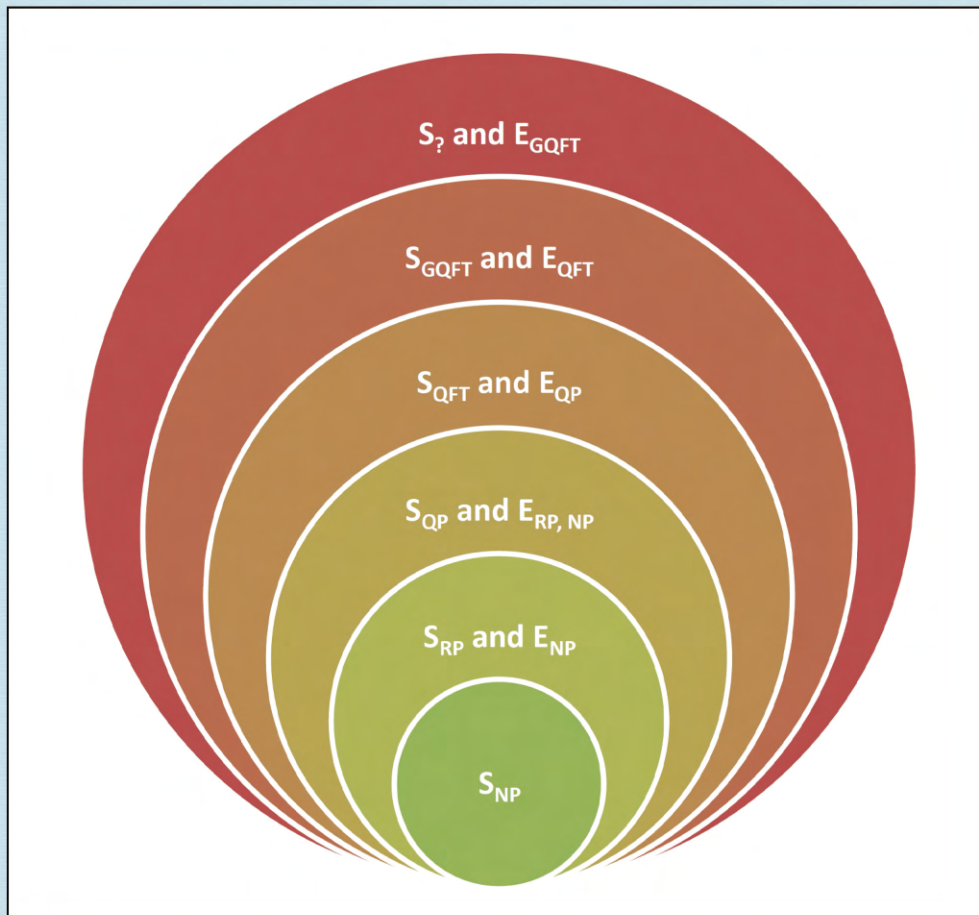


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# Geometric Interpretation of the Origin of the Universe

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## Abstract

An approach to the theory of geometrization of the Universe is proposed. The wave function of the Universe is represented by the Clifford number with the transfer rules that have the structure of the Dirac equation for any manifold. Solutions of this equation may be obtained in terms of the geometric interpretation. A new model is proposed that can explain the manifestation of the dark energy and dark matter in the Universe as a geometrical entity with a mechanism involving the spontaneous symmetry breaking.

## Keywords

Clifford Algebra, Wave Function, Geometric Interpretation, Structural Equations

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## 1. Introduction

The problem of origin of the Universe is far from being solved. Modern ideas (rather hypotheses) about the cause of the formation of the state of the universe suggest the instability of some fundamental scalar fields associated with the quantum nature of the matter [1]. The reasons and physical mechanism of the Universe origination remain open. In the paper [2] an approach was proposed to describe the causes and physical mechanism of the universe origination in terms of the first principles of statistical mechanics and quantum field theory. With this approach we can answer the question concerning the probable occurrence of an additional physical field, but nothing can be said about its geometric nature, except the assumption that everything has arose from a state of vacuum that corresponds to the lowest value of energy. In the case of spontaneous generation of an additional field in vacuum, the energy of the ground state of the “new” vacuum for the fields of different nature should be lower than the energy of the

ground state of the “initial” vacuum. The interaction of the new field with fluctuations of a field of different nature ensures the decreasing of energy for the new state. There may occur a transition from the zero-field state to a state with the final spontaneously generated field. The new field interacts with the fluctuations of the vacuum, and in the presence of the nonlinear self-interaction caused by the fluctuations of different nature, a nonzero value of this field may occur.

We make an attempt to describe the fundamental field in terms of some physical entity, to derive the laws of its changes, and to find a mathematical apparatus that would describe these changes. The question arises about the geometric nature of this fundamental field. It may be scalar as well as have other geometric images. It is natural that its geometric characteristics are determined by the space that is created as the result of the distribution of matter. Without matter there is no point in talking about the geometry. In terms of physical characteristics, the most suitable at the moment is the Clifford number [2] [3]. The main idea of this paper is to describe the origination of the Universe in terms of the Clifford numbers and to find a probable explanation for the dark matter and energy, as well as to explain the observed meaning of the visible matter. To do this, we first focus on the basic properties of the Clifford algebra and show its advantages for the physical situation under consideration.

First of all we suggest that the spinor representation of the wave function of the universe as a quantum object is not very suitable for our case [4]. Cartan [5] showed that for the dimensional representation of spinors the complete linear coordinate transformation does not exist. Dirac spinors do not preserve the structure of the ring although they preserve the structure of the linear vector space. The allowed states are exhausted because it is impossible to calculate the behavior of the wave function during the parallel transfer and, moreover, it is impossible to determine the state of the ensemble of particles. In [6], a theorem is proved that states that associative algebra with the partition over the field of real numbers is real, complex, or Clifford algebra that uses the Clifford numbers and has the structure of the ring [7]. This is a vector space over the field of real numbers that is represented as an additive group where the multiplication of elements is distributive rather than commutative. This ring has ideals that may be obtained as a relevant projection on a specially selected element [7]. Such ideals are Dirac spinors in the standard approach. The representation of the Clifford algebra contains more information about physical properties than spinors. The geometric properties of the Clifford algebra may be naturally introduced into the theory of the Universe [1] [8] [9] [10] and employed to extend its physical meaning. We will try to show that the representation of Clifford’s algebra best fits the description of the initial state of the Universe and provides more opportunities to explain both dark matter and baryon asymmetry.

As has been shown earlier [11] [12] [13] [14] [15], the application of the Clifford algebra contains all standard functions of the quantum mechanics and provides [3] [4] a unified basis for the physical knowledge including the theory of general relativity and electromagnetism. When we introduce the Clifford num-

ber into the scheme of quantum mechanics [13], we should take into account the peculiarities of this formulation. In this case, we obtain a quantum-mechanics theory that considers only the algebraic structure and does not contain any specific requirements. The idea of this paper is to present the wave function of the universe by a geometric entity, *i.e.*, the Clifford numbers, with the rules of transformation by the Dirac equation for any variety. The solutions of these equations may be obtained in terms of the geometric interpretation. Thus, the physical essence is described in terms of a geometric object with relevant transformation rules and the structure of the ring with respect to all algebraic operations. This makes it possible to highlight the contributions of the fields of different geometric nature in determining the energy and mass of the Universe.

## 2. Clifford Algebra. Differentiable Manifold

First from all we briefly describe the basic principles of Clifford's algebra with a view to their practical use. We use the basic idea [3] [4] [7], of the correspondence between matrices and basic elements of an algebra and thus define the space for the Clifford algebra. In the special theory of relativity the Dirac matrix  $\gamma_\mu$  acts as a unit vectors. An arbitrary linear combined product of these matrices has all the properties of the structure of the Clifford algebra with three complex units, starting with the time matrix  $\gamma_0^2 = 1$  and three spatial matrices  $\gamma_\mu^2 = -1$ . Therefore, we may reproduce any element belonging to the induced vector space as a direct sum of all probable tensor representations. In this case, an arbitrary function may be written in terms of the direct sum of a scalar, vector, bivector, trivector, and pseudo scalar that is given by

$$\Psi = \psi_0 \oplus \Psi_\mu \gamma_\mu \oplus \Psi_{\mu\nu} \gamma_\mu \gamma_\nu \oplus \Psi_{\mu\nu\lambda} \gamma_\mu \gamma_\nu \gamma_\lambda \oplus \Psi_{\mu\nu\lambda\rho} \gamma^\mu \gamma_\nu \gamma_\lambda \gamma_\rho \quad (1)$$

When we change the direction of the basis vectors to the opposite, we obtain  $\bar{\Psi} = \Psi_s \ominus \Psi_v \oplus \Psi_b \ominus \Psi_t \oplus \Psi_p$ . Another element of symmetry is the change of multiplication of the basis vectors to the inverse order in the representation of Clifford numbers, which yields  $\tilde{\Psi} = \Psi_s \oplus \Psi_v \ominus \Psi_b \ominus \Psi_t \oplus \Psi_p$ . We introduce the notation  $i \equiv \gamma_5 \equiv \gamma_0 \gamma_1 \gamma_2 \gamma_3$  (we denote the complex number as  $\tilde{i}$ ) and then we have another symmetry element, *i.e.*, the multiplication by  $i$ , presented as  $i\Psi$  that is not equivalent to  $\Psi i$ . Having introduced the elements of symmetry, we need to propose a mathematical operation over the field of Clifford numbers. The direct sum of the tensor subspace may be given a ring structure by means of a direct tensor product in the symbolic notation, *i.e.*,  $\Psi\Psi = \Psi \cdot \Phi + \Psi \wedge \Phi$ , where  $\Psi \cdot \Phi$  is an inner product or convolution that decreases the number of basis vectors and  $\Psi \wedge \Phi$  is an external product that increases the number of basis vectors. If each Clifford number is multiplied by a fixed matrix that has one column with one element and all other zeros, then we may obtain a Dirac spinor with four elements. This column may be used to reproduce the spinor representation of each Clifford number. There is a complete correspondence between the spinor column and the elements of the external algebra introduced here pre-

viously.

In the next step we have to find the rule of comparison of two Clifford numbers at different points of the probable manifold [4]. To do this, we have to determine the change of the geometric object under the action of a complete linear group of coordinate transformations, *i.e.*, the deformation of the coordinate system and the rule of parallel displacement on various probable manifolds. An arbitrary deformation of the coordinate system may be expressed in terms of deformations of the basis vectors  $e_\mu = \gamma_\mu X$ , where  $X$  is the Clifford number that describes arbitrary changes in the basis (including arbitrary displacements and rotations) that do not violate its normalization, *i.e.*, under the condition  $\tilde{X}X = 1$ . It is not difficult to verify because  $e_{\mu\nu}^2 = \tilde{X}\gamma_{\mu\nu}X\tilde{X}\gamma_{\mu\nu}X = \gamma_{\mu\nu}^2\tilde{X}X = I$  and this does not violate the definition of the basis norm [7]. Now, for an arbitrary basis, we define at each point in the space a single complete linearly independent form that is a geometric entity that characterizes this manifold point. Such a geometric entity may be specified using

$$\Psi = \Psi_0 \oplus \Psi_\mu e_\mu \oplus \Psi_{\mu\nu} e_\mu e_\nu \oplus \Psi_{\mu\nu\lambda} e_\mu e_\nu e_\lambda \oplus \Psi_{\mu\nu\lambda\rho} e_\mu e_\nu e_\lambda e_\rho \tag{2}$$

If this point of manifold is occupied by the matter, then its geometric characteristics may be described by the coefficients of this representation, including the coordinate basis  $e_\mu = dx_\mu$ . A product of arbitrary forms of this type is given by a similar form with new coefficients, thus providing the ring structure. This approach makes it possible to consider the mutual relation of fields of different physical nature [3] [16]. In what follows we may consider a new concept of the description of a particle and the characteristics of a manifold as a geometric entity.

Defining the characteristics of a manifold as a function of a point implies associating each point of the set with the Clifford number and its value. If this function is differentiated with respect to its argument, then we have to introduce a differentiation operation [3]. To determine the transfer operation on an arbitrary manifold, we have to determine the operator of derivative. This operation may be defined as  $D = \gamma_\mu \frac{\partial}{\partial x_\mu}$ , where  $\frac{\partial}{\partial x_\mu}$  is associated with the change along the curves passing through a given point in space. The act of this operator at any Clifford number may be represented as

$$D\Psi = D \cdot \Psi + D \wedge \Psi \tag{3}$$

where  $D \cdot \Psi$  and  $D \wedge \Psi$  may be regarded as the “divergence” and “rotor” of the relevant Clifford number. According to the definition of the differentiated manifold, a single coordinate system is insufficient for covering a manifold whose topology differs from the topology of an open set in the Euclidean space

The structure of such a geometric construction should be supplemented by the correlation between the values of the transferred forms at different points of the manifold [3]. When assigning internal values to the characteristics of the manifold, we should introduce the transformation of Clifford numbers by changing the coordinate system. It may be identified by displaying the relevant Clifford numbers under the action of a certain group associated with the corres-



ponding transformation. The conversion is possible if it is caused by any geometric characteristics changing the coordinate system accordingly, as well as by transforming the geometric objects. This requires the full use of the Clifford algebra as elements of the group of the internal vector space (group  $Sp(n)$ )  $XY = Z$ , where  $X, Y, Z$  have similar preliminary representation. A certain group of transformations converts each Clifford number by the law  $\Psi' = \Psi X$ , where  $X$  determines the elements of the reflection of the Clifford algebra in our case and satisfies the condition  $\tilde{X}X = 1$ . For this algebra, we may write the first structure equation that defines the covariant derivative [3] as given by:

$$\Omega = d\Psi - \omega\Psi \quad (4)$$

with the law of calibration transformation for the connectivity

$$\omega' = X\omega\tilde{X} + Xd\tilde{X} \quad (5)$$

for the conservation co-variant transformation according to the similar law  $\omega' = \omega X$ . This equation is referred to as the first structure equation, but now it acquires the meaning in the Clifford algebra. In this case, an arbitrary Clifford number may always be reduced to a canonical form though the local deformations of the eigenbase become, however, unobservant because the Tetrude form  $Xd\tilde{X}$  corresponds to the second term of the calibration transformation. Then the second structure equation that defines the “curvature” form may be written as

$$F = d\omega - \omega\omega \quad (6)$$

with the law of transformation under the algebra being given by  $F' = XF\tilde{X}$ . The transfer equation for the curvature tensor with the transformation law may be written in the form

$$dF - F\omega + \omega F = J \quad (7)$$

where  $J$  is the flow form with the analogous general representation that complies the transformation  $J' = XJ\tilde{X}$ . The equation thus obtained may be regarded as the field equation, its form is apparently similar to the analogous equation for the connectivity form obtained in Lie algebra [3] [4]. Those equations possess a more general character as their structure contains interrelation of the geometric characteristics whose tensor nature is different. In this presentation we may write the fourth structure equation that demonstrates the dependence between the covariant derivation and the curvature, *i.e.*,  $d\Omega - \Omega\omega + \Psi F = 0$ .

It is natural to assume that each elementary formation at an arbitrary point of the manifold may be described by a Clifford number. Then the wave function of the elementary formation is represented by a complete geometric object, *i.e.*, the sum of probable direct forms of the induced space of the Clifford algebra. Moreover, by attributing a geometric interpretation to the wave function, we may obtain correct transfer rules for an arbitrary manifold [3] and new results related to the geometric nature of the wave function [7]. According to [7], each even Clifford number  $\Psi = \bar{\Psi}$  under the condition  $\Psi\tilde{\Psi} \neq 0$ , in the Euclidean

space may be presented in the canonical form, *i.e.*,  $\Psi = \{\rho(x)\exp(i\beta)\}^{\frac{1}{2}} X$ , where  $X\tilde{X} = 1$  describes all coordinate transformations. It is clear that  $\int \tilde{\Psi}\Psi d\tau$  is scalar and in the physical interpretation of this geometric entity it is rather evident inasmuch as  $\rho(x)$  may be associated with the probability density of finding a particle in an arbitrary spatial point, and  $\beta$  is the angle that determines the eigenvalue of a particle with positive or negative energy. We can take  $\beta = 0$  for the matter and  $\beta = \pi$  for the antimatter. Thus it becomes possible to describe the intermediate states of the particle since the form of the wave function of an arbitrary particle ensemble is similar [10]. It is important that  $\rho \equiv \tilde{\Psi}\Psi = (\Psi_s + \Psi_p)^2 + \Psi_b^2$  are represented by the products of different tensor representations of the general type of the wave function that have different geometric interpretations and correspond not only to the scalar field but may have different physical origins of the fields of different nature. It is proved in the book [7] that the odd part of the general Clifford number may be presented as the even part multiplied by a separate element of this algebra and thus it is not difficult to manipulate with the full Clifford number. Now for the wave function as a geometric entity, we may write the first structure equation in the standard form, *i.e.*,

$$d\Psi - \omega\Psi = m\Psi \quad (8)$$

that formally reproduces the Dirac equation but has wider meaning than in the spinor representation. The question of describing the wave function as a geometric entity was considered earlier in the article [4]. Among these results, we indicate that the Dirac equation in the geometric representation in the general theory of relativity is nothing but the equation of transfer on an arbitrary manifold, therefore, its solution may be interpreted purely geometrically. Moreover, the geometric representation of the wave function yields other results that simply reveal the geometric nature of the wave function [17]. Next, these equations will be derived from the principle of least action in the geometric interpretation. The presented equations first of all solve the problem of transformation of a finite-dimensional representation of the wave function under the action of a complete linear group of coordinate transformations [3] [7].

### 3. Geometrical Origin of the Universe

Next we assume that the occurrence in the vacuum of the fundamental scalar field that is generated spontaneously and interacts with the fluctuations of all other fields may be associated with a phase transition that owes to the decrease of the vacuum ground state energy [2] [18]. Moreover, evolution of the Universe formed by the fluctuations of physical fields may be described in terms of the Clifford number  $\Psi$  [4]. The probable stationary distributions of the fundamental field are generated by the multiplicative noise produced by the nonlinear interaction. After that, the standard cosmological model may be modified. The fundamental field in the form of all probable geometric representations interacts

with fluctuations through the change of the parameter of coupling of the given field with vacuum. Such fluctuations may be considered as a source of the multiplicative vacuum noise. In this case, such noise not only changes the value of the field, but also changes the shape of the effective potential due to the changes in the state of the system. This effect, in turn, changes the conditions for the formation of bubbles of a new phase and determines the evolution of the Universe. The generator of this noise is the vacuum itself in the form of a wave function for each point of the manifold with the Planck size. This model differs from the known scenario of stochastic inflation of the universe [1] that takes into account the fluctuations of the fundamental field but disregards the fluctuations of the unstable vacuum due to the fluctuations in the coupling parameter. The internal fluctuations of the manifold generate the stochastic behavior of the system that may induce changes of its stationary state. The most significant point here is that now the fundamental field is described by the Clifford number rather than scalar and contains all the geometric characteristics of the space that may be born as the result of the emergence of the matter. Only the distribution of the matter can describe the space that arises.

We start with the assumption that phase transition from the “initial” vacuum with only fluctuations of different fields to a new state of vacuum generates a new non-zero fundamental field. This means that the presence of a new field makes the “new” vacuum different from the “primary” vacuum for any field of arbitrary geometric characteristics that may exist. The resulting field should reduce the energy of the “new” vacuum with respect to the energy of the “primary” vacuum. Therefore, the energy density of the ground state of the “new” vacuum may be supplied through  $\varepsilon = \varepsilon_v - \frac{\mu_0^2}{2} \tilde{\Psi}\Psi$ , where the second part is the field energy in the term associated with the wave function with the geometrical presentation in terms of the Clifford numbers; the coefficient  $\mu_0^2$  describes the coupling of the new field and the “primary” vacuum, *i.e.*, the self-consistent interaction of the new field with the probable fluctuations that may exist in the “primary” vacuum. Here we have to make two remarks. The first one concerns the decrease in the initial energy of the ground state with the appearance of the new field, and the second one is related to the coupling coefficient that is now positive and thus explanations of the appearance of such a sign used in the standard approach are not required. The energy of the new system may be presented in the form given by

$$E = E_v - \int \frac{\mu_0^2}{2} \tilde{\Psi}\Psi d\tau, \quad (9)$$

If we want to describe the evolution of the system  $\langle out | \exp iHt | in \rangle$ , we still need to average all probable fluctuations with which the new field can interact. For this purpose it is sufficient to present the nonlinear coupling in the form  $\mu_0^2 = \mu^2 + \xi$ , where  $\langle \xi(t)\xi(0) \rangle = \sigma^2$  and  $\sigma^2$  is the dispersion of the coupling coefficient fluctuations which allows averaging over all possible fluctuations

$$\begin{aligned} \langle out | \exp i H t | in \rangle &\sim \int D\Psi \int D\xi \exp i \left\{ E_v - \frac{1}{2} \mu^2 \tilde{\Psi} \Psi + \frac{1}{2} \xi \tilde{\Psi} \Psi + \frac{\xi^2}{\sigma^2} \right\} \\ &\sim \int D\Psi \exp i \left\{ E_v - \frac{1}{2} \mu^2 \tilde{\Psi} \Psi + \frac{\sigma^2}{4} (\tilde{\Psi} \Psi)^2 \right\} \end{aligned} \tag{10}$$

This implies that we have a system with the effective energy (averaged over the fluctuations of the other field coupled with the wave function) given by

$$E = E_v - \int \left[ \frac{1}{2} \mu^2 \tilde{\Psi} \Psi - \frac{\sigma^2}{4} (\tilde{\Psi} \Psi)^2 \right] d\tau \tag{11}$$

where  $V(\Psi) = -\frac{1}{2} \mu^2 \tilde{\Psi} \Psi + \frac{\sigma^2}{4} (\tilde{\Psi} \Psi)^2$  is the well-known expression for the energy of the fundamental field [1] with the nonlinear coupling coefficient determined by the dispersion of fluctuations. This implies that with no new field  $\Phi = 0$ ,  $E = E_v$  while for  $\rho = \tilde{\Psi} \Psi = \frac{\mu^2}{\sigma^2}$  the expression for the effective

ground state energy of the “new” vacuum reduces to  $E = E_v - \frac{\mu^4}{4\sigma^2} \tau$ . The last

relation suggests the conclusion that the energy of the “new” vacuum is lower than the energy of the primary vacuum, *i.e.*, the phase transition results in the formation of a new vacuum ground state with non-zero additional field that has new geometric presentations. If  $\sigma^2$  tends to infinity, then the energy of the new state tends to the initial energy of the ground state. The energy of the new state can vanish for  $E_v = \frac{\mu^4}{4\sigma^2}$ . This relation may be applied to estimate the

maximum dispersion of vacuum fluctuations. In addition, the effective potential can now be given in terms of the probability density of the material field  $V(\rho) = -\frac{1}{2} \mu^2 \rho + \frac{\sigma^2}{4} \rho^2$ , which may be useful for the interpretation of different compositions of energy and matter as a result of spontaneous breaking symmetry. It should be noted that this is the total probability density of the material field, and whether it is “dark” depends on the tensor characteristics of the field in which we feel it. It may be invisible in the vector electromagnetic field but will definitely be felt in the gravitational and possibly in the fields of another tensor presentation.

#### 4. Geometrical Description Evolution of the Universe

Now we can offer a slightly different manifestation of the birth of the universe based on the representation of its wave function in terms of geometric essence. What arises as a result of the birth of matter must contain a geometric image. From the point of view of geometry, only the distribution of matter can be interpreted. This role can be played by Clifford’s number with the appropriate physical interpretation. An additional field is required for the emergence of matter, the spontaneous excitation of which leads to the emergence of elementary particles. In our case, such a field is the wave function  $\Psi$  in terms of dif-

ferent tensor representations, *i.e.*, it has all probable tensor representations with the dimension of the space to be created. That is, the geometry is laid down from the very beginning in the characteristics of the point of the variety on which we describe it.

Having minimized the expression for the energy of the system 9 by independent functions  $\Psi$  and  $\tilde{\Psi}$ , we obtain for the wave function in the homogeneous case the Gross-Pitaevskii equation with all the physical consequences for solving such an equation.

$$\frac{\delta E}{\delta \tilde{\Psi}} = \left[ -\mu^2 + \sigma^2 (\tilde{\Psi}\Psi) \right] \Psi = 0 \quad (12)$$

Equations similar in content but richer in nature may be obtained from the dynamics of changes of the wave function in the geometric interpretation. To do this, we consider the dynamical action recorded for the wave function of the universe in the presence of matter. As has been mentioned earlier [3], the action in terms of the geometric invariant may be presented as

$$S = \int d\tau \left\{ \frac{1}{2} (F\tilde{F} + \tilde{F}F) + m\Psi\tilde{\Psi} \right\} \quad (13)$$

The Lagrange multiplier  $m$  takes into account the normalization condition for the wave function  $\int d\tau \{ \Psi\tilde{\Psi} \} = 1$ . The “general” curvature in the presentation of Clifford numbers takes the form;  $F = d\Psi - \lambda\Psi\Psi$  where the coefficient  $\lambda$  takes into account the dependence of the connectivity field on the wave function itself  $\omega = \lambda\Psi$ . Minimization of this functional yields an equation that is at the same time the second structure equation for the Clifford algebra, *i.e.*,

$$dF - F\lambda\Psi + \tilde{\lambda}\Psi F = J \quad (14)$$

for the change of the “curvature” under the parallel transfer under the influence of the full group of transformations of the coordinate system. In the homogeneous case  $d\Psi = 0$  such equation reduces to the above Gross-Pitaevskii equation:  $\left[ -\mu^2 + \sigma^2 (\tilde{\Psi}\Psi) \right] \Psi = 0$  with  $\mu^2 = m$  and  $\sigma^2 = \tilde{\lambda}\dot{\lambda}$ .

To apply this approach to the description of the universe, we have to make a natural assumption. The new vacuum contains nothing except the born formation. For this reason, all changes associated with the wave function are due only to its changes in the vacuum in its presence. Therefore, its behavior can be influenced by only one characteristic of the new vacuum, namely this wave function. In this case, the wave function itself acts as a field that changes its characteristics, or as a connectivity of the space with the new vacuum. The equation required for the wave function is natural in the form

$$d\Psi - \lambda\Psi\Psi = F, \quad dF - F\lambda\Psi + \lambda\Psi F = J \quad (15)$$

when the first structure equation is at the same time the second structure equation for the “curvature”  $F$ . It is assumed that  $\omega \sim \lambda\Psi$ , which corresponds to our previous assumption where  $\lambda$  determines the relationship of the wave function to manifold.

On the other hand, it is easy to verify that if we consider odd Clifford numbers

under the assumption  $\Psi_\mu \sim A_\mu$  and  $\Psi_{\mu\nu\rho} \sim \Gamma_{\mu\nu\rho}$ , then we simultaneously reproduce both Maxwell and Einstein equations for the curvature for different components of the last equation, provided that the covariant derivative of the even part  $\Psi_{\mu\nu} \sim g_{\mu\nu}$  yields zero. As example

$F_{\mu\nu} = A_{\mu,\nu} - A_{\nu,\mu} + \rho A_\mu A_\nu + \lambda \Gamma_{\mu\nu\lambda,\lambda} + \Psi_{\mu\nu\rho\sigma} g_{\rho\sigma} + \lambda \Gamma_{\mu\nu\rho} A_\rho$  where  
 $(1 + \Psi_s) \Gamma_{\mu\nu\rho} = g_{\nu\lambda,\mu} + g_{\lambda\mu,\nu} + g_{\mu\nu,\rho} + \lambda g_{\mu\nu} A_\rho + g_{\rho\nu} A_\mu + \lambda g_{\rho\mu} A_\nu + \lambda \Psi_{\mu\nu\rho\sigma} A_\sigma$ . All these elementary calculations are not given here because of their cumbersome-ness [3].

It should be noted that the physical entity described by the wave function in the form of the Clifford number does not belong to certain quantum statistics and contains elements of both fermions and bosons. In the case of spontaneous symmetry breaking, a part of such an object turns into particles and a part remains a field, each of which corresponds to certain elements of symmetry. It is possible that in Clifford's algebra it is possible to write a more general relation which will take away information on a condition of the condensed part but at present it could not be found though it is possible to use the approach offered in article [19].

