

The Origins of Bosons and Fermions

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Abstract

This paper proposes that all **Bosons** and all **Fermions** originate from even more elementary constituents, which called Spin Angular Momentum Vacuum (**SAMV**).

SAMV is filled with Primitive Spin Particles (**PSP**). The total square spin angular momentum of each **PSP** is negative, less than zero.

Those **PSP** labeled by index $-\frac{1}{2}$ of Casimir Operator, are called Vacuum Spin Particle (**VSP**), which could be contracted into so-called Vacuum Bubbles (**VB**).

VB are **identical bubbles**, are "sub-observable physical quantities".

VB are paired up into Vacuum Bubble Pair **VBP**.

VSP ω_j (or ω^+ , ω^-) results from Self-identical vacuum bubble interaction $\omega_{S=0}^{+-}(k)$ through the zero order Phase Transition **PT**.

When the 1st, 2nd, 3rd,... order **PT** of **VBP** occur, then **VBP** turn into **Bosons** and **Fermions**, excited out of sea level of **SAMV** ocean.

Keywords

Spin Angular Momentum Vacuum **SAMV** ; Primitive Spin Particles **PSP** ;
Vacuum Spin Particles (**VSP**) ; Abnormal Casimir Operator **ACO** ;
Vacuum Contractions **VC** ; Vacuum Bubbles **VB** ; Vacuum Bubble Pair **VBP** ;
Phase Transitions **PT** ; Bosons, Fermions ;
The Third Kind Of Particles **TKP** ; Chaos Spin Hierarchy **CSH** ;
The Equivalence of Vacuum Bubbles ;
Locality and Nonlocality of Vacuum Contractions ;
Local and Nonlocal Angular Momentum Commutations

1. Introduction

Why in universe there exist those particles, **Bosons** with integer value spin $0\hbar, 1\hbar, 2\hbar, \dots$ and **Fermions** with half-integer spin $\hbar/2, 3\hbar/2, 5\hbar/2, \dots$?

Where they come from?

There are all types of Bosons and Fermions in nature, whatever happen with them in any physical interaction processes as known yet, these particles bear themselves quite different, but always keep their spin values to be either integers or half-integers !

The phenomenons show there must be something abstruse, abstruse philosophy.

Spin and energy are fundamental physical quantities used to describe the behaviour of particles. The difference between physical dimension of spin and that of energy is only the frequency factor. Contrary to energy amounts of a particle, always be variable, the spin angular momentum values of a particle are very limited strictly. It seems angular momentum may be more fundamental than energy.

When appreciating the mathematical elegance of "The Third Kind Of Particles", **TKP**, be struck by an idea that:

Analog of an electron, which could possess its negative energy sea introduced by Dirac's intuitive feelings cite: [1], why not a spin particle possesses its negative angular momentum ocean ? and see what happen.

The values of spin angular momentum values of a particle, maybe too, variable from positive region to negative region, angular momentum and energy are in common in front of physical picture.

Unfortunately, conventional quantum mechanics could not provide a math frame of angular momentum, that can construct a total square angular momentum with negative eigenvulues.

That's all due to finite demensional and Hermitian matrix representation in conventional quantum mechanics.

That's all due to total square angular momentum operator is always a positive Casimir operator in conventional quantum mechanics.

If strive for mathematical elegance balance between angular momentum and energy, perhaps, the enlightments from Dirac could give us some suggestions.

Turn to **TKP**, **TKP** can offer "Abnormal Casimir Operator, **ACO**", whose labels of irreducible representation of Lie group with the index of negative values.

CSH and **ACO** may unveil the mysterious origins of Bosons and Fermions.

Chaos Spin Hierarchy CSH

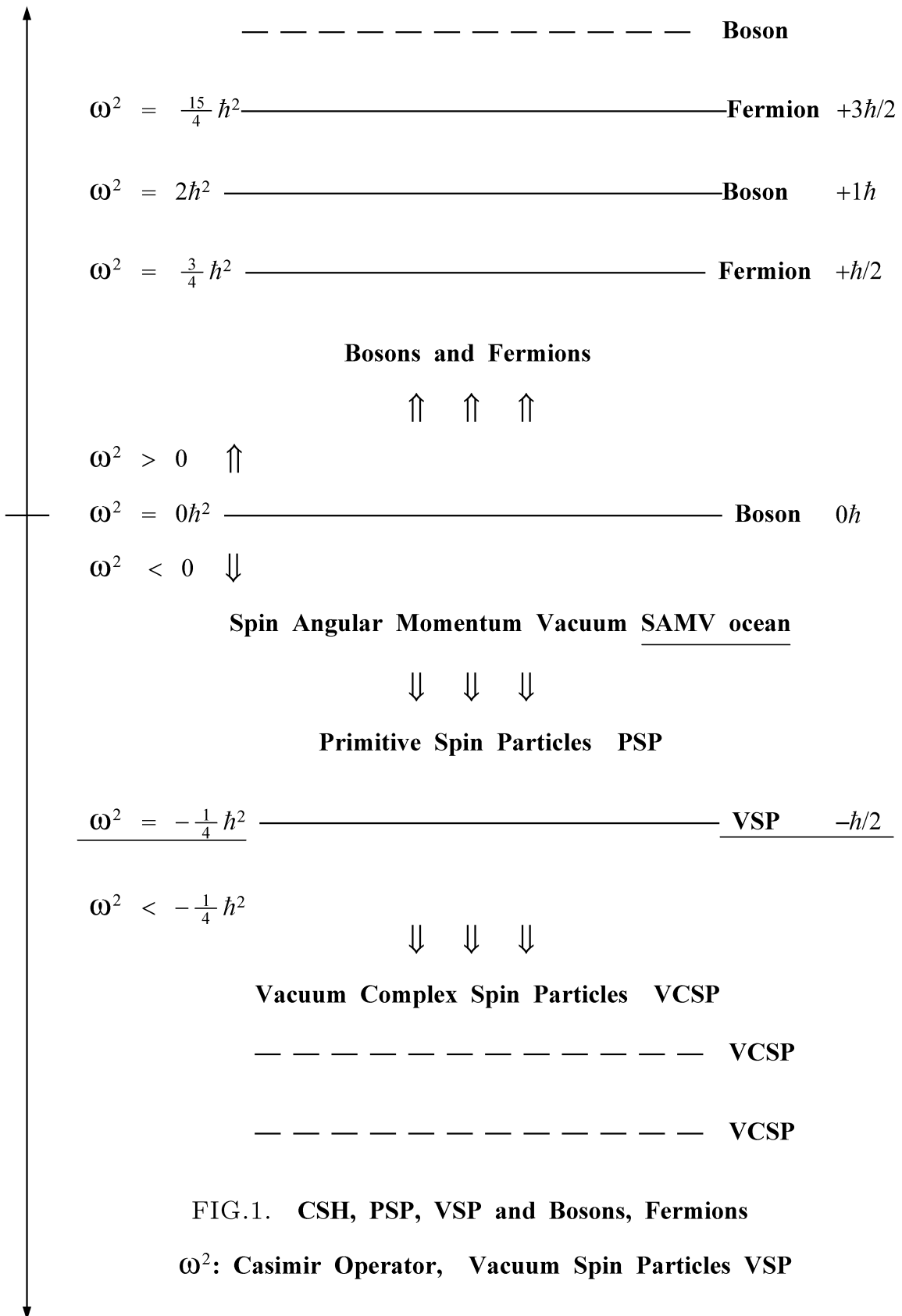


FIG.1. CSH, PSP, VSP and Bosons, Fermions

ω^2 : Casimir Operator, Vacuum Spin Particles VSP

2. Bosons and Fermions in TKP

Angular momentum commutation rules are given as

$$\Omega_i \Omega_j - \Omega_j \Omega_i = i\hbar \Omega_k, \quad (1)$$

$i, j, k = 1, 2, 3$ are circulative. Or by raising operator and lowing operator

$$\Omega^+ = \Omega_1 + i\Omega_2 \quad (2)$$

$$\Omega^- = \Omega_1 - i\Omega_2 \quad (3)$$

Then (1) be turned into (4), (5), (6)

$$\Omega_3 \Omega^+ - \Omega^+ \Omega_3 = +\Omega^+ \quad (4)$$

$$\Omega_3 \Omega^- - \Omega^- \Omega_3 = -\Omega^- \quad (5)$$

$$\Omega^+ \Omega^- - \Omega^- \Omega^+ = 2\Omega_3 \quad (6)$$

Now a group of new symbols is introduced as

$$\Omega^{+,-} = \Omega^+ \Omega^- \quad (7)$$

$$\Omega^{-,+} = \Omega^- \Omega^+ \quad (8)$$

Then (6) be expressed as

$$\Omega^{+,-} - \Omega^{-,+} = 2\Omega_3 \quad (9)$$

$\Omega^{+,-}$ and $\Omega^{-,+}$ are matrix products, which are diagonal matrices. We call them matrix contractions.

The types of math symbols (7), (8), (9) are frequently used in this paper.

