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Presence of Multifractality in High-Energy Nuclear Collisions

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Abstract

In the present study an attempt is made to examine multifractality in multiparticle production in relativistic nuclear collisions; multifractality is investigated in 14.5 A GeV/c ^{28}Si-nucleus collisions. For this, G_q-moments are calculated and variations of $\ln\langle G_q\rangle$ with $-\ln\delta\eta$ are looked into. Values of mass exponents, t_q, and generalised dimensions, D_q, are obtained. Analysis of multifractal moments reveals that multiplicity fluctuations are of dynamical nature.

Keywords

Multifractal Moments, Dynamical Fluctuations, Quark-Gluon Plasma, Emulsion

1. Introduction

Analysis of high-energy heavy-ion collisions [1]-[4] offers a unique opportunity to investigate occurrence of dynamical fluctuations [5]-[7] in A-A collisions. To understand the real dynamics of multiparticle production, multifractality is envisaged to become an important tool for both theoretician and experimentalist. Intermittency and multifractality in turbulent fluids have been extensively studied [8]. It was suggested that multifractal analysis is carried out by calculating multifractal moments, G_q, of the multiplicity distributions in a given pseudorapidity (η) space. The main purpose of adopting multifractal moments, G_q, approach is to explain multifractality and self-similarity in multiparticle production in relativistic nucleus-nucleus collisions. However, G_q-moments are greatly influenced by statistical fluctuations in the case of events having lower multiplicities. It is worth mentioning that if the particle production process exhibits self-similar behavior, a modified form of G_q-moment is used by introducing a step function [2], which leads to power-law dependence on the phase space bin size. Importance of multifractal analysis of high-energy nuclear collision data lies in the fact that these moments

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can be calculated for the negative values of the order of moments, q, also, whereas factorial moments are defined only for positive integral values of the order of moments. High-energy heavy-ion collisions are considered to be an ideal site for creating the conditions for producing Quark-Gluon Plasma (QGP) and dynamical fluctuations are one of the most reliable signals of QGP formation. Fluctuations in multiplicity and pseudorapidity distributions [9] are the most significant approaches to study nuclear matter produced in these collisions. Using the calculated values of various moments, non-statistical fluctuations in high-energy nuclear interactions can be investigated. It may be noted that multifractality may play an important role for searching the existence of dynamical fluctuations in the multipaticle production.

2. Mathematical Formalism

In order to study multifractality, a selected pseudorapidity interval, $\Delta\eta$ is partitioned into M bins of equal size $\delta\eta = \Delta\eta/M$. Let n_j be the number of particles lying in j^{th} bin, then multifractal moments, G_q, may be calculated [1] [10] using:

$$G_q = \sum_j \left(p_j\right)^q \tag{1}$$

where quantity p_j is defined as $p_j = n_j/n$ and n be the total number of particles.

In the above expression the summation is carried over non-empty bins only. For a given data sample, averaging is done over all the events comprising the total number of events, N_{evt}, the average value of multifractal moments, $\langle G_q \rangle$ is calculated from:

$$\langle G_q \rangle = \frac{1}{N_{\text{evt}}} \sum_1^{N_{\text{evt}}} G_q \tag{2}$$

If rapidity distribution possesses fractal nature, a power-law behavior of $\langle G_q \rangle$ of the following type should be observed over a small pseudorapidity range, $\delta\eta$:

$$\langle G_q \rangle \propto (\delta\eta)^{t_q} \tag{3}$$

where t_q are known as mass exponents.

The resulting linear dependence of $\ln\langle G_q \rangle$ on $-\ln\delta\eta$ may be used to determine the values of t_q making use of the following relationship:

$$t_q = \lim_{\delta\eta \to 0} \frac{\Delta \ln\langle G_q \rangle}{\Delta \ln \delta\eta} \tag{4}$$

The generalized dimensions, D_q, are considered to contain useful property regarding fractals occurring in multiparticle production in relativistic nuclear collisions. Generalized dimensions are defined as:

$$D_q = \frac{t_q}{q-1} \tag{5}$$

Increase in the value of D_q with q is said to describe the pattern as multi fractal, whereas constancy of D_q would point towards monofractality.