## 5. Conclusions

As the result, we propose a probable scenario for the formation of the universe. We assume that in the presence of a spontaneously generated fundamental field with different geometric presentation, the vacuum energy is lower than the ground state energy of the primary vacuum and that the ground field energy is influenced by its nonlinear interactions with the fluctuations of the physical fields of different nature. To avoid the problem of the influence of gravity on the evolution of the universe at the stage of spontaneous emergence of the fundamental field, we note that the energy of the primary vacuum is not contained in the Einstein equation and the evolution of the universe is determined only by the energy of the fundamental field.

Only the distribution of matter in turn determines the geometry. The birth of matter is determined only by a non-zero fundamental field that contains contributions of fields of different physical and thus geometric nature. The presentation of probability densities of material entities contains fields whose geometric characteristics do not overlap and therefore cannot be observed within the behavior of individual components. For example, the change in the electromagnetic field may not be affected by the field connectivity described by the tensor characteristic of the third rank. For our universe, the vacuum is different from the primary one and its state depends on the fundamental field that possesses different tensor representation. In addition to the scalar part of the fundamental field, there are fields of other tensor dimensions that may be involved in the influence of the dark matter.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# Mars Blue Clearing and Allais Effect

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## Abstract

Starting from the so-called “blue clearing” phenomenon, this paper establishes a link between disturbances of the Martian gravitational potential, the Allais effect of syzygy, astral influences and the Raman Stokes effect. This phenomenon is apparently peculiar to the Martian atmosphere. Photographs of Mars taken in blue light normally show only the atmosphere itself and clouds high above the surface. On occasion of oppositions, however, blue photographs will penetrate in varying degrees to the surface of Mars. Curiously, a burst of brightness and storms then occur on Mars. The atmosphere and clouds can be seen and photographed at short wavelengths by Earth-based telescopes equipped with a Wratten 47 filter. It happens that the blue screen of the filter suddenly begins to disappear and that the Martian surface becomes visible. The exact mechanism that produces blue clearing when Earth is between the Sun and Mars is highly speculative. We believe that the “Allais syzygy effect” may explain this phenomenon. The opposition would generate a “gravito-electromagnetic tension”, which would spawn fluctuations in the gravitational potential of Mars, accompanied and linked to an electromagnetic effect. The outcome would be to trigger dust storms and exacerbate a disorderly excitement of molecules in the atmosphere. The thermal agitation facilitates the absorption of energy and the formation of small condensations that cause light scattering. Assuming that the Martian gravity decreased slightly, a Stokes Raman scattering would manifest at intramolecular level of the Martian atmosphere: the emitted photon has a lower energy than the absorbed photon. Therefore, it is mainly the waves corresponding to the spectral regions yellow, orange or red that are diffused, what eliminates short wavelengths. We deduce that the size of the inhomogeneities resulting from thermal excitation turns out to be greater than the length of the light waves of blue or purple regions of the spectrum.

## Keywords

Blue Clearing, Allais Syzygy Effect, Astral Influences, Gravito-Electromagnetic Tension, Theory of Relation, Stokes Raman Effect



## 1. Introduction

Mars, the first outer planet, possesses an orbit bigger than that of the Earth. Every two years, the celestial body becomes very brilliant and its apparent diameter is then widely up to 10 arcseconds, and may even reach the exceptional value of 25 arcseconds. This blaze in brightness occurs when Mars is in opposition, *i.e.* when the Earth is between the Sun and Mars. The opposition is the period when Mars is closest to Earth, which explains the high brightness of the planet and its high apparent diameter. At perihelion, the opposition occurs at a distance of 56 million kilometers from Earth, against 102 million kilometers at aphelion. Between an opposition in aphelion and in perihelion, the apparent diameter of Mars is halved. Because of the eccentricity of its orbit, and to a lesser extent, of the one of the Earth, the transition at the smallest distance can effectively take place up to 8 and a half days before or after opposition. The perihelic oppositions, which are most favorable for observation, are rare and only reproduce every 15 to 17 years on average. Mars, in opposition at perihelion, is 3.25 times as bright as when it is in opposition at aphelion [1] [2].

At aphelion, the distance from Mars to the Sun is 249 million kilometers, while it is only 207 million kilometers at perihelion. Therefore, the interval between two oppositions is not exactly 780 days, it is 810 days between two neighboring oppositions of the perihelion and 764 days for two neighboring oppositions of the aphelion. In the vicinity of an opposition, the celestial body began his retrograde motion. When Earth and Mars are diametrically opposed with regard to the sun, the distance between them may exceed the 400 million kilometers. The red planet is then easily confused with a star. Dust storms tend to develop precisely at perihelic opposition. The transparent state of the Martian atmosphere plays at least a role as important in the observation of the size of the disc; the dust in suspension may obscure many details of the surface. Although the size of the Martian disc reaches peaks during these oppositions, dust storms can degrade the viewing conditions, the surface of Mars appears then faded and blurred.

In addition to the spectacular dust storms, the generally clear layer of gases allows the observation of the clouds which sometimes stand out in a way contrasted on the Martian disc, and this in spite of their delicacy and their vaporous nature. If they are enough brilliant, these clouds can compete with the polar cap, sometimes becoming a source of confusion. Filters are found almost indispensable to differentiate atmospheric phenomena. Blue filter (W38A or W80) or purple (W47) are recommended to study them. The surface of Mars has a tendency to fade in favor of the thin atmosphere in this part of the spectrum. An altitude cloud will reach its maximum brightness with a blue filter, and will appear less prominent with a green filter (W58) or orange (W21). In contrast, while green or blue-green filters (W64) can bring out the mist, the ice surface and ice associated with the polar cap, and while the orange filter (W23A) or the red filter (W25) make appear more brilliant the deserts and distinguish the storms of dust,

we notice that these phenomena are mitigated with a blue filter [3] [4].

#### **Martian blue clearing**

A notable exception, however, occurs when the “blue clearing” happens, a poorly understood phenomenon that American specialists have dubbed in this way and which surprised all those who saw it. From time to time, when Mars is in opposition with the Earth (Earth relatively to Mars occupies the same position as the Moon compared to Earth during a solar eclipse) every 2 years and 50 days on an average, the blue screen that hides all the details of the planet suddenly begins to disappear. The clearing can last for periods of several days, may be limited to one hemisphere and can vary in intensity from 0 (no surface features detected) to 3 (surface features can be seen as well as in the white light) [5].

The albedo of Mars is at best very feeble and, at worst, invisible through a blue filter, and the atmosphere and white clouds appear much brighter. The albedo appears vague through light blue filters, such as the Wratten 80A. With a dark blue filter (W47) or violet (380 - 420 nm), the disc usually appears featureless except for clouds, hazes, and the polar regions [6]. The anomaly becomes an occasion where the markings on the surface of Mars can be seen clearly and photographed in the blue and violet light by Earth-based telescopes equipped with a Wratten 47 filter which is the standard for studying a blue clearing. Nobody seems to know why the Martian surface then becomes visible, why the atmosphere turns transparent into the blue and violet wavelengths.

#### **Overview of the first variations of the atmospheric transparency of Mars**

The mystery has a Martian history accompanied by a photographic progression that dates back to the beginning of the twentieth century. The author pays homage to the paper by Martz, Jr. E.P [7] which is the source of the previous related papers [8]-[20]. Researchers G. A. Tikhov, Pluvinel and Baldet pointed out the difference between photographs of Mars at long and short wavelengths, due to the darkening properties of the Martian atmosphere. W.H. Wright [8] has demonstrated it in detail. E. C. Slipher [9] first specifically noted the marked variation in transparency of the Martian atmosphere, to light of short wavelengths, at certain times. W. H. Pickering [10] also noted variation in the obscuring properties of the Martian atmosphere, on his early photographs of the planet. S. L. Hess [11] has pointed out that such variations in atmospheric transparency may have a very interesting effect on the color of the dark green surface markings. In 1941 he found a positive correlation between atmospheric transparency to blue light, and halting of the normal seasonal color changes, and an uncertain relationship in 1939, due to lack of a completely continuous series of photographs. R. S. Richardson [12] has discussed the general problem of atmospheric transparency on Mars, and the proposal of Hess, in an interesting popular article. G. P. Kuiper [13] discusses in some detail the problem of the Martian atmospheric haze and the seasonal variations of color as related to the nature of the surface markings.

G. de Vaucouleurs [14] reviews previously available observational material in regard to the atmospheric transparency on Mars and discusses the possible ori-

gin of the obscuring medium. He points out that the variable obscuration can hardly arise from the variation in the molecular diffusion of the gaseous atmosphere and also that it is not likely to be due to variation in water droplets, surface dust, volcanic dust, or meteoritic dust suspended in the Martian atmosphere. S. L. Hess [15] has suggested that the variation of the blue haze may be due to forming and evaporation of high altitude clouds of frozen carbon dioxide crystals. G. P. Kuiper [13] indicates that the blue haze may be more likely due to frozen water crystals at somewhat lower levels. It has also been proposed that the blue haze may be due to day lit auroral emission in the upper Martian atmosphere.

During 1937 and 1939 E.P. Martz, Jr. [16] conducted extensive programs of photography of Mars (and other planets), by red (6500 A.U.), yellow (5600 A.U.), green (5300 A.U.), and blue-violet (4400 A.U.) light, from Mount Wilson, Griffith, and Steward Observatories. N.N. Sytinskaya [17] has discussed the variation in atmospheric turbidity across the disk of Mars, at the equator, for several different dates in 1939. E. C. Slipher [18] has noted that small variations in atmospheric transparency on Mars, to short wavelengths, occur from time to time, but that the major increases in transparency occur nearly coincidentally with opposition date of the planet. R. Wildt [19], E.P. Martz, Jr. [20] and others have suggested that the blueviolet Martian atmospheric haze may arise from variation in daylit-type auroral or fluorescence phenomena, rather than to particle scattering. Rosen has suggested that the blue-violet haze may arise from carbon-smoke type particles.

As can be seen, numerous hypotheses have been suggested as explanation of the phenomenon, like a “blue haze” on Mars concealing the surface at blue wavelengths. All such hypotheses have been found untenable until 1972 when a new explanation was offered simultaneously in the USSR and the U.S [21] [22]. It has been suggested that dust clouds may form above light regions because of an increase in the general circulation of the Martian atmosphere. This suggestion was confirmed in 1975 and led to a link between the appearance of these clouds and the intensity of blue clearing.

#### **Allais eclipse effect and Allais syzygy effect**

However, even with this explanation, the phenomenon remains poorly understood and the mystery remains unsolved. We consider that the anomaly of the blue clearing is similar to the anomalous movements exhibited by Maurice Allais pendulum at the time of a solar eclipse [23] and anticipate that a relationship must exist between the anomaly, the development of dusty clouds and the Allais effect [24].

Motion disturbances of a pendulum during a solar eclipse were observed for the first time by chance by Maurice Allais June 30, 1954, during measures of the azimuth of the plan of oscillation of a paraconical pendulum [24]. He observed a similar disturbance in 1959. This sudden change of the speed of precession of the plan of oscillation of the pendulum during an eclipse constitutes the Allais effect, also known as “Allais eclipse effect”. He made two non-stop experiments during

two periods of many planetary alignments, from 20 Nov. to 15 Dec. 1959 and from 15 March to 15 April 1960. In both experiments the azimuth had increased. On the other hand, Saxl and Allen (1970) observed with a torsion pendulum that these anomalies also applied to syzygies [25]. The latter are the phases of the lunar movement which find the Earth, the Moon and the Sun almost on the same line, that is to say, full Moon and new Moon. They bring the maximal height of tides. In 2002 and 2003, D. Olenici et S.B. Olenici, following the methodology of Maurice Allais, performed two non-stop experiments during periods of many planetary alignments [26]. All these experiments confirmed that the Allais effect appears during syzygies as well as during solar eclipses. An alignment without eclipse with a minimum of three celestial bodies is called a syzygy. The anomaly during alignment without eclipse with a minimum of three celestial bodies can be termed Allais syzygy effect [27]. The Allais effect becomes a general term that encompasses the Allais eclipse effect and the Allais syzygy effect (conjunction and opposition). The Allais eclipse effect is a special case of the Allais syzygy effect. We assume that when the Earth is between the Sun and Mars, the Allais effect (including syzygy and eclipse) causes fluctuations in the gravitational potential of Mars, what would be at the origin of the storms of dust and the blue clearing.

In the next discussion, we provide some precisions on the astral influences and the Allais syzygy effect. It is justified to the extent that it manages to establish on experimental bases the correspondences between celestial bodies and the Earth and makes touch of the finger the scientific reality. The last section takes us back to the hypothesis of the Allais syzygy effect: the effect would cause fluctuations in the gravitational potential of Mars, what would explain the blue clearing seen from Earth and the Stokes Raman effect of Martian atmosphere.

## 2. Discussion: Allais Syzygy Effect and Astral influences

If the hypothesis is correct, it depends on what is called “the phenomenon of astral influences”. This phenomenon reveals the influences of the Sun on Earth and planets between them. It seems certain that they exist even if they are more complicated and confusing than imagined before. Our gravitational theories do not take these influences into account, as if the bottom of things had escaped us. To begin, let us say that in 1954, Einstein and the majority of the astronomers were opposed to the idea that the space was crossed by magnetic fields, that the Sun and the planets had an electric charge and that the electromagnetism could play a role in the celestial mechanics. In 1950, Immanuel Velikovsky, a psychoanalyst doctor, published a book in which he asserted that the space is not “empty” and that electromagnetism plays a fundamental role in the solar system. In early 1955, astronomers got radio signals from Jupiter. When Einstein, a few days before its death, learnt the news he used his influence to ensure that we experimentally verify the theories of Velikovsky [28] [29] [30]. From 1951, one had already noticed that the quality of the radio reception depends among others on the solar activity while leaving an unexplained residue. John H. Nelson, *pro-*

*grammation analyst* at R.C.A. Communications thought that this residue could be explained by the heliocentric position taken by the planets, *i.e.* with respect to the Sun. According to him, some very particular planetary configurations would disrupt the reception of radio waves: the days when the planets appear, relative to the sun, either at right angle to each other, either in conjunction or in opposition [31] [32]. In 1963, JA Roberts wrote an article in *Planetary Space Science Research*, showing that Venus, Jupiter and Saturn are the source of powerful radio wave emissions that the Earth is able to capture [33]. Since, artificial satellites have brought a revolution in the designs that one had of empty space and it has been shown that all the planets in the solar system have an electromagnetic field, even if those of Mercury, Mars and Venus are weak [34] [35].

Therefore, we can consider that the planets are giant electromagnets that describe with prodigious speeds their revolutions around the Sun, the central electromagnet. The Sun around which they describe their orbits, has a considerable static electricity charge to give rise an “electrostatic field” around the celestial bodies of the solar system. As in general, the core of each of the planets is quite a good electrical conductor, this core behaves, by moving in the magnetic field, as the induced of a dynamoelectric machine or of a magneto turning between poles of its magnet. It follows that, according to the classical laws of electromagnetic induction, any cause which will make vary the intensity of the field, or any cause which will make vary the speed of displacement of the conductive body in the field, will modify the intensity of the observed effects [36] [37] [38].

The theory of general relativity is accountable to both the nature of matter and motion of stars by hanging on to the experimental reality. Matter a curvature of space creates gravity. However, the gravitational field, framed with precision in the network of its formulas, lets escape the energy, or the electromagnetic radiation [39]. The Cosmos is made of matter and energy, and the solar system is no exception. Matter evokes inertia while radiant energy is active. Universal gravitation causes the bodies to exert an attraction on each other. In our solar universe, the radiant energy originally engendered the matter. This one became a reservoir of forces and movements. Celestial bodies, grouped into overlapped entities, act by gravitation on each other and maintain their respective movements. In turn these movements facilitate and diversify transformations that radiant energy is likely to operate on all of them. Viewed in isolation, stars acquire their own dynamics that have double character of gravity and electromagnetism [40] [41] [42]. Although relativity gives a picture of the physical universe while being unable to account for the energy, we can remark that the curvature of the light of the theory concerns the electromagnetism in connection with a gravitational potential. This is not a purely gravitational effect like for the advance of the perihelion of Mercury or the gravitational redshift [43]. We also note that the great majority of the gravitational systems which we meet in the physical universe are systems in constant total mass. In this context, according to the general relativity, the electromagnetism goes hand in hand with the gravitational: light is bent in proportion by matter (mass); more the gravitational po-

tential is big, more the light is bending.

Nevertheless, when there are disturbances of move of gravitational measurement instruments that raise or lower the gravitational potential, contemporary physicists have come to deny the facts to dodge the discomfort of these mysterious disruptions that do not fit the established theories. It is nevertheless necessary to become aware that if these disturbances are connected to physical phenomena observed during the syzygies and the solar eclipses, and if they are applied on both the mass and the light, we have to involve gravitation and electromagnetism in the fundamental principle of a “gravito-electromagnetic dynamics”.

Let us remind that disturbances of movement of a pendulum during a solar eclipse were observed for the first time in a fortuitous way by Maurice Allais on June 30th, 1954, during measures of the azimuth of the plane of oscillation of a paraconical pendulum [24]. This sudden change of the speed of precession of the plane of oscillation of the pendulum during an eclipse constitutes the Allais effect, also called “Allais eclipse effect”. On the other hand, Saxl and Allen observed with a torsion pendulum that these anomalies could also be applied to the syzygies [25]. The latter are the phases of the lunar movement which find again Earth, Moon and Sun more or less on the same line, that is to say Full moon and New Moon. They bring back the maximum height of the tides. An alignment without eclipse with a minimum of three celestial bodies can be called Allais syzygy effect. The experiments of Maurice Allais of November-December, 1959 and March-April, 1960 with the paraconical pendulum confirmed the existence of inexplicable periodic structures within the framework of Newtonian mechanics and relativistic mechanics, amplitudes hundred million times larger than the amplitudes calculated with the current theory. They also allowed to demonstrate the existence of a direction of variable anisotropy over time and to specify the azimuth at any time. M. Allais interprets this anisotropy of space as corresponding to an anisotropy of inertia according to the considered direction of the anisotropy of space and resulting of astronomical influences. These promising experiments were not able to continue due to the closure of his laboratory at Saint-Germain on June 1960, following a cabal [24].

If anomalies of the paraconical pendulum must be highlighted during a configuration opposing the Sun and Mars, they would serve to demonstrate that the influence of the Sun and the Earth on Mars entails an anisotropic space, variable with time in direction and in intensity. And to determine if there is an abnormal “gravitational tension” which affects the gravitational potential and engenders the observed hurricanes.

Let us add that we were interested in the discoveries and deductions of M. Allais because they were in agreement with the theory of the Relation [44] [45] [46]. According to this theory, under the principle of compensation, the electromagnetic space of inertia which, globally, decreases is offset by an increase of the gravific space. When we talk of inertia of space, it is an inertia that opposes gravity (like that of special relativity) and not of the inertial forces that accom-

pany the gravitational forces in the same direction. Locally, within a gravific space which becomes suddenly non-static, the energy of an electromagnetic space of inertia augments, activates and is counterbalanced by a gravific space that abates. In the event of an eclipse or a syzygy which would disrupt momentarily the celestial body in the point to make slightly vary downward the reduction its gravitational potential, we shall say that the “electromagnetic” mass of the photon (we know that the photon has no energy of rest) the equation  $m = hv/c^2$  increased and that the electromagnetic activity within the celestial body grew at the expense of the gravity. According to Einstein’s equivalence principle, the photon has a “gravitational mass” equivalent to an “inertial mass” and equal to  $hv/c^2$ . According to the relation  $v' = v(1 + gl/c^2)$ , a photon emitted by Mars will have a frequency a little higher at his arrival at the Earth’s surface, with mass  $hv/c^2$ , supposedly constant during the journey [47]. That this mass and also the “proper” time seem modified during the state of eclipse or syzygy, it is the proof that we are dealing with an anomaly similar to the Pioneer effect that makes vary the “invariance”.

### 3. Blue Clearing, Allais Syzygy Effect and Raman Stokes Effect

#### Blue clearing seen from Earth and Allais syzygy effect

After these explanations which are other thing than the knowledge of appearances, we can return to our supposition: when the Earth is between the Sun and the Mars, the Allais effect causes fluctuations in the gravitational potential of Mars ( $\Phi = v^2 = gl$ ), what would be at the origin of swirls of dust and of the blue clearing.

With a blue filter, the Earth perceives Mars with a light that corresponds to short waves in the blue region of the spectrum. During the blue clearing—*i.e.*, the disappearance of the blue screen of the filter—the Earth sees Mars with a light that corresponds to longer wavelengths in the yellow, orange or red areas of the spectrum.

In fact, if the light collected through the blue filter of the telescopes on Earth was envisaged as radio signals sent by Mars, the interval between two signals would be longer, giving a slight redshift which can be interpreted as a slight decrease of the attraction of the Sun, or the Sun-Earth tandem. By Doppler effect, we could believe that Mars suddenly goes away, that there is a kind of redshift, as if the Earth, or rather the Sun behind, attracted less Mars. The observer on Earth has the impression that during the time  $t = l/c$  Mars moves of the distance  $l$  outwards with the acceleration  $g$ . This is effectively the inverse of the Pioneer anomaly [46]. In term of gravitational frequency shift, there is a tiny abnormal blueshift.

If we consider that in normal times the quantum of energy received by the Earth from Mars is  $\varepsilon = hv$  and if we assume that the light is emitted from Mars at the distance  $l$  (distance Mars-Earth) [48], the total energy of a photon of frequency  $\nu$  and energy  $h\nu$ , reaching the earth’s surface becomes

$$hv' = hv + hvgl/c^2 . \quad (1)$$

The receiver located on the Earth's surface detects a frequency  $\nu'$  superior to  $\nu$  from the Martian source ( $g$  is the Earth's gravitational field):

$$\nu' = \nu \left[ 1 + gl/c^2 \right]. \quad (2)$$

We can say that during the blue clearing, Mars acts as if it was moving away of a distance  $l$  from the Earth, because of a potential loss of attraction. The Earth's gravitational field  $g$  amounted to  $g - g'$ . When a photon emitted by Mars reaches the Earth's surface, it has lost the potential energy  $(hv/c^2)(g - g')l$  and won the kinetic energy  $(hv/c^2)(g - g')l$ . Its total energy has become

$$h(\nu' - \nu'') = hv + (hv/c^2)(g - g')l . \quad (3)$$

The frequency  $\nu' - \nu''$  of the photon at his arrival at the Earth's surface is less red-shifted relative to its initial frequency according to the relation

$$(\nu' - \nu'') = \nu \left[ 1 + (g - g')l/c^2 \right]. \quad (4)$$

During the phenomenon, the receiver disposed on the Earth's ground, detects a frequency  $\nu' - \nu''$  slightly smaller than  $\nu'$  without the blue clearing. This means a small blueshift for Mars.

#### **Allais effect and Stokes Raman effect of the atmosphere of Mars**

We assume that the gravitational potential of Mars varies when the Earth is between the Sun and Mars, what would have the effect of triggering dust storms, heavy atmospheric variations and cloud formations, which are related to the time of these oppositions. This change would exacerbate a disorderly animation of molecules in the atmosphere. The thermal agitation would encourage creation of tiny rarefactions or condensations and it is them that cause the light scattering, because they disturb the optical homogeneity of the atmosphere. The optical medium becomes heterogeneous and the incident light is scattered laterally.

Atom or molecule stores energy in their excited state. A molecule can be excited to a very high energy state. The amount of energy necessary to reach this state is  $h\nu_o$ . That energy is released when the molecule returns to a lower state. The return to the ground-state vibrational energy level  $\nu = 0$  results in the emission of a light that has the frequency  $\nu_o$ . This emission is usually observed in the visible spectral region and is called Rayleigh scattering. Nevertheless, we think that what is happening to the intramolecular level of the atoms, that constitute the molecules of the Martian atmosphere which diffuses waves, can be likened to a Stokes Raman effect [49] [50].

The scattering of light on the optical modes is called Raman scattering. It is different from the Rayleigh scattering because the scattered light changes the frequency of the spectrum active vibration. Historically, the effect was first observed with molecules. Molecules vibrate, and each molecular vibration corresponds to a certain amount of energy. In the scattering process, this energy is added or subtracted from the incident light. A Stokes Raman effect occurs when the molecule absorbs the incident light of frequency  $\nu_o$  and reemits light at a



lower frequency.

Thus, during the time of opposition and blue clearing, the gravitational potential  $\Phi$  of Mars would be perturbed. The molecule would then be relaxed from the excited state and would not completely relaxed at the level of energy minimal,  $\nu = 0$ , but would stop at  $\nu = 1$ , or even at an upper energy level. The energy of the emitted photon would be lower, we would have  $E_s = h(\nu_o - \nu_1)$ , where  $S$  stands for Stokes. The energy emitted in this process would be decreased of  $h\nu_1$ . Spectral lines with frequencies smaller than  $\nu_o$  are called Stokes lines [51] [52] [53].

In this way, if the Martian gravity varies in decreasing a bit, the tiny rarefactions lose in condensation, enlarge and the dimensions of the inhomogeneities resulting from the thermal excitation are proving to be greater than the length of light waves. From then on, it is mainly the waves corresponding to the yellow, orange or red regions of the spectrum which are diffused, what would have consequence to rule out the waves corresponding to purple or blue regions of the spectrum [54].

#### 4. Conclusions

This paper establishes for the first time a link between the phenomenon of “blue clearing”, violent storms and the Allais effect of syzygy. American specialists have given the name of blue clearing to an upheaval that violently surprised them: when Mars is in opposition (Earth between Mars and Sun), the blue screen that hides the details of the planet in some filters, suddenly begins to disappear, during a few days. However, some confusion seems to surround the term “blue clearing of Mars”. The tumult has nothing to do with strong dust storms giving way to a calm clarity or to an unexpected illumination of the planet atmosphere. It simply eliminates the blue color of the filter of an observation instrument placed on Earth or in space.

At the end of the Introduction we argued that the blue clearing is caused in the first instance by the Allais syzygy effect. We see a close relationship between the episode of Mars blue clearing and the eclipse effect during solar eclipses observed by Maurice Allais. In the first case, we observe on the blue filter a brutal change of wavelength; the filter becomes fortuitously a sort of measure of the wavelength or frequency. In the second case, the paraconical pendulum is an instrument to study certain compartments of Earth’s gravity. If anomalies of the paraconical pendulum must be highlighted during a configuration opposing the Sun and Mars, they would serve to demonstrate that the influence of the Sun and the Earth on Mars entails an anisotropic space, variable with time in direction and in intensity, and to determine if there is an abnormal “gravitational tension” which affects the gravitational potential and engenders the observed hurricanes.

The surface of Mars becomes visible when the blue color corresponding to a certain wavelength disappears from the blue filter which allows exclusive obser-

vation of the Martian atmosphere. This frequency shift signifies an alteration of the gravitational potential of Mars. It often occurs during syzygy at the time of unusual strong storms which are symptomatic of disturbances within the planet and can affect its rotation on itself. Since these disturbances are exerted on both mass and light, the blue clearing phenomenon makes it possible to realize that we must involve gravity and electromagnetism in the fundamental principle of “gravito-electromagnetic dynamics”.

In a broader way, during the Discussion, we glimpsed that if the hypothesis is correct, it falls under what is called “the phenomenon of astral influences” which reveals the influences of the Sun on the Earth and of the planets between them. It seems certain that these influences exist even if they are more complicated and confusing than previously imagined. Our gravitational theories do not take this into account. No spectral analysis of the “reflectors” that are the planets has never really been delivered, and no scientist has ever dared to involve universal gravitation and electromagnetism in an exhaustive study of the impulse of celestial bodies. Of course, a “gravitational tension” was noticed when the planets get closer or go away as well as an “electromagnetic tension” related to solar activity, but this paper is referring to a gravito-electromagnetic tension. The bodies of the solar system generate magnetic fields which intersect, constantly react by induction on one another, amending at each instant their vibratory modulations according to the displacements of their respective positions. In brief, there would be a physical influence of celestial bodies linked up with “gravitational and electromagnetic currents”. Each planet would exert a specific action and there would be angular relationships between the planets or luminaries with each other and with the Earth.