Base on (7), (8), the marks of $0\hbar$ boson, $\hbar/2$ fermion, $1\hbar$ boson and $3\hbar/2$ boson are given as below:

▲ $0\hbar$ boson $\Omega_{j,+0}$

$$\Omega_{+0}^{+,-} = \Omega_{+0}^+ \Omega_{+0}^-, \quad \Omega_{+0}^{-,+} = \Omega_{+0}^- \Omega_{+0}^+; \quad \Omega_{3,+0}, \quad \Omega_{+0}^2 \quad (10)$$

▼ $\hbar/2$ fermion $\Omega_{j,+1/2}$

$$\Omega_{+1/2}^{+,-} = \Omega_{+1/2}^+ \Omega_{+1/2}^-, \quad \Omega_{+1/2}^{-,+} = \Omega_{+1/2}^- \Omega_{+1/2}^+; \quad \Omega_{3,+1/2}, \quad \Omega_{+1/2}^2 \quad (11)$$

♠ $1\hbar$ boson $\Omega_{j,+1}$

$$\Omega_{+1}^{+,-} = \Omega_{+1}^+ \Omega_{+1}^-, \quad \Omega_{+1}^{-,+} = \Omega_{+1}^- \Omega_{+1}^+; \quad \Omega_{3,+1}, \quad \Omega_{+1}^2 \quad (12)$$

◆ $3\hbar/2$ fermion $\Omega_{j,+3/2}$

$$\Omega_{+3/2}^{+,-} = \Omega_{+3/2}^+ \Omega_{+3/2}^-, \quad \Omega_{+3/2}^{-,+} = \Omega_{+3/2}^- \Omega_{+3/2}^+; \quad \Omega_{3,+3/2}, \quad \Omega_{+3/2}^2 \quad (13)$$

The following **2.1, 2.2, 2.3, 2.4** are what we have learned from TKP cite: [2]

And then from paragraphy **3.**, we are going to explore the origins of bosons $\Omega_{j,+0}$ ▲, $\Omega_{j,+1}$ ♠ and fermions $\Omega_{j,+1/2}$ ▼, $\Omega_{j,+3/2}$ ◆.

2.1. For Boson $0\hbar$, we have \blacktriangle

$$\Omega_{+0}^+ = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & i\sqrt{12} & & & & & & & & \mathbf{0} & \cdot \\ \cdot & 0 & 0 & i\sqrt{6} & & & & & & & & \cdot \\ \cdot & & 0 & 0 & i\sqrt{2} & & & & & & & \cdot \\ \cdot & & & 0 & 0 & 0 & & & & & & \cdot \\ \cdot & & & & 0 & 0 & 0 & & & & & \cdot \\ \cdot & & & & & 0 & 0 & i\sqrt{2} & & & & \cdot \\ \cdot & & & & & & 0 & 0 & i\sqrt{6} & & & \cdot \\ \cdot & & & & & & & 0 & 0 & i\sqrt{12} & & \cdot \\ \cdot & \mathbf{0} & & & & & & & 0 & 0 & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (14)$$

$$\Omega_{+0}^- = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & & & & & & & & \mathbf{0} & \cdot \\ \cdot & i\sqrt{12} & 0 & 0 & & & & & & & & \cdot \\ \cdot & & i\sqrt{6} & 0 & 0 & & & & & & & \cdot \\ \cdot & & & i\sqrt{2} & 0 & 0 & & & & & & \cdot \\ \cdot & & & & 0 & 0 & 0 & & & & & \cdot \\ \cdot & & & & & 0 & 0 & 0 & & & & \cdot \\ \cdot & & & & & & i\sqrt{2} & 0 & 0 & & & \cdot \\ \cdot & & & & & & & i\sqrt{6} & 0 & 0 & & \cdot \\ \cdot & \mathbf{0} & & & & & & & i\sqrt{12} & 0 & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (15)$$

And four diagonal matrices

$$\Omega_{+0}^{+-} = \Omega_{+0}^+ \Omega_{+0}^- = - \text{diag} (\dots, 12, 6, 2, 0, 0, 2, 6, 12, 20, \dots) \quad (16)$$

$$\Omega_{+0}^{-+} = \Omega_{+0}^- \Omega_{+0}^+ = - \text{diag} (\dots, 20, 12, 6, 2, 0, 0, 2, 6, 12, \dots) \quad (17)$$

$$\Omega_{3,+0} = + \text{diag} (\dots, 4, 3, 2, 1, 0, -1, -2, -3, -4, \dots) \quad (18)$$

$$\Omega_{+0}^2 = + \text{diag} (\dots, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots) \quad (19)$$

2.3. For Boson $1\hbar$, we have ♠

$$\Omega_{+1}^+ = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & i\sqrt{10} & & & & & & & \mathbf{0} & \cdot \\ \cdot & 0 & 0 & i\sqrt{4} & & & & & & & \cdot \\ \cdot & & 0 & 0 & 0 & & & & & & \cdot \\ \cdot & & & 0 & 0 & \sqrt{2} & & & & & \cdot \\ \cdot & & & & 0 & 0 & \sqrt{2} & & & & \cdot \\ \cdot & & & & & 0 & 0 & 0 & & & \cdot \\ \cdot & & & & & & 0 & 0 & i\sqrt{4} & & \cdot \\ \cdot & & & & & & & 0 & 0 & i\sqrt{10} & \cdot \\ \cdot & \mathbf{0} & & & & & & & & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (26)$$

$$\Omega_{+1}^- = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & & & & & & & & \mathbf{0} & \cdot \\ \cdot & i\sqrt{10} & 0 & 0 & & & & & & & & \cdot \\ \cdot & & i\sqrt{4} & 0 & 0 & & & & & & & \cdot \\ \cdot & & & 0 & 0 & 0 & & & & & & \cdot \\ \cdot & & & & \sqrt{2} & 0 & 0 & & & & & \cdot \\ \cdot & & & & & \sqrt{2} & 0 & 0 & & & & \cdot \\ \cdot & & & & & & 0 & 0 & 0 & & & \cdot \\ \cdot & & & & & & & i\sqrt{4} & 0 & 0 & & \cdot \\ \cdot & \mathbf{0} & & & & & & & i\sqrt{10} & 0 & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (27)$$

And four diagonal matrices

$$\Omega_{+1}^{+-} = \Omega_{+1}^+ \Omega_{+1}^- = - \text{diag} (\dots, 10, 4, 0, -2, -2, 0, 4, 10, 18, \dots) \quad (28)$$

$$\Omega_{+1}^{-+} = \Omega_{+1}^- \Omega_{+1}^+ = - \text{diag} (\dots, 18, 10, 4, 0, -2, -2, 0, 4, 10, \dots) \quad (29)$$

$$\Omega_{3,+1} = + \text{diag} (\dots, 4, 3, 2, 1, 0, -1, -2, -3, -4, \dots) \quad (30)$$

$$\Omega_{+1}^2 = + \text{diag} (\dots, 2, 2, 2, 2, 2, 2, 2, 2, 2, \dots) \quad (31)$$

The Flow of This Paper

1. Introduction

2. Bosons and Fermions in TKP

3. Vacuum Spin Particles VSP

4. Vacuum Bubbles VB and Vacuum Contractions VC

5 Phase Transitions PT of Vacuum Bubbles

6. The formations of Bosons and Fermions

7. Vacuum Bubble Pair VBP $\omega_{S=|i-j|}^{+,-}(k)$ of the k th generations of CSH

8. Formation of bosons, fermions, TKP and VSP of 2nd generation of CSH

9. Formation of bosons, fermions, TKP and VSP of 3rd generation of CSH

10. The S th order phase transitions of Vacuum Bubble Pair in SAMV ocean

11. *Spin* particles with Casimir Operator ω_S^2 formed of Vacuum Bubble Pair $\omega_S^{+,-}$

12. Local and Nonlocal Spin Angular Momentum Commutation Rules

13. Conclusions

3. Vacuum Spin Particles VSP

Spin Angular Momentum Vacuum SAMV consist of Primitive Spin Particles PSP. SAMV is very deep ocean. About SAMV what This paper concerns is in the shallow water region, its Casimir Operator ω^2 , which labeled by the negative index $-\frac{1}{2}$, is only negative one-fourth, $-\frac{1}{4}\hbar^2$ deep.

VSP is the special and the important case of PSP with index $-\frac{1}{2}$.

Vacuum Spin Particles VSP obey angular momentum commutation rules.

Symbols $\omega_1(38)$, $\omega_2(39)$, $\omega_3(40)$, $\omega^2(47)$, (48) are the representation of VSP.

The Casimir Operator ω^2 of VSP is "abnormal", because of its negative $-\frac{1}{4}$ value, instead of those, which labeled by positive or zero values seen in conventional quantum mechanics.

From (49), (50) get

$$\mathfrak{W}_1\mathfrak{W}_2 - \mathfrak{W}_2\mathfrak{W}_1 = \frac{1}{2}i \text{diag} (\dots, 7, 5, 3, 1, -1, -3, -5, -7, \dots) = i\mathfrak{W}_3 \quad (59)$$

Obviously (59), (54), (58) show $\mathfrak{W}_1, \mathfrak{W}_2, \mathfrak{W}_3$ satisfy angular momentum commutation rules.

$$\mathfrak{W}_j\mathfrak{W}_k - \mathfrak{W}_k\mathfrak{W}_j = i\mathfrak{W}_l, \quad j, k, l = 1, 2, 3 \quad (60)$$

Further, $\mathfrak{W}_1, \mathfrak{W}_2, \mathfrak{W}_3$ are angular momentum operators,

From (49), (50) get

$$\mathfrak{W}_1\mathfrak{W}_2 + \mathfrak{W}_2\mathfrak{W}_1 = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & 3i & & & & & \mathbf{0} & \cdot \\ \cdot & 0 & 0 & 0 & i & & & & & \cdot \\ \cdot & -3i & 0 & 0 & 0 & 0 & & & & \cdot \\ \cdot & & -i & 0 & 0 & 0 & 0 & & & \cdot \\ \cdot & & & 0 & 0 & 0 & 0 & i & & \cdot \\ \cdot & & & & 0 & 0 & 0 & 0 & 3i & \cdot \\ \cdot & & & & & -i & 0 & 0 & 0 & \cdot \\ \cdot & \mathbf{0} & & & & & -3i & 0 & 0 & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \quad (61)$$

a) As the label $-\frac{1}{2}$ (47) of irreducible representation, further $\mathfrak{W}_1, \mathfrak{W}_2, \mathfrak{W}_3$ are named $-\frac{1}{2}\hbar$ spin particle.

b) As $-\frac{1}{4} < 0$ (46) of Casimir operator \mathfrak{W}^2 , and momentum commutation rules (23), further (38), (39), (40) are called Vacuum Spin Particles, **VSP**.