3. Experimental Details

We have analyzed data set comprising 555 events produced in 14.5 A GeV/c ^{28}Si-nucleus collisions. Data sample include collisions with $n_h \geq 0$, where n_h represent the number of charged particles produced with relative velocities, $\beta \leq 0.7$. Experimental results have been compared with the corresponding results for the data generated using Lund model, FRITIOF and Monte Carlo simulation.

4. Results and Discussion

4.1. Study of $\ln\langle G_q \rangle$ as a Function of $-\ln(\delta\eta)$

Figure 1 shows the variations of $\ln\langle G_q \rangle$ with $-\ln\delta\eta$ for the experimental data on 14.5 A GeV/c ^{28}Si-nucleus collisions; these variations are studied for three groups of targets namely, CNO, emulsion and AgBr groups of nuclei for various classes of interactions. It may be emphasized that multifractal moments show linearly increasing trend with decreasing bin width, $\delta\eta$, for all the three groups of the targets; this linearly increasing behavior which is shown over a large interval of $-\ln\delta\eta$ for positive order of the moments, q, in comparison to the ones for negative q values. For negative values saturating trend is discernible with decreasing $\delta\eta$. The only reason for this behavior appears to be the fact that particle multiplicity will decrease with decreasing bin width, $\delta\eta$ [11]. Multifractal moments for CNO group of targets saturates a little earlier in comparison to those for AgBr nuclei. This linearly increasing nature demonstrated by multifractal moments in the rapidity space indicates the presence of self-similarity. To investigate the dynamical fluctuations using multifractal moments, experimental results are compared with the Monte Carlo generated data sets for 14.5 A GeV/c ^{28}Si-nucleus collisions. **Figure 2** compares the behaviors of $\ln\langle G_q\rangle$ vs $-\ln\delta\eta$ plots for the experimental, FRITIOF and Monte Carlo generated data sets. The variation for the MC simulated data is very smooth as compared to those for the experimental and FRITIOF data samples.