In a more restricted way, we saw in the section “Blue clearing, Allais syzygy effect and Raman Stokes effect” that in the event of the case of the opposition, whereas Mars is induced by the Sun and the Earth, one would have an effect gravito-electromagnetic of syzygy: the energy of the vibrations of the molecules would be disturbed and modified. Thenceforward that the Martian gravity decreases slightly, we can conceive that the energy of the emitted photon can be weaker by Stokes Raman scattering and that the blue of the spectrum can give way to longer wavelengths, which would remove the blue filter screen and make all the details of the planet appear. If we bring clarifications on the physical meaning of the variation of the gravitational potential and on a Raman effect which involves a scattered light above or below the standard frequencies, it will be possible to better understand the influence of the celestial bodies with each other.

### **Conflicts of Interest**

The author declares no conflicts of interest regarding the publication of this paper.

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# A Standard Model Approach to Inflation

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## Abstract

By assuming the cosmological principle includes the Pauli Exclusion Principle (PEP) and that the initial singularity existed within Planck time and length scales, a model for inflationary expansion is argued using only standard model physics without any changes to general relativity. All Fermionic matter is forced by the PEP to make a quantum transition to minimally orthogonal states in sequential Planck time intervals. This results in an initial inflation effect due to nearest neighbor quantum transitions which is then exacerbated by matter and antimatter creation effects due to collisions giving rise to the observational effects of universal inflation. The model provides a mechanistic explanation for primordial expansion using only physics from the standard model, specifically utilizing the PEP as a repulsion force between indistinguishable fermions. The present theory offers the benefit of not requiring any particles or fields outside of the standard model nor utilizing changes to general relativity. More succinctly, this theory goes beyond simply offering a mathematical representation (or fit) of the functional dependence but rather offers a mechanistic model to drive inflation using only standard model physics.

## Keywords

Inflation, Cosmological Expansion, Standard Model, Pauli Exclusion Principle, Genesis

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## 1. Introduction

The inflationary model has enjoyed great success in describing modern cosmological observations of homogeneity and isotropy along with a flat space-time [1] [2]. Difficulties with any mechanistic origin of the ad-hoc inflaton [3] have resulted in numerous alternative descriptions of the initial rapid expansion of the universe. These models include unique general relativity cases such as bouncing [3] [4] [5] [6], varying speed of light requirements [7] [8] [9], string theory [10]

[11] along with multiple other alternatives [12] [13] [14] [15]. Still, other models provide functional representations of both inflation and dark energy [16] [17] [18]. The current work diverts from the traditional functional representations of inflation and proposes an entirely mechanistic model such that only modern physics is sufficient to require inflationary origins.

The effect from going back in time predicted by general relativity requires that all matter began in a singularity without a sufficient time dependent cosmological constant to reverse the process early on [19]. By assuming the singularity, the proposed theory is able to describe a mechanism to initiate inflationary expansion at genesis. It is reasonable to assume continuity of all the known physical laws even at the Planck scales [20] [21] backwards in time to include this primordial singularity. Here, particular attention is placed on requiring the Pauli Exclusion Principle (PEP) to also be in full effect at the Planck scale from which the proposed mechanism is derived.

The minimum stable state for baryonic matter which can be associated with adhering to the PEP requirement is postulated to scale with that of either a neutron star (NS) or atomic nuclei. This means that to a first approximation, all indistinguishable Fermionic matter (*i.e.*, quarks) which had been present in the big bang (BB) singularity are forced at a minimum to push their nearest neighbor Fermions away on the order of this maximum packing density for nucleonic matter. The principle being that by combining the fundamental assumption of existence at the Planck scale in the singularity, it can then be argued that PEP also applies at the Planck scale and forces minimal physical separation in one Planck unit of time as a standard quantum transition from one state to another.

Given that all leptons, quarks and baryons of the standard model are composed of fermions, the anisotropy of their respective wavefunctions forces the PEP to uniformly distribute them into their minimally orthogonal and lowest energy states upon existence with the quantum transition taking place over the assumed Planck time scale. This, because of overlap of identical antisymmetric wavefunctions would result in cancellation of some of the particle's probability density function and so violate conservation of lepton and baryon numbers resulting in an effective separation force to maintain conservation of lepton number [22].

One of the most fundamental observations arising from PEP in measurements, is the repulsive force it provides when placing materials under pressure. It is the PEP which keeps crystalline materials at fixed interatomic distances despite the Coulombic attraction between the oppositely charged particles. When two objects are pushed together, it is the PEP repulsive force which prevents the exterior valence electrons of the two objects from overlapping and so serves as the equal and opposite force to their being pushed together [23].

## 2. Analysis and Results

The standard FLRW metric given by Carroll *et al.* [24] is  $H^2 = \frac{8\pi G}{3} \rho_M + \frac{\Lambda}{3} - \frac{k}{a^2}$  assuming  $k = 0$  describes the current model for universal expansion (where the

standard symbol definitions apply *i.e.*,  $H^2 = (\dot{a}/a)^2$ ). When the Hubble length  $a$  approaches zero, the proper time from general relativity  $d\tau^2 = dt^2 - dx^2 d\Omega^2$  becomes ill defined where both the spatial component  $dx$  and the temporal component  $dt$  approach zero as the FLRW model matter density  $\rho_M$  goes to infinity at  $\tau = 0 = a$ . Rather, we will assume that at the beginning of time, the BB singularity evolves at the Planck scale.

## 2.1. Inflation

### 2.1.1. Expansion Initiation

The initial singularity scaling  $a \approx 0$  is taken to be on the order of the Planck length  $l_p = 1.6e-35 \text{ m} = \hbar m_p^{-1} \cdot c^{-2}$  which is calculated using the definition of Planck mass  $m_p = 2.177e-8 \text{ kg} = (\hbar c/G)^{1/2}$  where  $c$  and  $G$  have their customary definitions of the speed of light and the gravitational constant respectively. These assumptions are also taken to occur in the initial time interval of the Planck time  $t_p = 5.4e-44 \text{ s} = l_p/c$  which then provides a means to predict the effects from the PEP to all fermionic matter at its genesis.

The scale assumed here for quark density is taken to be similar to that associated with a NS or barionic nuclei. The minimally orthogonal baryon number density of  $0.16 \text{ fm}^{-3}$  [25] then provides some initial condition predictions after the passage of the first unit of Planck time. At zero time, we begin with any arbitrary number of fermions in the singularity.

### 2.1.2. The 1<sup>st</sup> Planck Time Interval

The overlapping fermion wave functions in the initial singularity simply make a quantum transition to an adjacent location to conserve fermion number. This fundamentally accepts the assumption that the requisite PEP separation has to take place within a single interval of the Planck time and so allows a calculation of the momentum transfer imparted to fermionic matter due to its genesis.

Using the scaling from that of a neutron in a NS [25], the resulting relative displacement  $L$  between nearest neighbors for each quark would then be  $L \approx 2e-15 \text{ m}$  in the initial time interval  $\sim 5e-44 \text{ s}$ . This quantum transition then culminates in an apparent violation of special relativity as the initial relative speed of any two adjacent fermions becomes  $v \sim 2e-15 \text{ m}/5e-44 \text{ s} \approx 1e20 \text{ c}$  (although this is really just a simple quantum transition for each fermion in a single unit of Planck time). In this 1<sup>st</sup> Planck interval, each fermion has just transitioned outside the horizon of its nearest neighbors making them no longer causally connected at that moment.

With the initial dimensions of fundamental particles assumed to be Planck length going to a nearest neighbor distance  $L$  of  $2e-15 \text{ m}$ , this provides an expansion of 20 orders of magnitude during that 1<sup>st</sup> Planck time interval alone. With minimally orthogonal states being required, this initiates a homogenous, though hyper-chaotic, initial condition.

If these first generation quarks were the lowest energy state available upon existence, this means each bare quark mass can be approximated as  $m \approx 5 \text{ MeV}/c^2$  [26]. The resulting kinetic energy  $KE$  from the initial quantum transition of nearest neighbors can then be calculated from  $\sqrt{p^2 c^2 + m^2 c^4} - m^2 c^4 \approx pc$ . Al-

though relativistic values become imaginary at such speeds, being that this is really just a quantum transition, we will use for relative interparticle momentum, the value of  $p = mv = 1e22 \text{ MeV}/c$ , giving a contribution to the  $KE$  per quark of  $1e22 \text{ MeV}$ . This places the total energy of the transition more than 20 orders of magnitude greater than the initial rest mass per fermion.

This process is depicted in **Figure 1** where reflecting boundaries are assumed on the rightmost portion of the image.

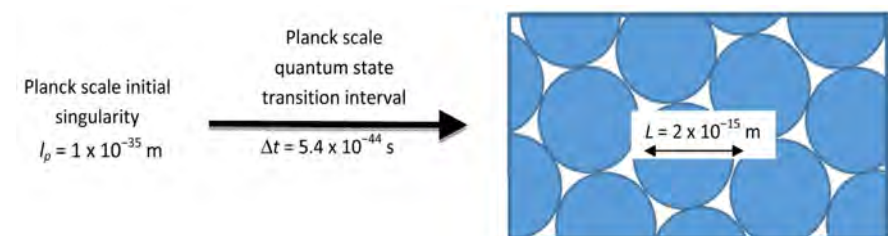
### 2.1.3. The 2<sup>nd</sup> Planck Time Interval

Within this closed packed configuration from the 1<sup>st</sup> Planck time interval, the subsequent fermions can also be assumed to obey the uncertainty principle  $\Delta E \sim \hbar/\Delta t \approx 7e-22 \text{ MeV} \cdot \text{s}/5e-44 \text{ s} \sim 1e22 \text{ MeV}$  placing this kinetic energy component effectively equal to that caused by the PEP imposed on the existence criteria. This means the kinetic energy during the second Planck time is approximately equally divided between and expansion motion and random motion for all particles. This provides a convenient mechanism to insure effective thermal equilibrium at existence without the need for any empirical coupling between disjoint regions outside each other's horizon. To the extent that this homogeneity continues due to these random expansion forces, uncoupled macroscopic regions would evolve in a similar chaotic manner with virtually indistinguishable phase space distributions.

The combined expansion energy and random kinetic energy coupled with extreme closed packing then implies fermion collisions are taking place. In other words, minimal orthogonality means that adjacent particles are touching an expected neighbor but with spatial offsets to allow distinguishable quantum numbers for each fermion. The resultant extreme high energy impacts between adjacent fermions having an average of  $1e22 \text{ MeV}$  of kinetic energy will create a vast sea of matter and antimatter particles, bringing these into existence even when they were not required in the initial singularity.

### 2.1.4. The 3<sup>rd</sup> Planck Time Interval

To a good approximation, the uncertainty principle energy alone is sufficiently large to assume equal fractions of massless and massive particles and antiparticles all being formed in subsequent Planck time intervals [27]. Those which were created in the 2<sup>nd</sup> time interval would largely be annihilated in the 3<sup>rd</sup> Planck time interval due to spatial overlap.



**Figure 1.** Quantum transition of a Planck scale singularity to a maximum packing density fermion sea.



The remaining fermions from any prior Planck time interval would then be replaced by these new fermions which did not recombine with their antiparticles. Those in the current Planck time would again be subject to the same physics previously described and so would continue the process of creating new particles until the attractive forces are able to catch up to the kinetics of the ensuing rapidly evolving inflationary epoch.

In each Planck time interval, the number of residual remnant created Fermions would scale with the statistical fluctuations in the number of particles created in each volume. To provide an initial estimate of this value, assume the average standard model particle rest mass is 100 MeV such that the  $1e22$  MeV energy per fermion is creating an additional  $1e20$  particles at each fermion location. The statistical fluctuations from this swarm of particles packed into this  $0.16 \text{ fm}^{-3}$  would then be  $1e10$  additional fermions. These fermions which then remain unto the 4<sup>th</sup> and further Planck time intervals would then undergo additional PEP quantum transitions exacerbating the inflationary process further. This process would continue through successive time intervals until attenuated by other means.

### 2.1.5. Subsequent Planck Time Intervals

The newly created particles with each prior Planck time interval (which were not annihilated by their antiparticles) will require PEP transitions which again will further exacerbate these inflationary effects.

After a time period of  $L/c = 2e-15 \text{ m}/3e8 \text{ m}\cdot\text{s}^{-1} \sim 1e-23 \text{ s}$  or approximately  $3e24$  Planck intervals, this is when gravity (and photons) will first start to appreciably be felt by nearest neighbors. By this time, each  $0.16 \text{ fm}^{-3}$  of volume will have created approximately  $1e10$  fermions for every preceding Planck interval ( $\sim 3e24$ ). Each new fermion then which had PEP transitioned to a new adjacent location then holds to the same rules as those prior, creating an average of  $1e10$  new fermions and so forth for an apparent runaway particle generation mechanism.

With each successive generation of particle creation giving rise to  $1e22$  MeV of new energy, each of these will send off gravitational waves along with their associated gluons, pseudoscalar and vector mesons to eventually provide adhesion forces. The photons will be able to undergo energy to mass transitions but not so for the gravitons as these would pass through each other as pure waves (consistent with the latest LIGO findings [28]). This means the gravity will continually build up and eventually be felt by adjacent and even distant newly created particles.

This gravitational pull will have been being built up by all preceding Planck interval particles and so by around  $1e25$  intervals, it can be assumed that contraction forces would start to be felt. By this time, the number of created fermions alone would be on the order of  $(1e25)e10$  or  $1e250$  fermions not to mention the associated melee of exotic particles in the mix. This number is approximately 120 orders of magnitude larger than that known in the observable un-

iverse [29] and so scales how vast the physical universe may actually be outside of our current horizon.

It is generally accepted that all matter present today is an arbitrarily small fraction of this overabundance of matter compared to antimatter present in the initial mix, other conditions are possible [30] but these are all assumed here not to violate the PEP and so are fully consistent with the proposed model offered here for inflation.

## 2.2. Isotropy and Homogeneity

The spatial location of any fermion at any time is irrelevant to the inflationary mechanism proposed leaving their evolution in phase space as effectively identical as the forces are identical but chaotic. This means that despite the lack of causal connectedness, the model still leaves each region to evolve in identical conditions subject to deviations due to random motions and statistical variations. In this sense, the expected outcome is effectively a homogenous distribution on the large scale after local coalescing effects are taken into consideration. The massive inflationary expansion also explains the expected flatness of space on the large scale as the effective stretching from the initial chain reaction of particle production would have imposed this condition.

## 3. Discussion

Utilizing the PEP, the uncertainty principle and the conservation laws, sequential Planck time units for quantum transitions create a massive chain reaction of particle creation sufficient to explain inflationary origins. This model effectively places inflation at the very initial moments of the BB eventually ( $\sim 1e-20$  s later) followed by random deceleration and cooling from subsequent attractive interactions. Standard BB cosmological models then continue to evolve using currently understood particle physics and general relativity models. This was accomplished using an arbitrary number of starting particles in the initial singularity.

The proposed inflation mechanism also has a certain elegance in that it only requires making almost intuitive and basic assumptions regarding initial existence at the Planck scale along with an axiomatic adherence to the PEP, from these, inflation is postulated effectively at genesis. Specifically, it is assumed that within the Planck time at the BB singularity, the PEP forces any arbitrary number of primordial adjacent like fermions apart sufficient to enable minimally distinct particle wavefunctions. The resultant energy from expansion and uncertainty momentum then creates a large number of other fermions whose remnant (which statistically did not recombine with antiparticles) then creates a subsequent generation of fermions to carry on the process. This continues until gluons, photons, and gravity (including the gravitational effect of neutrinos) can eventually start to coalesce the massive expansion forces.

This Pauli force effectively provides the initial starting energy of expansion by

requiring all Fermions to have these distinct quantum states. All identical Fermions then start with all others outside their horizon with random motion driving subsequent evolution. This model accounts for why the universe is so smooth on large scales, the requisite minimally orthogonal states at the initial Planck time forces this to be the initial condition everywhere. Likewise, flatness is postulated to be due to a massive scale and a subsequent purely random walk in all directions of all particles preventing curvature on large scales while still allowing clumping due to the same mechanism on small scales.

Conservation of lepton and boson number can be obtained within each horizon for the initial inflationary period but is not addressed further in this work but certainly warrants future attention. That conservation laws are due to symmetries in nature as demonstrated by Noether's theorem [29] implies that the fundamental driver for PEP can actually be traced back to symmetry in nature itself. Where symmetry itself came from or why the primordial singularity from classical general relativity was present in the first place is not addressed and potentially cannot even be addressed further in this manner insofar as either any testable or repeatable observation might be offered. This assumes symmetry itself is irreducible as might be the singularity.

#### 4. Conclusions

By imposing the initial singularity from General Relativity to exist at the Planck scale when  $t = 0$ , sequential quantum transitions in Planck time intervals result in an inflationary expansion with an arbitrary number of starting fermions. Particle generation rates of  $\sim 1E10$  particles per  $\sim 1E25$  Planck time intervals give rise to  $\sim 1E250$  fermion particles alone being required to come into existence in the first  $\sim 1E-20$  s. After this, gluon, photon, and graviton effects (assumed to propagate at light speed) just start to take effect and attenuate the process.

The resultant effects from these basic standard model and relativistic assumptions give rise to a mechanistic expectation of observable cosmology in terms of homogeneity and isotropy on large scales while maintaining equivalent phase space dynamics in regions formed outside the horizon of comparable disjoint regions. In this way, the contrived inflaton is not required to explain big bang cosmology but rather standard model physics, constrained at the Planck scale.

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#### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# About Degeneration of Landau's Levels

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## Abstract

Through the non separable solution of the eigenvalue problem associated to the problem of a charged particle in a flat box and a constant transversal magnetic field, with Landau and symmetric gauges, it is found that the Landau's levels are numerably degenerated in both cases. A mathematical proposition is proven to carry out this statement.

## Keywords

Landau's Gauge, Symmetric Gauge, Quantum Hall Effect, Degeneration

## 1. Introduction

The quantum Hall effect has had a great deal of physical and experimental importance since its discovery [1] [2] [3] [4], and one of the basic elements to understand this effect is the Landau's levels (eigenvalues of the eigenvalue problem), which has shown being correct even if the eigenfunctions are not totally right since the eigenvalue problem is not separable in all of its variables [5]. These eigenfunctions have been also found in different form [6], but both of them result to be equivalents [7]. However, a correct non separable solution of the eigenvalue problem has already been given on references [5] [8], where it is shown that for the eigenvalue problem with the Hamiltonian

$$\hat{H} = \frac{(\mathbf{p} - q\mathbf{A}/c)^2}{2m}, \quad (1)$$

with the Landau's gauge  $\mathbf{A} = (-By, 0, 0)$  ( $B$  is constant) and with the symmetric gauge  $\mathbf{A} = B(y, -x, 0)/2$  (inverse magnetic field), the Landau's levels are gotten

$$E_n = \hbar\omega_c (n + 1/2), \quad \omega_c = qB/mc, \quad (2)$$

the magnetic flux  $\Phi$  is quantized

$$\frac{\Phi}{\Phi_0} \in \mathcal{Z}, \quad \Phi_0 = 2\pi\hbar c/2q, \tag{3}$$

and the eigenfunctions for Landau’s gauge (ignoring the  $z$ -variable) are given by

$$\Phi_n^L(x, y) = \sqrt{\frac{\beta}{L_y}} e^{-i\beta^2 xy} \psi_n(\beta x), \quad \beta = \sqrt{m\omega_c/\hbar}, \tag{4}$$

where  $L_y$  represents the length of the box in the  $y$ -direction, and  $\psi_n$  is the harmonic oscillator solutions. For the symmetric gauge the eigenfunctions are

$$\Phi_n^S(x, y) = A_n e^{-\alpha(x^2+y^2)-\lambda(x-iy)} (2\alpha(x+iy)+\lambda)^n, \quad \alpha = qB/4\hbar c, \tag{5}$$

where  $\lambda$  is a complex constant, and  $A_n = e^{-|\lambda|^2/4\alpha} / \sqrt{\pi n! (2\alpha)^{n-1}}$  is a normalized constant. In addition, for the Landau’s gauge case one has  $[\hat{p}_x, \hat{H}] = 0$ , and for the symmetric gauge case one has  $[\hat{L}_z, \hat{H}] = 0$ . These facts allow to have the following additional generated functions

$$\hat{p}_x \Phi_n^L = m\omega_c (ix - y) \Phi_n^L - i\hbar\beta\sqrt{2n} \Phi_{n-1}^L \tag{6}$$

and

$$\hat{L}_z \Phi_n^S = \hbar\lambda z^* \Phi_n^S + \hbar\sqrt{2\alpha n} z \Phi_{n-1}^S, \quad z = x + iy, \tag{7}$$

which are also eigenfunctions of the Hamiltonian

$$\hat{H}(\hat{p}_x \Phi_n^L) = E_n (\hat{p}_x \Phi_n^L) \tag{8}$$

and

$$\hat{H}(\hat{L}_z \Phi_n^S) = E_n (\hat{L}_z \Phi_n^S). \tag{9}$$

from these relations, it was thought that Landau’s levels were doubly degenerated. However, we will see that this result is deeper than it was first thought since it allows to have numerably degeneration for the Landau’s levels. The reason for example (6) is also an eigenfunction, as shown in (8), is the following: from the expression  $[\hat{p}_x, \hat{H}] = 0$ , one has

$$0 = [\hat{p}_x, \hat{H}] \Phi_n^L = \hat{p}_x (\hat{H} \Phi_n^L) - \hat{H} (\hat{p}_x \Phi_n^L) = E_n (\hat{p}_x \Phi_n^L) - \hat{H} (\hat{p}_x \Phi_n^L), \tag{10}$$

and the result (8) follows.

## 2. Analysis of the Degeneration

Let us make first some mathematical statements that will help to understand the situation. Let  $\mathcal{E}$  be our Hilbert space, normally the set of quadratic integrable function in some set  $\Omega$  contained in some dimensional space  $L^2(\Omega) = \int_{\Omega} |f|^2 d\mu$ , and let  $\mathcal{L}(\mathcal{E})$  the set of linear operators acting in the space  $\mathcal{E}$ . Thus, one has the following proposition.

**Prop. 1.-** Let  $A, H \in \mathcal{L}(\mathcal{E})$  be linear operators such that  $[A, H] = 0$ , and let  $\{E_n, \phi_n\}_{n \in \mathcal{Z}}$  be the solutions of the eigenvalue problem  $H\phi = E\phi$ . If  $A\phi_n$  is not proportional to  $\phi_n$ , then  $A^j \phi_n, j \in \mathcal{Z}^+$  is an eigenfunction of  $H$  with the same eigenvalue  $E_n$  (Therefore, the spectrum is numerably degenerated).

**Proof:** The fact  $[A, H] = 0$  implies that  $[A^j, H] = 0$  for  $j \in \mathbb{Z}^+$ , where  $A^j$  means  $j$ -applications of the operator  $A$  ( $A \circ A \circ \dots \circ A$ ). Therefore one has that  $H(A^j \phi_n) = E_n(A^j \phi_n)$ . Since  $A\phi_n$  is not proportional to  $\phi_n$ , it represents a new function, and by induction  $A^j \phi_n$  represents a new function for  $j \in \mathbb{Z}^+$ . So, defining  $f_n^0 = \phi_n$  and  $f_n^j = A^j \phi_n$ , one has a set functions  $\{f_n^j\}_{n,j \in \mathbb{Z}^+}$  which are eigenfunctions with the same eigenvalue

$$Hf_n^j = E_n f_n^j, \quad n, j \in \mathbb{Z}^+ \bullet \tag{11}$$

In addition to the above proposition, one has the following: If  $H \in \mathcal{L}(\mathcal{E})$  is an Hermitian operator, that is  $\langle Hf, g \rangle = \langle f, Hg \rangle$  with the inner product  $\langle f, g \rangle = \int_{\Omega} f^* g d\mu$ , one has the known proposition

**Prop. 2.-** Let  $H \in \mathcal{L}(\mathcal{E})$  be an Hermitian operator, and let  $\{E_n, f_n^j\}$  the set of solutions of the eigenvalue problem  $H\phi = E\phi$ , where the spectrum is degenerated (this degeneration is represente by the index “ $j$ ”). Then, the functions  $\{f_n^j\}$  are orthogonal with respect the index “ $n$ ”, but the orthogonality is undetermined with respect the index “ $j$ ”.

**Proof:** The relation  $\langle Hf_{n_1}^{j_1}, f_{n_2}^{j_2} \rangle = \langle f_{n_1}^{j_1}, Hf_{n_2}^{j_2} \rangle$  implies that  $(E_{n_2} - E_{n_1}) \langle f_{n_1}^{j_1}, f_{n_2}^{j_2} \rangle = 0$ . Then, for  $n_1 \neq n_2$  one has necessarily that  $\langle f_{n_1}^{j_1}, f_{n_2}^{j_2} \rangle = 0$  (orthogonality, independently of  $j_1$  and  $j_2$ ), but if  $n_1 = n_2 = n$  the expression  $\langle f_{n_1}^{j_1}, f_{n_2}^{j_2} \rangle$  is undetermined  $\bullet$

Of course, given a non orthogonal set of functions  $\{f_n^j\}$ , one can construct an orthogonal set  $\{\tilde{f}_n^j\}$  through the Gram-Schmidt process [9]. Now, the results presented in (4), (5), (6), (7), (8) and (9) state exactly the conditions for the application of the Prop. 1 above. Therefore, the Landau’s levels are numerably degenerated in both cases with the Landau and symmetric gauges. The states associated to each Landau’s level are

$$\{f_{n,j}^L\}_{j \in \mathbb{Z}^+}, \quad f_{n,0}^L(x, y) = \Phi_n^L(x, y) \quad \text{and} \quad f_{n,j}^L = \hat{p}_x^j \Phi_n^L \tag{12}$$

and

$$\{f_{n,j}^S\}_{j \in \mathbb{Z}^+}, \quad f_{n,0}^S(x, y) = \Phi_n^S(x, y) \quad \text{and} \quad f_{n,j}^S = \hat{L}_z^j \Phi_n^S. \tag{13}$$

It is easy to see, for example, from (4) and (6) that  $\langle f_n^0 | f_n^1 \rangle \not\sim \delta_{0,1}$ . Therefore, the set defined by (12) is non orthogonal, and the same happens with the set (13). Of course, the general solution of the Schödinger’s equation ( $i\hbar \partial \Psi / \partial t = \hat{H} \Psi$ ) for this problem should be written for the Landau’s gauge (ignoring the  $z$ -variable) as

$$\Psi^L(x, y, t) = \sum_{n,j=0}^{\infty} C_{n,j}^L f_{n,j}^L(x, y) e^{-iE_n t / \hbar} \tag{14}$$

and for the symmetric gauge as

$$\Psi^S(x, y, t) = \sum_{n,j=0}^{\infty} C_{n,j}^S f_{n,j}^S(x, y) e^{-iE_n t / \hbar}, \tag{15}$$

being  $C_{n,j}^L$  and  $C_{n,j}^S$  constant, and  $E_n$  is the Landau’s levels (2).



Now, from the result (11) and the expression (12) it is not difficult to see that one has the following relation

$$f_n^{j+1} = m\omega_c \left( \hbar j f_n^{j-1} + (ix - y) f_n^j \right) - i\sqrt{2nm\omega_c} \hbar f_{n-1}^j, \quad j \geq 0, \quad (16)$$

and

$$\hat{H}f_n^{j+1} = E_n f_n^{j+1} + m\omega_c \left( \hbar j f_n^{j-1} + ix f_n^j + \frac{i}{m\omega_c} \hat{p}_y f_n^j \right). \quad (17)$$

Thus, from the result (11), one must have that

$$\hbar j f_n^{j-1} + ix f_n^j + \frac{i}{m\omega_c} \hat{p}_y f_n^j = 0, \quad (18)$$

which it is not difficult to check it directly.