Recall conventional Fermion, example of electron, its label of irreducible representation is $+\frac{1}{2}$ below

$$\mathfrak{W}_{electron}^2 = +\frac{3}{4}I_0 = +\frac{1}{2}\left(+\frac{1}{2} + 1\right)I_0 \quad (62)$$

$$\mathfrak{W}^2 = +\frac{3}{4} > 0 \quad (63)$$

c) As the label $+\frac{1}{2}$ (62) of irreducible representation, further electron is named $+\frac{1}{2}\hbar$ spin particle.

By the way, (61) is different from (53) and (57), the former is an infinite Hermitian matrix and the latter two are infinite anti-Hermitian matrices.

4. Vacuum Bubbles VB and Vacuum Contractions VC

$\mathfrak{W}^{+,-}$, $\mathfrak{W}^{-,+}$ are Vacuum Bubbles *and* Vacuum Contractions

Let us perform the following change of bases (38), (39) of **VSP**

$$\mathfrak{W}^+ = \mathfrak{W}_1 + i\mathfrak{W}_2 \quad (64)$$

$$\mathfrak{W}^- = \mathfrak{W}_1 - i\mathfrak{W}_2 \quad (65)$$

Then get raising operator \mathfrak{W}^+ and lowering operator \mathfrak{W}^- .

$$\begin{aligned} \mathfrak{W}^+ = & \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 3i & & & & & & & \mathbf{0} & \cdot \\ \cdot & 0 & 0 & 2i & & & & & & & \cdot \\ \cdot & & 0 & 0 & i & & & & & & \cdot \\ \cdot & & & 0 & 0 & 0 & & & & & \cdot \\ \cdot & & & & 0 & 0 & -i & & & & \cdot \\ \cdot & & & & & 0 & 0 & -2i & & & \cdot \\ \cdot & & & & & & 0 & 0 & -3i & & \cdot \\ \cdot & \mathbf{0} & & & & & & 0 & 0 & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}, & \mathfrak{W}^- = & \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & 0 & 0 & & & & & & & \mathbf{0} & \cdot \\ \cdot & 3i & 0 & 0 & & & & & & & \cdot \\ \cdot & & 2i & 0 & 0 & & & & & & \cdot \\ \cdot & & & i & 0 & 0 & & & & & \cdot \\ \cdot & & & & 0 & 0 & 0 & & & & \cdot \\ \cdot & & & & & -i & 0 & 0 & & & \cdot \\ \cdot & & & & & & -2i & 0 & 0 & & \cdot \\ \cdot & \mathbf{0} & & & & & & -3i & 0 & & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{aligned} \quad (66) \quad (67)$$

And have the following useful relations:

$$\mathfrak{W}^+\mathfrak{W}^- = \mathfrak{W}_1^2 + \mathfrak{W}_2^2 + \mathfrak{W}_3 \quad (68)$$

$$\mathfrak{W}^-\mathfrak{W}^+ = \mathfrak{W}_1^2 + \mathfrak{W}_2^2 - \mathfrak{W}_3 \quad (69)$$

$$\mathfrak{W}_1^2 + \mathfrak{W}_2^2 = \frac{1}{2} (\mathfrak{W}^+\mathfrak{W}^- + \mathfrak{W}^-\mathfrak{W}^+) \quad (70)$$

$$\mathfrak{W}_3 = \frac{1}{2} (\mathfrak{W}^+\mathfrak{W}^- - \mathfrak{W}^-\mathfrak{W}^+) \quad (71)$$

$$\mathfrak{W}^2 = \mathfrak{W}_1^2 + \mathfrak{W}_2^2 + \mathfrak{W}_3^2 \quad (72)$$

From (66), (67) obtain

$$\mathfrak{W}^+\mathfrak{W}^- = - \text{diag} (\dots, 9, 4, 1, 0, 1, 4, 9, 16, \dots) \quad (73)$$

$$\mathfrak{W}^-\mathfrak{W}^+ = - \text{diag} (\dots, 16, 9, 4, 1, 0, 1, 4, 9, \dots) \quad (74)$$

Using (73), (74), from (71), obtain

$$\begin{aligned}\Omega_3 &= \frac{1}{2} \text{diag} (\dots, 7, 5, 3, 1, -1, -3, -5, -7, \dots) \\ &= \text{diag} (\dots, \frac{7}{2}, \frac{5}{2}, \frac{3}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{3}{2}, -\frac{5}{2}, -\frac{7}{2}, \dots)\end{aligned}\quad (75)$$

$$\Omega_3^2 = + \text{diag} (\dots, \frac{49}{4}, \frac{25}{4}, \frac{9}{4}, \frac{1}{4}, \frac{1}{4}, \frac{9}{4}, \frac{25}{4}, \frac{49}{4}, \dots)\quad (76)$$

Using (73), (74), from (70), obtain

$$\begin{aligned}\Omega_1^2 + \Omega_2^2 &= -\frac{1}{2} \text{diag} (\dots, 25, 13, 5, 1, 1, 5, 13, 25, \dots) \\ &= - \text{diag} (\dots, \frac{50}{4}, \frac{26}{4}, \frac{10}{4}, \frac{2}{4}, \frac{2}{4}, \frac{10}{4}, \frac{26}{4}, \frac{50}{4}, \dots)\end{aligned}\quad (77)$$

Then, (72) and (76), (77) give the Casimir Operator Ω^2 labeled by the negative index $-\frac{1}{2}$

$$\Omega^2 = \text{diag} (\dots, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, -\frac{1}{4}, \dots)\quad (78)$$

$$= -\frac{1}{4} I_0 = -\frac{1}{2} (-\frac{1}{2} + 1) I_0\quad (79)$$

Expressions (75), (79) are just (40), (47).

$\Omega^+\Omega^-$ (73), $\Omega^-\Omega^+$ (74) are in the form of diagonal of matrix products, whose values are negative. What are the roles of (73), (74) in SAMV ocean ?

4.1 Vacuum Bubbles VB and Vacuum Contractions VC of VSP

The above process hints the matrix products $\Omega^+\Omega^-$ (73), $\Omega^-\Omega^+$ (74) are the marvellous concepts connected with diagonal matrices.

We call them matrix contractions, or Vacuum Contractions **VC** or Vacuum Bubbles **VB**.

Now use compact notions (80), (81) to define **Vacuum Contraction of a drop of Vacuum Bubbles** in SAMV ocean.

$$\text{Vacuum Contraction } \Omega^{+,-} \equiv \text{Matrix Product } \Omega^+\Omega^-\quad (80)$$

$$\text{Vacuum Contraction } \Omega^{-,+} \equiv \text{Matrix Product } \Omega^-\Omega^+\quad (81)$$

In SAMV ocean, there are many, many... drops of **Vacuum Bubbles** which resulted from the contracted Vacuum Spin Particles **VSP**.

These bubbles are the individual bubbles, are so-called "identical bubbles" just as "identical particles" in statistical physics world.

Symbol j is introduced to indicate the amounts of many, many...individual bubbles.

4.2 Self-identical bubble Vacuum contractions

We use single subscript j to express Vacuum Spin Particles \mathfrak{W}_j . And double subscripts j,j to label the identical bubbles $\mathfrak{W}_{j,j}^{+,-}$ (84) and $\mathfrak{W}_{j,j}^{-,+}$ (85) composed of Vacuum Spin Particle \mathfrak{W}_j

$$\mathfrak{W}_j^+ = \mathfrak{W}_{j,1} + i\mathfrak{W}_{j,2} \quad (82)$$

$$\mathfrak{W}_j^- = \mathfrak{W}_{j,1} - i\mathfrak{W}_{j,2} \quad (83)$$

Where $\mathfrak{W}_{j,1}$ and $\mathfrak{W}_{j,2}$ are the first component and the second component of Vacuum Spin Particle \mathfrak{W}_j .

Further, contractions (80), (81) can be extended to the following expressions

$$\text{Vacuum Contraction of identical bubble } j \quad \mathfrak{W}_{j,j}^{+,-} \equiv \mathfrak{W}_j^+ \mathfrak{W}_j^- \quad (84)$$

$$\text{Vacuum Contraction of identical bubble } j \quad \mathfrak{W}_{j,j}^{-,+} \equiv \mathfrak{W}_j^- \mathfrak{W}_j^+ \quad (85)$$

The appearance of subscript j in (84), (85) will bring many magical and profound ideas to modern physics.

Vacuum contractions (84), (85) also be called **Self-identical bubble Vacuum contractions**, because the two subscripts appear in $\mathfrak{W}_{j,j}^{+,-}$ (84) and $\mathfrak{W}_{j,j}^{-,+}$ (85) are the same symbol j .

Due to symbol j of (84) and (85) is referred to any drop of Vacuum Bubbles in **SAMV** ocean, obviously, symbol j can be all the positive number, zero, negative number. From now on, symbol $j = 0$ is chosen to represent the indicate Self-identical bubble Vacuum contractions we are caring about.

