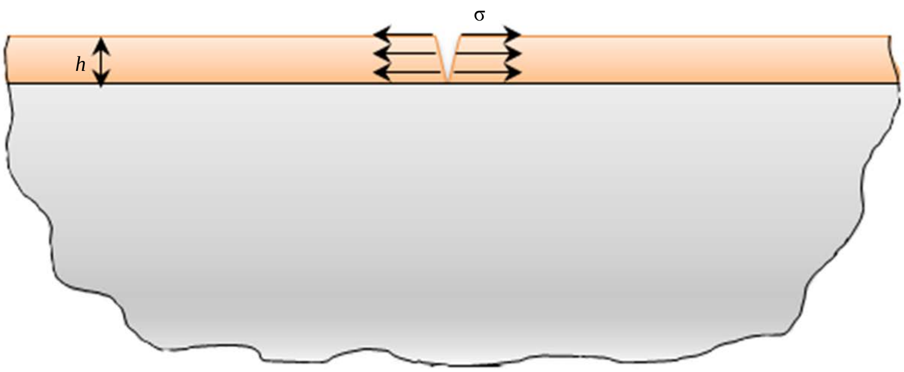

Figure 6. Fully cracked film under tensile stress.

Figure 7 shows the geometry and the boundary conditions of the plane-strain problem. The crack is represented by the line CD. The thickness of the film is h. The substrate has an infinite thickness. The model is fully fixed along AB. The vertical boundary EF is subjected to an initial tensile stress (σ = 1 Pa) and other boundaries are traction free. At equilibrium, the film and the substrate deform so that the tractions along the crack faces vanish and the crack opens. For each set of material properties of the film and the substrate, solutions were sought with various values of E_f/E_s and v_f/v_s in order to obtain the asymptotic solution for an isolated single crack with a semi-infinite substrate.

The finite element meshes are generated as follows. First divide the whole domain into two regions, as indicated in **Figure 8**. In the upper region, the one with the crack, a uniform mesh (number: 101 * 11) is generated with the plane strain solid continuum four-node bilinear quadrilateral elements (CPE4R). In the lower region, semi-infinite substrate, the mesh (number: 101 * 1) is generated with the plane strain solid continuum infinite four-node linear quadrilateral elements (CINPE4). The meshes of the regions are compatible in their intersection, and also alignment of the crack with the elements is convenient for the computation of the opening displacement.

4. Results and Discussion

For the two-dimensional analysis, the two type of SIF (K_{I} and K_{II}) are related to the energy release rate, G, as follow [18],

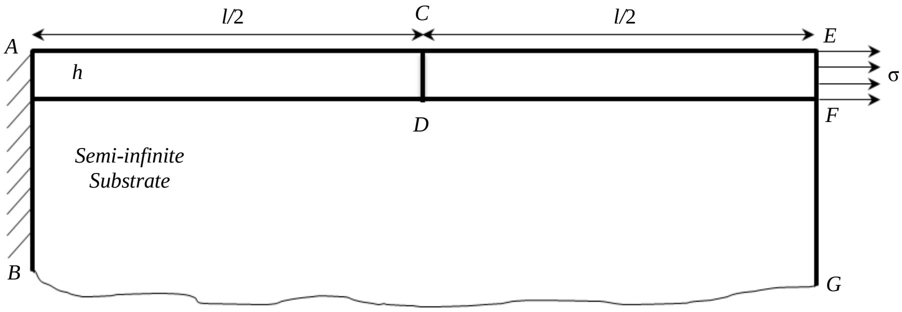

Figure 7. The FEM model of the plane-strain problem: geometry and boundary conditions.

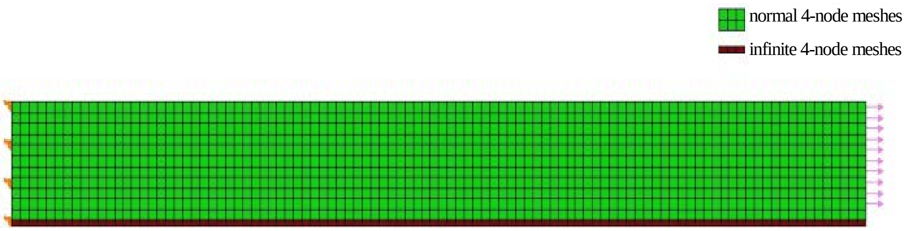

Figure 8. The FEM model of the plane-strain problem:assigning mesh to the film and the semi-infinite substrate.

$$G = \frac{1}{\overline{E}}\left(K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2\right) \tag{8}$$

In the previous studies of cracking in thin films (e.g., [1]), a unifying dimensionless number Z has been defined such that the energy release rate for a crack is,

$$G = Z\frac{\sigma_0^2 h}{\overline{E}} \tag{9}$$

where $\overline{E}$ is the plane-strain modulus of the film. The number Z is a dimensionless driving force, depending on the cracking pattern. Huang *et al*. [2], modeled dimensionless energy release rate of channeling cracks by XFEM and obtained that energy release rate has increased by increasing α.

For the channeling crack in the present study, the first type of SIF (K_{I}) of a two bonded elastic layers was calculated using finite element method. Different Poisson's ratios, v_f/v_s, of 0.5, 0.9, 1, 2, 3, 4, 5 and the elastic modulus ratios, E_f/E_s, of 0.1, 0.2, 0.3, ⋯, 8, 9, 10 were choosing for calculation, because all different materials can be located in this range.