Similarly, from (5) and (7), one can get

$$f_n^{j+1} = \hbar \lambda z^* \sum_{m=0}^j c_m^j f_n^m + \hbar \sqrt{2\alpha n} z \sum_{m=0}^j d_m^j f_{n-1}^m, \quad (19)$$

where one has defined the constants  $c_m^j$  and  $d_m^j$  as  $c_m^j = \binom{j}{m} (-\hbar)^{j-m}$  and

$d_m^j = \binom{j}{m} \hbar^{j-m}$ , being  $\binom{j}{m} = j! / m!(j-m)!$  the binomial coefficient. In addition, one has the following action

$$\hat{H}f_n^{j+1} = E_n f_n^{j+1} - \frac{2\hbar^2}{m} \sum_{m=0}^j \left( c_m^j \hbar \lambda \partial_z f_n^m + c_m^j \alpha \hbar \lambda z^* f_n^m + d_m^j \hbar \sqrt{2\alpha n} \partial_z f_{n-1}^m + d_m^j \alpha \hbar z \sqrt{2\alpha n} f_{n-1}^m \right). \quad (20)$$

Then, using (11), it follows that

$$\sum_{m=0}^j \left( c_m^j \hbar \lambda \partial_z f_n^m + c_m^j \alpha \hbar \lambda z^* f_n^m + d_m^j \hbar \sqrt{2\alpha n} \partial_z f_{n-1}^m + d_m^j \alpha \hbar z \sqrt{2\alpha n} f_{n-1}^m \right) = 0, \quad (21)$$

which it is also not difficult to verify directly.

### 3. Conclusion

Due to previous results (6), (7), (8) and (9), obtained in [8], and the Prop. 1, we must conclude that the Landau's levels are numerably degenerated. This degeneration may have important consequences in the quantum dynamics of the quantum Hall Effect and topological insulators.

### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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# One Dimensional Conservative System with Quadratic Dissipation and Position Depending Mass

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## Abstract

For a 1-D conservative system with a position depending mass within a dissipative medium, its effect on the body is to exert a force depending on the squared of its velocity, a constant of motion, Lagrangian, generalized linear momentum, and Hamiltonian are obtained. We apply these new results to the harmonic oscillator and pendulum under the characteristics mentioned about, obtaining their constant of motion, Lagrangian and Hamiltonian for the case when the body is increasing its mass.

## Keywords

Dissipation, Position Mass Depending, Constant of Motion, Hamiltonian

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## 1. Introduction

Variable mass problems without dissipation have a long history and are known as Gylden-Meshcherskii problems [1] [2] [3] [4] [5]. As it is known, Newton's equation with position mass depending is not invariant under Galileo's transformation [6] [7], and Sommerfeld gave a modification of this equation to overcome this problem [8]. However, this modification has a fundamental problem when external force is zero, and that is why one considers Newton's equation of motion as a good equation of motion for these types of problems [9] [10]. This approach was used for 1-D conservative systems with position depending mass [11], binary stars with mass exchanged [12] [13], binary galaxies with mass exchanged [14], and fluid dynamics [15]. On the other hand, 1-D systems with constant mass and quadratic dissipation have also been studied [16]. Therefore, in this paper both situations are considered at the same time, position mass de-

pending and quadratic dissipation on 1-D conservative systems, and for these systems one will find a Constant of Motion, Lagrangian, Generalized Linear Momentum, and Hamiltonian. The results will be applied to the study on the dynamics of the harmonic oscillator and pendulum systems with this dissipation and with increasing of mass behavior.

## 2. Analytical Approach

Newton's equation for 1-D conservative systems, characterized by an external force  $F(x)$ , with position depending mass,  $m(x)$ , and a quadratic dissipation force,  $-\alpha v^2$  (being  $\alpha$  a nonnegative real constant, and  $v \geq 0$ ), is given by

$$\frac{d}{dt}(m(x)v) = F(x) - \alpha v^2, \quad v \geq 0, \quad (1)$$

where  $v$  represents the velocity,  $v = dx/dt = \dot{x}$ , of the body, and  $\alpha$  is a constant. One will consider that  $m(0) = m_0$  represents the initial mass of the system at the point  $x = 0$ . Equation (1) can be written as an autonomous dynamical system defined in  $\mathfrak{R}^2$  as

$$\dot{x} = v, \quad (2a)$$

$$\dot{v} = \frac{F(x) - (\alpha + m_x)v^2}{m(x)}, \quad (2b)$$

where  $m_x$  has been defined as  $m_x = dm/dx$ . A constant of motion for this system is a function  $K = K(x, v)$  such that  $dK/dt = 0$ , that is, it must satisfy the following first order partial differential equation

$$v \frac{\partial K}{\partial x} + \frac{F(x) - (\alpha + m_x)v^2}{m(x)} \frac{\partial K}{\partial v} = 0, \quad (3)$$

which can be solved by the characteristics method [17], where the equations for the characteristics are

$$\frac{dx}{v} = \frac{m(x)dv}{F(x) - (\alpha + m_x)v^2} = \frac{dK}{0}. \quad (4)$$

The last term just tell us that the function  $K$  must be an arbitrary function of the characteristic  $C$  obtained from the others two terms,  $K = G(C)$  where  $G$  is arbitrary. From the others two terms, one can write the following equation

$$\frac{m(x)}{2} \frac{dv^2}{dx} + (\alpha + m_x)v^2 = F(x). \quad (5)$$

Defining a new variable  $\xi$  as  $\xi = v^2$  and rearranging terms, this equation is written as

$$\frac{d\xi}{dx} + \frac{2(\alpha + m_x)}{m(x)} \xi = \frac{2F(x)}{m(x)}. \quad (6)$$

Now, multiplying this equation by  $\exp\left(\int^x \frac{2(\alpha + m_x)}{m(s)} ds\right)$ , the resulting equa-

tion can be written as

$$\frac{d}{dx} \left( \xi e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}} \right) = \frac{2F(x)}{m(x)} e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}}, \quad (7)$$

which can easily be integrated, and one gets the following expression in terms of the variable  $v$

$$v^2 e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}} = 2 \int \frac{F(x)}{m(x)} e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}} dx + A, \quad (8)$$

where  $A$  is the constant of integration. Then, one chooses the characteristic curve as  $C = m_o A/A$  and chooses the function  $G$  as  $G(C) = C$  to get the constant of motion

$$K_\alpha^{(+)}(x, v) = \frac{m_o v^2}{2} e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}} - m_o \int \frac{F(x)}{m(x)} e^{\int^x \frac{2(\alpha+m_x)ds}{m(s)}} dx. \quad (9a)$$

Using the following identity

$$e^{\int^x \frac{2m_x(s)ds}{m(s)}} = \left( \frac{m(x)}{m_o} \right)^2, \quad (9b)$$

the expression (9a) is written finally as

$$K_\alpha^{(+)}(x, v) = \frac{m^2(x)v^2}{2m_o} e^{2\alpha \int^x \frac{ds}{m(s)}} - \frac{1}{m_o} \int^x m(x)F(x) e^{2\alpha \int^x \frac{ds}{m(s)}} dx. \quad (10)$$

This expression is of the form

$$K_\alpha^{(+)}(x, v) = T_\alpha(x, v) + V_\alpha(x) \quad (11)$$

where  $T_\alpha$  is some type of effective kinetic energy of the system,

$$T_\alpha(x, v) = \frac{m^2(x)v^2}{2m_o} e^{2\alpha \int^x \frac{ds}{m(s)}} \quad (12)$$

and  $V_\alpha$  is just the effective potential

$$V_\alpha(x) = -\frac{1}{m_o} \int^x m(x)F(x) e^{2\alpha \int^x \frac{ds}{m(s)}} dx. \quad (13)$$

Then, one can say that  $K_\alpha^{(+)}$  represents the **effective energy** of the system.

## 2.1. Special Cases

Let us note the following:

First, one has the following limit

$$\lim_{\alpha \rightarrow 0} K_\alpha^{(+)}(x, v) = \frac{m^2(x)v^2}{2m_o} - \frac{1}{m_o} \int^x m(x)F(x) dx, \quad (14a)$$

which is the expression obtained in reference [11].

Second, assuming the mass as constant,  $m(x) = m_o$ , one gets

$$K_{\alpha}^{(+)}(x, v) = \frac{m_o v^2}{2} e^{2\alpha x/m_o} - \int^x F(x) e^{2\alpha x/m_o} dx, \tag{14b}$$

which is the expression obtained in references [16] [18] (for the non relativistic case).

Third, for  $\alpha = 0$  and  $m(x) = m_o$ , one gets the usual energy of a conservative system

$$K(x, v) = \frac{m_o}{2} v^2 + V(x), \tag{14c}$$

where  $V(x)$  is the potential of the system,  $V(x) = -\int F(x) dx$ .

### 2.2. Lagrangian and Hamiltonian

Now, since  $K_{\alpha}^{(+)}(x, v)$  is a constant of motion, a Lagrangian of the system can be found through the relation [19] [20] [21]

$$L_{\alpha}^{(+)}(x, v) = v \int^v \frac{K_{\alpha}^{(+)}(x, \xi) d\xi}{\xi^2}.$$

In this way and considering (13), one gets

$$L_{\alpha}^{(+)}(x, v) = \frac{m^2(x)v^2}{2m_o} e^{2\alpha \int^x \frac{ds}{m(s)}} - V_{\alpha}(x) \tag{15}$$

The generalized linear momentum is

$$p_{\alpha}^{(+)}(x, v) = \frac{m^2(x)v}{m_o} e^{2\alpha \int^x \frac{ds}{m(s)}}. \tag{16}$$

With this expression and the Legendre's transformation,  $H(x, p) = v(x, p)p - L(x, v(x, p))$ , the Hamiltonian of the system is given by

$$H_{\alpha}^{(+)}(x, p) = \frac{m_o p^2}{2m(x)} e^{-2\alpha \int^x \frac{ds}{m(s)}} + V_{\alpha}(x) \tag{17}$$

If we apply the above observations (11) on the expressions (10), (15), (16), and (17), one gets the corresponding correct expression for these cases.

Let us notice from (1) that the dissipation for  $v < 0$  can be obtained by making the change  $\alpha \rightarrow -\alpha$  on the expressions already found. Therefore, the constant of motion, Lagrangian, generalized linear momentum, and Hamiltonian when  $v < 0$  are given by

$$K_{\alpha}^{(-)}(x, v) = K_{-\alpha}^{(+)}(x, v), \tag{18a}$$

$$L_{\alpha}^{(-)}(x, v) = L_{-\alpha}^{(+)}(x, v), \tag{18b}$$

$$p_{\alpha}^{(-)}(x, v) = p_{-\alpha}^{(+)}(x, v), \tag{18c}$$

and

$$H_{\alpha}^{(-)}(x, p) = H_{-\alpha}^{(+)}(x, p). \tag{18d}$$

However, notice from (13) that the potential  $V_{-\alpha}(x)$  can be very different

from  $V_\alpha(x)$ , as it will be seen on below examples.

### 3. Mass Linear Dependence on Position

In this case, one has the following dependence of the mass with respect the position of the body

$$m(x) = m_o + \beta x, \quad (19)$$

where  $\beta$  is a constant. Then, it follows that

$$e^{2\alpha \int^x \frac{ds}{m(s)}} = \left( \frac{m_o + \beta x}{m_o} \right)^{2\alpha/\beta}. \quad (20)$$

So, from the expressions (10), (15), (16), and (17), one obtains

$$K_\alpha^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 + V_\alpha(x), \quad (21a)$$

$$L_\alpha^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 - V_\alpha(x), \quad (21b)$$

$$P_\alpha^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{m_o^{1+2\alpha/\beta}} v, \quad (21c)$$

and

$$H_\alpha^{(+)}(x, p) = \frac{m_o^{1+2\alpha/\beta}}{2(m_o + \beta x)^{2+2\alpha/\beta}} p^2 + V_\alpha(x), \quad (21d)$$

where the effective potential  $V_\alpha$  is given by

$$V_\alpha(x) = -\frac{1}{m_o^{1+2\alpha/\beta}} \int^x F(x)(m_o + \beta x)^{1+2\alpha/\beta} dx. \quad (21e)$$

#### 3.1. Harmonic Oscillator

For the harmonic oscillator, one has that  $F(x) = -kx$ , and using the following integration

$$\int x(m_o + \beta x)^{1+2\alpha/\beta} dx = \frac{1}{\beta^2} (m_o + \beta x)^{2+2\alpha/\beta} (2\beta x + 2\alpha x - m_o), \quad (22)$$

the effective potential is

$$V_\alpha(x) = \frac{k(m_o + \beta x)^{2+2\alpha/\beta} (2\beta x + 2\alpha x - m_o)}{\beta^2 m_o^{1+2\alpha/\beta} (3 + 2\alpha/\beta)(2 + 2\alpha/\beta)}, \quad (23)$$

where the constant term  $-km_o^2(1/4\alpha^2 + 1/6\beta^2)$  could be added to get the right limits ( $\lim_{\alpha \rightarrow 0} \lim_{\beta \rightarrow 0} V_\alpha = \lim_{\beta \rightarrow 0} \lim_{\alpha \rightarrow 0} V_\alpha = kx^2/2$ ), and one obtains the constant of motion, Lagrangian, generalized linear momentum, and Hamiltonian for the harmonic oscillator with linear position dependence on its mass and quadratic dissipation as

$$K_\alpha^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 + V_\alpha(x) \quad (24a)$$

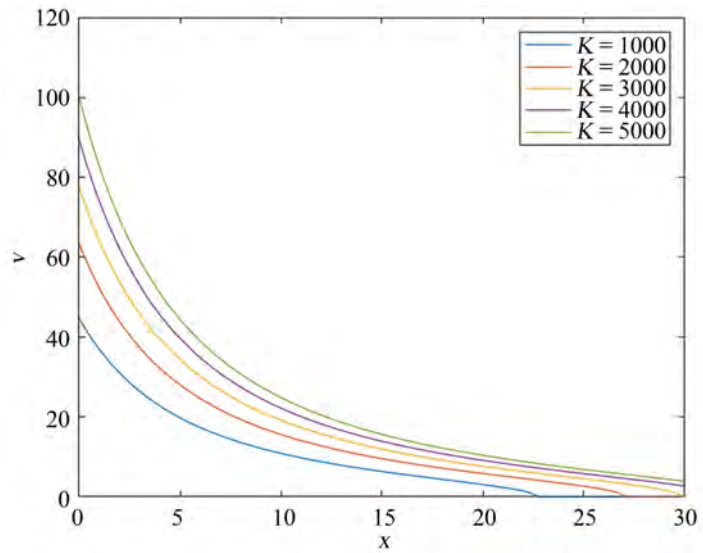
$$L_{\alpha}^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 - V_{\alpha}(x) \tag{24b}$$

$$p_{\alpha}^{(+)}(x, v) = \frac{(m_o + \beta x)^{2+2\alpha/\beta}}{m_o^{1+2\alpha/\beta}} v, \tag{24c}$$

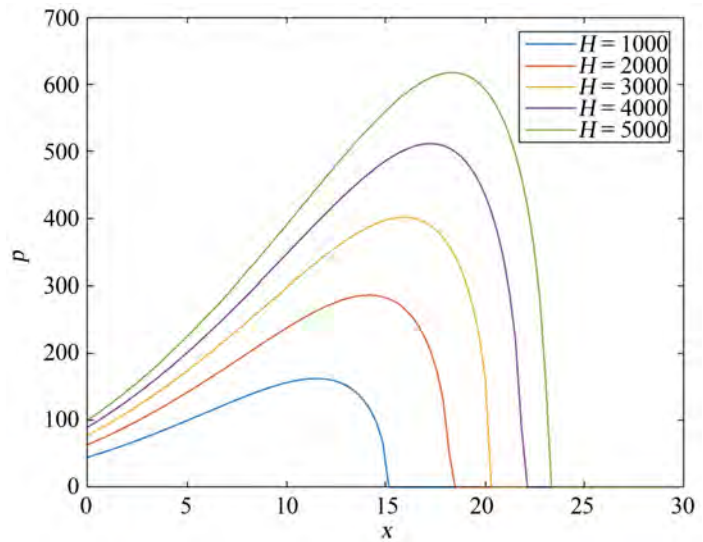
and

$$H_{\alpha}^{(+)}(x, p) = \frac{m_o^{1+2\alpha/\beta} p^2}{2(m_o + \beta x)^{2+2\alpha/\beta}} + V_{\alpha}(x). \tag{24d}$$

**Figure 1** shows the behavior of the body in the one quarter of the phase spaces  $(x, v)$  and  $(x, p)$  for several values of the parameter  $K$ , with  $m_o = 1 \text{ kg}$ ,



(a)



(b)

**Figure 1.** Behavior through the constant of motion and the hamiltonian. (a)  $\alpha = \beta = 0.1 \text{ kg/m}$ ; (b)  $\alpha = \beta = 0.1 \text{ kg/m}$ .



and  $k = 1 \text{ N/m}$ . Note that since  $\beta \geq 0$ , the system is acquiring mass as the position is increasing. Because of this, and due that one has dissipation in the system, the body will perform a damping spiral behavior on the phase spaces  $(x, v)$  and  $(x, p)$ , which is not shown here.

To determine this spiral damping behavior and assuming always and increasing of mass, one would have to divide the phase space  $(x, v)$  in four regions: 1)  $v > 0$  and  $x > 0$ , 2)  $v < 0$  and  $s \geq 0$ , 3)  $v < 0$  and  $s < 0$ , 4)  $v > 0$  and  $s \leq 0$ . On the upper plane ( $v > 0$ ) one uses  $K_\alpha^{(+)}$ , and in the lower plane one uses  $K_\alpha^{(-)}$ . Once  $v = 0$  on the region (1), the effective energy  $K_\alpha^{(-)}$  is determined by the value of the effective potential at the point  $x_1$  where this happens, and the mass changes on the region (2) of the form  $m(s) = m_o + \beta s_1 + \beta(s_1 - s)$  in the interval  $s \in [s_1, 0]$ . On the region (3) the mass must vary as  $m(s) = m_o + 2\beta s_1 + \beta|s|$  until the body reaches again a velocity  $v = 0$  at the point  $s_2$  (negative). At this point the effective energy  $K_\alpha^{(+)}$  is defined by the value of the effective potential at this point, and the mass varies on this region (4) as  $m(s) = m_o + 2\beta s_1 + \beta|s_2| + \beta(|s_2| - |s|)$  until the body reaches  $s = 0$ , completing on cycle of the spiral motion. The same would be repeated with the other cycles of the spiral motion. The reason of this complication is due to the fact that during the whole motion the body is increasing its mass, otherwise one would have mass oscillation depending whether  $x$  is positive or negative. The same idea is applied for the Hamiltonian and the phase space  $(x, p)$ , and note the great different behavior of body on the phase space  $(x, p)$  with respect the phase behavior on the phase space  $(x, v)$ , due to the position dependence of the generalized linear momentum (24c).

### 3.2. Pendulum

The position on the pendulum is determined by its displacement  $s$  respect its equilibrium position at the angle  $\theta = 0$ , that is  $s = l\theta$ , where  $l$  denotes the length of the cord. The force acting on the body, of mass  $m(s)$ , hanged at the end of the cord is given by  $F(s) = -gm(s)\sin(s/l)$ , being  $g$  the constant acceleration due to gravity. Using the following integration [22]

$$f(s) = \int F(s)(1 + \beta s)^{1+2\alpha/\beta} ds = -\frac{g(\beta l)^{3+2\alpha/\beta}}{2\beta} e^{i(m_o/\beta l - \pi\alpha/\beta)} \times \left\{ \gamma\left(3 + \frac{2\alpha}{\beta}, -i\frac{m_o + \beta s}{\beta l}\right) + (-1)^{3+2\alpha/\beta} \gamma\left(3 + \frac{2\alpha}{\beta}, i\frac{m_o + \beta s}{\beta l}\right) \right\}, \quad (25a)$$

where  $\gamma$  is the uncompleted gamma function [22] (page 940). If we select the mass as  $m_o = 1 \text{ kg}$  and  $\alpha/\beta = n$  and integer number, the function  $f(s)$  can be given by

$$f(s) = -g \sum_{k=0}^{2+n} \frac{(2+n)!(\beta l)^k}{(2+n-k)!} \sum_{j=0}^k \frac{s^{k-j}}{l^{k-j}(k-j)!} \cos(s/l + j\pi/2). \quad (25b)$$

Therefore, the effective potential is

$$V_\alpha(s) = \frac{f(s)}{m_o^{1+2\alpha/\beta}} \tag{25c}$$

The Constant of Motion, Lagrangian, generalized linear momentum, and Hamiltonian are

$$K_\alpha^{(+)}(s, v) = \frac{(m_o + \beta s)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 + V_\alpha(s) \tag{26a}$$

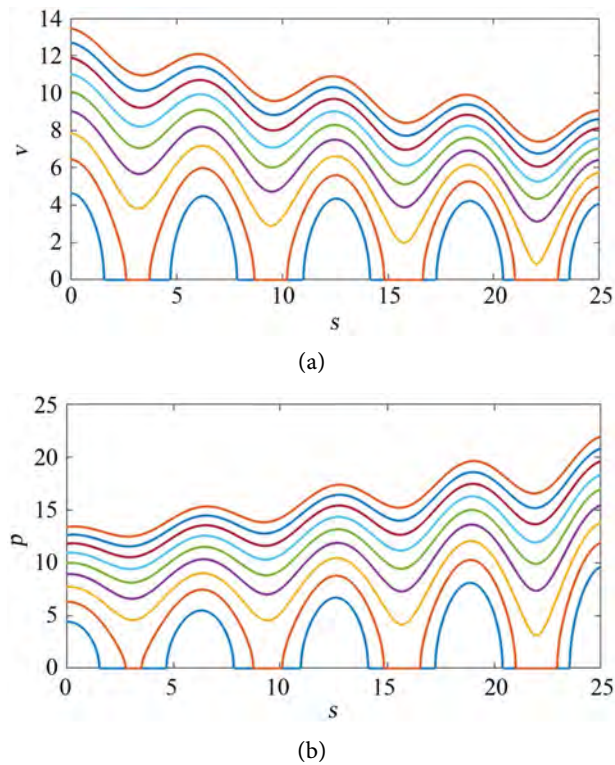
$$L_\alpha^{(+)}(s, v) = \frac{(m_o + \beta s)^{2+2\alpha/\beta}}{2m_o^{1+2\alpha/\beta}} v^2 - V_\alpha(s) \tag{26b}$$

$$p_\alpha^{(+)}(s, v) = \frac{(m_o + \beta s)^{2+2\alpha/\beta}}{m_o^{1+2\alpha/\beta}} v, \tag{26c}$$

and

$$H_\alpha^{(+)}(s, p) = \frac{m_o^{1+2\alpha/\beta}}{2(m_o + \beta s)^{2+2\alpha/\beta}} p^2 + V_\alpha(s), \tag{26d}$$

where  $v$  represents the velocity of the body,  $v = ds/dt$ . **Figure 2** shows the behavior of the body in the first quadrant ( $s \geq 0$ ,  $v \geq 0$ , and  $p \geq 0$ ) of the phase spaces  $(s, v)$  and  $(s, p)$  for the values of the parameters  $K$  and  $H$  given by 0, 10, 20, 30, 40, 50, 60, 70, and 80 (blue, orange, yellow, and so on) with  $m_o = 1$  kg, and  $l = 1$  m and  $f(s)$  taken as the expression (25b). The inner blue and orange lines represent the oscillatory spiral damping behavior of the body



**Figure 2.** Behavior through the constant of motion and the hamiltonian. (a) Phase space  $(x, v)$ ; (b) Phase space  $(x, p)$ .

due to increasing of mass during its motion and the damping factor. The upper lines represent the rotational spiral damping behavior of the body due to the same reason (this spiral damping behavior is not shown on these plots. To get this behavior one would need to proceed similarly as it was explained for the harmonic oscillator part).

The effective potential  $V_\alpha$  has an oscillatory increasing behavior as a function of the displacement  $s$ . Therefore, it does not matter which value of the effective energy  $K$  or  $H$  takes, due to the increasing of mass and damping factor, the body will perform an oscillatory damping behavior, that is, the origin of the phase space is an attractor of the dynamics of the body (as it happened with the first example). On **Figure 2(b)** one sees an apparent increasing of the generalized linear momentum as the body is rotating. However, eventually will reach the return point of the potential and the generalized linear momentum will be zero (as the yellow line indicates).

#### 4. Conclusions and Comments

In general, we have constructed constant of motion, Lagrangian, generalized linear momentum, and Hamiltonian for a 1-D conservative system with position depending mass and embedded in a medium where the body feels a dissipative force which depends quadratically on its velocity. In particular, we made the analysis for the case when the body increases its mass linearly on its displacement, where the dynamics in the phase spaces  $(x, v)$  and  $(x, p)$  is plotted on one quadrant of these spaces, which could be very important if one wants to use quantum mechanics for these system, and we have shown the damping effect on the motion of the body for the harmonic oscillator and pendulum systems due to dissipative force and the increasing of its mass.

We want to comment something for the case of mass lost, we have seen from our model (19) with  $\beta < 0$  that the motion is limited to a displacement given by  $x_{\max} = m_o/|\beta|$  (zero mass), where the potential function of the harmonic oscillator is zero, and this value would represent a singularity in the velocity behavior for positive generalized energies (for generalized energies higher than  $V_\alpha(x_{\max})$ ), but it would represent a zero motion in space  $(x, p)$  for the Hamiltonian.

#### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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## List of Terminology

$\alpha$  : Dissipation parameter;

$\beta$  : Variation of mass with respect to “ $x$ ” parameter;

$F(x)$  : Conservative force;

$K_{\alpha}^{\pm}(x, v)$  : Constant of motion ( $+(v > 0)$  and  $-(v < 0)$ ) or Effective energy;

$T_{\alpha}(x, v)$  : Effective Kinetic Energy;

$V_{\alpha}(x)$  : Effective Potential;

$L_{\alpha}^{\pm}(x, v)$  : Lagrangian ( $+(v > 0)$  and  $-(v < 0)$ );

$p_{\alpha}^{\pm}(x, v)$  : Generalized Linear Momentum ( $+(v > 0)$  and  $-(v < 0)$ );

$H_{\alpha}^{\pm}(x, p)$  : Hamiltonian ( $+(p > 0)$  and  $-(p < 0)$ );

$\gamma(a, b)$  : Uncompleted gamma function.

# Quantum Mechanics of Moving Bound States

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## Abstract

The quantum mechanics of bound states with discrete energy levels is well understood. The quantum mechanics of scattering processes is also well understood. However, the quantum mechanics of moving bound states is still debatable. When it is at rest, the space-like separation between the constituent particles is the primary variable. When the bound state moves, this space-like separation picks up the time-like separation. The time-separation is not a measurable variable in the present form of quantum mechanics. The only way to deal with this un-observable variable is to treat it statistically. This leads to rise of the statistical variables such entropy and temperature. Paul A. M. Dirac made efforts to construct bound-state wave functions in Einstein's Lorentz-covariant world. In 1927, he noted that the c-number time-energy relation should be incorporated in the relativistic world. In 1945, he constructed four-dimensional oscillator wave functions with one time coordinate in addition to the three-dimensional space. In 1949, Dirac introduced the light-cone coordinate system for Lorentz transformations. It is then possible to integrate these contributions made by Dirac to construct the Lorentz-covariant harmonic oscillator wave functions. This oscillator system can explain the proton as a bound state of the quarks when it is at rest, and explain the Feynman's parton picture when it moves with a speed close to that of light. While the un-measurable time-like separation becomes equal to the space-like separation at this speed, the statistical variables become prominent. The entropy and the temperature of this covariant harmonic oscillator are calculated. It is shown that they rise rapidly as the proton speed approaches that of light.

## Keywords

Bound States in Einstein's World, Bohr and Einstein on the Hydrogen Atom, Quark-Parton Puzzle, Lorentz Group

## 1. Introduction

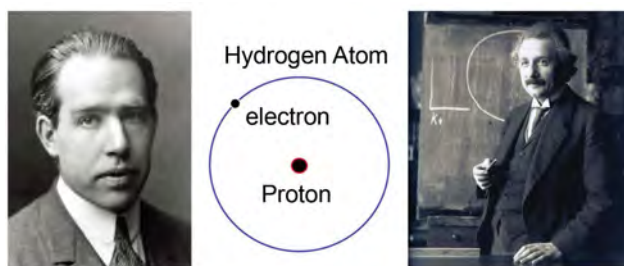
Let us start with **Figure 1**. During the early years of the 20-th Century, Niels

Bohr was worrying about the electron orbit of the hydrogen atom, while Albert Einstein was interested in how things appear to moving observers. Bohr's concern led to the present form of quantum mechanics where the hydrogen atom is a standing wave localized within a finite region. Einstein formulated his special theory of relativity based on the Lorentzian geometry of space and time applicable to Maxwell's theory of electromagnetism.