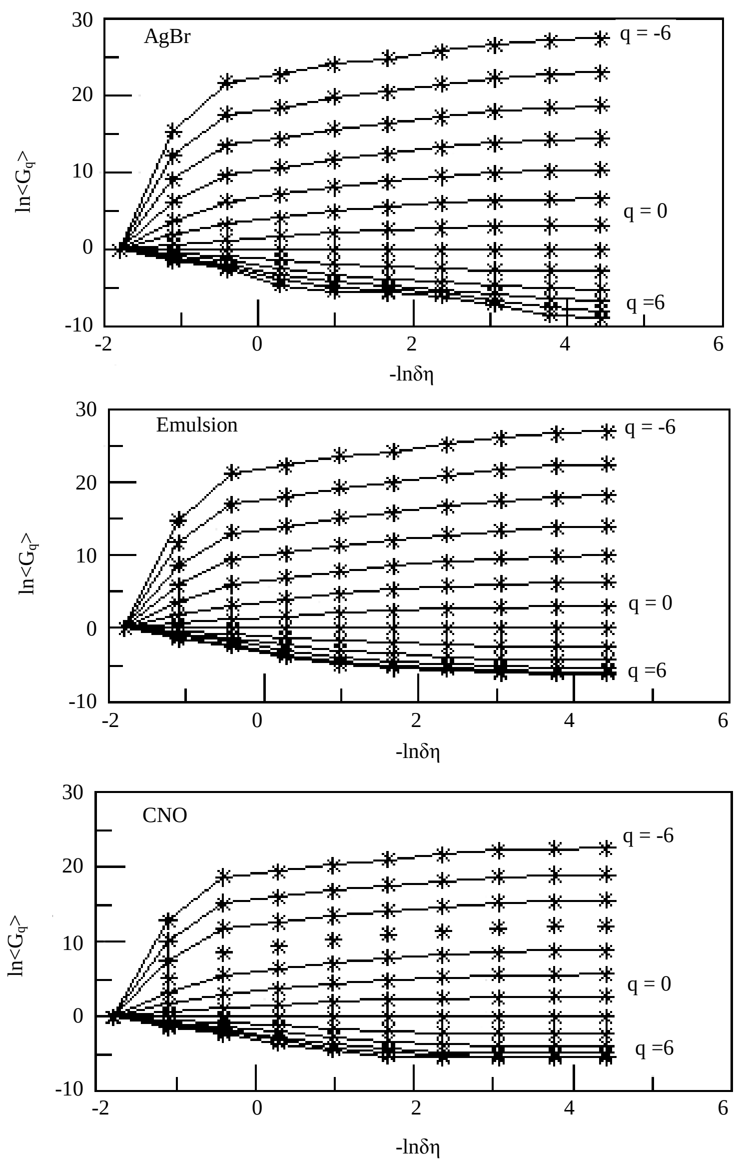

Figure 1. Variations of $\ln\langle G_q\rangle$ with $-\ln\delta\eta$ for the experimental data on 14.5 A GeV/c ^{28}Si-nucleus collisions.

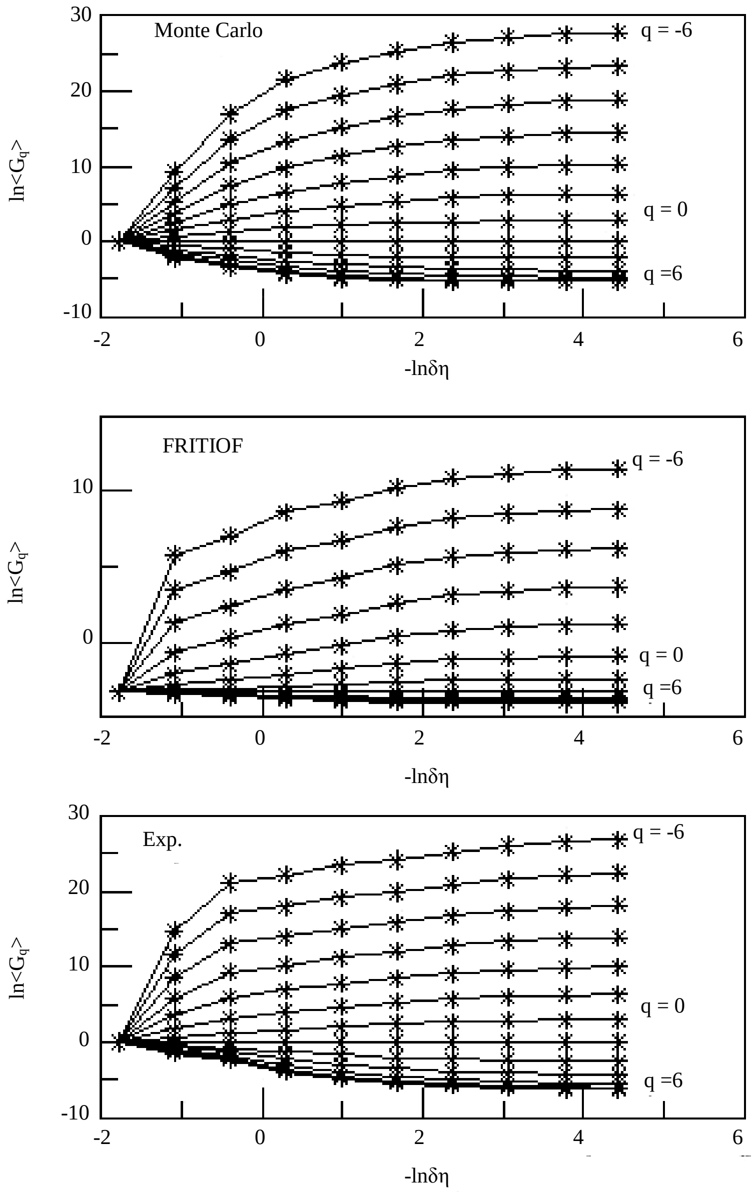

Figure 2. Variations of $\ln\langle G_q\rangle$ with $-\ln\delta\eta$ for the experimental and simulated data samples.