The variation of the SIF for different elastic ratios is presented in **Figure 9**, **Figure 10**. It can be seen, the change of the K_{I} value for different modulus ratios decreases by decreasing Poisson's rations. In the case of no elastic mismatch ($\alpha = \beta = 0$), the stress singularity reduces to the square root singularity of a crack tip in a homogeneous elastic material, *i.e.* $s = 0.5$ (Equation (7)), and K_{I} has the minimum values (**Figure 9**). When the substrate is stiffer than the film ($\alpha < 0$), the singularity is weaker, *i.e.* $s < 0.5$, and K_{I} values are lower. When the substrate is more compliant than the film ($\alpha > 0$), the singularity is stronger, *i.e.* $s > 0.5$, and K_{I} values are higher. For an extremely compliant substrate ($\alpha \to 1$), the singularity exponent approaches $1 (s \to 1)$ and K_{I} has the maximum value.

The results can be tabulated in **Table 1**.

5. Conclusion

The infinite elements, for an elastic fracture mechanic problem, have been used to characterize the cracking of thin films bonded to thick substrate materials. The SIF has been extracted from the simulations. The SIF of the plane-strain problem depends on the elastic mismatch between the film and the substrate. The result demonstrates that the infinite elements can be applied to model problems with different elastic properties of films and

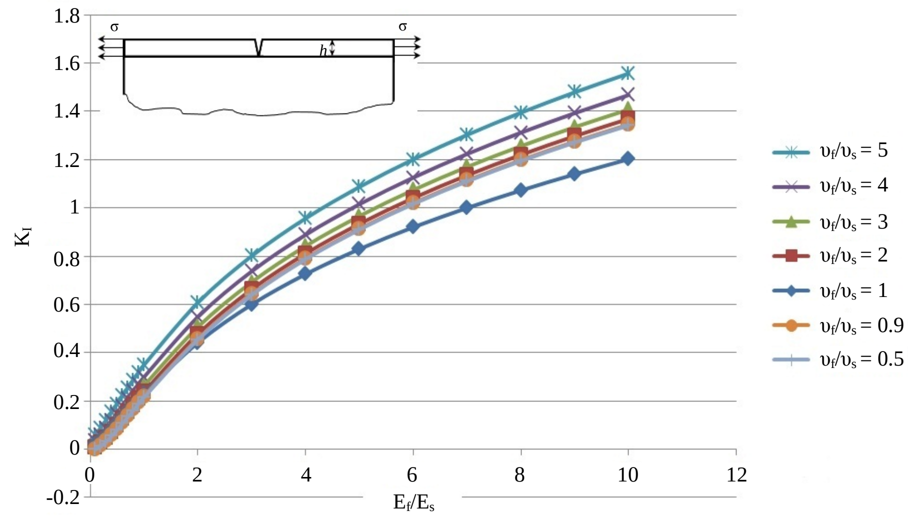

Figure 9. Variation of K_I with different Poisson's ratios, v_f/v_s, and Young's modulus ratios.

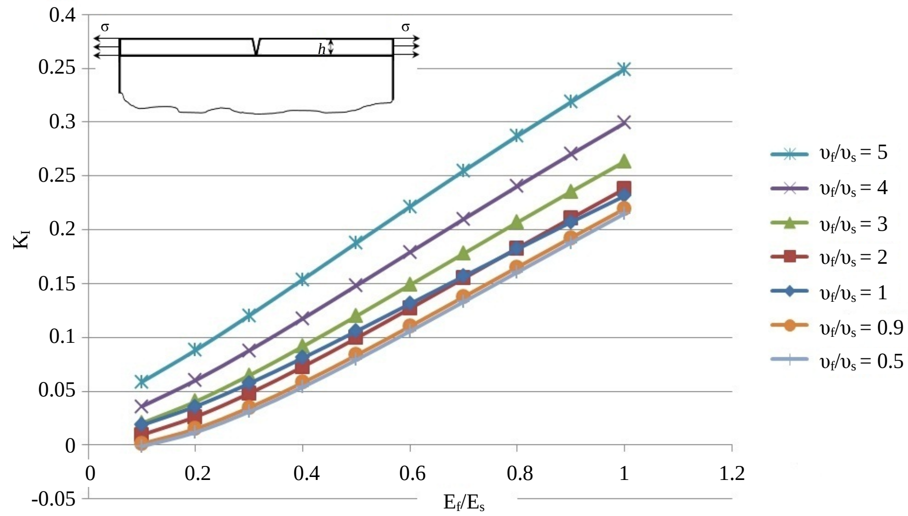

Figure 10. Variation of K_I with different Poisson's ratios, v_f/v_s, and Young's modulus ratios, E_f/E_s, of 0.1 - 1 for detail.

Table 1. Summary of the results.

Property		Dundurs' parameters		S	SIF	Non-dimensional energy release rate [2]
$E_s\uparrow$	E_f-	$\alpha\downarrow$	β-	$s\downarrow$	$K_I\downarrow$	$\omega_I\downarrow$
$v_s\uparrow$	v_f-	$\alpha\downarrow$	β-	$s\downarrow$	$K_I\downarrow$	$\omega_I\downarrow$
$E_f\uparrow$	E_s-	$\alpha\uparrow$	β-	$s\uparrow$	$K_I\uparrow$	$\omega_I\uparrow$
$v_f\uparrow$	v_s-	$\alpha\uparrow$	β-	$s\uparrow$	$K_I\uparrow$	$\omega_I\uparrow$

substrates. SIF for channeling crack has obtained as a function of elastic mismatch ratio between the substrate and the film. Results show that K_I has the minimum value in $E_f/E_s = 0.1$ and $v_f/v_s = 0.5$ condition and it has the maximum value in $E_f/E_s = 10$ and $v_f/v_s = 5$. In general view K_I has the minimum value when $v_f = v_s$. Because there is no result in this form, qualitative comparisons with the available previous studies (*i.e*. Non-dimensional energy release rate [2]) show good general agreements.

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