It is known that Bohr and Einstein met occasionally to discuss physics. However, there are no written records to indicate that they ever discussed how moving hydrogen atoms appear to a stationary observer. If they did not discuss this problem, it is understandable because there are no observable hydrogen atoms moving with relativistic speeds. Yet, this Bohr-Einstein issue defines an important problem in quantum mechanics. The bound state in quantum mechanics with discrete energy levels is well understood. However, how would those energy levels appear to moving observers? What will happen to the size of the bound state? Indeed, the Bohr-Einstein issue of the hydrogen atom leads to the problem of moving quantum bound states in Einstein's Lorentz-covariant world.

There are a number of key questions on the moving bound state. In the Lorentz-covariant world, the time variable is linearly mixed with the longitudinal coordinate. There is also the time-energy uncertainty relation. How is this relation mixed with Heisenberg's uncertainty for momentum and space? Paul A. M. Dirac raised these questions in 1927, and attempted to find a solution using harmonic oscillator wave functions in 1945. In addition, in 1949, he introduced the light-cone coordinate system for squeeze transformations in the two-dimensional space of the time and longitudinal coordinate.

The Bohr radius is a spatial separation between the proton and electron in the hydrogen atom. If this atom is boosted, this spatial separation picks up its time-like component. However, this time-like separation is not included in the present form of quantum mechanics. On the other hand, it is still possible to regard this time separation as an un-observable variable and treat it statistically, using the density matrix [1]-[11]. Then, there comes the question of entropy and temperature from this statistical treatment.



**Figure 1.** Niels Bohr and Albert Einstein with the hydrogen atom. One hundred years ago, Bohr was worrying about why the radius of the electron in the hydrogen atom orbit cannot be smaller than the finite value known today as the Bohr radius. Einstein was interested in how things appear to moving observers. Then, how would the hydrogen atom appear to moving observers? This question defines the subject of quantum mechanics of bound states in the Lorentz-covariant world.

While there are no observable hydrogen atoms, these days, high-energy accelerator produce protons moving with speeds close to that of light. Furthermore, thanks to Gell-Mann's quark model [12], the proton was found to be a quantum bound state just like the hydrogen atom. Its constituents are the quarks. Since the proton and hydrogen atom share the same bound-state quantum mechanics, it is possible to study moving hydrogen atoms by looking at moving protons.

In 1969, Feynman observed that the ultra-fast proton appears like a collection of an infinite-number of free particles with a wide-spread momentum distribution [13] [14] [15]. Feynman called them *partons*. The question then is whether Gell-Mann's quarks and Feynman's partons are two different ways of observing the same entity. Indeed, the problem of moving hydrogen atom becomes the quark-parton puzzle. The Bohr-Einstein issue of moving hydrogen atom can be addressed in terms of the quark-parton puzzle of Gell-Mann and Feynman, as illustrated in **Figure 2**.

In this paper, we review first efforts made in the past to resolve this quark-parton issue [16] [17], using the Lorentz-covariant oscillator wave function. We then use the same wave function to study the problem arising from the un-observable time-separation variable.

Paul A. M. Dirac made his lifelong efforts to construct a localized wave function in Einstein's Lorentz-covariant world. For this purpose, Dirac published



100 years ago, Bohr was worrying about the orbit of the hydrogen atom.

Einstein was interested in how things look to moving observers. Then how the hydrogen atom would look to moving observers? This was a metaphysical question for them.



50 years ago, the proton became a bound state of the quarks sharing the same quantum mechanics as that for the hydrogen atom, according to Gell-Mann. If it moves with a speed close to that of light, the proton appears as a collection of partons, according to Feynman.

**Question.** Does the proton appear like a collection of Feynman's partons to a moving observer?



Photo of Gell-Mann by Y.S.Kim (2010), all others photos are from the public domain.

**Figure 2.** Bohr and Einstein, and then Gell-Mann and Feynman. Did Bohr and Einstein discuss how the hydrogen appears to moving observers? We do not know. After 1950, with particle accelerators, the physics world started producing protons with relativistic speeds. Furthermore, the proton became a quantum bound state of the quarks like the hydrogen atom. The problem of fast-moving hydrogen atoms became the problem of protons moving with relativistic speeds. How would the proton appear when it moves with a speed close to that of light? This quark-parton puzzle addresses the Bohr-Einstein issue of moving hydrogen atom.



four important papers [18] [19] [20] [21]. By integrating the first three of these four papers, it is possible to construct the harmonic oscillator wave functions which can be Lorentz-transformed [22] [23] [24].

In order to carry out this integration, we need the mathematical instrument constructed by Eugene P. Wigner in his 1939 paper on the inhomogeneous Lorentz group [25]. In his paper, Wigner pointed out a particle in the Lorentz-covariant world has its four-momentum. In addition, this particle has internal space-time symmetries.

Thus in Section 2, we review the aspects of Wigner's paper applicable to the internal space-time symmetries in Einstein's Lorentz-covariant world.

In Section 3, we list the first three papers Dirac published from 1927 to 1945 [18] [19] [20], and integrate them. The result is a harmonic oscillator wave function which can be Lorentz-boosted. The time-separation variable plays a prominent role in this Lorentz-covariant wave function.

In Section 4, we examine Feynman's attempts to construct Lorentz-covariant oscillator wave function starting from a Lorentz-invariant wave equation [26]. Let us consider a hadron (bound state of the quarks) consisting of two quarks. This hadron has two space-time coordinate systems. One is for the hadron moving freely, and the other is for the motion of quarks inside the hadron. For the hadronic coordinate, the Klein-Gordon equation and its solutions are applicable. For the internal coordinate, we can use the harmonic oscillator wave functions constructed from the integration of Dirac's three papers discussed in Section 3.

In Section 5, this covariant harmonic oscillator is applied to the physics of hadrons. This wave function allows us to Lorentz-boost the hadron at rest to its speed very close to that of light. The hadron at rest is like a quantum bound state like the hydrogen atom according to Gell-Mann's quark model [12]. However, the same hadron appears like a collection of free massless particles called partons. This aspect is called Feynman's parton picture of the hadron [13] [14] [15]. The question then is whether the quarks and partons are two different ways of looking at the same entity. We resolve this issue using the Lorentz-covariant oscillator wave functions constructed in Sections 3 and 4. This Lorentz-covariant wave function depends on the time-separation variable which becomes more prominent as the hadron gains its speed.

In Section 6, it is noted that there is a time-separation variable between the quarks. This variable becomes more prominent when the hadron becomes faster. This time-separation is not a measurable dynamical variable in the present form of quantum mechanics. However, the density matrix tells us how to deal with this unobservable variable. It allows us to translate our inability to measure this variable into entropy and temperature. This problem was discussed in the literature [27] [28]. It is shown there that the hadron, when Lorentz boosted, experiences the rise in entropy and also the rise in temperature. It is possible to calculate them as functions of the hadronic speed using the density matrix.

## 2. Wigner’s Little Groups for Internal Space-Time Symmetries

In 1939, Eugene Paul Wigner published his paper entitled *On unitary representations of the inhomogeneous Lorentz group* [25]. In this paper, Wigner spells out the internal space-time symmetries of particles in the Lorentz-covariant world [22]. Let us consider a particle in this world. It has its four-momentum. If this particle is at rest, it has its rotational degree of freedom. If its spin is 1/2, the symmetry group is  $SU(2)$  like (locally isomorphic to)  $O(3)$  (three-dimensional rotation group). If its spin is one, its symmetry group is  $O(3)$ . This aspect is well known.

Massless particles cannot be brought to their rest frames. According to Wigner [25], the little group for the massless particle is like  $E(2)$  or the two-dimensional Euclidean group, with one rotational degree of freedom plus two translational degrees of freedom. The rotational degree of freedom can easily be identified with the helicity of the massless particle. However, the two translational degrees have a stormy history until 1987, when Kim and Wigner noted that the  $E(2)$  group is like the cylindrical group where both the translations perform up-down translations on the cylindrical surface. This allows us to identify this up-down translation as with the gauge transformation [29].

In the Lorentz-covariant world, Einstein’s momentum-energy relation is applicable to both massive and massless particles, as shown in **Table 1**. When the massive particle is Lorentz-boosted, its energy-momentum becomes that of the massless particle when its speed becomes very close to that of light.

We are thus led to the question of whether there exists one little group which can be the  $O(3)$ -like group when the particle is at rest and the  $E(2)$ -like cylindrical group when the particle moves with the speed very close to that of light.

Let us outline the procedure in which the  $O(3)$  symmetry becomes that of the cylindrical group. The three-dimensional rotation group is generated by

$$L_x = -i \left( y \frac{\partial}{\partial z} - z \frac{\partial}{\partial y} \right), \quad L_y = -i \left( z \frac{\partial}{\partial x} - x \frac{\partial}{\partial z} \right), \quad L_z = -i \left( x \frac{\partial}{\partial y} - y \frac{\partial}{\partial x} \right), \quad (1)$$

**Table 1.** Lorentz covariance of particles with internal space-time symmetries. The first row tells that Einstein’s energy relation is applicable to both massive and massless particles. Likewise, the second row is Wigner’s little groups. They are like  $O(3)$  for massive particles, and are like  $E(2)$  for massless particles, where  $E(2)$  means the two-decisional Euclidean group which is isomorphic to the cylindrical group.

	Massive, Slow	COVARIANCE	Massless, Fast
Energy-Momentum	$E = p^2/2m$	Einstein’s $E = \sqrt{(cp)^2 + (mc^2)^2}$	$E = cp$
Internal Space-time Symmetry	$S_3$ $S_1, S_2$	Wigner’s Little Groups	$S_3$ Gauge Transformation

satisfying the commutation relations

$$[L_x, L_y] = iL_z, \quad [L_y, L_z] = iL_x, \quad [L_z, L_x] = iL_y. \quad (2)$$

Let us go to **Figure 3**. The circle in this figure illustrates the  $O(3)$ -like little group for the massive particle at rest.

If it is boosted along the  $z$ -direction, the  $z$  coordinate picks up the time-like component, and the geometry is four-dimensional. While this geometry is described in detail in the 1987 paper of Kim and Wigner [29], we give here a simplified version.

When the particle is boosted along the  $z$  direction, the  $z$  component of this circle becomes expanded. If the speed becomes close to that of light, the sphere becomes a cylinder, as indicated in **Figure 3**.

On the surface of this cylinder, there are no variations of the  $x$  and  $y$  components, and thus

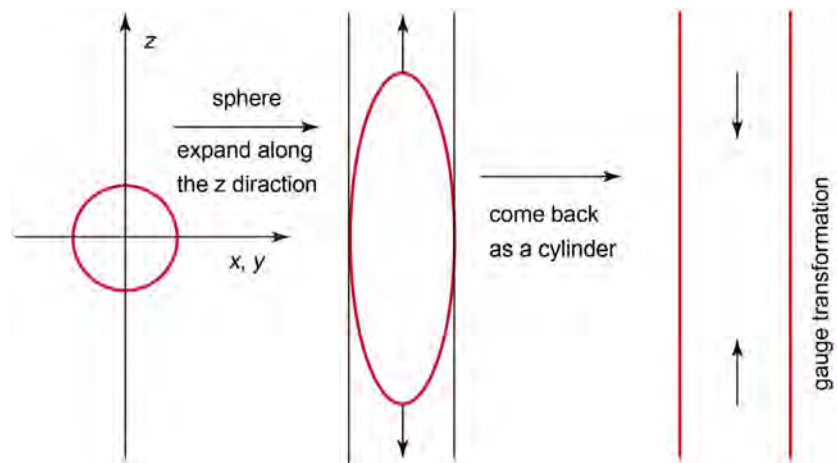
$$L_x \rightarrow P_y = -i\left(y \frac{\partial}{\partial z}\right), \quad L_y \rightarrow -P_x = i\left(x \frac{\partial}{\partial z}\right), \quad (3)$$

and  $L_z$  remains unchanged. Both  $P_x$  and  $P_y$  generate translations along the  $z$  direction. These new operators satisfy the commutation relations

$$[P_x, P_y] = 0, \quad [L_z, P_x] = iP_y, \quad [L_z, L_y] = -iP_x. \quad (4)$$

It is appropriate to call the group generated by these three operators the *cylindrical group*.

This set of commutation relation identical with that for  $E(2)$  or the two-dimensional Euclidean group with  $L_z$  as the generator of rotations and  $P_x$  and  $P_y$  as the generators of translations along the  $x$  and  $y$  directions respectively. The translation generators take the form



**Figure 3.** Lorentz covariance of the internal space-time symmetry. The symmetry is like  $O(3)$  or a sphere when the particle is at rest. This sphere becomes elongated when the particle gains speed along the  $z$  direction. It becomes a cylinder when the speed approaches the speed of light. The rotational degree of freedom of this cylinder corresponds to the helicity of the massless particle, and up-down translation leads to the gauge degree of freedom.

$$P_x = -i \frac{\partial}{\partial x}, \quad P_y = -i \frac{\partial}{\partial y}. \quad (5)$$

Thus, the cylindrical group is like (locally isomorphic to) the  $E(2)$  group. In Wigner's original paper, the little group for massless particles is like the  $E(2)$  group. The  $E(2)$  group can now be replaced with the cylindrical group [29] [30].

Let us go back to **Figure 3**. We would expect the sphere, when Lorentz-boosted, becomes contracted like a pancake according to Einstein's space contraction. However, this figure shows the opposite effect. The Lorentzian geometry is four-dimensional and the time-like direction should also be included. In this geometry, the Lorentz boost leads to both pancake-like contraction and football-like elongation. The contraction produces the  $E(2)$  geometry and the elongation produces the cylindrical geometry as shown in **Figure 3**. The cylindrical geometry leads to the correct interpretation of the internal space-time symmetry of massless particles.

### 3. Dirac's Efforts to Construct Relativistic Quantum Mechanics

Paul A. M. Dirac made his lifelong effort to formulate quantum mechanics consistent with Einstein's special relativity. The Dirac equation of electrons and positrons is a case in point. This equation is well known.

In addition, he made efforts to formulate a mathematical device to deal localized quantum distributions, such as the hydrogen atom, in Einstein's Lorentz-covariant world. For this purpose, he published the following four papers.

1) In 1927, Dirac pointed out that the time-energy uncertainty should be considered if the system is to be Lorentz-covariant [18].

2) In 1945, Dirac said the Gaussian form could serve as a representation of the Lorentz group [19].

3) In 1949, when Dirac introduced both his instant form of quantum mechanics and his light-cone coordinate system [20], he clearly stated that finding a representation of the inhomogeneous Lorentz group was the task of Lorentz-covariant quantum mechanics.

4) In 1963, Dirac used the symmetry of two coupled oscillators to construct the  $O(3,2)$  de Sitter group, namely the Lorentz group applicable to the three-dimensional  $(x, y, z)$  space plus two time variables [21]. This paper serves as a prelude to the synthesis of quantum mechanics and special relativity [24] [31] [32].

Dirac's papers are poetic, mathematically transparent, and easy to understand. This does not necessarily mean that there is nothing to add to his papers. His papers do not have figures. Thus it is profitable to translate his poems into figures and illustrations. Each of the above four papers is independent. Thus it is profitable to connect his 1945 paper to his early paper of 1927, and his 1949 paper to his earlier papers of 1927 and 1945. Furthermore, Eugene Wigner was his brother-in-law. Wigner published an important paper in 1939 providing the

mathematical tool for Dirac's problems, but he never used Wigner's mathematics in any meaningful ways.

Let us consider what we can add to his papers in order to construct quantum mechanics valid in Einstein's Lorentz-covariant world. We are particularly interested in how to Lorentz-boost localized wave functions.

### 3.1. Dirac's C-Number Time-Energy Uncertainty Relation

In 1972 [33], Eugene Paul Wigner drew attention to the fact that time-energy uncertainty relation, known from the transition time and line broadening in atomic spectroscopy, existed before 1927 when Heisenberg formulated his uncertainty principle.

In 1927 [18], Dirac studied the uncertainty relation which was applicable to the time and energy variables. When the uncertainty relation was formulated by Heisenberg for the position and momentum variables, Dirac considered the possibility of whether a Lorentz-covariant uncertainty relation could be formulated with these two uncertainty relations [18].

Dirac then noted that the time variable is a c-number and thus there are no excitations along the time-like direction. However, there are excitations along the space-like longitudinal direction starting from the position-momentum uncertainty. Since the space and time coordinates are mixed up for moving observers, Dirac wondered how this space-time asymmetry could be made consistent with Lorentz covariance. This was indeed a major difficulty for him.

However this difficulty does not exist. Wigner's little group for massive particles at rest is the three-dimensional rotation group, without time-dependence. The concept of the little group did not exist in 1927.

### 3.2. Dirac's Four-Dimensional Oscillators

Since the language of special relativity is the Lorentz group, and harmonic oscillators provide a starting point for the present form of quantum mechanics, Dirac considered the possibility of using harmonic oscillator wave functions to construct representations of the Lorentz group [19].

Thus in his 1945 paper [19], Dirac considers the Gaussian form

$$\exp\left(-\frac{1}{2}[x^2 + y^2 + z^2 + t^2]\right). \quad (6)$$

The  $x$  and  $y$  variables can be dropped from this expression, as we are considering a Lorentz boost only along the  $z$  direction. We can thus write the above equation as:

$$\exp\left(-\frac{1}{2}[z^2 + t^2]\right). \quad (7)$$

Since  $(z^2 - t^2)$  is a Lorentz-invariant quantity, this expression may seem strange for those who believe in Lorentz invariance [26], but it is normalizable in the  $t$  variable, and accommodates the time-energy uncertainty relation. This non-invariant form will change when it is boosted along the  $z$  direction.

However, there are no excitations along the time-like direction. This space-time asymmetry was noted in Dirac's own paper of 1927 [18]. This asymmetry problem was resolved by Wigner's  $O(3)$ -like little group for massive particles discussed in Section 2. Without time-like excitations, the oscillator wave function should take the form

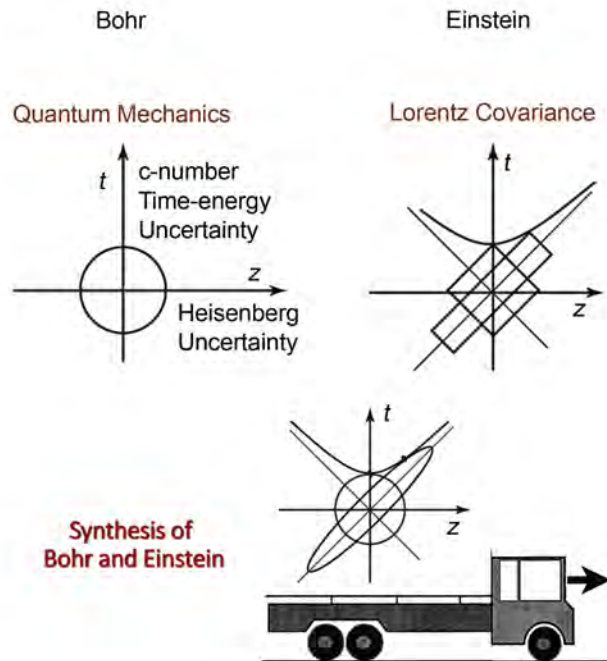
$$\psi^n(z, t) = \left(\frac{1}{\pi n! 2^n}\right)^{1/2} H_n(z) \exp\left(-\left[\frac{z^2 + t^2}{2}\right]\right), \quad (8)$$

for the  $n$ -th excited state, where  $H_n(z)$  is the Hermite polynomial for excitations along the  $z$  direction. This expression does not contain the Hermite polynomial in the  $t$  variable.

Since the localization of this wave function is dominated by the Gaussian form, let us concentrate our efforts on the ground state. For this ground state, this wave function become

$$\psi(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left(-\left[\frac{z^2 + t^2}{2}\right]\right). \quad (9)$$

This corresponds to the circular distribution in Figure 4. Since the form is not Lorentz-invariant, the circle in Figure 4 will appear differently to moving observers. This question was addressed in Dirac's 1949 paper [20].



**Figure 4.** Integration of Dirac's three papers [18] [19] [20]. In 1927, Dirac noted there exists the time-energy uncertainty, in addition to Heisenberg's position-momentum relation [18]. He attempted to combine them with a Gaussian form in 1945 [19]. In 1949, Dirac noted that the Lorentz boost squeezes space-time along the light cones. This allows us to synthesize the circle and rectangle to an ellipse for the moving oscillator. This figure provides the resolution to the Bohr-Einstein issue of the moving hydrogen atom. The remaining question is whether we can observe this effect in laboratories.

### 3.3. Forms of Relativistic Dynamics

In 1949, the Reviews of Modern Physics celebrated Einstein's 70th birthday by publishing a special issue. This issue included Dirac's paper entitled *Forms of Relativistic Dynamics* [20]. There, Dirac introduced his light-cone coordinate system. In this system a Lorentz boost is seen to be a squeeze transformation, where one light-cone axis expands while the other contracts in such a way that their product remains invariant as shown in **Figure 4**.

Also in this 1949 paper [20], Dirac introduced his *instant form* of relativistic quantum mechanics. This has the condition

$$x_0 \approx 0. \quad (10)$$

What did his approximate equality mean? We can interpret this as his  $c$ -number nature of the time-energy uncertainty relation which he discussed in his 1927 paper [18]. In the language of harmonic oscillators [19], there are no excited states along the time axis, as is shown in Equation (8).

In the same 1949 paper, Dirac introduced the light-cone coordinate system. Starting from the formula's for the Lorentz boost along the  $z$  direction:

$$\begin{pmatrix} z' \\ t' \end{pmatrix} = \begin{pmatrix} \cosh(\eta) & \sinh(\eta) \\ \sinh(\eta) & \cosh(\eta) \end{pmatrix} \begin{pmatrix} z \\ t \end{pmatrix}. \quad (11)$$

Dirac defined his light-cone variables as [20]

$$u = \frac{z+t}{\sqrt{2}}, \quad v = \frac{z-t}{\sqrt{2}}. \quad (12)$$

Then the Lorentz boost of Equation (11) becomes diagonal:

$$\begin{pmatrix} u' \\ v' \end{pmatrix} = \begin{pmatrix} e^\eta & 0 \\ 0 & e^{-\eta} \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix}. \quad (13)$$

It is then apparent that  $u$  variable becomes expanded, but the  $v$  variable becomes contracted. This aspect was illustrated also in **Figure 4**. The product then becomes

$$uv = u'v' = \frac{1}{2}(z+t)(z-t) = \frac{1}{2}(z^2 - t^2) \quad (14)$$

which remains invariant. The Lorentz boost is therefore a squeeze transformation, and the Gaussian form of Equation (9) is transformed to.

$$\psi_\eta(z, t) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left(-\frac{1}{4}\left[e^{-2\eta}(z+t)^2 + e^{2\eta}(z-t)^2\right]\right). \quad (15)$$

This is of course the elliptic distribution as noted in **Figure 4**.

In addition, in his 1949 paper [20], Dirac stated that the task of constructing relativistic quantum mechanics is that of constructing a *representation of the inhomogeneous Lorentz group*, which is also known as the Poincaré group [22]. This group has ten generators for three rotations, three Lorentz boosts, and four space-time translations.

It is well known that the present form of quantum field theory based on the

scattering matrix is a representation of the Poincaré group. The question is whether it is possible to formulate the bound state problem as a representation of the same group.

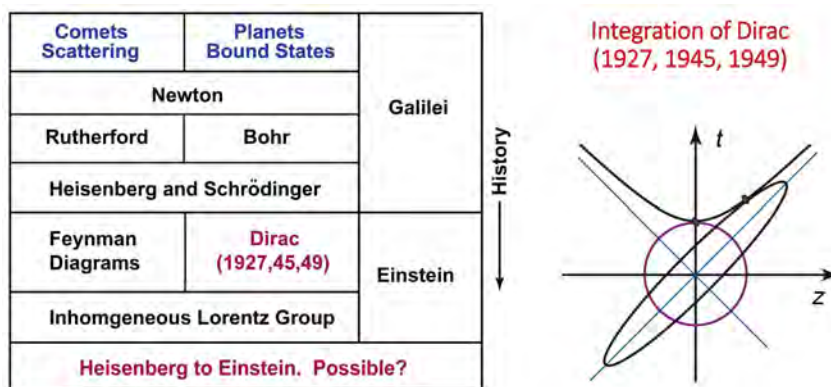
It is now clear that Dirac was interested in using harmonic oscillators to construct a representation of the inhomogeneous Lorentz group [22]. **Figure 5** indicates that the integration of Dirac's first three papers [18] [19] [20] lead to a representation of Wigner's  $O(3)$ -like little group for the massive particle. Since it is well known that quantum field theory is a representation of the inhomogeneous Lorentz group, both the field theory and the oscillator formalism are two different representations of the same inhomogeneous Lorentz group satisfying Dirac's requirement [20].

### 3.4. Dirac's Two Oscillators

In 1963, Dirac started with two harmonic oscillators, and he ended up with ten generators [21]. These generators satisfy the closed set of commutators for the  $O(3,2)$  group, namely the Lorentz group applicable to three space-like coordinates and two time-like coordinates. This group has ten generators, like the Poincaré group. Like the Poincaré group, it has the subgroup  $O(3,1)$  Lorentz group when we consider only one of the two time coordinates. There are four generators involving the second time coordinates, namely three boost generators with respect to three space coordinates and one rotation generator with respect to the first time variable.

The harmonic oscillator is the language of quantum mechanics, while the group  $O(3,2)$  is the language of Lorentz transformations. Thus, we are led to the question of deriving special relativity from quantum mechanics.

According to Dirac [20], the task of constructing relativistic quantum mechanics is that of constructing a representation of the inhomogeneous Lorentz



**Figure 5.** History of physics as a series of synthesis. Newton synthesized scattering and bound states with his differential equation. Schrödinger and Heisenberg synthesized the Rutherford scattering and the Bohr atom. There then comes Einstein's world. Quantum field theory is for scattering problem while the covariant harmonic oscillator is for bound states. They both are representations of the inhomogeneous Lorentz group. Next, are quantum mechanics and special relativity are derivable from the same basket of equations? This question is addressed in the literature [24] [31] [32].



group with ten generators, as specified in **Figure 5**. Dirac's  $O(3,2)$  group also has ten generators. It is thus a challenge to see whether this  $O(3,2)$  group can be converted into the inhomogeneous Lorentz group. This question was discussed in detail in the literature [24] [31] [32].

#### 4. Scattering and Bound States

In Section 3, we studied the quantum bound state in the Lorentz-covariant world using harmonic oscillator wave functions. This wave function is a representation of Wigner's little group, which is a subgroup of Dirac's inhomogeneous Lorentz group.

For free particles in the covariant world, we use the Klein-Gordon equation. If the particle has a space-time extension, it is possible to use harmonic oscillators as we did in Section 3. The question then is whether it is possible to write an equation for both. Indeed, this problem was recognized by Feynman, Kislinger, and Revndal in 1971 [26].

Let us start with two quarks. We are quite familiar with the Klein-Gordon equation for a free particle in the Lorentz-covariant world. We shall use the four-vector notations

$$x_\mu = (x, y, z, t), \quad \text{and} \quad x_\mu^2 = x^2 + y^2 + z^2 - t^2. \quad (16)$$

Then the Klein-Gordon equation becomes

$$\left( -\left[ \frac{\partial}{\partial x_\mu} \right]^2 + m^2 \right) \phi(x) = 0. \quad (17)$$

The solution of this equation takes the familiar form

$$\exp[\pm i(p_1 x + p_2 y + p_3 z \pm Et)], \quad (18)$$

with  $E = \sqrt{p_1^2 + p_2^2 + p_3^2 + m^2}$ .