As contractions (73), (74) are attributed to Self-identical bubble Vacuum contractions, further they can be rewritten as below

$$\mathfrak{W}_{0,0}^{+,-} = \mathfrak{W}_0^+ \mathfrak{W}_0^- = - \text{diag} (... , 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25, ...) \quad (86)$$

$$\mathfrak{W}_{0,0}^{-,+} = \mathfrak{W}_0^- \mathfrak{W}_0^+ = - \text{diag} (... , 36, 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, ...) \quad (87)$$

Using vacuum contractions (86), (87), all the informations of irreducible representation labeled by the negative index $-\frac{1}{2}$, can be obtained.

4.3 Dual-identical bubble Vacuum contractions

How about the bubble Vacuum contractions, if " From the self-action of one drop of **Bubbles** to the dual-action of all drops of **Bubbles** " ?

Provide that subscript $i \neq j$, then (84), (85) turn to

$$\text{Contraction between bubble } i \text{ and bubble } j \quad \mathfrak{W}_{ij}^{+-} \equiv \mathfrak{W}_i^+ \mathfrak{W}_j^- \quad (88)$$

$$\text{Contraction between bubble } j \text{ and bubble } i \quad \mathfrak{W}_{ji}^{-+} \equiv \mathfrak{W}_j^- \mathfrak{W}_i^+ \quad (89)$$

In process that similar to Self-identical bubble Vacuum contractions (86), (87), then four Dual-identical bubble Vacuum contractions are given in following. More details can be founded from cite: [3]

$$\mathfrak{W}_{+1,0}^{+-} = \mathfrak{W}_{+1}^+ \mathfrak{W}_0^- = -\text{diag} (\dots, 20, 12, 6, 2, 0, 0, 2, 6, 12, 20, 30, \dots) \quad (90)$$

$$\mathfrak{W}_{0,+1}^{-+} = \mathfrak{W}_0^- \mathfrak{W}_{+1}^+ = -\text{diag} (\dots, 30, 20, 12, 6, 2, 0, 0, 2, 6, 12, 20, \dots) \quad (91)$$

$$\mathfrak{W}_{+1,-1}^{+-} = \mathfrak{W}_{+1}^+ \mathfrak{W}_{-1}^- = -\text{diag} (\dots, 24, 15, 8, 3, 0, -1, 0, 3, 8, 15, 24, \dots) \quad (92)$$

$$\mathfrak{W}_{-1,+1}^{-+} = \mathfrak{W}_{-1}^- \mathfrak{W}_{+1}^+ = -\text{diag} (\dots, 35, 24, 15, 8, 3, 0, -1, 0, 3, 8, 15, \dots) \quad (93)$$

$$\mathfrak{W}_{+2,-1}^{+-} = \mathfrak{W}_{+2}^+ \mathfrak{W}_{-1}^- = -\text{diag} (\dots, 18, 10, 4, 0, -2, -2, 0, 4, 10, 18, 28, \dots) \quad (94)$$

$$\mathfrak{W}_{-1,+2}^{-+} = \mathfrak{W}_{-1}^- \mathfrak{W}_{+2}^+ = -\text{diag} (\dots, 28, 18, 10, 4, 0, -2, -2, 0, 4, 10, 18, \dots) \quad (95)$$

$$\mathfrak{W}_{+2,-2}^{+-} = \mathfrak{W}_{+2}^+ \mathfrak{W}_{-2}^- = -\text{diag} (\dots, 21, 12, 5, 0, -3, -4, -3, 0, 5, 12, 21, \dots) \quad (96)$$

$$\mathfrak{W}_{-2,+2}^{-+} = \mathfrak{W}_{-2}^- \mathfrak{W}_{+2}^+ = -\text{diag} (\dots, 32, 21, 12, 5, 0, -3, -4, -3, 0, 5, 12, \dots) \quad (97)$$

Next, the above four groups of contractions or bubbles will be used to synthesize the known bosons and fermions of today's physics by means of phase transitions of Vacuum Bubbles.

5 Phase Transitions PT of Vacuum Bubbles

Referring to Phase transitions of Vacuum Bubbles, it means the processes in which the sub-observable physical quantities, vacuum contractions \mathfrak{W}_{ij}^{+-} and \mathfrak{W}_{ji}^{-+} , are combined into observable physical quantities, $\mathfrak{W}_{1;ij}^2 + \mathfrak{W}_{2;ij}^2$ and $\mathfrak{W}_{3;ij}$, $\mathfrak{W}_{3;ij}^2$, by (70), (71), (72).

In Dual-identical bubble Vacuum contractions, Subscript i and subscript j are satisfied with the following formula (**refer.** paragraph 6.)

$$s = |i - j| = 1, 2, 3, 4, 5, \dots \quad (98)$$

For Self-identical bubble Vacuum contractions, $s = 0$.

From next paragraph, we begin to explain how bosons and fermions are formed from phase transitions of Dual-identical Bubbles ($s \neq 0$).

6. The formations of Bosons and Fermions

6.1 Formation of $0\hbar$ bosons result in the first order Vacuum Bubble phase transition by $\omega_{+1,0}^{+,-}$ and $\omega_{0,+1}^{-,+}$

From

Table1: Vacuum Contractions (90),(91)

$$\begin{aligned} \omega_{+1,0}^{+,-} &= \\ - \text{diag} (\dots 20, 12, 6, 2, 0, 0, 2, 6, 12, 20, 30, \dots) \\ \omega_{0,+1}^{-,+} &= \\ - \text{diag} (\dots 30, 20, 12, 6, 2, 0, 0, 2, 6, 12, 20, \dots) \end{aligned}$$

Obtain

Table2: $\omega_{1,+1,0}^2 + \omega_{2,+1,0}^2$

$$\begin{aligned} \omega_{1,+1,0}^2 + \omega_{2,+1,0}^2 &= \frac{1}{2} (\omega_{+1,0}^{+,-} + \omega_{0,+1}^{-,+}) = \\ - \frac{1}{2} \text{diag} (\dots 50, 32, 18, 8, 2, 0, 2, 8, 18, 32, 50, \dots) \\ &= \\ \text{diag} (\dots -25, -16, -9, -4, -1, 0, +1, -4, -9, -16, -25, \dots) \end{aligned}$$

And

Table3: $\omega_{3,+1,0}, \omega_{3,+1,0}^2$

$$\begin{aligned} \omega_{3,+1,0} &= \frac{1}{2} (\omega_{+1,0}^{+,-} - \omega_{0,+1}^{-,+}) = \\ + \frac{1}{2} \text{diag} (\dots 10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10, \dots) \\ &= \text{diag} (\dots +5, +4, +3, +2, +1, 0, -1, -2, -3, -4, -5, \dots) \\ \omega_{3,+1,0}^2 &= \\ \text{diag} (\dots +25, +16, +9, +4, +1, 0, +1, +4, +9, +16, +25, \dots) \end{aligned}$$

Then

Table4: $\omega_{+1,0}^2$

$$\begin{aligned} \omega_{+1,0}^2 &= \omega_{1,+1,0}^2 + \omega_{2,+1,0}^2 + \omega_{3,+1,0}^2 = \\ \text{diag} (\dots 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, \dots) \\ &= 0I_0 = 0(0 + 1)I_0 \end{aligned}$$

Table3 $\omega_{3,+1,0}$ and Table4: $\omega_{+1,0}^2$ are just the known $0\hbar$ boson of **CSH** !

$$\blacktriangle \quad \omega_{+1,0}^2 = \Omega_{+0}^2 \quad \omega_{3,+1,0} = \Omega_{3,+0} \quad !!! \quad (99)$$

6.2 Formation of $\hbar/2$ fermions result in the second order Vacuum Bubble phase transition by $\omega_{+1,-1}^{+,-}$ and $\omega_{-1,+1}^{-,+}$

From

Table5: Vacuum Contractions (92),(93)

$$\begin{aligned} \omega_{+1,-1}^{+,-} &= \\ - \text{diag} (\dots & 24, 15, 8, 3, 0, -1, 0, 3, 8, 15, 24, \dots) \\ \omega_{-1,+1}^{-,+} &= \\ - \text{diag} (\dots & 35, 24, 15, 8, 3, 0, -1, 0, 3, 8, 15, \dots) \end{aligned}$$

Obtain

Table6: $\omega_{1,+1,-1}^2 + \omega_{2,+1,-1}^2$

$$\begin{aligned} \omega_{1,+1,-1}^2 + \omega_{2,+1,-1}^2 &= \frac{1}{2} (\omega_{+1,-1}^{+,-} + \omega_{-1,+1}^{-,+}) = \\ - \frac{1}{2} \text{diag} (\dots & 49, 39, 23, 11, 3, -1, -1, 3, 11, 23, 39, \dots) \\ &= \\ + \frac{1}{4} \text{diag} (\dots & -98, -78, -46, -22, -6, 2, 2, -6, -22, -46, -78, \dots) \end{aligned}$$

And

Table7: $\omega_{3,+1,-1}, \omega_{3,+1,-1}^2$

$$\begin{aligned} \omega_{3,+1,-1} &= \frac{1}{2} (\omega_{+1,-1}^{+,-} - \omega_{-1,+1}^{-,+}) = \\ + \frac{1}{2} \text{diag} (\dots & 11, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9, \dots) \\ \omega_{3,+1,-1}^2 &= \\ + \frac{1}{4} \text{diag} (\dots & 121, 81, 49, 25, 9, 1, 1, 9, 25, 49, 81, \dots) \end{aligned}$$

