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Erratum: The Gravitational Radiation Emitted by a System Consisting of a Point Particle in Close Orbit around a Schwarzschild Black Hole

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ABSTRACT

We correct from the previous paper: the first, second and third order derivatives of the Bondi metric function *J* at the ISCO of the binary system consisting of a Schwarzschild black hole and a point particle. Previously, these derivatives where not correctly determined and that resulted in the incorrect determination of the emitted gravitational radiation at null infinity. The now correctly calculated gravitational radiation is now in full agreement with that obtained by the standard 5.5 PN formalism to about ninety eight percent. The small percentage difference observed is due to the slow convergence property of the PN formalism as compared to the null cone formalism, otherwise the results are basically the same.

Keywords: Black Hole; Particle; Gravitational Radiation; Null Infinity

1. Errors

1) Equation (13) in [1] should be correctly read as

$$\frac{\mathrm{d}v}{\mathrm{d}x} = 1 + \frac{2v}{x^2(1-2x)}\left((x-v)\left(2+\frac{iv}{x}\right) - x(7x+8v)\right) \quad (1)$$

where v is the orbital frequency of the system.

2) Also in the original paper by the author [1], there was an inherent numerical error due to the incorrect determination of the first, second, and third order derivatives of the Bondi metric function $J_{0+}(x)$ and $J_{0-}(x)$ in

$$\begin{aligned} J_+(x) &= c4 + c1x + c2J_{0+}(x) \\ J_-(x) &= c9 + c6x + c7J_{0-}(x). \end{aligned} \quad (2)$$

The correct derivatives are now here given by

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}x}J_{0+}(x) &= \left(\frac{s-v_+(s)}{sv_+(s)}\right), \\ \frac{\mathrm{d}^2}{\mathrm{d}x}J_{0+}(x) &= \left(-\frac{\frac{\mathrm{d}v_+(s)}{\mathrm{d}s}}{sv_+(s)}\right) + \left(\frac{s-v_+(s)}{sv_+(s)}\right)^2, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\mathrm{d}^3}{\mathrm{d}x}J_{0+}(x) = {} & \left(\frac{-\frac{\mathrm{d}^2v_+(s)}{\mathrm{d}s}v_+(s) + \left(\frac{\mathrm{d}v_+(s)}{\mathrm{d}s}\right)^2}{sv_+^2(s)}\right) - 2\left(\frac{s-v_+(s)}{sv_+(s)}\right)\left(\frac{\frac{\mathrm{d}v_+(s)}{\mathrm{d}s}}{sv_+(s)}\right) + \left(\frac{-\frac{\mathrm{d}v_+(s)}{\mathrm{d}s}}{sv_+(s)}\right) \\ & + \left(\frac{s-v_+(s)}{sv_+(s)}\right)^2\left(\frac{s-v_+(s)}{sv_+(s)}\right), \end{aligned} \quad (4)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}J_{0-}(x)=\frac{1}{v_-(s)},$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x}J_{0-}(x)=\left(\frac{1-\frac{\mathrm{d}v_-(s)}{\mathrm{d}s}}{v_-^2(s)}\right),$$

$$\frac{\mathrm{d}^3}{\mathrm{d}x}J_{0-}(x)=-\left(\frac{\frac{\mathrm{d}^2v_-(s)}{\mathrm{d}s}}{v_-^2(s)}\right)-\left(\frac{1-\frac{\mathrm{d}v_-(s)}{\mathrm{d}s}}{v_-^3(s)}\right) \quad (5)$$

where $x \to s = 1-2x$ at the black hole horizon (*i.e.* regular singularity). We used the Matlab ode 45 solver to solve the initial value Ricatti type Equation (1) for v. After the transformation $x = 1/r$, and now with v having been numerically calculated, the above derivatives then simplifies to

$$\frac{\mathrm{d}}{\mathrm{d}x}J_{2+}(x)=7.04456881148929 -1.31528646137769i, \quad (6)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x}J_{2+}(x)=41.54717973225140 -18.10743764931648i, \quad (7)$$

$$\frac{\mathrm{d}^3}{\mathrm{d}x}J_{2+}(x)=3.312074783567341\times10^3 -9.909330338546977\times10^1 i, \quad (8)$$

and

$$\frac{\mathrm{d}}{\mathrm{d}x}J_{2-}(x)=-21.88051906545720 -3.5914983000297i, \quad (9)$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x}J_{2-}(x)=4.419194136304895\times10^2 +6.628089504318726\times10^1 i, \quad (10)$$

$$\frac{\mathrm{d}^3}{\mathrm{d}x}J_{2-}(x)=2.280190335720033\times10^3 +4.074905896176351\times10^2 i. \quad (11)$$