In 1971, Feynman *et al.* considered two particles  $a$  and  $b$  bound together by a harmonic oscillator potential, and wrote down the equation [26]

$$\left\{ -\left[ \frac{\partial}{\partial x_{a\mu}} \right]^2 - \left[ \frac{\partial}{\partial x_{b\mu}} \right]^2 + (x_{a\mu} - x_{b\mu})^2 + m_a^2 + m_b^2 \right\} \phi(x_{a\mu}, x_{b\mu}) = 0. \quad (19)$$

The bound state of these two particles is one *hadron*. The constituent particles are called *quarks*. We can then define the four-coordinate vector of the hadron as

$$X = \frac{1}{2}(x_a + x_b), \quad (20)$$

and the space-time separation four-vector between the quarks as

$$x = \frac{1}{2\sqrt{2}}(x_a - x_b). \quad (21)$$

Then Equation (19) becomes

$$\left\{ -\left[ \frac{\partial}{\partial X_\mu} \right]^2 + m_0^2 + \left( -\left[ \frac{\partial}{\partial x_\mu} \right]^2 + x_\mu^2 \right) \right\} \phi(X, x) = 0. \tag{22}$$

This differential equation can then be separated into

$$\left( -\left[ \frac{\partial}{\partial X_\mu} \right]^2 + m_0^2 \right) \phi(X, x) = -\left( -\left[ \frac{\partial}{\partial x_\mu} \right]^2 + x_\mu^2 \right) \phi(X, x), \tag{23}$$

with

$$\phi(X, x) = f(X)\psi(x), \tag{24}$$

where  $f(X)$  and  $\psi(x)$  satisfy their own equations:

$$\left( -\left[ \frac{\partial}{\partial X_\mu} \right]^2 + m_a^2 + m_b^2 + \lambda \right) f(X) = 0 \tag{25}$$

and

$$\frac{1}{2} \left( -\left[ \frac{\partial}{\partial x_\mu} \right]^2 + x_\mu^2 \right) \psi(x) = \lambda \psi(x). \tag{26}$$

Here, the wave function takes the form

$$\phi(X, x) = \psi(x) \exp\left[ \pm i(P_x X + P_y Y + P_z Z \pm ET) \right], \tag{27}$$

where  $P_x, P_y, P_z$  are for the hadronic momentum, and

$$E^2 = P_x^2 + P_y^2 + P_z^2 + M^2, \quad \text{with } M^2 = m_a^2 + m_b^2 + \lambda. \tag{28}$$

Here the hadronic mass  $M$  is determined by the parameter  $\lambda$ , which is the eigenvalue of the differential equation for  $\psi(x)$  given in Equation (26).

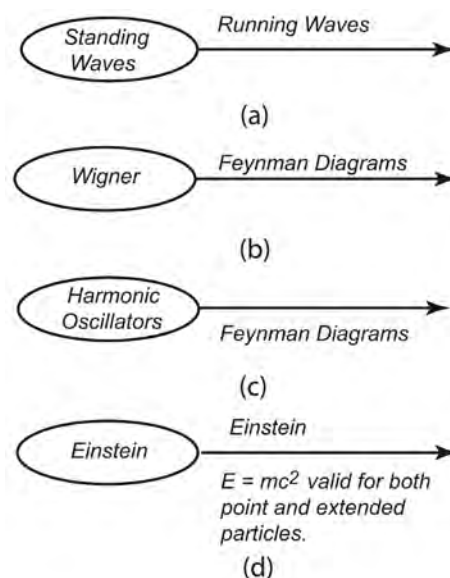
Considering Feynman diagrams based on the S-matrix formalism, quantum field theory has been quite successful. It is, however, only useful for physical processes where, after interaction, one set of free particles becomes another set of free particles. The questions of localized probability distributions and their Lorentz covariance is not addressed by the present form of quantum field theory. In order to tackle this problem and address these questions, Feynman *et al.* suggested harmonic oscillators [26]. In **Figure 6**, we illustrate this idea.

However, for their wave function  $\psi(x)$ , Feynman *et al.* uses a Lorentz-invariant exponential form

$$\exp\left( -\frac{1}{2} [x^2 + y^2 + z^2 - t^2] \right). \tag{29}$$

This wave function increases as  $t$  becomes large. This is not an acceptable wave function. They overlooked the normalizable exponential form given by Dirac in Equation (6). They even overlooked the same normalizable in the paper of Fujimura *et al.* [34] which was quoted in their own paper.

Thus, we are fully justified in replacing the meaningless Gaussian form of Equation (29) with the Gaussian form developed in Section 3.



**Figure 6.** Scattering and bound states as a single representation of the inhomogeneous Lorentz group. In an effort to combine quantum mechanics with special relativity, Feynman gave us this road-map. In (a), we start with a running wave and standing wave. In (b), running waves are for Feynman diagrams, and standing waves are representations of Wigner's little group. In (c), we can use harmonic oscillator wave functions for standing waves. If the oscillator wave functions are Lorentz-covariant, Einstein's Lorentz covariance is valid for the entire system, as specified in (d).

## 5. Lorentz-Covariant Quark Model

Early successes in the quark model include the calculation of the ratio of the neutron and proton magnetic moments [35], and the hadronic mass spectra [26] [36]. These are based on hadrons at rest. We are interested in this paper how the hadrons in the quark model appear to observers in different Lorentz frames.

These days, modern particle accelerators routinely produce protons moving with speeds very close to that of light. Therefore, the question is whether the covariant wave function developed in Section 4 can explain the observed phenomena associated with those protons moving with relativistic speed.

The idea that the proton or neutron has a space-time extension had been developed long before Gell-Mann's proposal for the quark model [12]. Yukawa [37] developed this idea as early as 1953, and his idea was followed up by Markov [38].

Hofstadter [39] [40], by using electron-proton scattering to measure the charge distribution inside the proton, made the first experimental discovery of the non-zero size of the proton. If the proton were a point particle, the scattering amplitude would just be a Rutherford formula. However, Hofstadter found a tangible departure from this formula which can only be explained by a spread-out charge distribution inside the proton.

Indeed, the first success of the Lorentz-covariant oscillator was demonstrated in the calculation of the Hofstadter effect. Using this wave function, Markov made his calculation in 1956 even before Gell-Mann formulated his quark model

in 1964 [38]. After the quark model, many authors made their calculations of the Hofstadter effect using the same wave function [34] [41] [42] [43] [44] [45]. These papers amply demonstrate the elliptic deformation of the Gaussian distribution shown in **Figure 4** and **Figure 5**.

Next, we are facing a more fundamental question. Let us go back to **Figure 2**. The quark model and its Lorentz-covariant wave function allow us to address the issue of the quark-parton puzzle, and thus the Bohr-Einstein issue of how the hydrogen atom appears to moving observers.

### 5.1. Feynman's Parton Picture

As we did in Sections 3 and 4, we continue using the Gaussian form for the wave function of the proton. If the proton is at rest, the  $z$  and  $t$  variables are separable, and the time-separation can be ignored, as we do in non-relativistic quantum mechanics. If the proton moves with a relativistic speed, the wave function is squeezed as described in **Figure 4** and **Figure 5**. If the speed reaches that of light, the wave function becomes concentrated along positive light cone with  $t = z$ . The question then is whether this property can explain the parton picture of Feynman when a proton moves with a speed close to that of light.

It was Feynman who, in 1969, observed that a fast-moving proton can be regarded as a collection of many *partons*. The properties of these partons appear to be quite different from those of the quarks [13] [14] [15]. For example, while the number of quarks inside a static proton is three, the number of partons appears to be infinite in a rapidly moving proton. The following systematic observations were made by Feynman:

- 1) When protons move with velocity close to that of light, the parton picture is valid.
- 2) Partons behave as free independent particles while the interaction time between the quarks becomes dilated.
- 3) Partons have a widespread distribution of momentum as the proton moves quickly.
- 4) There seems to be an infinite number of partons or a number much larger than that of quarks.

The question is whether the Lorentz-squeezed wave function described in **Figure 4** can explain all of these peculiarities.

Each of the above phenomena appears as a paradox, when the proton is believed to be a bound state of the quarks. This is especially true of (b) and (c) together. How can a free particle have a wide-spread momentum distribution.

To resolve this paradox, we construct the momentum-energy wave function corresponding to Equation (15). We can construct two independent four-momentum variables [26] if the quarks have the four-momenta  $p_a$  and  $p_b$ .

$$P = p_a + p_b, \quad q = \sqrt{2}(p_a - p_b). \quad (30)$$

Since  $P$  is the total four-momentum, it is the four-momentum of the proton. The four-momentum separation between the quarks is measured by  $q$ . We can

then write the light-cone variables as

$$q_+ = \frac{q_0 + q_z}{\sqrt{2}}, \quad q_- = \frac{q_0 - q_z}{\sqrt{2}}. \tag{31}$$

This results in the ground-state momentum-energy wave function

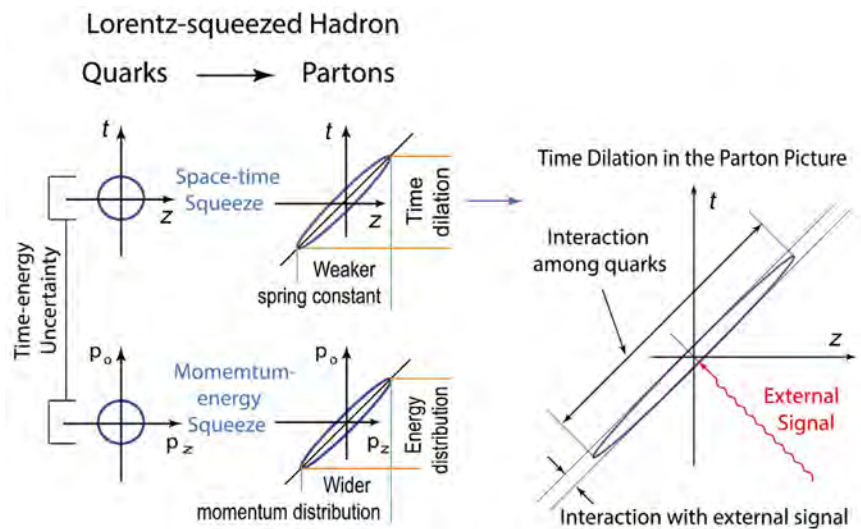
$$\phi_\eta(q_z, q_0) = \left(\frac{1}{\pi}\right)^{1/2} \exp\left\{-\frac{1}{2}\left[e^{-2\eta}q_+^2 + e^{2\eta}q_-^2\right]\right\}. \tag{32}$$

Since the harmonic oscillator is being used here, the momentum-energy wave function has the mathematical form identical to that of the space-time wave function of Equation (15). These wave functions have the same Lorentz squeeze properties [16] [17]. These Lorentz-squeeze properties are illustrated in **Figure 7**.

From this figure, we can see that both wave functions behave like those for the static bound state of quarks when the proton is at rest with  $\eta = 0$ . However, as  $\eta$  increases, the wave functions become concentrated along their respective positive light-cone axes. This means that the quarks become like massless particles with wide space and momentum distributions. This is the property of Feynman’s parton picture [13] [14].

Another puzzle is that quarks are coherent when the proton is at rest but the partons appear as incoherent particles. Does this mean that the Lorentz boost destroys coherence? Obviously, the answer to this question is NO.

When the proton is boosted, its matter becomes squeezed, as shown in **Figure 7**. The result is that the wave function for the proton becomes concentrated in the elliptic region along the positive light-cone axis, which is expanded in length by  $\exp(\eta)$ . As a consequence, the minor axis is contracted by  $\exp(-\eta)$ .



**Figure 7.** Lorentz-squeezed wave functions in space-time and in momentum-energy variables. Both wave functions become concentrated along their respective positive light-cone axes as the speed of the proton approaches that of light. All the peculiarities of Feynman’s parton picture are presented in these light-cone concentrations.

Thus, the interaction time for the quarks among themselves becomes dilated. Thus, the quarks appear to be free particles to external signals. As the ellipse becomes more squeezed, the quarks become light-like massless particles, as illustrated in **Figure 8**.

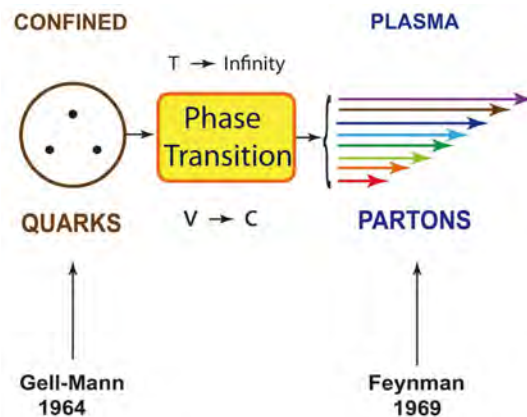
As indicated also in **Figure 7**, the probing signal is moving in the direction opposite to the direction of the proton, it travels along the negative light-cone axis with  $t = -z$ . As the proton contracts along this negative light-cone axis, the interaction time decreases by  $\exp(-\eta)$ . Then the ratio of the interaction time to the oscillator period becomes  $\exp(-2\eta)$ . Each proton, produced by the Fermilab accelerator used to have an energy of 900 GeV. This then means that the ratio is  $10^{-6}$ . Because this is such small number, the external signal cannot sense the interaction of the quarks among themselves. The quarks, appearing like partons, are free independent particles as also observed by Feynman [13] [14].

The momentum distribution becomes wide spread, also as is indicated in **Figure 7**. As it becomes concentrated along the positive light-cone axis, the quarks become light-like massless particles. As in the case of the Black-body radiation, the number of particles is infinite with a continuous momentum (thus energy) distribution, also as noted by Feynman [13] [14].

This resolution of the quark-parton puzzle is tabulated in **Table 2** along with Einstein's energy-momentum relation, and Wigner's little group for internal space-time symmetries. Indeed, the quarks and partons are two different way of looking at the same entity in Einstein's Lorentz-covariant world.

## 5.2. Lorentz-Invariant Uncertainty Products

In the harmonic oscillator regime, the energy-momentum wave functions take the same mathematical form, and the uncertainty relation in terms of the uncertainty products is well understood. However, in the present case, the oscillator wave functions are deformed when Lorentz-boosted, as shown in **Figure 7**.



**Figure 8.** The effect of the Lorentz squeezes appearing in the real world. The Lorentz-squeezed wave functions shown in **Figure 7** appear in the world as Feynman's parton picture in this figure. There are infinite-number of massless partons, with wide-spread momentum distribution, as in the case of photons in the black-body radiation.

**Table 2.** Lorentz covariance of hadrons. The little group of Wigner unifies the internal space-time symmetries for massive and massless particles. This issue was addressed earlier in **Table 1**. Wigner’s little groups allow us to give a unified picture for particles with internal space-time structures such as Gell-Mann’s quark model and Feynman’s parton picture.

	Massive, Slow	COVARIANCE	Massless, Fast
Energy-Momentum	$E = p^2/2m$	Einstein’s $E = \sqrt{(cp)^2 + (mc^2)^2}$	$E = cp$
Internal Space-time Symmetry	$S_3$ $S_1, S_2$	Wigner’s Little Groups	$S_3$ Gauge Transformation
Relativistic Extended Particles	Quark Model	Integration of Dirac’s papers 1927, 1945, 1949	Parton Model

According to this figure, both the space-time and momentum-energy wave functions become spread along their longitudinal directions. Does this mean that the Lorentz boost increases the uncertainty?

In order to address this question, let us write the momentum-energy wave function as a Fourier transformation of the space-time wave function:

$$\phi(q_z, q_0) = \frac{1}{2\pi} \int \psi(z, t) \exp\{i(q_z z - q_0 t)\} dt dz. \tag{33}$$

The transverse  $x$  and  $y$  components are not included in this expression. The exponent of this expression can be written as

$$q_z z - q_0 t = q_+ v + q_- u, \tag{34}$$

with

$$q_{\pm} = \frac{1}{\sqrt{2}}(q_z \pm q_0). \tag{35}$$

In terms of these variables, the Fourier integral takes the form

$$\frac{1}{2\pi} \int \psi(z, t) \exp\{i(q_+ v + q_- u)\} dt dz. \tag{36}$$

In this case, the variable  $q_+$  is conjugate to  $v$ , and  $q_-$  is to  $u$ . Let us go back to **Figure 7**. The major (minor) axis of the space-time ellipse is conjugate to the minor (major) axis of the momentum-energy ellipse. Thus the uncertainty products

$$\langle u^2 \rangle \langle q_-^2 \rangle \quad \text{and} \quad \langle v^2 \rangle \langle q_+^2 \rangle \tag{37}$$

remain invariant under the Lorentz boost.

### 6. Entropy and Temperature of Moving Hadrons

The entropy is a measure of our ignorance and is computed from the density matrix [1]-[11]. The density matrix is needed when the experimental procedure

does not analyze all relevant variables to the maximum extent consistent with quantum mechanics. The purpose of the present section to discuss a concrete example of the entropy arising from our ignorance in relativistic quantum mechanics formulated in Sections 3 and 4.

Let us consider a bound state of two particles, or the hadron consisting of two quarks bound together by a harmonic oscillator potential. Then there is a Bohr-like radius measuring the space-like separation between the quarks. There is also the time-like separation in the Lorentz-covariant regime. If the hadron is at rest, the time dependence is purely Gaussian with no excitations. Thus, this un-observable variable can be integrated out without affecting the space-like separation.

If the hadron moves along the  $z$  direction, this time-separation variable becomes more prominent, but there are no ways to measure this variable in the present form of quantum mechanics. We thus have to regard this variable as un-measurable variable, and treat it statistically.

As in the case of Section 4, let us consider a hadron consisting of two quarks. If the space-time position of two quarks are specified by  $x_a$  and  $x_b$  respectively, the system can be described by the variables

$$X = \frac{x_a + x_b}{2}, \quad \text{and} \quad x = \frac{x_a - x_b}{2\sqrt{2}}. \quad (38)$$

The four-vector  $X$  specifies where the hadron is located in space and time, while the variable  $x$  measures the space-time separation between the quarks. In the convention of Feynman *et al* [26], the internal motion of the quarks bound by a harmonic oscillator potential can be described by the Lorentz-invariant equation

$$\frac{1}{2} \left\{ \left[ -\frac{\partial^2}{\partial z^2} + z^2 \right] - \left[ -\frac{\partial^2}{\partial t^2} + t^2 \right] \right\} \psi(z, t) = \lambda \psi(z, t). \quad (39)$$

For simplicity, we do not consider the transverse coordinates  $x$  and  $y$ .

It is possible to construct a representation of Dirac's inhomogeneous Lorentz group [22] from the solutions of the differential equation of Equation (39). If the hadron is at rest, the solution should take the form of Equation (8). Let us rewrite this solution as the wave function

$$\psi_0^n(z, t) = \left( \frac{1}{\sqrt{\pi n! 2^n}} \right)^{1/2} H_n(z) \exp \left\{ - \left[ \frac{z^2 + t^2}{2} \right] \right\}. \quad (40)$$

The subscript 0 means that the wave function is for the hadron at rest. The above expression is not Lorentz-invariant, and its localization undergoes a Lorentz squeeze as the hadron moves along the  $z$  direction as shown in **Figure 4**.

For this Lorentz-covariant system, it is convenient to use the light-cone variables

$$u = \frac{z+t}{\sqrt{2}}, \quad v = \frac{z-t}{\sqrt{2}}, \quad (41)$$



introduced in Equation (12). The Lorentz-boost along the  $z$  axis leads to

$$u \rightarrow u' = e^\eta u, \quad v \rightarrow v' = e^{-\eta} v, \tag{42}$$

where  $\eta$  is the boost parameter and is  $\tanh \eta = v/c$ . In terms of these light-cone variables, the wave function of Equation (40) can be written as

$$\psi_0^n(x, t) = \left( \frac{1}{\pi n! 2^2} \right)^{1/2} H_n \left( \frac{u+v}{\sqrt{2}} \right) \exp \left( - \left[ \frac{u^2 + v^2}{2} \right] \right). \tag{43}$$

If the system is boosted, the wave function becomes

$$\psi_\eta^n(x, t) = \left( \frac{1}{\pi n! 2^2} \right)^{1/2} H_n \left( \frac{e^{-\eta} u + e^\eta v}{\sqrt{2}} \right) \exp \left( - \left[ \frac{e^{-2\eta} u^2 + e^{2\eta} v^2}{2} \right] \right). \tag{44}$$

This wave function can be expanded as [24]

$$\psi_\eta^n(z, t) = \left( \frac{1}{\cosh \eta} \right)^{(n+1)} \sum_k \left[ \frac{(n+k)!}{n!k!} \right]^{1/2} (\tanh \eta)^k \chi_{n+k}(z) \chi_k(t), \tag{45}$$

where  $\chi_n(z)$  is the  $n$ -th excited-state oscillator wave function.

Here comes the fundamental problem. If the hadron is at rest, this wave function is separable in  $z$  and  $t$ . If the  $t$  variable is integrated out, the rest is the present form of non-relativistic quantum mechanics.

However, if the hadron moves and gains speed, the  $t$  dependence becomes non-separable, and we have to resort to density matrix. From the wave function of Equation (6), we can construct the pure-state density matrix

$$\rho_\eta^n(z, t; z', t') = \psi_\eta^n(z, t) [\psi_\eta^n(z', t')]^*, \tag{46}$$

where  $[\psi_\eta^n(z', t')]^* = \psi_\eta^n(z', t')$ . This pure-state density matrix satisfies the condition  $\rho^2 = \rho$ :

$$\rho_\eta^n(z, t; x', t') = \int \rho_\eta^n(z, t; x'', t'') \rho_\eta^n(x'', t''; z', t') dz'' dt''. \tag{47}$$

However, there are at present no measurement theories which accommodate the time-separation variable  $t$ . Thus, we can take the trace of the  $\rho$  matrix with respect to the  $t$  variable. Then the resulting density matrix is

$$\rho_\eta^n(z, z') = \int \psi_\eta^n(z, t) \psi_\eta^n(z', t) dt \tag{48}$$

$$= \left( \frac{1}{\cosh \eta} \right)^{2(n+1)} \sum_k \frac{(n+k)!}{n!k!} (\tanh \eta)^{2k} \chi_{n+k}(z) \chi_{n+k}(z'). \tag{49}$$

The trace of this density matrix is one, but the trace of  $\rho^2$  is less than one, as

$$Tr(\rho^2) = \int \rho(z, z') \rho(z', z) dz' dz \tag{50}$$

$$= \left( \frac{1}{\cosh \eta} \right)^{4(n+1)} \sum_k \left[ \frac{(n+k)!}{n!k!} \right]^{2k} (\tanh \eta)^{4k}. \tag{51}$$

which is less than one. This is due to the fact that we do not know how to deal with the time-like separation in the present formulation of quantum mechanics. Our knowledge is less than complete.

The standard way to measure this ignorance is to calculate the entropy defined as [1] [2] [3] [27]

$$S = -Tr[\rho \ln(\rho)]. \quad (52)$$

This formula is known as the *Shannon entropy* in the current literature on quantum computation and quantum information [11].

If we pretend to know the distribution along the time-like direction and use the pure-state density matrix given in Equation (46), then the entropy is zero. However, if we do not know how to deal with the distribution along the time separation  $t$ , then we should use the density matrix of Equation (48) to calculate the entropy, and the result is [27]

$$S = 2(n+1) \left[ (\cosh \eta)^2 \ln(\cosh \eta) - (\sinh \eta)^2 \ln(\sinh \eta) \right] \quad (53)$$

$$- \left( \frac{1}{\cosh \eta} \right)^{2(n+1)} \sum_k \frac{(n+k)!}{n!k!} \left[ \ln \left[ \frac{(n+k)!}{n!k!} \right] \right] (\tanh \eta)^{2k}. \quad (54)$$

Let us go back to the wave function given in Equation (6). As is illustrated in **Figure 4**, its localization property is dictated by the Gaussian factor which corresponds to the ground-state wave function. For this reason, we expect that much of the behavior of the density matrix or the entropy for the  $n$ -th excited state will be the same as that for the ground state with  $n = 0$ . For this state, the density matrix and the entropy are

$$\rho(z, z') = \left( \frac{1}{\pi \cosh(2\eta)} \right)^{1/2} \exp \left\{ -\frac{1}{4} \left[ \frac{(z+z')^2}{\cosh(2\eta)} - (z-z')^2 \cosh(2\eta) \right] \right\}, \quad (55)$$

and

$$S = (\cosh^2 \eta) \ln(\cosh^2 \eta) - (\sinh^2 \eta) \ln(\sinh^2 \eta), \quad (56)$$

respectively. The quark distribution  $\rho(z, z)$  becomes

$$\rho(z, z) = \left[ \frac{1}{\pi \cosh(2\eta)} \right]^{1/2} \exp \left( \frac{-z^2}{\cosh(2\eta)} \right). \quad (57)$$

The width of the distribution becomes  $\cosh(2\eta)$ , and becomes wide-spread as the hadronic speed increases. Likewise, the momentum distribution becomes wide-spread, as in the case of Feynman's parton picture described in Subsection 5.1. This simultaneous increase in the momentum and position distribution widths is called the parton phenomenon in high-energy physics [13] [14] [15]. The position-momentum uncertainty becomes  $\cosh^2 \eta$ . This increase in uncertainty is due to our ignorance about the physical but unmeasurable time-separation variable. This does not violate the fundamental law of the uncertainty as described in Subsection 5.2.

The use of an unmeasurable variable as a *shadow coordinate* is not new in physics [7] [8]. Feynman's book on statistical mechanics contains the following paragraph [5].

When we solve a quantum-mechanical problem, what we really do is divide the universe into two parts—the system in which we are interested and the rest of the universe. We then usually act as if the system in which we are interested comprised the entire universe. To motivate the use of density matrices, let us see what happens when we include the part of the universe outside the system.

In the present paper, we have identified Feynman’s rest of the universe as the time-separation coordinate in a relativistic two-body problem. Our ignorance about this coordinate leads to a density matrix for a non-pure state, and consequently to an increase of entropy. **Figure 9** shows the entropy as a function of the hadronic speed or  $\tanh \eta$ . The entropy is zero when the hadron is at rest. It increases rapidly as the hadronic speed approaches the speed of light.

Finally, let us examine how the ignorance will lead to the concept of temperature [28]. For the Lorentz-boosted ground state, the density matrix of Equation (48) becomes

$$\rho_\eta(z, z') = \left( \frac{1}{\cosh \eta} \right) \sum_k (\tanh \eta)^{2k} \chi_k(z) \chi_k(z'). \tag{58}$$

We can now compare this expression for the oscillator system in the thermally excited state. In terms of the temperature  $T$ , the density matrix takes the form [46] [47].

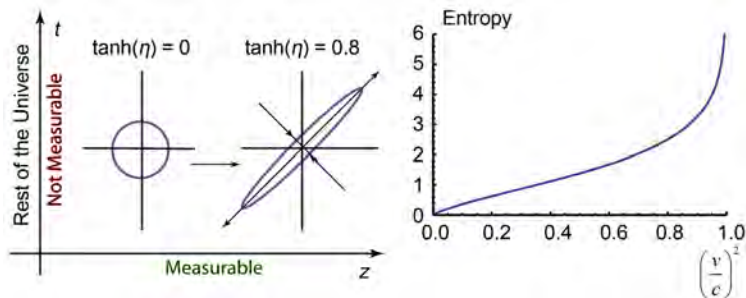
$$\rho_T(z, z') = (1 - e^{-1/T}) \sum_n e^{-n/T} \chi_n(z) \chi_n(z'), \tag{59}$$

where  $T$  means  $kT/\hbar\omega$ , with  $k$  and  $\omega$  as Boltzmann’s constant and the frequency of oscillation respectively. If we compare this expression with Equation (58). Then

$$e^{-1/T} = (\tanh \eta)^2. \tag{60}$$

This leads to

$$T = \frac{-1}{2 \ln(\tanh \eta)}. \tag{61}$$



**Figure 9.** Feynman’s rest of the universe and entropy. For the hadron, the space-like extension is measurable, but the time-like separation is not. It is in the rest of the universe. This non-measurable variable should be treated statistically. It leads to the increase in entropy.

The temperature rises rapidly as the hadronic speed approaches the speed of light, as indicated in **Figure 8**. As we noted in Subsection 5.1, the hadron becomes a plasma state as in the case of Feynman's parton picture.

## 7. Concluding Remarks

The primary purpose of this paper was to study the entropy and temperature of bound states in the Lorentz-covariant world. The result of study is given in Section 6.

This problem arises because the time separation between the constituent particles is not a measurable quantity in the present form of quantum mechanics. On the other hand, it is possible to treat this unmeasurable variable statistically. The time separation is negligible when the bound state is at rest, but it becomes as significant as the space separation (like the Bohr radius) when the bound state moves.