Then

Table8: $\omega_{+1,-1}^2$

$$\begin{aligned} \omega_{+1,-1}^2 &= \omega_{1,+1,-1}^2 + \omega_{2,+1,-1}^2 + \omega_{3,+1,-1}^2 = \\ + \frac{1}{4} \text{diag} (\dots & 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, \dots) \\ &= \frac{3}{4} I_0 = \frac{3}{2} (\frac{3}{2} + 1) I_0 \end{aligned}$$

Table7 $\omega_{3,+1,-1}$ and Table8 $\omega_{+1,-1}^2$ are just the known $\hbar/2$ fermion of **CSH** !

$$\blacktriangledown \quad \omega_{+1,-1}^2 = \Omega_{+1/2}^2 \quad \omega_{3,+1,-1} = \Omega_{3,+1/2} \quad !!! \quad (100)$$

6.3 Formation of $1\hbar$ bosons result in the third order Vacuum Bubble phase transition by $\omega_{+2,-1}^{+,-}$ and $\omega_{-1,+2}^{-,+}$

From

Table9: Vacuum Contractions (94),(95)

$$\begin{aligned} \omega_{+2,-1}^{+,-} &= \\ - \text{diag} (\dots 18, 10, 4, 0, -2, -2, 0, 4, 10, 18, 28, \dots) \\ \omega_{-1,+2}^{-,+} &= \\ - \text{diag} (\dots 28, 18, 10, 4, 0, -2, -2, 0, 4, 10, 18, \dots) \end{aligned}$$

Obtain

Table10: $\omega_{1;+2,-1}^2 + \omega_{2;+2,-1}^2$

$$\begin{aligned} \omega_{1;+2,-1}^2 + \omega_{2;+2,-1}^2 &= \frac{1}{2} (\omega_{+2,-1}^{+,-} + \omega_{-1,+2}^{-,+}) = \\ - \frac{1}{2} \text{diag} (\dots 46, 28, 14, 4, -2, -4, -2, 4, 14, 28, 46, \dots) \\ &= \\ \text{diag} (\dots -23, -14, -7, -2, 1, 2, 1, -2, -7, -14, -23, \dots) \end{aligned}$$

And

Table11: $\omega_{3;+2,-1}, \omega_{3;+2,-1}^2$

$$\begin{aligned} \omega_{3;+2,-1} &= \frac{1}{2} (\omega_{+2,-1}^{+,-} - \omega_{-1,+2}^{-,+}) = \\ + \frac{1}{2} \text{diag} (\dots 10, 8, 6, 4, 2, 0, -2, -4, -6, -8, -10, \dots) \\ &= \text{diag} (\dots 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, \dots) \\ \omega_{3;+2,-1}^2 &= \\ \text{diag} (\dots 25, 16, 9, 4, 1, 0, 1, 4, 9, 16, 25, \dots) \end{aligned}$$

Then

Table12: $\omega_{+2,-1}^2$

$$\begin{aligned} \omega_{+2,-1}^2 &= \omega_{1;+2,-1}^2 + \omega_{2;+2,-1}^2 + \omega_{3;+2,-1}^2 = \\ \text{diag} (\dots 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, \dots) \\ &= 2I_0 = 1(1+1)I_0 \end{aligned}$$

Table11 $\omega_{3;+2,-1}$ and Table12 $\omega_{+2,-1}^2$ are just the known $1\hbar$ boson of CSH !

♠

$$\omega_{+2,-1}^2 = \Omega_{+1}^2 \quad \omega_{3;+2,-1} = \Omega_{3,+1} \quad !!! \quad (101)$$

6.4 Formation of $3\hbar/2$ fermions result in the fourth order Vacuum Bubble phase transition by $\mathfrak{W}_{+2,-2}^{+,-}$ and $\mathfrak{W}_{-2,+2}^{-,+}$

From

Table13: Vacuum Contractions (96),(97)

$$\begin{aligned} \mathfrak{W}_{+2,-2}^{+,-} &= \\ &- \text{diag} (\dots 21, 12, 5, 0, -3, -4, -3, 0, 5, 12, 21, \dots) \\ \mathfrak{W}_{-2,+2}^{-,+} &= \\ &- \text{diag} (\dots 32, 21, 12, 5, 0, -3, -4, -3, 0, 5, 12, \dots) \end{aligned}$$

Obtain

Table14: $\mathfrak{W}_{1,+2,-2}^2 + \mathfrak{W}_{2,+2,-2}^2$

$$\begin{aligned} \mathfrak{W}_{1,+2,-2}^2 + \mathfrak{W}_{2,+2,-2}^2 &= \frac{1}{2} (\mathfrak{W}_{+2,-2}^{+,-} + \mathfrak{W}_{-2,+2}^{-,+}) = \\ &- \frac{1}{2} \text{diag} (\dots 53, 33, 17, 5, -3, -7, -7, -3, 5, 17, 33, \dots) \\ &= \\ &+ \frac{1}{4} \text{diag} (\dots -106, -66, -34, -10, 6, 14, 14, 6, -10, -34, -66, \dots) \end{aligned}$$

And

Table15: $\mathfrak{W}_{3,+2,-2}, \mathfrak{W}_{3,+2,-2}^2$

$$\begin{aligned} \mathfrak{W}_{3,+2,-2} &= \frac{1}{2} (\mathfrak{W}_{+2,-2}^{+,-} - \mathfrak{W}_{-2,+2}^{-,+}) = \\ &+ \frac{1}{2} \text{diag} (\dots 11, 9, 7, 5, 3, 1, -1, -3, -5, -7, -9, \dots) \\ \\ \mathfrak{W}_{3,+2,-2}^2 &= \\ &+ \frac{1}{4} \text{diag} (\dots 121, 81, 49, 25, 9, 1, 1, 9, 25, 49, 81, \dots) \end{aligned}$$

Then

Table16: $\mathfrak{W}_{+2,-2}^2$

$$\begin{aligned} \mathfrak{W}_{+2,-2}^2 &= \mathfrak{W}_{1,+2,-2}^2 + \mathfrak{W}_{2,+2,-2}^2 + \mathfrak{W}_{3,+2,-2}^2 = \\ &+ \frac{1}{4} \text{diag} (\dots 15, 15, 15, 15, 15, 15, 15, 15, 15, 15, \dots) \\ &= \frac{15}{4} I_0 = \frac{3}{2} \left(\frac{3}{2} + 1 \right) I_0 \end{aligned}$$

Table15 $\mathfrak{W}_{3,+2,-2}$ and Table16 $\mathfrak{W}_{+2,-2}^2$ are just the known $3\hbar/2$ fermion of **CSH** !

$$\blacklozenge \quad \mathfrak{W}_{+2,-2}^2 = \Omega_{+3/2}^2 \quad \mathfrak{W}_{3,+2,-2} = \Omega_{3,+3/2} \quad !!! \quad (102)$$

6.5 Concluding Summations

- Table17: Vacuum Spin Particle, **VSP** with spin $-\frac{1}{2}$
 formed from 0th order phase transition
 by $\omega_{0,0}^{+,-}$ and $\omega_{0,0}^{-,+}$. $\omega_{0,0}^2 = -\frac{1}{2}(-\frac{1}{2} + 1) = -\frac{1}{4}$

$$\omega_{3;0,0}$$

$$= \text{diag} (... 11/2, 9/2, 7/2, 5/2, 3/2, 1/2, -1/2, -3/2, -5/2, -7/2, -9/2, ...)$$

- ▲ Table18: $0\hbar$ bosons formed from 1st order phase transition
 by $\omega_{+1,0}^{+,-}$ and $\omega_{0,+1}^{-,+}$: $\omega_{+1,0}^2 = 0(0 + 1) = 0$

$$\omega_{3;+1,0}$$

$$= \text{diag} (... 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, ...)$$

- ▼ Table19: $\hbar/2$ fermions formed from 2nd order phase transition
 by $\omega_{+1,-1}^{+,-}$ and $\omega_{-1,+1}^{-,+}$: $\omega_{+1,-1}^2 = +\frac{1}{2}(+\frac{1}{2} + 1) = \frac{3}{4}$

$$\omega_{3;+1,-1}$$

$$= \text{diag} (... 11/2, 9/2, 7/2, 5/2, 3/2, 1/2, -1/2, -3/2, -5/2, -7/2, -9/2, ...)$$

- ♠ Table20: $1\hbar$ bosons formed from 3rd order phase transition
 by $\omega_{+2,-1}^{+,-}$ and $\omega_{-1,+2}^{-,+}$: $\omega_{+2,-1}^2 = 1(1 + 2) = 2$

$$\omega_{3;+2,-1}$$

$$= \text{diag} (... 5, 4, 3, 2, 1, 0, -1, -2, -3, -4, -5, ...)$$

- ◆ Table21: $3\hbar/2$ fermions formed from 4th order phase transition
 by $\omega_{+2,-2}^{+,-}$ and $\omega_{-2,+2}^{-,+}$: $\omega_{+2,-2}^2 = \frac{3}{2}(\frac{3}{2} + 1) = \frac{15}{4}$

$$\omega_{3;+2,-2}$$

$$= \text{diag} (... 11/2, 9/2, 7/2, 5/2, 3/2, 1/2, -1/2, -3/2, -5/2, -7/2, -9/2, ...)$$