4.2. Mass Exponents

Values of the mass exponents, t_q, are determined by fitting the $\ln\langle G_q\rangle$ versus $-\ln\delta\eta$ plots in the region which exhibits linear behavior. The slopes of these fits give the values of the mass exponents. **Figure 3** show the variations of t_q, t_q^{stat} and t_q^{dyn} with the order of the moments, where t_q^{stat} represent the slopes for the Monte Carlo generated data and t_q^{dyn} are the dynamical component of the fluctuations; t_q^{dyn}, t_q^{stat}, t_q and q satisfy the following relationship:

$$t_q^{\text{dyn}} = t_q - t_q^{\text{stat}} + q - 1 \tag{5}$$

Figure 3 shows that the values of t_q increases with increasing *q*. The increasing trends for positive and negative values of *q* are quite dissimilar. These values depends on $q > 0$ and $q < 0$. t_q^{dyn} are observed to be quite different from t_q. **Figure 4** compares the values of the mass exponents, t_q, for the experimental and

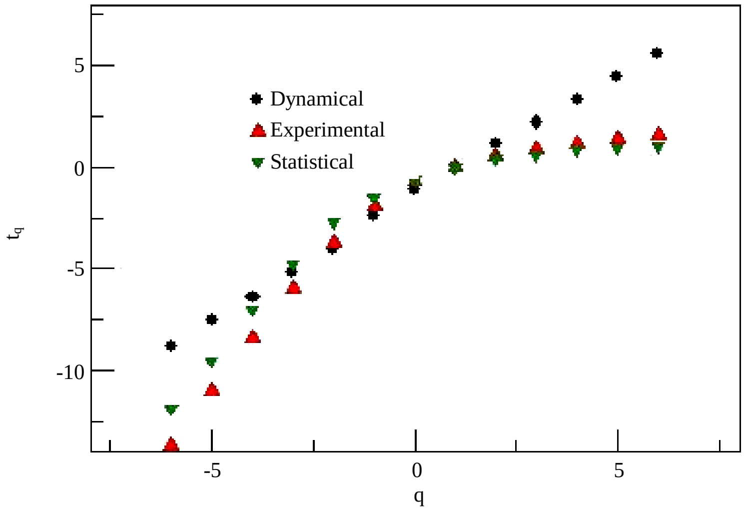

Figure 3. Variations of t_q, t_q^{stat} and t_q^{dyn} with q.

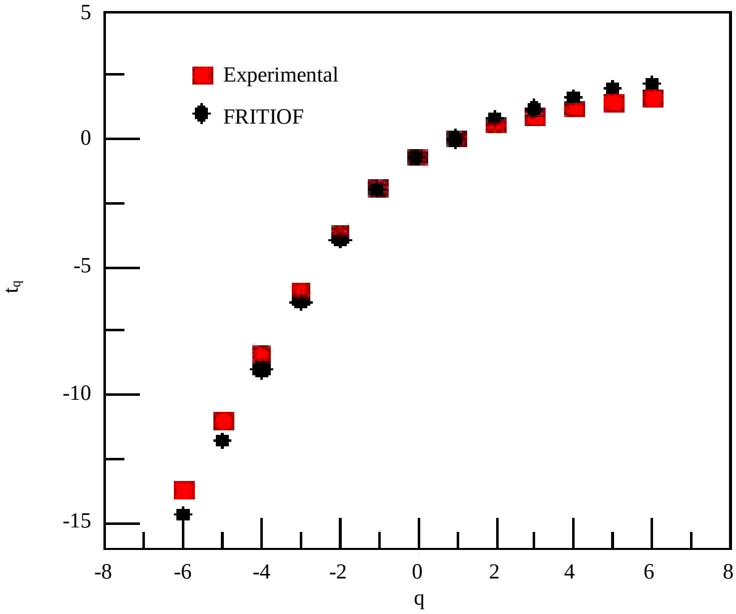

Figure 4. Variations of t_q with q for the experimental and FRITIOF data.

FRITIOF generated data. It is noticed that the variations are similar for both the data sets. Again target dependence of the mass exponents, t_q, which are shown in **Figure 5**, indicates that the values of mass exponents t_q, for $q < 1$, have lower values for the heavier targets as compared to those for the lighter ones, for which $q > 1$.