In order to study this effect, we need at least one Lorentz-covariant model for bound states. Dirac and Feynman made their efforts to construct such a wave function. Much of the present paper is devoted to the integration of their efforts to construct wave functions for moving bound states in Einstein's Lorentz-covariant world.

It was Paul A. M. Dirac who made efforts to construct wave functions for moving bound states. Dirac's papers and books are like beautiful poems, but they do not contain figures. It was a challenge to convert his poems [18] [19] [20] into the circle and the rectangle given in **Figure 4**. Then it is easy to integrate those two figures. A more detailed explanation is given in a recent book entitled *Physics of the Lorentz Group, 2nd Edition* by Başkal, Kim, and Noz [24].

We used in this paper the Lorentz-covariant wave function which provides the resolution to the question whether the quarks and partons are two different ways of looking at the same entity in the Lorentz-covariant world, as illustrated in **Figure 8**. This also provides the answer to the Bohr-Einstein issue of moving hydrogen atoms.

## Acknowledgements

This review paper is largely based on the papers and books I, the author of this paper, published with Marilyn Noz since 1973. I thank her for this prolonged collaboration. I thank also my younger colleagues who co-authored with me many papers I published since 1978. Their names are D. Han, P. Hussar, S. H. Oh, and D. Son. My later co-authors include S. Başkal, and E. Georgieva.

In July of 1962, I became an assistant professor of physics at the University of Maryland. Paul A. M. Dirac visited the University for one week in October of the same year at the invitation of John S. Tall who was the chairman of the physics department. Toll assigned me as the personal assistant to Dirac. This gave me an excellent opportunity to learn physics directly from Dirac.

Dirac told me that physicists in general do not understand the difference be-

tween the Lorentz invariance and the Lorentz covariance. This difference was illustrated in terms of the hyperbola and the squeezed circle **Figure 4**. This allowed me to integrate Dirac's attempts to construct the wave function in the Lorentz-covariant world.

Another fundamental issue is the unification of the internal space-time symmetries. In his paper of 1939 [25], he stated that those symmetries are like  $O(3)$  and  $E(2)$  respectively. The question then is whether the  $E(2)$  symmetry can be obtained continuously as the mass of particle becomes zero. This question has a stormy history [48]. Because the transition from  $O(3)$  to  $E(2)$  is not an analytic continuation. I am grateful to Professor Eugene Paul Wigner for spending time with me to settle this issue as described in Section 2 of the present paper.

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix

In Section 4, we noted that the bound state has its internal space-time coordinates, and we considered a bound state equation in Equation (26). Let us write this oscillator equation:

$$\frac{1}{2} \left( - \left[ \frac{\partial}{\partial x_\mu} \right]^2 + x_\mu^2 \right) \psi(x) = \lambda \psi(x). \tag{62}$$

This differential equation is separable in all space-time variables. Thus we can concentrate on the longitudinal and time coordinates. The equation then becomes

$$\frac{1}{2} \left[ \left( - \left[ \frac{\partial}{\partial z} \right]^2 + z^2 \right) - \left( - \left[ \frac{\partial}{\partial t} \right]^2 + t^2 \right) \right] \psi(z, t) = \lambda \psi(z, t). \tag{63}$$

Since the time excitations are not allowed, the solution of this equation takes the form

$$\psi^n(z, t) = \left( \frac{1}{\pi} \right)^{1/4} \exp\left( \frac{-t^2}{2} \right) \xi_n(z), \tag{64}$$

where  $\xi_n(z)$  is the oscillator wave function for the n-th excited state. The differential equation of Equation (63) is invariant under the Lorentz boost:

$$z \rightarrow \frac{z + \beta t}{\sqrt{1 - \beta^2}}, \quad t \rightarrow \frac{t + \beta z}{\sqrt{1 - \beta^2}}, \tag{65}$$

and the boosted wave function becomes

$$\psi_\beta^n(z, t) = \left( \frac{1}{\pi} \right)^{1/4} \exp\left( \frac{-t'^2}{2} \right) \phi_n(z'), \tag{66}$$

with

$$z' = \frac{z - \beta t}{\sqrt{1 - \beta^2}}, \quad t' = \frac{t - \beta z}{\sqrt{1 - \beta^2}}. \tag{67}$$

The wave function of Equation (66) becomes  $\psi^n(z, t)$  given in Equation (64) when  $\beta = 0$ . There are no excitations along the  $t$  direction because of the c-number time-energy uncertainty relation [18].

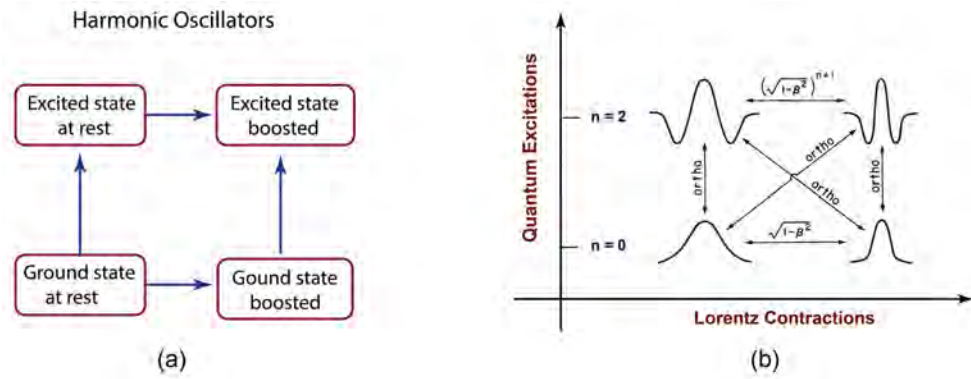
It is then of interest to evaluate the integral [24] [49], and the result is

$$\int \psi^n(z, t) \psi_\beta^{n'}(z, t) dx dt = \left( \sqrt{1 - \beta^2} \right)^{(n+1)} \delta_{nn'}. \tag{68}$$

The orthogonality relation and the contraction property contained in this formula are illustrated in **Figure A1**. The stationary ground state wave function is orthogonal to all excited states. This ground state is contracted by  $\sqrt{1 - \beta^2}$ . This is consistent with our understanding of Einstein's Lorentz contraction of a rod.

Then why is  $\left[ \sqrt{1 - \beta^2} \right]^{(n+1)}$  for the  $n$ -th excited states? It is because the (*wave-function*)<sup>2</sup> has  $(n + 1)$  humps. Then why are they multiplicative?





**Figure A1.** Orthogonality and Lorentz contractions of the covariant harmonic wave functions, with words for (a) and graphs for (b). The orthogonality is maintained for all excited states. The Lorentz-contraction factor of  $\sqrt{1-\beta^2}$  for the ground state is consistent with Einstein’s Lorentz contraction. For the  $n$ -th excited state, the  $|wavefunction|^2$  has  $(1 + n)$  humps. Thus the net contraction thus should  $[\sqrt{1-\beta^2}]^{(n+1)}$ .

In order to answer this question, let us use the bra-and-ket notation for the harmonic oscillators, where the ground state is  $|0\rangle$  and  $|n\rangle$  is for the  $n$ -th excited state. We use  $a$  and  $a^\dagger$  for step-down and step-up operators respectively. As is well known, the  $n$ -th excited state becomes

$$|n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle. \tag{69}$$

Thus, each additional hump is produced through the multiplication process.

# Causality, Uncertainty Principle, and Quantum Spacetime Manifold in Planck Scale

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## Abstract

In causal set theory, there are three ambiguous concepts that this article tries to provide a solution to resolve these ambiguities. These three ambiguities in Planck's scale are: the causal relationship between events, the position of the uncertainty principle, and the kinematic. Assuming the interaction between events, a new definition of the causal relationship is presented. Using the principle of superposition, more than one world line is attributed to two events that are interacting with each other to cover the uncertainty principle. Using these achievements, it is shown that kinematics has no place in the Planck dimension and that quantum spacetime manifold should be used instead.

## Keywords

Causal Set Theory, Causality, Uncertainty Principle, Kinematic, Manifold

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## 1. Introduction

Scientists are pursuing three main areas of researches to achieve the theory of quantum gravity. The oldest is the theory of quantum strings, which is due to the impossibility of evaluating its results in the laboratory, advances in this field are considered more from the perspective of mathematics [1]. The second theory is Loop Quantum Gravity, which does not seem to be a comprehensive theory at Planck scale [1]. The theory of causal sets is the third important branch of researches that has been considered by scientists today due to the use of simple and fundamental assumptions [2]. The important point for the complete success of these theories in the field of quantum gravity seems to be the concept of time in physics which remains unresolved up to now. In the other words, one of the main unsolved problems of physics is the true nature of time. Some scientists be-

lieve that all that exists are things that change. Things do not change in time; the change of things is time and time is simply a complex of rules that govern the change. Time is inferred from things [3]. Others believe that everything that is true and real is such in a moment that is one of a succession of moments. Space is emergent and approximate and the laws of nature evolve in time and may be explained by their history. Time is the most real aspect of our perception of the world [4]. In our previous article, we have concluded that [5]:

- 1) The world is composed by events that change.
- 2) We sense the changes of events as the passage of time.
- 3) All events which are in mutual or multi-interaction with each other compose a system and other non-related events compose its environment. A boundary exists between each system and its environment.
- 4) In each application domain of a physical theory, there are some main conceptual paradigms. During the transition between the different application domains through the boundaries, one should pay enough attention to the conceptual paradigm shift.

It should be noted that before formulating the theory of causal sets in the form that is now available to us, important and fundamental researches have been done by scientists. Robb has defined null, parallel lines and plane and proved numerous theorems involving them and described the relativity using the discrete spacetime (*i.e.*, casual structure) [6] [7]. Hawking *et al.*, [8] and Malament [9] have proved that the casual structure of a spacetime, together with a conformal factor, determines the metric of a Lorentzian spacetime, uniquely. It has been shown that one can recover the conformal metric by using the before and after relations amongst all events [10]. Now, if one has a measure for the conformal factor, he/she can recover the entire metric and spacetime [10]. Of course, 't Hooft [11] and Myrheim [12] have independently found the causal set theory too.

Of course, other efforts are being made by scientists to introduce the theory of quantum gravity by attention to the locality and causality. One of them is causal dynamic triangulation (CDT) [13]. Near the Planck scale, the structure of spacetime itself is supposed to be constantly changing due to quantum fluctuations and topological fluctuations. CDT theory uses a triangulation process which varies dynamically and follows deterministic rules, to map out how this can evolve into dimensional spaces similar to that of our universe [13]. Diel [14] has assumed that the elementary structure of spacetime is a derivative of causal dynamical triangulation and, at the elementary level, space consists of a (discrete) number of interconnected space points, each of which is connected to a small number of neighbouring space points. He has shown that emergence and propagation of quantum fields (including particles) are mapped to the emergence and propagation of space changes by utilizing identical paths of in/out space point connections [14]. Also, it is well known that in Einstein's theory of general relativity, events are placed on the system world line, and Schrodinger's time-dependent equation emphasizes the existence of a causal relationship between

events. On the other hand, based on the particle approach in quantum mechanics, as well as describing the quantum field theory and many body physics by particle creation/annihilation operators of particles/quasi-particles, the issue of locality can be considered as an important subject. In other words, by considering the causality and locality, it is possible to develop an alternative causal model of quantum theory and quantum field theory, in which quantum objects are the basic units of causality and locality [15]. In this model, not only the quantum objects are embedded in space and move within space, but also the dynamics of space is triggered by the dynamics of the quantum objects. The causal model of QT/QFT assumes discretized spacetime similar to the spacetime of causal dynamical triangulation [15].

In this paper, we try to answer three ambiguous concepts in the theory of causal sets at Planck scale. These three problems are determining the type of causal relationship between events, explaining the position of the uncertainty principle and its importance in quantizing the theory of causal sets and the place of kinematics in this theory. First, with a brief review of the theory of causal sets, we enumerate the basic features of this theory. Then, with a brief review of the concept of causality in physics, we explain the type of causal relationship between events in the theory of causal sets to use in the rest of this article. By reviewing the effect of the constant speed of light on the theory of special relativity and its relation to the concept of time in physics, we show that kinematics can have no place in the Planck scale. Finally, considering the position of the uncertainty principle in quantum physics and reviewing published articles in the field of quantum manifold, we will compile and introduce the general structure of the quantum spacetime manifold.

The structure of the article is as follows: in Section 2, we review the discrete spacetime as causal sets. A short review about the special causality in physics is presented in Section 3 and in Section 4 the kinematical and dynamical models are discussed. The property of quantum spacetime manifold is provided in Section 5 and the summary is presented in Section 6.

## 2. Discrete Spacetime as Causal Set

In a causal set  $C$  including the elements  $\{a_1, a_2, a_3, \dots, a_{n-1}, a_n\}$  the relation  $a_i < a_j$  for  $i \leq j$  is satisfied. The pair  $(C, \leq)$  is reflexive, antisymmetric, transitive, and locally finite. Therefore, the causal matrix  $C$  can be defined by

$$C_{a_i, a_j} = \begin{cases} 1, & a_i < a_j \\ 0 & \text{Otherwise} \end{cases} \quad (1)$$

Also, a nearest neighbor relation (called link) is a relation  $a_i < a_j$  such that there exists no  $a_k \in C$  with  $a_i < a_k < a_j$ . The elements  $a_i$  and  $a_j$  are the nearest neighbors and their relation is shown as  $a_i < *a_j$ . Now, the link Matrix  $L$  can be defined by

$$L_{a_i, a_j} = \begin{cases} 1, & a_i < *a_j \\ 0 & \text{Otherwise} \end{cases} \quad (2)$$

It is obvious that both  $C$  and  $L$  matrices are strictly upper triangular and a causal set is a partially ordered set. By attention to the relativistic causality [16] [17], one can construct a causal set from a Lorentzian manifold  $(M, g)$ . The manifold  $M$  represents the collection of all spacetime events and the metric  $g$  is a symmetric non-degenerate tensor on  $M$  of signature  $(+, -, -, -)$ . We know, the infinitesimal displacement is given by

$$ds^2 = -dt^2 + \delta_{ij} dx^i dx^j \quad (3)$$

where,  $i, j = 1, 2, 3, \dots, d$  and here  $d = 1$ . We can rewrite Equation (3) as

$$ds^2 = -(dt + dx)(dt - dx) + \delta_{ij} dx^i dx^j \quad (4)$$

where,  $i, j = 1, 2, 3, \dots, d-1$ . By defining,  $x^+ = \frac{x+t}{\sqrt{2}}$  and  $x^- = \frac{t-x}{\sqrt{2}}$ , we can write

$$ds^2 = -2dx^+ dx^- + \delta_{ij} dx^i dx^j \quad (5)$$

By comparing Equation (5) with Equation (3), it can be concluded that both  $x^+$  and  $x^-$  act as time-coordinate. It is called the lightcone coordinate. One nice thing about the lightcone coordinate is that the causal structure is partially included into the coordinate system itself. Therefore, for two points  $x_1 = (x_1^+, x_1^-)$  and  $x_2 = (x_2^+, x_2^-)$  we have  $x_1 \leq x_2$  if and only if  $x_1^+ \leq x_2^+$  and  $x_1^- \leq x_2^-$ . Now, if the length of diamond in lightcone coordinate be equal to  $S$ , one can find the  $n$  random points in the  $(1+1)$  dimensional space by

$$P = S \times \text{Random number}(x^-, x^+) \times \text{Rotation matrix}(45^\circ) \quad (6)$$

It should be noted that

$$\begin{pmatrix} t \\ x \end{pmatrix} = \begin{pmatrix} \sqrt{2}/2 & \sqrt{2}/2 \\ -\sqrt{2}/2 & \sqrt{2}/2 \end{pmatrix} \begin{pmatrix} x^- \\ x^+ \end{pmatrix} = \begin{pmatrix} \cos 45^\circ & \sin 45^\circ \\ -\sin 45^\circ & \cos 45^\circ \end{pmatrix} \begin{pmatrix} x^- \\ x^+ \end{pmatrix} \quad (7)$$

For example, we found 1000 points in a  $(1+1)$ -dimensional space by

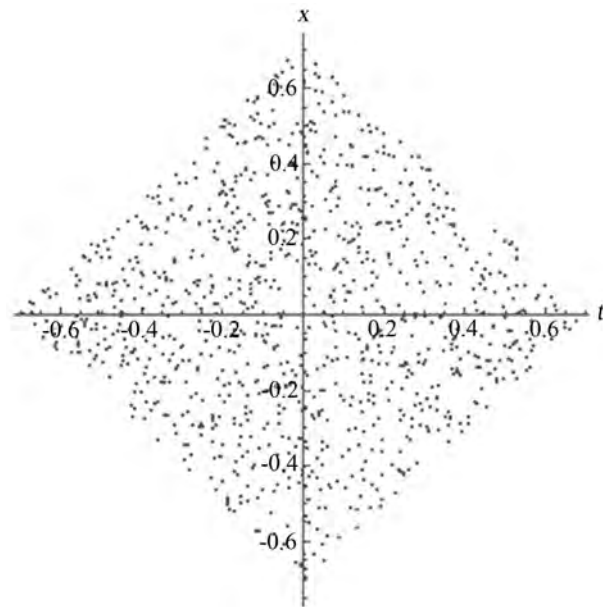
$$P = 1 \times \text{Random number}(-0.5, +0.5) \times \text{Rotation matrix}(45^\circ) \quad (8)$$

and shown them in **Figure 1**, after sorting.

However, in  $(1+1)$ -dimensional there are one temporal (unidirectional) dimension and one spatial (bidirectional) dimension. Since, the proper time is given by

$$d\tau^2 = -dt^2 + dx_i^2 \quad (9)$$

For,  $dt > 0$  and  $d\tau^2 > 0$ , the points will be placed in future timelike region. It means that not only the spatial distance ( $dx_i^2$ ) should be greater than the temporal distance ( $dt^2$ ) but also  $dt > 0$ . Therefore, the element of the casual matrix  $C$  will be equal to one if the both conditions are satisfied simultaneously for two elements  $a_i$  and  $a_j$  of the causal set and otherwise it will be equal to zero. Using the method, one can find the causal matrix  $C$ . By keeping the non-zero elements of  $C$ -matrix when  $a_i$  and  $a_j$  are only the nearest neighbor elements and replacing the other non-zero elements by zero number, the link



**Figure 1.** (Color online) 1000 random points in  $(1 + 1)$ -dimensional space.

matrix  $L$  can be found. The above explained method which is used for finding the causal set, the causal matrix and the link matrix from a Lorentzian manifold is called sprinkling method.

Since, the points of a casual set are placed in the future timelike region, it can be concluded that there is a priority (time precedence) between points respect to the time of occurrence. In the other words, a finite path of length  $n$  (maximum chain) is a sequence of distinct elements  $a_1 < *a_2 < *a_3 < * \dots < *a_{n-1} < *a_n$  in the future timelike region. Therefore, the priority in occurrence is called the causality in casual set theory. The causal set which is found from a Lorentzian manifold by sprinkling method is invariant under the boost transformation in spite of the lattice model. Therefore, the causal set based physical theory is Lorentzian invariant at Planck scale in spite of the other physical theories about spacetime at Planck scale. In next section, we will discuss about the causality in physics and show that the priority in occurrence is the sufficient condition and the interaction between each two relates  $a_i < *a_j$  is the necessary condition for assigning the causal relation to two relates  $a_i$  and  $a_j$ .

### 3. Causality in Physics

In Newtonian physics, one can exactly determine the future if he/she knows the initial and boundary conditions. The process is called a deterministic process. Time-dependent Schrodinger equation is a deterministic equation *i.e.*, if one knows the initial and boundary conditions at time  $t$  he/she will be able to find the state function of the system at time  $t + 1$ . But, in quantum physics, the total state of a system is specified by the superposition of substates (superposition principle). Based on the principle, nobody knows the exact final state of the system before observation. After observation, one of the superposed substates will

create the output of observation. The process is called a probabilistic process. In probabilistic process the output of observation can be created by one of the many superposed substates and in deterministic process the output of observation is created by the exact initial state of the system. Therefore, there is an interaction process between output and input of observation such that the output is created by input while we cannot exactly specify the input before appearing the output in the probabilistic process. The interaction between output and input is called causality. In Newtonian physics, there is the deterministic causality and in quantum physics there is the probabilistic causality. Therefore, in deterministic causality, the elements of the casual world line have two properties: causality and priority in occurrence (time precedence). But in probabilistic causality, we encounter many world lines theoretically (before observation) such that the elements of each causal worldline have the causality and priority properties. It means that, a causal set which is found by sprinkling method and have a specific finite path has only the priority properties and cannot be considered as a deterministic causality. For classical point particle, we assign a specific path

$a_1 < *a_2 < *a_3 < \dots < *a_{n-1} < *a_n$  to the system in the future timelike region. Therefore, the specific path in a causal set which is found by sprinkling method is not a suitable candidate for the probabilistic causality. For a quantum point particle, we should consider all chains between  $a_1$  and  $a_n$  and then use the discrete path integral method for finding the amplitude for the whole trajectory [13]. But we did not consider the causality between events in this case, and in consequence we lost some important information or added some non necessary information to the final state of the closed system including observer. It means that the current sprinkling method for arising the causal set is only suitable for the deterministic causality (classical systems) if the causal relation will be added to it.

Also, causality is an interaction process between input and output although it has a certain concept between folks. Usually, folks have some intuitions about causality. The raised question is: whether there is something in the world that realizes the intuition of folk about the causality? The question has to be answered empirically, and thus commonly depends on the natural science. It is called Canberra methodology [18]. The Canberra methodology includes two stages [18] [19]. At first stage, we specify something which we interested to analyze them from philosophical point of view. Then we collect together the platitudes concerning our subject matter and finally conjoin them for defining a theoretical role for the things we are interested in. At the second stage, we look at our theory of the world to tell us what, if anything, plays the rule so defined [18] [19]. Of course, there is another methodology which is called naturalism [18]. The methodology is often divided into a descriptive and a normative part [20] [21]. In the descriptive part it is studied how we acquire knowledge within science and in normative part the justification for this knowledge is given [18] [20] [21]. The naturalistic approach to causation has become well known as the

empirical analysis of causation [18]. It has been shown that there is no difference between two methodologies about causation if we consider the causation as interaction between relates and pay attention to the fact that output of observation is created by its input [18]. Therefore, the elements of causality worldline have two important properties. First, there is an occurrence priority between them and second the prior relata causes the next relata. It means that we should omit the non-causal elements from the worldline for finding the causal worldline. The causal world line shows the history of system evolutions in the future timelike region. If we deal with the quantum physics, we have to consider all causal world lines between two relates before observation for showing the probabilistic history of system evolutions in the future timelike region due to the superposition principle. Of course, from the Heisenberg uncertainty principle point of view, we have to consider more than a causal world line before observation, too. Therefore, for quantum point particle we should use the discrete path integral method for finding the amplitude for the whole trajectory [22]. Since, we consider the causal world line in our closed system including observer the time passes as changes in relates. But in kinematic model of causal set theory since we only consider the priority in occurrence theoretically the time does not pass because no changes happen in the relates. By attention to the new concept of time (as change in relates) we review the kinematic and dynamic models in the next section.

#### 4. Kinematical or Dynamical Models

In physics, the kinematic is referred to the time independent case. If time is sensed as the change of things, kinematic will be equal to the no change case. In the other words, if no change is sensed no time will pass and in consequence defining the time is meaningless. It can be shown that the special relativity can be deduced from the assumption that the velocity of light does not depend on the observer and it is the maximum velocity of things in vacuum [23]. In order to make the concept of time clearer let us, assume two frame of references  $A$  and  $B$  move with velocity  $v$  respect to each other. The observers on both references have no sense about time in own reference frame but when they see the other frame since its position changes, he/she sense the time. Also, let us, assume two rulers are placed in each frame. If they want to measure the length of ruler in own frame, they can use two light flashes. The time difference between received flashes from the back and the front of the ruler multiplied by the velocity of light  $C$  in own frame is equal to the length of the ruler. It should be noted that, in the closed system including ruler, light flashes and observer the change in position of light flashes is sensed and therefore time passes. For measuring the length of moving ruler, they should measure the time difference between received flashes from the back and the front of the ruler, again. But, whether the rate in the change of the flash positions is equal to the previous case? *i.e.*, whether the velocity of light  $C$  does not depend on observer frame of reference? Why?



Let us, assume  $C$  is constant (note that it is only an assumption). **Figure 2** shows the spacetime diagram of two moving reference frames respect to each other. At time  $T$ , the observer in nonmoving frame sends a light flash toward the moving frame. The observer in moving frame receives the flash at time  $t_2$ . The light flash is reflected toward the nonmoving frame by a mirror and the observer receive it at time  $k^2T$ . The equation of moving of light flash (red arrow) is

$$t - T = \frac{x}{C} \rightarrow x = C(t - T) \tag{10}$$

and the equation of moving observer is

$$t = \frac{x}{v} \rightarrow x = vt \tag{11}$$

In the triangle with two red arrows, the dashed blue line is the middle-perpendicular line and in consequence one can write

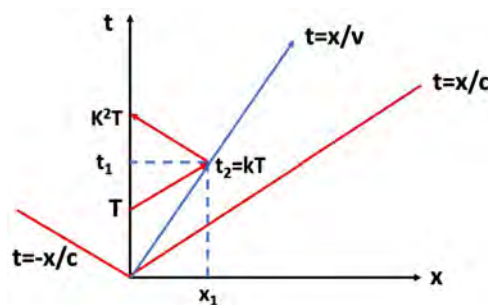
$$\frac{t_1}{t_2} = \frac{C\sqrt{C-v}}{(C-v)\sqrt{C+v}} = \frac{1}{\sqrt{1-v^2/C^2}} \tag{12}$$

It means that the assumption of independency of light velocity to reference frame causes the time dilation. Now, if the length of ruler in moving frame is  $L_0$  (the ruler is at the rest) its length in nonmoving frame (ruler is moving) can be calculated as

$$L = \frac{Cv}{C-v} \frac{L_0}{kv} = \frac{CL_0}{C-v} \frac{\sqrt{C-v}}{\sqrt{C+v}} = \frac{L_0}{\sqrt{1-v^2/C^2}} \tag{13}$$

Therefore, the assumption of independency of light velocity to the reference frame causes the length contraction.

But, in relativity, proper time ( $\tau$ ) along a timelike world line is defined as the time as measured by a clock following that line. It is thus independent of coordinates, and is a Lorentz scalar. The proper time interval between two events on a world line is the change in proper time. This interval is the quantity of interest, since proper time itself is fixed only up to an arbitrary additive constant, namely the setting of the clock at some event along the world line. The proper time interval between two events depends not only on the events but also the world line



**Figure 2.** (Color online) The spacetime diagram of two moving reference frames respect to each other. Red arrows show the light flashes and the blue arrow shows the causal world line of moving frame.

connecting them, and hence on the motion of the clock between the events. It is expressed as an integral over the world line (analogous to arc length in Euclidean space). An accelerated clock will measure a smaller elapsed time between two events than that measured by a non-accelerated (inertial) clock between the same two events. The twin paradox is an example of this effect.

Therefore, up to now, we used two main assumptions and one definition:

1) If nothing changes in a closed system, the time definition is meaningless. It is called the dynamical assumption.

2) If the velocity of light is constant and maximum velocity of things in vacuum, we expect to see time dilation and length contraction phenomena. It is called the velocity of light assumption [23].

3) The proper time interval between two events depends not only on the events but also the world line connecting them.

Then, from special relativity point of view the below questions can be asked:

1) What is about the dynamical assumptions at the Planck scale?

2) Whether it is correct that the causal set dynamic is found from a kinematic version of a causal set if the kinematic version, which includes no time, cannot exist at the Planck scale?

3) What is about the velocity of light assumption at the Planck scale?

4) Whether it is expected that we see some physical phenomena related to the non-variable velocity of light at the Planck scale?

Although it is not possible to answer all of these questions by using the above explanations, but even if we consider the proper time, since kinematics means time independency and quantum gravity theory is supposed to explain spacetime on the Planck scale, this theory cannot be based on a kinematical theory and should be developed based on a dynamical theory from the beginning.