7. Vacuum Bubble Pair **VBP** $\omega_{S=|i-j|}^{+,-}(k)$ of the k th generations of **CSH**

In phase transition processes, contraction $\omega_{i,j}^{+,-}$ and contraction $\omega_{j,i}^{-,+}$ always appear to pair up together, then a mapping called Vacuum Bubble Pair **VBP** is introduced:

$$\omega_{i,j}^{+,-}(k) \text{ and } \omega_{j,i}^{-,+}(k) \rightarrow \{ \omega_{i,j}^{+,-}(k), \omega_{j,i}^{-,+}(k) \} \in \omega_{S=|i-j|}^{+,-}(k): \text{VBP} \quad (103)$$

$$S = |i - j|; \quad k = 1, 2, 3, \dots \quad (104)$$

Where $\omega_{S=|i-j|}^{+,-}(k)$ are the sets used to represent

$$\omega_{S=|i-j|}^{+,-}(k): \text{the } s\text{th order phase transition of Vacuum Bubble Pair} \quad (105)$$

In the previous discussions:

a) Mapping (106) indicates Self-identical bubble Vacuum contractions $\omega^{+,-}(73)$, $\omega^{-,+}(74)$ or $\omega_{0,0}^{+,-}(86)$, $\omega_{0,0}^{-,+}(87) \in \omega_{S=0}^{+,-}(k=1)$.

b) Mapping (107)–(110) indicate the four groups of Dual-identical bubble Vacuum contractions $\omega_{+1,0}^{+,-}(90)$, $\omega_{0,+1}^{-,+}(91)$; $\omega_{+1,-1}^{+,-}(92)$, $\omega_{-1,+1}^{-,+}(93)$; $\omega_{+2,-1}^{+,-}(94)$, $\omega_{-1,+2}^{-,+}(95)$; $\omega_{+2,-2}^{+,-}(96)$, $\omega_{-2,+2}^{-,+}(97) \in \omega_{S \neq 0}^{+,-}(k=1)$.

In above paragraph 6. what follows mean:

$$\omega_{0,0}^{+,-}, \omega_{0,0}^{-,+} \text{ (86),(87)} \rightarrow \{ \omega_{0,0}^{+,-}(1), \omega_{0,0}^{-,+}(1) \} \in \omega_{S=0}^{+,-}(1) \quad (106)$$

$$\omega_{+1,0}^{+,-}, \omega_{0,+1}^{-,+} \text{ 6.1} \rightarrow \{ \omega_{+1,0}^{+,-}(1), \omega_{0,+1}^{-,+}(1) \} \in \omega_{S=1}^{+,-}(1) \quad (107)$$

$$\omega_{+1,-1}^{+,-}, \omega_{-1,+1}^{-,+} \text{ 6.2} \rightarrow \{ \omega_{+1,-1}^{+,-}(1), \omega_{-1,+1}^{-,+}(1) \} \in \omega_{S=2}^{+,-}(1) \quad (108)$$

$$\omega_{+2,-1}^{+,-}, \omega_{-1,+2}^{-,+} \text{ 6.3} \rightarrow \{ \omega_{+2,-1}^{+,-}(1), \omega_{-1,+2}^{-,+}(1) \} \in \omega_{S=3}^{+,-}(1) \quad (109)$$

$$\omega_{+2,-2}^{+,-}, \omega_{-2,+2}^{-,+} \text{ 6.4} \rightarrow \{ \omega_{+2,-2}^{+,-}(1), \omega_{-2,+2}^{-,+}(1) \} \in \omega_{S=4}^{+,-}(1) \quad (110)$$

In following paragraph 8. what follows will be used:

$$\dots \{ \omega_{-1,-1}^{+,-}(2), \omega_{-1,-1}^{-,+}(2) \}; \{ \omega_{0,0}^{+,-}(2), \omega_{0,0}^{-,+}(2) \}; \{ \omega_{+1,+1}^{+,-}(2), \omega_{+1,+1}^{-,+}(2) \} \dots \in \omega_{S=0}^{+,-}(2) \quad (111)$$

$$\dots \{ \omega_{0,-1}^{+,-}(2), \omega_{-1,0}^{-,+}(2) \}; \{ \omega_{+1,0}^{+,-}(2), \omega_{0,+1}^{-,+}(2) \}; \{ \omega_{+2,+1}^{+,-}(2), \omega_{+1,+2}^{-,+}(2) \} \dots \in \omega_{S=1}^{+,-}(2) \quad (112)$$

$$\dots \{ \omega_{+1,-1}^{+,-}(2), \omega_{-1,+1}^{-,+}(2) \}; \{ \omega_{+2,0}^{+,-}(2), \omega_{0,+2}^{-,+}(2) \}; \{ \omega_{+3,+1}^{+,-}(2), \omega_{+1,+3}^{-,+}(2) \} \dots \in \omega_{S=2}^{+,-}(2) \quad (113)$$

$$\dots \in \omega_{S=3}^{+,-}(2) \quad (114)$$

In following paragraph 9. what follows will be used:

$$\dots \{ \omega_{-1,-1}^{+,-}(3), \omega_{-1,-1}^{-,+}(3) \}; \{ \omega_{0,0}^{+,-}(3), \omega_{0,0}^{-,+}(3) \}; \{ \omega_{+1,+1}^{+,-}(3), \omega_{+1,+1}^{-,+}(3) \} \dots \in \omega_{S=0}^{+,-}(3) \quad (115)$$

$$\dots \{ \omega_{0,-1}^{+,-}(3), \omega_{-1,0}^{-,+}(3) \}; \{ \omega_{+1,0}^{+,-}(3), \omega_{0,+1}^{-,+}(3) \}; \{ \omega_{+2,+1}^{+,-}(3), \omega_{+1,+2}^{-,+}(3) \} \dots \in \omega_{S=1}^{+,-}(3) \quad (116)$$

$$\dots \{ \omega_{+1,-1}^{+,-}(3), \omega_{-1,+1}^{-,+}(3) \}; \{ \omega_{+2,0}^{+,-}(3), \omega_{0,+2}^{-,+}(3) \}; \{ \omega_{+3,+1}^{+,-}(3), \omega_{+1,+3}^{-,+}(3) \} \dots \in \omega_{S=2}^{+,-}(3) \quad (117)$$

$$\dots \in \omega_{S=3}^{+,-}(3) \quad (118)$$

8. Formation of bosons, fermions and TKP and VSP
of the second generation of **CSH**

Table22: Primitive Spin Particle, **VSP** with spin $-\frac{1}{2}$
formed from 0th order phase transition
of the second generation of **CSH**

$$\text{by } \omega_{S=0}^{+-}(2): \quad \omega_{S=0}^2(2) = -\frac{1}{2} \left(-\frac{1}{2} + 1\right) = -\frac{1}{4}$$

$$\omega_{3, S=0}^{+-}(2)$$

$$= \text{diag} (... \ 3, \quad 5/2, \quad 2, \quad 3/2, \quad 1, \quad 1/2, \quad 0, \quad -1/2, \quad -1, \quad -3/2, \quad -2, \quad ...)$$

Table23: $-\frac{1}{4}$ spin **TKP**
formed from 1st order phase transition
of the second generation of **CSH**

$$\text{by } \omega_{S=1}^{+-}(2): \quad \omega_{S=1}^2(2) = -\frac{1}{4} \left(-\frac{1}{4} + 1\right) = -\frac{3}{16}$$

$$\omega_{3, S=1}^{+-}(2)$$

$$= \text{diag} (... \ 11/4, \ 9/4, \ 7/4, \ 5/4, \ 3/4, \ 1/4, \ -1/4, \ -3/4, \ -5/4, \ -7/4, \ -9/4, \ ...)$$

Table24: $0\hbar$ bosons formed from 2nd order phase transition
of the second generation of **CSH**

$$\text{by } \omega_{S=2}^{+-}(2): \quad \omega_{S=2}^2(2) = 0(0 + 1) = 0$$

$$\omega_{3, S=2}^{+-}(2)$$

$$= \text{diag} (... \ 3, \quad 5/2, \quad 2, \quad 3/2, \quad 1, \quad 1/2, \quad 0, \quad -1/2, \quad -1, \quad -3/2, \quad -2, \quad ...)$$

Table25: $\hbar/4$ **TKP** formed from 3rd order phase transition
of the second generation of **CSH**

$$\text{by } \omega_{S=3}^{+-}(2): \quad \omega_{S=3}^2(2) = \frac{1}{4} \left(\frac{1}{4} + 1\right) = \frac{5}{16}$$

$$\omega_{3, S=3}^{+-}(2)$$

$$= \text{diag} (... \ 11/4, \ 9/4, \ 7/4, \ 5/4, \ 3/4, \ 1/4, \ -1/4, \ -3/4, \ -5/4, \ -7/4, \ -9/4, \ ...)$$

Table26: $\hbar/2$ fermions formed from 4th order phase transition
of the second generation of **CSH**

$$\text{by } \omega_{S=4}^{+-}(2): \quad \omega_{S=4}^2(2) = \frac{1}{2} \left(\frac{1}{2} + 1\right) = \frac{3}{4}$$

$$\omega_{3, S=4}^{+-}(2)$$

$$= \text{diag} (... \ 3, \quad 5/2, \quad 2, \quad 3/2, \quad 1, \quad 1/2, \quad 0, \quad -1/2, \quad -1, \quad -3/2, \quad -2, \quad ...)$$

9. Formation of bosons, fermions and TKP and VSP
of the third generation of **CSH**

Table27: Vacuum Spin Particle, **VSP** with spin $-\frac{1}{2}$
formed from 0th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{S=0}^{+,-}(3): \quad \omega_{S=0}^2(3) = -\frac{1}{2}(-\frac{1}{2} + 1) = -\frac{1}{4}$$

$$\omega_{3, S=0}^{+,-}(3)$$

$$= \text{diag} (... 11/6, \quad 3/2, \quad 7/6, \quad 5/6, \quad 1/2, \quad 1/6, \quad -1/6, \quad -1/2, \quad -5/6, \quad -7/6, \quad -3/2, \quad ...)$$