From which we get the simplified expressions for J_+ and J_- in Equation (2). We use these expressions in the remainder of the computation as discussed by the author in detail in [1], and they are given by

$$\begin{aligned}J_+(r)&=c4+c1/r+c2\big(1+(7.04456881148929-1.31528646137769i)(1/r-1/6)\\&+(20.77358986-9.053718825i)(1/r-1/6)^2+(552.0124641-16.51555057i)(1/r-1/6)^3\big),\end{aligned} \quad (12)$$

$$\begin{aligned}J_-(r)&=c9+c6/r+c7\big(1-(21.88051906545720+3.59149830002971i)(1/r-1/6)\\&+(220.9597068+3.314044752i)(1/r-1/6)^2+(380.0317227+67.91509828i)(1/r-1/6)^3\big).\end{aligned} \quad (13)$$

After further computations as outlined in [1], we were able to get the following system of equations from the junction conditions at r_0 (the ISCO)

$$\begin{aligned}&0.1666666667c1+c2-0.07000563429im+0.3749650946m\\&+0.1662592015c6+0.007642559603ic6=0,\\[6pt]&-0.4786910378c2-0.09845907046ic2+0.06481481480c1+0.001154184775m\\&-0.02366533918c6+0.0001409996558ic6-0.02244086291im=0,\\[6pt]&0.6877634271ic6+0.4080000000ic1+c1+131.2878882c2-197.5267733ic2\\&+4.384639047m+1.675659905c6+2.049946578im=6.864684247m\end{aligned} \quad (14)$$

with

$$\rho=-\frac{1}{4}\frac{m\sqrt{15}\left[\mathrm{Re}\{\mathrm{e}^{2vui}\}Z_{2,2}-\mathrm{Re}\{\mathrm{ie}^{2vui}\}Z_{2,-2}\right]}{36\sqrt{\pi}}. \quad (15)$$

We solve the above system of equations for the constants $c1$, $c2$, and $c6$. The theory of how these constants and those expressed in the Appendix in [1] come about is explained by the author in that paper. The correct numerical expressions for $c1$, $c2$, and $c3$ are now given by

$$c1=(-0.5722276842+0.4849522675i)m, \quad (16)$$

$$c2=(0.01759330111+0.01961301970i)m, \quad (17)$$

$$c6=(-1.792120050-0.1006643777i)m. \quad (18)$$

From the above corrections we were able to find the following correct graphs of the Bondi metric variables J, U, and ω.

Theoretically, the metric functions J and U are smooth throughout the entire computational domain as outlined in [1], and this behavior is indeed confirmed in **Figures 1** and **2**. The metric function ω does not have this property as can be observed in **Figure 3**, but it is crucial in the calculation of the gravitation radiation in the entire domain. All other metric functions are intergrated radially from Γ to $\mathcal{I}^+$. The above results indicate that the junction conditions at $r0 = 6$ were implemented correctly and that our numerical methods worked properly.

Then finally, we were able to find the gravitational news function as

$$\mathcal{N} = \mathrm{Re}\left(0.01066946485 + 0.07007936942i\right)m \cdot \left[\mathrm{Re}\left(\mathrm{e}^{2i\nu u}Z_{22}\right) + \mathrm{Re}\left(-\mathrm{i}\mathrm{e}^{2i\nu u}Z_{2,-2}\right)\right], \tag{19}$$

from which the Bondi mass loss is given by

$$\frac{\mathrm{d}m}{\mathrm{d}u} = -\frac{1}{4\pi}\int_{\text{sphere}}\left|\mathcal{N}\right|^2 = -\frac{1}{4\pi}\left(0.010669464m\right)^2 \tag{20}$$

$$= -0.000849049m^2. \tag{21}$$

We finally validate our gravitational radiation result in Equation (21) by comparing it to the results of Poisson [2]

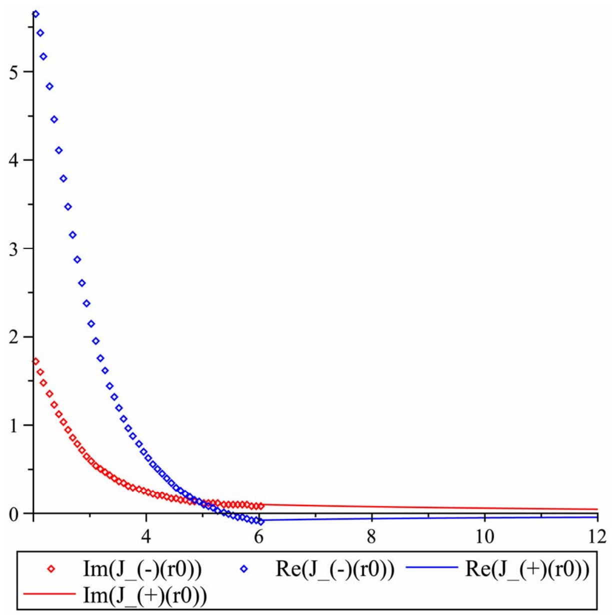

Figure 1. The graph of $\mathrm{Re}\left(J_-(r0)\right)$, $\mathrm{Im}\left(J_-(r0)\right)$ and $\mathrm{Re}\left(J_+(r0)\right)$, $\mathrm{Re}\left(J_+(r0)\right)$ for the Schwarzschild space-time. $\nu = 0.07$ and $\ell = 2$.