4.3. Generalized Dimensions

Figure 6 exhibits the variations of the generalized dimensions, D_q, with q for the three categories of targets. From the figure it is clear that the values of D_q for higher values of q are relatively lower in comparison to those for the lower values of q. For different orders of the moments, q, the values of generalized dimensions are positive and decrease with increasing q. This behavior supports the predictions of the multifractal cascade model. It is clear from the **Figure 6** that the generalized dimensions have higher values for heavier targets. The high values of generalized dimensions for heavier targets may be attributed to the fact that multiplicity for heavier targets is higher.

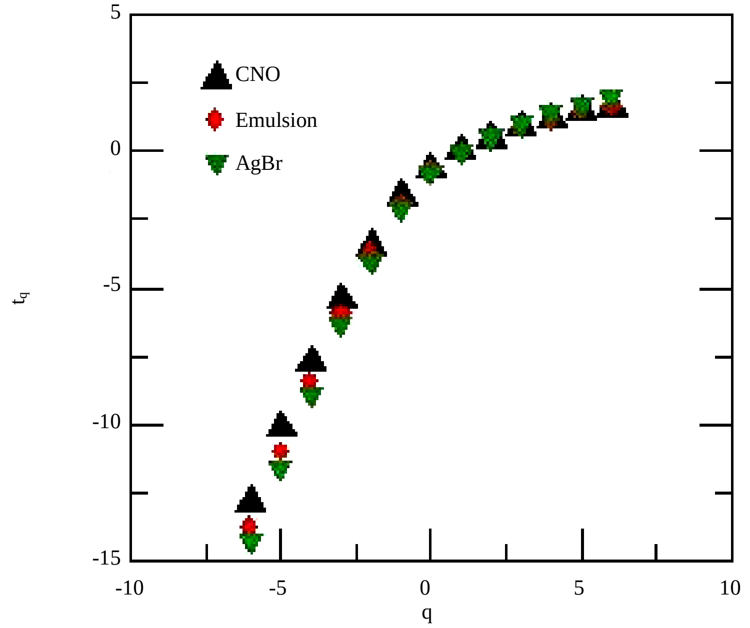

Figure 5. Variations of mass exponents, t_q, with q for the experimental data on 14.5 A GeV/c ^{28}Si-nucleus interactions.

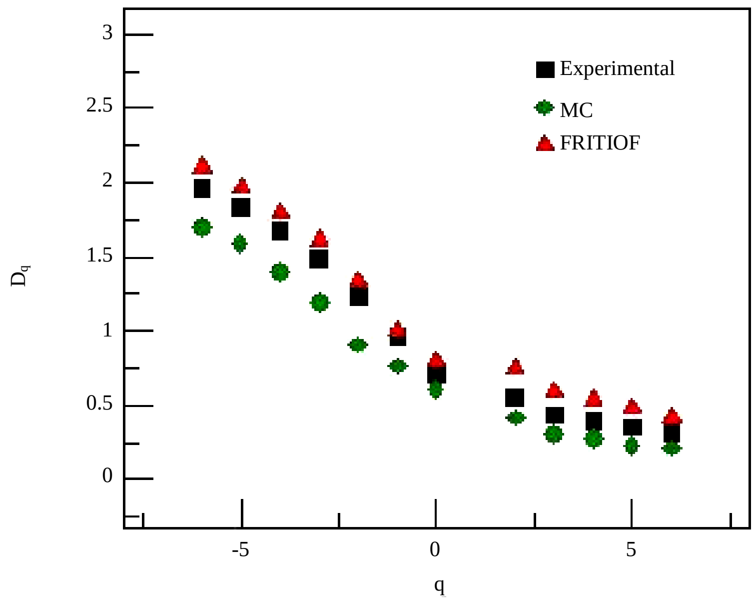

Figure 6. Variations of generalized dimensions, D_q, with q for the experimental, FRITIOF and Monte Carlo generated data.

5. Conclusion

The results of the present study are quite important for drawing meaningful conclusions regarding occurrence of multifractality in multiparticle production in high-energy nucleus-nucleus collisions. It is clearly observed that $\ln\langle G_q\rangle$ first increases linearly with $-\ln\delta\eta$ and then saturates as η resolution increases. The nature of variation clearly hints towards power-law behavior of multifractal moments as a function of $\delta\eta$, thereby indicating the presence of multifractality. The variations of mass exponents t_q, and generalized dimensions D_q, with q also support the presence of multifractality in the collisions considered in the present study.

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