Table28: $-\frac{1}{3}$ spin **TKP**
formed from 1st order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{S=1}^{+,-}(3): \quad \omega_{S=1}^2(3) = -\frac{1}{3}(-\frac{1}{3} + 1) = -\frac{2}{9}$$

$$\omega_{3, S=1}^{+,-}(3)$$

$$= \text{diag} (... 5/3, \quad 4/3, \quad 1, \quad 2/3, \quad 1/3, \quad 0, \quad -1/3, \quad -2/3, \quad -1, \quad -4/3, \quad -5/3, \quad ...)$$

Table29: $-\frac{1}{6}$ spin **TKP**
formed from 2nd order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{S=2}^{+,-}(3): \quad \omega_{S=2}^2(3) = -\frac{1}{6}(-\frac{1}{6} + 1) = -\frac{5}{36}$$

$$\omega_{3, S=2}^{+,-}(3)$$

$$= \text{diag} (... 11/6, \quad 3/2, \quad 7/6, \quad 5/6, \quad 1/2, \quad 1/6, \quad -1/6, \quad -1/2, \quad -5/6, \quad -7/6, \quad -3/2, \quad ...)$$

Table30: $0\hbar$ bosons formed from 3rd order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{S=3}^{+,-}(3): \quad \omega_{S=3}^2(3) = 0(0 + 1) = 0$$

$$\omega_{3, S=3}^{+,-}(3)$$

$$= \text{diag} (... 2, \quad 5/3, \quad 4/3, \quad 1, \quad 2/3, \quad 1/3, \quad 0, \quad -1/3, \quad -2/3, \quad -1, \quad -4/3, \quad ...)$$

Table31: $\hbar/6$ **TKP** formed from 4th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{s=4}^{+,-}(3): \quad \omega_{s=4}^2(3) = \frac{1}{6} \left(\frac{1}{6} + 1 \right) = \frac{7}{36}$$

$$\omega_{3, s=4}^{+,-}(3)$$

$$= \text{diag} (... 11/6, 3/2, 7/6, 5/6, 1/2, 1/6, -1/6, -1/2, -5/6, -7/6, -3/2, ...)$$

Table32: $\hbar/3$ **TKP** formed from 5th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{s=5}^{+,-}(3): \quad \omega_{s=5}^2(3) = \frac{1}{3} \left(\frac{1}{3} + 1 \right) = \frac{4}{9}$$

$$\omega_{3, s=5}^{+,-}(3)$$

$$= \text{diag} (... 5/3, 4/3, 1, 2/3, 1/3, 0, -1/3, -2/3, -1, -4/3, -5/3, ...)$$

Table33: $\hbar/2$ fermions formed from 6th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{s=6}^{+,-}(3): \quad \omega_{s=6}^2(3) = \frac{1}{2} \left(\frac{1}{2} + 1 \right) = \frac{3}{4}$$

$$\omega_{3, s=6}^{+,-}(3)$$

$$= \text{diag} (... 11/6, 3/2, 7/6, 5/6, 1/2, 1/6, -1/6, -1/2, -5/6, -7/6, -3/2, ...)$$

Table34: $2\hbar/3$ **TKP** formed from 7th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{s=7}^{+,-}(3): \quad \omega_{s=7}^2(3) = \frac{2}{3} \left(\frac{2}{3} + 1 \right) = \frac{10}{9}$$

$$\omega_{3, s=7}^{+,-}(3)$$

$$= \text{diag} (... 5/3, 4/3, 1, 2/3, 1/3, 0, -1/3, -2/3, -1, -4/3, -5/3, ...)$$

Table35: $5\hbar/6$ **TKP** formed from 8th order phase transition
of the third generation of **CSH**

$$\text{by } \omega_{s=8}^{+,-}(3): \quad \omega_{s=8}^2(3) = \frac{5}{6} \left(\frac{5}{6} + 1 \right) = \frac{55}{36}$$








$$\omega_{3, s=8}^{+,-}(3)$$

$$= \text{diag} (... 11/6, 3/2, 7/6, 5/6, 1/2, 1/6, -1/6, -1/2, -5/6, -7/6, -3/2, ...)$$

10. The sth order phase transitions of Vacuum Bubble Pair $\omega_{S=|i-j|}^{+,-}(k)$ in SAMV ocean.

The results of paragraphs 6., 7., 8., 9. are tabled as below.

Table36 Contributions of Vacuum Bubble Pair **VBP** to Phase Transition (**PT**) for the 1st, 2nd and 3rd of **CSH**

VBP $\omega_{S= i-j }^{+,-}(k)$	1st CSH		2nd CSH		3rd CSH	
	$\omega_S^2(1)$	<i>Spin</i> (1)	$\omega_S^2(2)$	<i>Spin</i> (2)	$\omega_S^2(3)$	<i>Spin</i> (3)
$\omega_{S=8}^{+,-}$	$\frac{63}{4}$	$\frac{7}{2}$	$\frac{15}{4} = \frac{60}{16}$	$\frac{6}{4} = \frac{3}{2}$	$\frac{55}{36}$	$\frac{5}{6}$
$\omega_{S=7}^{+,-}$	$12 = \frac{48}{4}$	$\frac{6}{2} = 3$	$\frac{45}{16}$	$\frac{5}{4}$	$\frac{10}{9} = \frac{40}{36}$	$\frac{4}{6} = \frac{2}{3}$
$\omega_{S=6}^{+,-}$	$\frac{35}{4}$	$\frac{5}{2}$	$2 = \frac{32}{16}$	$\frac{4}{4} = 1$	$\frac{3}{4} = \frac{27}{36}$	$\frac{3}{6} = \frac{1}{2}$
$\omega_{S=5}^{+,-}$	$6 = \frac{24}{4}$	$\frac{4}{2} = 2$	$\frac{21}{16}$	$\frac{3}{4}$	$\frac{4}{9} = \frac{16}{36}$	$\frac{2}{6} = \frac{1}{3}$
$\omega_{S=4}^{+,-}$	$\frac{15}{4}$ 	$\frac{3}{2}$	$\frac{3}{4} = \frac{12}{16}$	$\frac{2}{4} = \frac{1}{2}$	$\frac{7}{36}$	$\frac{1}{6}$
$\omega_{S=3}^{+,-}$	$2 = \frac{8}{4}$ 	$\frac{2}{2} = 1$	$\frac{5}{16}$	$\frac{1}{4}$	$0 = \frac{0}{36}$	$\frac{0}{6} = 0$
$\omega_{S=2}^{+,-}$	$\frac{3}{4}$ 	$\frac{1}{2}$	$0 = \frac{0}{16}$	$\frac{0}{4} = 0$	$-\frac{5}{36}$	$-\frac{1}{6}$
$\omega_{S=1}^{+,-}$	$0 = \frac{0}{4}$ 	$\frac{0}{2} = 0$	$-\frac{3}{16}$	$-\frac{1}{4}$	$-\frac{2}{9} = -\frac{8}{36}$	$-\frac{2}{6} = -\frac{1}{3}$
$\omega_{S=0}^{+,-}$	$-\frac{1}{4}$ 	$-\frac{1}{2}$	$-\frac{4}{16}$ 	$-\frac{2}{4}$	$-\frac{9}{36}$ 	$-\frac{3}{6}$
$\omega_{S=0}^{+,-}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$	$-\frac{1}{4}$	$-\frac{1}{2}$
VBP $\omega_{S= i-j }^{+,-}(k)$	1st CSH $\omega_S^2(1)$	<i>Spin</i> (1)	2nd CSH $\omega_S^2(2)$	<i>Spin</i> (2)	3rd CSH $\omega_S^2(3)$	<i>Spin</i> (3)