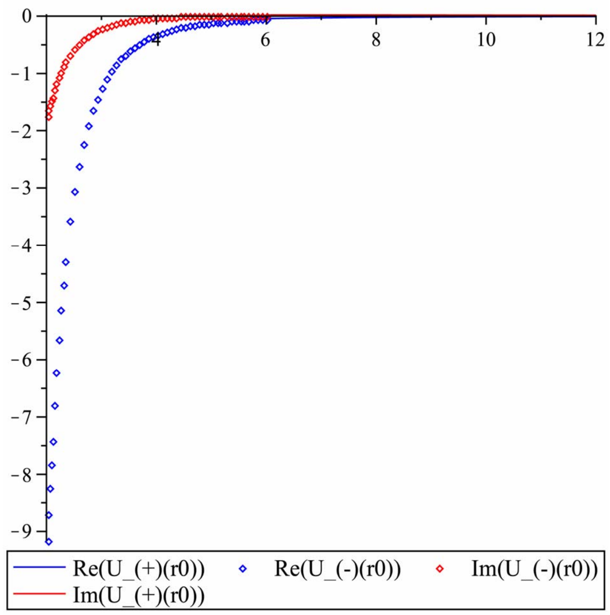

Figure 2. The graph of $\mathrm{Re}\left(U_-(r0)\right)$, $\mathrm{Im}\left(U_-(r0)\right)$ and $\mathrm{Re}\left(U_+(r0)\right)$, $\mathrm{Im}\left(U_+(r0)\right)$ for the Schwarzschild space-time. $\nu = 0.07$ and $\ell = 2$.

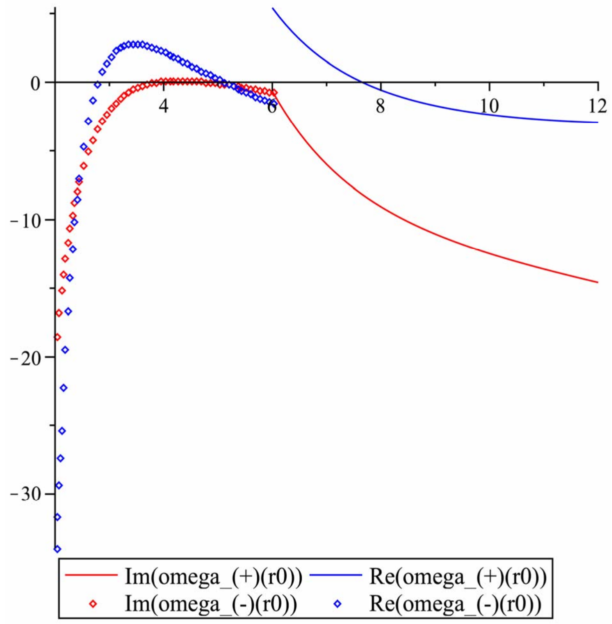

Figure 3. The graph of $\mathrm{Re}\left(\omega_-(r0)\right)$, $\mathrm{Im}\left(\omega_-(r0)\right)$ and $\mathrm{Re}\left(\omega_+(r0)\right)$, $\mathrm{Im}\left(\omega_+(r0)\right)$ for the Schwarzschild space-time. $\nu = 0.07$ and $\ell = 2$.

and Sasaki *et al*. [3], who used the 5.5 PN formalism to study the emitted gravitational radiation for the same problem as in this thesis. The 5.5 PN formula up to the eleventh order is given by

$$\begin{aligned}\frac{\mathrm{d}E}{\mathrm{d}t}=\left[\frac{32}{5}\Omega^{10/3}\mu^2\right]&\big(1-3.711309523809524u^2+12.56637061435917u^3-4.928461199294533u^4\\&-38.29283545469344u^5+\left(115.7317166756113-16.30476190476191\ln(u)\right)u^6\\&-101.5095959597416u^7+\left(-117.5043907226773+52.74308390022676\ln(u)\right)u^8\\&+\left(719.1283422334299-204.8916808741229\ln(u)\right)u^9\\&+\left(-1216.906991317042+116.6398765941094\ln(u)\right)u^{10}\\&+\left(958.9349701195667+473.6244781742307\ln(u)u^{11}\right)\big),\end{aligned}\tag{22}$$

where $\mu = Mm/(m+M) \approx Mm/M = m$ since $m \ll M$, and

$$u = r0\Omega = \frac{1}{r0^{1/2}} = \Omega^{1/3},\tag{23}$$

which implies that

$$\Omega = \frac{1}{r0^{3/2}}.\tag{24}$$

For $r0 = 6$, Equation (22) then simplifies to

$$\frac{\mathrm{d}E}{\mathrm{d}t} = -0.000898974m^2\tag{25}$$

The above comparison shows that our results are approximately consistent with those obtained from the PN formula. This also validates our approach to the gravitational radiation studies using null coordinates, as opposed to well known standard spherical coordinates.

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