11. Spin particles with Casimir operator Ω_S^2
formed of Vacuum Bubble Pair **VBP** $\Omega_S^{+, -}$

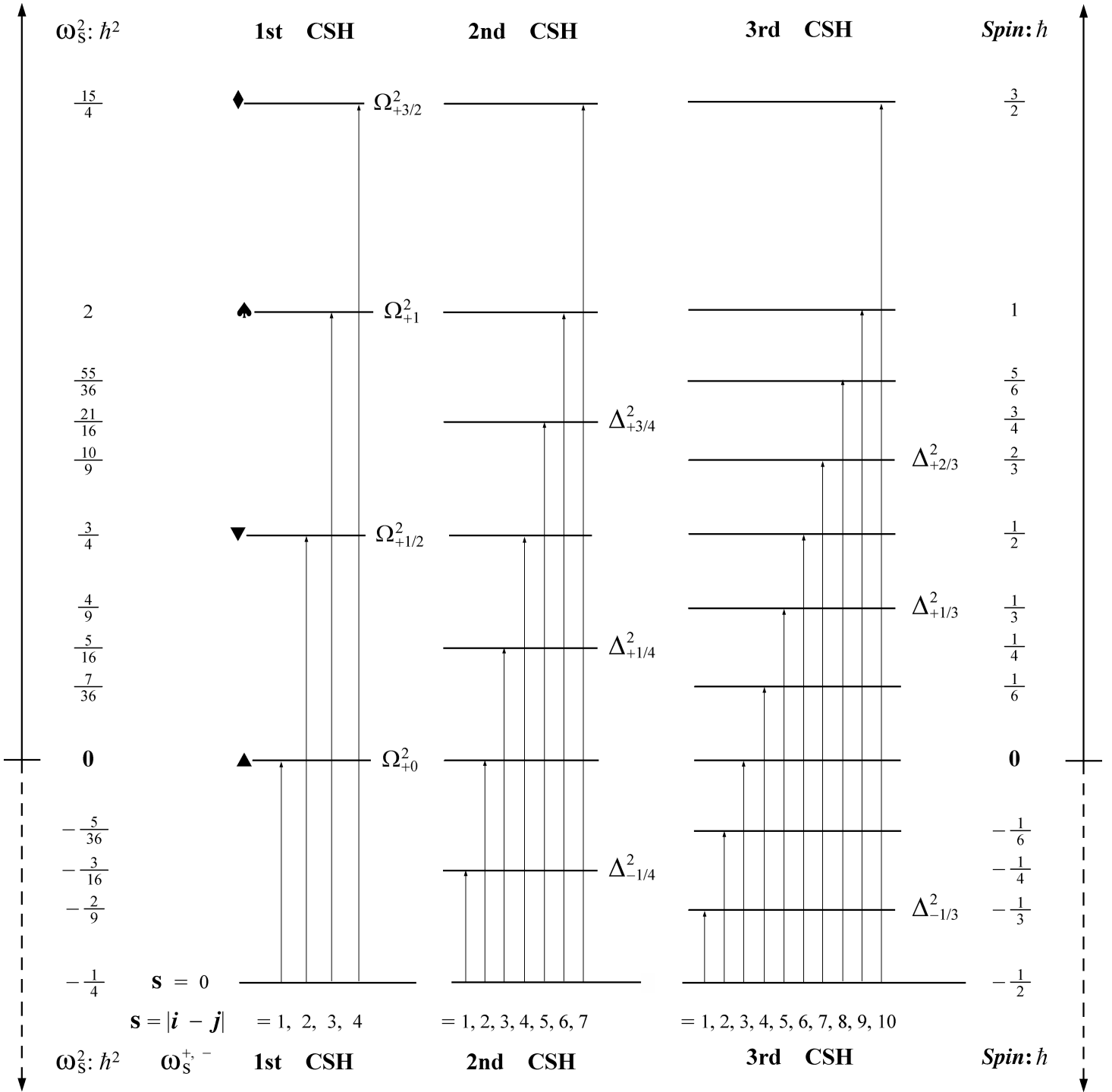


FIG.2. Phase transitions of Vacuum Bubble Pair $\Omega_S^{+, -} (k)$
from the sea floor one-fourth $-\frac{1}{4} h^2$ deep in SAMV ocean (not to scale)

12. Local and Nonlocal Spin Angular Momentum Commutation Rules

In conventional quantum mechanics, spin particles obey

$$[s_{ix}, s_{iy}]_- = is_{iz}, \quad x, y, z \text{ circulative} \quad (119.1)$$

$$[s_{jx}, s_{jy}]_- = is_{jz}, \quad (119.2)$$

$$[s_{ix}, s_{jy}]_- = 0, \quad i \neq j \text{ for any } x, y, z \quad (119.3)$$

s_{ix}, s_{jy} indicate x component of particle i , y component of particle j .

We call above commutation rules (119) Local Spin Particle Angular Momentum Commutation Rules **LR**

The commutation rules (60), of Spin $-\frac{1}{2}\hbar$ **VSP** particle what we discussed in paragraph 3, are **LR**.

LR are connected to Self-identical bubble vacuum contractions.

Using (64),(65) then (119) can be written as (120) below:

1) Local commutations, only for single particle i itself

$$s_{j,3}s_j^+ - s_j^+s_{j,3} = +s_j^+ \quad (120.1)$$

$$s_{j,3}s_j^- - s_j^-s_{j,3} = -s_j^- \quad (120.2)$$

$$s_j^+s_j^- - s_j^-s_j^+ = 2s_{j,3}^+ \quad (120.3)$$

2) Non-local commutations, for both particle i and particle j

$$s_{ij,3}s_i^+ - s_i^+s_{ij,3} = 0 \quad (121.1)$$

$$s_{ij,3}s_j^- - s_j^-s_{ij,3} = 0 \quad (121.2)$$

$$s_i^+s_j^- - s_j^-s_i^+ = 0 \quad (121.3)$$

The appearance of Dual-identical bubble vacuum contractions., lead to a new concept called Nonlocality of Vacuum Contractions.

$$(120) \text{ Locality: } \omega_j^+\omega_j^- - \omega_j^-\omega_j^+ = \omega_{j,j}^{+-} - \omega_{j,j}^{-+} \neq 0 \quad (122)$$

$$(121) \text{ NonLocality: } \omega_i^+\omega_j^- - \omega_j^-\omega_i^+ = \omega_{i,j}^{+-} - \omega_{j,i}^{-+} \neq 0 \quad (123)$$

And commutation rules (121) turn into

$$\omega_3\omega_i^+ - \omega_i^+\omega_3 = +\omega_i^+ \quad (124.1)$$

$$\omega_3\omega_j^- - \omega_j^-\omega_3 = -\omega_j^- \quad (124.2)$$

$$\omega_i^+\omega_j^- - \omega_j^-\omega_i^+ = 2\omega_3 \quad (124.3)$$

$$i \neq j \quad (124.4)$$

(124) are called **NonLocal Spin Angular Momentum Commutation Rules** or **NLR**.

When $i = j$, **NLR** degenerate to **LR**.

The more detailed proofs of **VSP NLR** are given in cite [3].

13. Conclusions

Bosons and Fermions are the offspring of Vacuum Spin Particles. **VSP**.

In this paper, **VSP** is $-\frac{1}{2}\hbar$ spin particle, labeled by $\omega_j = \Omega_{j,-1/2} = \omega_{j,S=0}(k)$, which lie on the sea floor of the shallow water region ($-\frac{1}{4}\hbar^2$ deep) in Spin Angular Momentum Vacuum **SAMV** ocean.

By means of Matrix Product $\omega^+\omega^-$ and $\omega^-\omega^+$, named Vacuum Contraction **VC** of Vacuum Spin Particles ω_j , then Vacuum Bubbles **VB** (see 4.) ω^{+-} and ω^{-+} are formed.

VB are identical bubbles. which could provide all types of Vacuum Bubble Pair **VBP** $\omega_S^{+-}(k)$ (see 7.)

There are many body interactions between these **VBP**, Such as Self-identical vacuum bubble interaction (see 4.2), Dual-identical vacuum bubble interaction (see 4.3),...

VSP ω_j (or ω^+ , ω^-) results from Self-identical vacuum bubble interaction $\omega_{S=0}^{+-}(k)$ through the zero order Phase Transition **PT**.

All known Bosons and all known Fermions result from Dual-identical vacuum bubble interaction $\omega_{S \neq 0}^{+-}(k)$ through mult-order Phase Transitions **PT** (see 7., 6., 8., 9.), which are shown in Table36, Table37 and FIG.

Table37

$k = 1, 2, 3, \dots$	$\omega_S^2(k)$	$Spin$	
$\omega_{S=4}^{+-}(k)$	$\frac{15}{4}$	$\frac{3}{2}$	$= \Omega_{+3/2}^{+-} \in \omega_{S=4}^{+-}(k) \blacklozenge$
$\omega_{S=3}^{+-}(k)$	2	1	$= \Omega_{+1}^{+-} \in \omega_{S=3}^{+-}(k) \spadesuit$
$\omega_{S=2}^{+-}(k)$	$\frac{3}{4}$	$\frac{1}{2}$	$= \Omega_{+1/2}^{+-} \in \omega_{S=2}^{+-}(k) \blacktriangledown$
$\omega_{S=1}^{+-}(k)$	0	0	$= \Omega_{+0}^{+-} \in \omega_{S=1}^{+-}(k) \blacktriangle$
$\omega_{S=0}^{+-}(k)$	$-\frac{1}{4}$	$-\frac{1}{2}$	$= \Omega_{-1/2}^{+-} \in \omega_{S=0}^{+-}(k) \blacksquare$

Where

$$\Omega_j^+ = \Omega_{1,j} + i\Omega_{2,j} \quad (125)$$

$$\Omega_j^- = \Omega_{1,j} - i\Omega_{2,j} \quad (126)$$

$$j = +0, +1/2, +1, +3/2, \dots \quad (127)$$

$$\Omega_j^{+-} = \Omega_j^+\Omega_j^- \quad (128)$$

$$\Omega_j^{-+} = \Omega_j^-\Omega_j^+ \quad (129)$$

The physical picture of Vacuum Bubbles **VB** Ω^{+-} and Ω^{-+} are described to be "sub-observable physical quantities", because they are "diagonalized". But not any reality of today's quantum world are corresponding to them.

The principle of Vacuum Bubble Pair **VBP** $\Omega_{S=|i-j|}^{+-}(k)$ is the key figure in the formations of bosons, fermions and all other potential spin particles that predicted.

VSP, $-\frac{1}{2}\hbar$ spin particle is the "critical particle" of Primitive Spin Particles **PSP**, if the depth of SAMV ocean is greater than $\Omega_S^2 = -\frac{1}{4}\hbar^2$, Primitive Spin Particles **PSP** would become the imaginary number or complex number spin particles. how about the physical pictures of them ?

The fundamental nature of them will become more mysterious, of course.

This paper explains how well-known Bosons and well-known Fermions in universe are constructed from more fundamental matter spin particles. And postulates the existence of **VSP**, $-\frac{1}{2}\hbar$ spin particle.

If reasonable, more or less, the detection of **TKP** seems to be the priority of all...

Hope the idea of this paper would be beneficial to the fellows of physics and Math.

This paper is the selections of cite: [3]

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