## 5. Quantum Manifold of Spacetime

It has been shown that two very different manifolds could not approximate the causal set, and in general, an arbitrary causal set may not embed in any Lorentzian manifold with a metric [24]. The question about how manifoldlike causal sets may arise from suitable dynamical laws has been justified, before [25] [26]. Generally, there are three types of dynamics that a causal set can have [26]. The classical dynamic can be used for explaining the continuum limit which is the general relativity. The dynamics of quantum matter and fields on a given “classical” causal set can be used for explaining the continuum limit which is the quantum field theory on a fixed curved spacetime. Finally, quantum dynamics of the causal set itself, which is the final aim in order to construct a quantum theory for gravity [26]. But, is there a kinematical discrete spacetime at Planck scale such that the both general relativity and quantum theory can be deduced from the spacetime? If one of the main aims of finding the quantum gravity theory is solving the existence of singularities in general relativity and renormalization requirement in quantum physics, why should one develop the classical

dynamic and dynamic of quantum matter? It seems that the quantum dynamics of the causal set itself should be the main branch of the future research program. In this research program, we should find a quantum spacetime manifold for deducing a suitable discrete causal set when the time is defined based on the changes in the elements of the causal set.

In above, we showed that for developing a causal set theory for quantum gravity at Planck scale, we should specify the importance and effectiveness of the below natural facts in our theory when we want to study the continuum limit:

- 1) The maximum velocity of things in vacuum which is the velocity of light.
- 2) Kinematic has no place in quantum gravity theory at Planck scale.
- 3) The uncertainty principle and superposition principle of quantum mechanics.

In the other words, the new quantum spacetime manifold should has some special properties for providing the above three requirements at least at continuum limits.

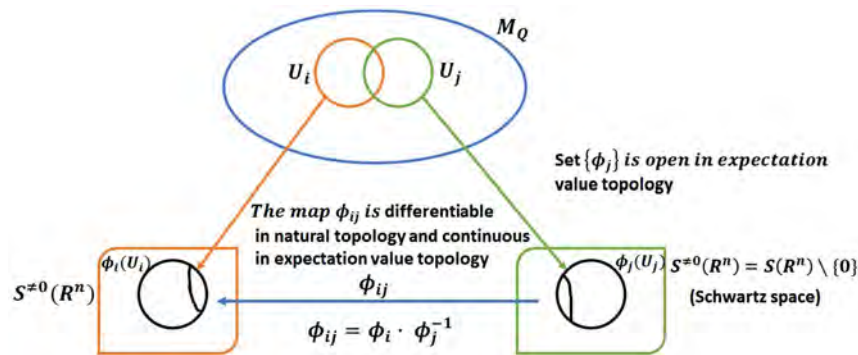
We know that the manifold geometry ( $\mathcal{M}$ ) is the heart of the general relativity and the observable operators on Hilbert space (Schwartz space ( $\mathcal{S}(R^n)$ )) are the main components of quantum mechanics. Since,  $R^n$  is the space of the position of classical events, it is expected that the background space  $R^n$  will be the limit of the  $\mathcal{M}$  and  $\mathcal{S}(R^n)$ . Now let us, assume that there is an infinite quantum manifold  $M_Q$ . It is well known that the expectation values of quantum observable operators follow the classical laws. Therefore, it may be possible one recovers the manifold geometry  $\mathcal{M}$  from  $M_Q$  by calculating the position expectation value [27] [28]. Also, in parallel,  $M_Q$  can be locally homomorphic to the  $\mathcal{S}(R^n)$  [27] [28]. But, the square-integrability is very important in quantum physics and in consequence we should only consider the family of all functions which have the below property

$$\|f\|_{\alpha,\beta} = \sup_{x \in R^n} |x^\alpha D_\beta f(x)| \quad (14)$$

For all multiindices  $\alpha$  and  $\beta$ , it is a family of seminorms which generates a topology on  $\mathcal{S}(R^n)$ . This topology is called the natural topology [27] [28]. Now, if we define the position expectation value as  $\bar{Q} = \frac{\langle f, Qf \rangle}{\langle f, f \rangle}$ , the open sets of expectation value topology ( $\bar{Q}^{-1}(W)$ ) exist and is defined as

$$\bar{Q}^{-1}(W) = \{f \in S^{\neq 0} \mid \bar{Q}(f) \in W\} \quad (15)$$

where,  $W \subset R^n$  is open in the standard topology on  $R^n$ . Thus, the expectation value topology is the coarsest topology in which the function  $\bar{Q}$  is continuous [27] [28]. By attention to the above definitions, it can be shown that the final quantum manifold will be a differentiable infinite dimensional manifold locally homeomorphic to  $S^{\neq 0}(R^n)$  and in contrast to the usual definition of an atlas, two different topologies called expectation value topology and natural topology should be introduced [27] [28]. **Figure 3** shows a quantum atlas, schematically.



**Figure 3.** (Color online) The schematic of a quantum atlas.

Now, a quantum manifold of dimension  $n$  is a set  $M_Q$  equipped with an equivalence class of quantum atlases of dimension  $n$ . The element of  $M_Q$  are called quantum points [27] [28]. If one finds a suitable method for arising the causal set from the quantum manifold, he/she will have a quantum causal set as the fundamental network of a spacetime at Planck scale. Of course, it can be a research program in future.

### 6. Summary

We have encountered some important problems with physics which three of them seem to be the most important: The singularities in general relativity, the renormalization requirements in quantum physics and the concept of time. Some bodies believe that if we can solve the problem of time, the other two remained problems will be solved. However, we have discussed about the nature of time in our previous article (Ref. [5]) and concluded that the time can be sensed as the changes in things. It means that under kinematic condition time cannot be defined, basically. Since, we are searching a unified theory between gravity and quantum for solving the above three mentioned main problems, at least, it seems that developing the dynamic of a causal set theory based on a kinematic causal set cannot help us much in this direction although, for studying some related classical problems at continuum level, it may help us. In the other words, we need a dynamical causal set at beginning. It means that a causal set should be raised from a quantum manifold. The quantum manifold is locally homomorphic to the Schwartz space and in parallel, the necessary manifold geometry of relativity can be recovered by using the quantum manifold. It should be noted that the causality relation differs from time precedence. In causality, two relations interact with each other and make change in each other but in time precedence, the priority is only important. Therefore, in a closed system including observer, we should consider a quantum manifold such that the causal world line, which is created by causal events between relates, appears in the manifold geometry of relativity. Also, we should pay enough attention to uncertainty and superposition principles for assigning a set of causal chains (paths) to each event instead of a specific exact path. Therefore, in Schwartz space, we should consider

a superposition of square integrable functions with different amplitudes when we want to study the homomorphic condition.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# Newton Did Not Invent or Use the So-Called Newton's Gravitational Constant; $G$ , It Has Mainly Caused Confusion

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## Abstract

Newton did not invent or use the so-called Newton's gravitational constant  $G$ . Newton's original gravity formula was  $F = \frac{Mm}{R^2}$  and not  $F = G \frac{Mm}{R^2}$ . In this paper, we will show how a series of major gravity phenomena can be calculated and predicted without the gravitational constant. This is, to some degree, well known, at least for those that have studied a significant amount of the older literature on gravity. However, to understand gravity at a deeper level, still without  $G$ , one needs to trust Newton's formula. It is when we first combine Newton's assumption, that matter and light ultimately consist of hard indivisible particles, with new insight in atomism that we can truly begin to understand gravity at a deeper level. This leads to a quantum gravity theory that is unified with quantum mechanics and in which there is no need for  $G$  and not even a need for the Planck constant. We claim that two mistakes have been made in physics, which have held back progress towards a unified quantum gravity theory. First, it has been common practice to consider Newton's gravitational constant as almost holy and untouchable. Thus, we have neglected to see an important aspect of mass; namely, the indivisible particle that Newton also held in high regard. Second, standard physics have built their quantum mechanics around the de Broglie wavelength, rather than the Compton wavelength. We claim the de Broglie wavelength is merely a mathematical derivative of the Compton wavelength, the true matter wavelength.

## Keywords

Newton Gravity, Newton's Gravitational Constant, Schwarzschild Radius, Quantum Gravity, Planck Length

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## 1. Newton Neither Invented Nor Used $G$

In his book, the Principia [1], Newton mentioned the gravitational force formula in words (see the Appendix) that create an equivalent to:

$$F = \frac{Mm}{R^2} \quad (1)$$

However, he did not make a single mention of any gravitational constant (with the notation of  $G$  or through any other notation), nor did he ever use such a constant himself. This appears to be something that few physicists or historians today know or acknowledge [2] [3]. In the Principia, Newton's focus was on relative masses, although he mentioned the word "mass" only once, but it is clear that he meant mass as an amount of matter. Based on easily-observable gravitational observations, such as the orbital time of satellites (moons and planets), he found the relative mass (weight) of Saturn, Jupiter, the Earth, and the sun; see also Cohen [4] for more detail on this. Cohen also pointed out that Newton's focus is on relative masses:

*"That is, since Newton is concerned with relative masses and relative densities, the test mass can take any unity"*

The kilogram definition of mass was invented more than 100 years after Newton published the Principia and thus came into being long after his death. Newton was, in several of his texts, clear on the idea that matter (and energy) at the deepest level is based on indivisible fully-hard particles with spatial dimension. He took this idea from atomism, a source that he referred to several times in his work [5] [6]. Newton was focused on atomism before he started to publish his work; this is evident from his unpublished notebook. He was also clear on this in Principia and, in particular, in his later book Opticks [7]. Newton thought that the amount of mass was related to the quantity of indivisible particles in the chosen mass. He even assumed that light was made up of such indivisible particles. He knew that it was impossible to find the number of indivisible particles in any observable mass at that time, an assertion that he mentioned in Principia. It was therefore natural for him to focus on relative masses when he worked with gravity. In short, to find the relative mass of two heavenly objects, Newton utilized satellite orbital time and the distance from the satellite to the center of the mass of which he wanted to find the relative mass; this is a method we return to shortly.

Newton also explained that weight is proportional to mass. In other words, twice the mass gives twice the weight in relation to two masses located the same distance from the gravitational object.

In 1798, Henry Cavendish [8] measured the density of the Earth using a torsion balance, also known as a Cavendish apparatus. The principles of such apparatus was already described by geologist John Michel [9] in 1784, but he died before he was able to use it, and Cavendish gives him full credit for the idea. Earlier, Newton had found the relative density between planets, and for this no Cavendish apparatus or similar was needed. However, when we want to find the



density of the Earth relative to a given substance, for example, water or iron, we need to know the gravity properties of a mass that we know is formed uniformly of the chosen substance. The Cavendish apparatus was needed to measure the gravitational effect from a small practical mass when one had knowledge of what substance it contained. Based on knowing the gravitational effect from such a known substance, one could compare that to the gravitational effect of the Earth and then know the density of the Earth relative to this substance.

Cavendish did not mention a gravitational constant nor did he have use for one. However, a Cavendish apparatus can indeed be used to find the gravitational constant  $G$ . The gravitational constant was needed when one decided on the kilogram definition of mass. Even if the kg definition was likely already introduced in 1796, it did not become widely used before around the 1870s. An important change here is that the Metre Convention was signed in 1875, leading to production of The International Prototype of the Kilogram. The kg definition of mass is, in our view, an incomplete definition of mass that needs  $G$  to become a complete mass measure that incorporates gravity effects from matter. The gravitational constant was likely mentioned for the first time by the French physicists Cornu and Baille [10] in 1873. Their paper mentioned the gravity force formula in the form  $F = fmm'/R^2$ , where  $f$  is the gravitational constant.

However, the idea took hold and in 1894, the gravitational constant was first called  $G$  (rather than  $f$ ) by Boys [11] in a proceeding at the Royal Society that followed shortly after he published in the prestigious journal *Nature*. To switch the notation from  $f$  to  $G$  is simply cosmetic<sup>1</sup>. Although, for example, Max Planck still used the notation  $f$  for the gravity constant in 1899, 1906, and 1928 [12] [13] [14], the use of  $G$  continued, and by the 1930s  $G$  had become the standard notation for the gravitational constant. Keep in mind that it took 200 years from the publication of Newton's gravitational theory to the first mention of the gravitational constant; thus it was, to some degree, a breakthrough, but from another perspective, it could also be seen as a disaster, as it led to an inferior definition of mass.

## 2. Newton's Gravity Formula; $F = \frac{\tilde{M}\tilde{m}}{R^2}$

As the original Newton formula is not compatible with the kg definition of mass (without adding a gravitational constant), we will call the Newton mass  $\tilde{M}$  to distinguish it from the modern kg definition of mass  $M$ . We will later explain why the mass we obtain from the original Newton formula is superior to the kg definition of mass.

The centripetal force in the Newtonian theory is given by  $\frac{\tilde{m}v^2}{R}$ . For a planet or moon to be in equilibrium within its orbit, the centripetal force must balance

<sup>1</sup>But Boys also had some interesting information in his paper on measurement methods in relation to  $G$ , for example.

with the gravitational force, so under the original Newton theory we must have:

$$\frac{\tilde{m}v^2}{R} - \frac{\tilde{M}\tilde{m}}{R^2} = 0 \quad (2)$$

Solved with respect to  $v$ , this gives an orbital velocity of:

$$v = \sqrt{\frac{\tilde{M}}{R}} \quad (3)$$

As we can see, this is quite different from the modern orbital velocity formula that is  $v = \sqrt{\frac{GM}{R}}$ . The difference is the Newton gravitational constant  $G$ , which, as we have noted, Newton himself never used. We can then ask, “*Does the formula work without the Newton gravitational constant?*” And, in fact, it does. Newton used the square of the orbital time and the distance between two masses to find the relative masses of heavenly objects. The orbital time is the circumference of the orbiting object (for example the moon) divided by the orbital velocity. In other words:

$$\begin{aligned} \frac{L}{v} &= \frac{L}{\sqrt{\frac{\tilde{M}}{R}}} \\ T &= \frac{L}{\sqrt{\frac{\tilde{M}}{R}}} \end{aligned} \quad (4)$$

This formula we can then solve with respect to mass, and we get:

$$\begin{aligned} \tilde{M} &= \frac{L^2 R}{T^2} \\ \tilde{M} &= \frac{(2\pi R)^2 R}{T^2} \\ \tilde{M} &= \frac{4\pi^2 R^3}{T^2} \end{aligned} \quad (5)$$

Assume we decide to measure orbital time in days (as Newton did) and distance in km (although naturally Newton used a different length measure). The distance to the sun can be found by parallax, and it is about 149.6 million km. The time it takes for the Earth to orbit the sun is 365 days. So now we can calculate the mass of the Sun as:

$$\tilde{M}_s = \frac{4\pi^2 149600000^3}{365^2} \approx 9.92 \times 10^{20} \text{ km}^3/\text{days}^2$$

As we can see, the mass has very strange notation and does not seem to be very recognizable or intuitive, but this is partly because we are accustomed to thinking of mass in terms of kg (or pounds). Next, let us calculate the mass of the Earth; for this we will use the orbital time of the moon, which is about 27.3 days. The distance from the Earth to the moon is about 384,400 km. The mass of the Earth must therefore be:

$$\tilde{M}_E = \frac{4\pi^2 384400^3}{27.3^2} \approx 3 \times 10^{15} \text{ km}^3/\text{days}^2$$

Again, this seems to be a strange mass that is hard for us relate to, but the mass of the sun relative to the Earth is now  $\frac{9.92 \times 10^{20}}{3 \times 10^{15}} \approx 329750$ . This is a number many of us do recognize; it is the mass of the sun relative to the Earth that we also obtain if we look at the modern kg definitions of the sun and the Earth. The  $4\pi^2$  will even cancel out in the relative mass formula, which can be described by:

$$\frac{R_1^3 T_2^2}{R_2^3 T_1^2} \quad (6)$$

Further, if the satellites were orbiting the objects we wanted to find the mass of at the same distance  $r_1 = r_2$ , then the relative mass is simply the orbital time squared divided by each other. This is very similar to Newton's reasoning in the Principia. As Newton pointed out, one could use any units one wanted (for distance or time) when the focus was on relative masses. When we say the sun's mass is 329,750 times that of the Earth's, then we have chosen the Earth as the unit mass. We could just as well have used the Earth mass as the unit mass when handling small objects on Earth. However, the mass of the Earth is enormous compared to any object we handle in our daily lives and so it would be hard to conceptualize it. Therefore, to have a better understanding of the mass, it makes sense to choose a smaller unit mass. The kg is a unit mass that is an arbitrarily-chosen mass, but it is practical—not so small so that it was hard to measure on an old-fashioned scale, and yet not so big that it could not be carried around. Weights, we must remember, were important to standardized trade, for example. So, we can say an almost arbitrary amount of weight (mass) was chosen as a kg. When we deal with a small practical mass, we can also quite easily know what substance it consists of—we can make a lead ball, gold ball, or iron ball, or we can simply fill a container with water. When we deal with planets, we know they likely consist of many types of elements, and it is harder to say for certain what their cores consist of completely.

Now to find the mass of the Earth in kg, we must first find a method to test gravity's effect on small practical masses, e.g., where we already know the kg mass of the object in question. Remember that to find the mass of the sun, Newton needed something orbiting the sun, but obviously there are plenty of planets to choose from. To find the mass of the Earth, he needed something that orbited the Earth, and indeed, the moon fit the bill. However, in order to measure a small practical mass, we need something "orbiting"<sup>2</sup> that is also very small (very small compared to planets, but still massive compared to atoms and molecules). This was a difficult task, and many attempts were undertaken, but it was first done accurately in 1798 by Henry Cavendish through what is known today as a "Cavendish apparatus" and consists of some small balls (made of lead or gold,

<sup>2</sup>Other methods were also considered here, with varied success.

for example) “orbiting” some larger (but still small) balls. Interestingly, the mass of a large lead ball in the Cavendish apparatus will have a Newton mass of:

$$\tilde{M} = \frac{2\pi^2 LR^2 \theta}{T^2} \quad (7)$$

where  $T$  is the oscillation time, and  $\theta$  is the deflection angle of the torsion balance from its rest position, and  $R$  is the distance from the small lead ball to the large lead ball, and  $L$  is the distance between the two small balls.

We know how to find this Newtonian type mass with the torsion balance, Formula (7). We do not need to know its kg mass or any other mass-measure for this. However, we can find its kg mass by comparing it with the kg standard by using a scale calibrated to kg. This now gives us a connection between the mysterious Newton mass and the kg (or pound). We can now also find the kg mass of the Earth, and the density of the Earth in terms of kg. The Cavendish apparatus, which was said to first find the gravitational constant indirectly, is both true and not true. Cavendish never mentioned a gravitational constant, and it is not needed under any circumstances, as we soon will see. The reason the Cavendish apparatus was required then was because one needed a way to measure the Newtonian type mass of a small object, so one could use the small unit (instead of the Earth, for example) as unit mass. The Cavendish apparatus also made it possible to accurately find the density of the Earth, not because of any gravitational constant, but because a small practical mass can be made of one substance where the density (weight) is known relative to other substances (e.g., gold versus water). In this way, one could find the density of the Earth very accurately relative to a given substance. If one had known a planet in our solar system consisted of a homogenous substance, take iron, for example, then there would have been no need for a Cavendish apparatus to find the density of the Earth relative to material objects. But we know of no such planet consisting of only one substance, and it would also be hard to check if that was really the case, even if it could be imagined. So, the breakthrough of the Cavendish apparatus was that one could find the gravity (Newtonian mass) of even a small practical mass. Naturally we can find the relative densities of different substances simply by using a scale.

Still, what we call the Newtonian mass,  $\tilde{M}$ , is difficult to fully understand, although it is no stranger than the kg. Up until now, we have used arbitrary units such as km for length, and Earth days as time. As we will see, it is when we first switch to more fundamental units and then explore the quantum world that we truly see the beauty of Newton’s formula.

#### Switching to more fundamental units

At this stage we can still choose any time unit we want: years, days, hours, or seconds. More important than the choice of time interval (time unit) is to link both time and length to something very fundamental in nature. This is light. We know from the writings of Aristotle (in his work *De Sensu*) that the Greek philosopher Empedocles, about 2500 BC, understood or at least assumed that the speed of light had a finite limit:

*Empedocles said that the light from the sun arrives first in the intervening space before it comes to the eye, or reaches the Earth. This might seem to be the case. For whatever is moved through space is moved from one place to another; hence, there must be a corresponding interval of time in which it is also moved from one place to the other.*

In 1676, Ole Christensen Rømer was likely the first to make a quantitative measurement of the speed of light and he concluded that it was finite. In 1704, in his book *Opticks* [7], Newton reported Rømer's calculations of the finite speed of light and gave a value of "seven or eight minutes" for the time it would take for light to travel from the sun to the Earth, an estimate that is not far from its real speed. So, Newton could have linked length to time through the speed of light, even if his calculations and predictions would have been somewhat inaccurate. In 1728, (one year after Newton's death) the English physicist James Bradley estimated the speed of light in a vacuum to be approximately 301,000 km per second, which is very close to today's defined value.

Here we will choose seconds as the time unit, and will link this to length through the speed of light. Our length unit will be the distance light travels in any given time unit. Here we choose the second; this is a well-known unit distance in modern physics, known as light-second (length); see, for example [15]. Now time and length units are suddenly related to something very fundamental. In modern physics, the speed of light is the same in every reference frame; it is known as  $c$  and per definition exactly 299,792,458 meters per second in vacuum. But here we have chosen the length unit that represents how long light travels in one second, so the speed of light will then be one light-second per second in this unit system. In other words, we can set  $c = 1$ , something that is often done in modern physics. What is important is that time and length are linked through something very fundamental, namely the speed of light.

Now the distance to from the Earth to the sun will be about  $R = 14960000000 \text{ m}/299792458 \text{ m/s} \approx 499$  light-seconds. The circumference of the orbit of the Earth around the sun is therefore about  $L = 2\pi \times 499$  light seconds. Further, we can find the mass of the sun

$$\tilde{M}_s = \frac{4\pi^2 R^3}{T^2} = \frac{4\pi^2 499^3}{(365 \times 24 \times 60 \times 60)^2} \approx 4.93 \times 10^{-6} \text{ Light-seconds} \quad (8)$$

This looks like a very unfamiliar mass, but soon we will see it makes much more sense than expressing the mass of the sun in kg. (The sun's mass in kg is approximately  $1.98 \times 10^{30}$ ).

Similarly, for the Earth we can use the moon's orbital time to find the mass of the Earth. The orbital time of the moon is about 27 days, or  $27 \times 24 \times 60 \times 60$  seconds. The distance to the moon is about 1.28 light-seconds. The mass of the Earth must therefore be:

$$\tilde{M}_E = \frac{4\pi^2 1.28^3}{(27 \times 24 \times 60 \times 60)^2} \approx 1.52 \times 10^{-11} \text{ Light-seconds} \quad (9)$$

This means the mass of the sun relative to the Earth must be approximately

$\frac{1.52 \times 10^{-11}}{4.93 \times 10^{-6}} \approx 324342$ . This is close to the actual modern accepted number.

Next let us use the orbital velocity formula  $v = \sqrt{\frac{\tilde{M}}{R}}$  to predict the orbital velocity of Saturn. The distance from the sun to Saturn is about 1.434 billion km, which is about 4783.3 light-seconds. The mass of the sun we have estimated to be  $4.93 \times 10^{-6}$  light seconds, and inputting the formula, we get:

$$v = \sqrt{\frac{4.93 \times 10^{-6}}{4783.3}} \approx 3.21 \times 10^{-5} \text{ Light-seconds per second}$$

That is, the orbital velocity is now on the dimensionless form; it is identical to  $\frac{v}{c}$ . In order to obtain meters per second, we need to multiply by  $c$  and this gives us about 9625 meters per second, which is the same as is observed in experiments. That our orbital velocity can actually be seen as  $\frac{v}{c}$  means it is a dimensionless number. For example, Langacker [15] in his book “*Can the Laws of Physics Be Unified?*” (2017) indicated that such dimensionless units as  $\frac{v}{c}$  could be more fundamental.

Actually, the mass we find in this way without depending on or knowing  $G$  is identical to half the Schwarzschild radius in meters divided by the speed of light, and exactly equal to the Haug radius [16];  $r_h = \frac{GM}{R} = \frac{1}{2}r_s$ , which is derived by taking into account relativistic mass that has been abandoned by general relativity theory [17] [18]. In other words, this is half the Schwarzschild radius in light-seconds or exactly the Haug radius in light-seconds. We propose that the Haug radius (divided by the speed of light) could be a much better model of mass than the kg-defined mass. However, no one should be fully convinced that light seconds are a better mass measure than kg just yet. It is when we get to the quantum aspects that this first becomes clear. As explained previously, we have demonstrated that we can predict relative masses, we can find the density of planets, and we can perform orbital velocity predictions, all with no knowledge of the gravitational constant. We will expand further on this before returning to look at the light-second mass from a quantum perspective.

### 3. Escape Velocity and Such Things as Time Dilation

Leibniz [19] already suggested the in 1688 that kinetic energy was given by  $mv^2$  a formula that “soon” was empirically confirmed by Gravesande [20] around 1720. We know today this should be corrected to  $E_k \approx \frac{1}{2}mv^2$  (ignoring relativistic effects, so valid for when  $v \ll c$ ). The escape velocity in Newton’s formula can be derived in the following way:

$$\frac{1}{2}\tilde{m}v_e^2 - \frac{\tilde{M}\tilde{m}}{R^2} \quad (10)$$

and when we solve with respect to  $v_e$ , this gives

$$v_e \approx \sqrt{\frac{2\tilde{M}}{R}} \tag{11}$$

We can also find expected gravitational time dilation by taking into account that the time of a clock at distance  $R_2$  must move faster than the clock at a distance of  $R_1$  ( $R_2 = R_1 + h$ , where  $h$  is the height about ocean level) from the center of the gravity object by:

$$\begin{aligned} \frac{T_2}{\sqrt{1-v_2^2}} &= \frac{T_1}{\sqrt{1-v_1^2}} \\ \frac{T_2}{\sqrt{1-\frac{2\tilde{M}}{R_2}}} &= \frac{T_1}{\sqrt{1-\frac{2\tilde{M}}{R_1}}} \\ T_2 &= T_1 \frac{\sqrt{1-\frac{2\tilde{M}}{R_2}}}{\sqrt{1-\frac{2\tilde{M}}{R_1}}} \end{aligned} \tag{12}$$

Assume the clock  $T_1$  is at sea level and clock  $T_2$  is 2,000 meters above sea level, which corresponds to  $r_1 \approx 6371000/c = 0.0212514$  light-seconds and  $r_2 = (6371000 + 2000)/c = 0.0212580$  light-seconds. For every second at the ocean level, following number of seconds will go by as observed from the mountain level:

$$T_2 = 1 \frac{\sqrt{1-\frac{2 \times 1.52 \times 10^{-11}}{0.0212580}}}{\sqrt{1-\frac{2 \times 1.52 \times 10^{-11}}{0.0212514}}} = 1.000000000000022 \text{ s} \tag{13}$$

which is the same as predicted by general relativity theory. The point is that here we have done it without any knowledge of  $G$ . What is even more important is our mass. The mass of the Earth, as we have said, is about  $1.52 \times 10^{-11}$  light-seconds. We can convert this to meters by multiplying by  $c = 299792458$  m/s. This means the mass of the Earth is  $1.52 \times 10^{-11} \times c = 0.0046$  m. This is actually half of the Schwarzschild radius of the Earth and identical to the Haug radius, which is no coincidence. From Newton's formula, one finds that the mass is the Haug radius of the Earth (when using length units linked to how far light travels in the arbitrary chosen time unit, here seconds). One gets the Haug radius by  $r_h = \frac{GM}{c^2}$ ;

however, modern physics has not recognized that half the Schwarzschild radius actually is a better definition of mass than the kilogram mass, but a new quantum gravity theory has taken advantage of this [21] [22]. Be aware that Michell [9] already, in 1784, got exactly the same radius for where the escape velocity was  $c$  as the much later Schwarzschild radius rooted in general relativity theory. So the Schwarzschild radius is not unique for general relativity theory [23]; they are the same.

## 4. Getting Down to the Quantum Level

Any rest-mass in terms of kg can be expressed as:

$$m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c} \quad (14)$$

where  $\hbar$  is the Planck constant,  $\bar{\lambda}$  is the reduced Compton length [24], and  $c$  is the well-known speed of light. This formula<sup>3</sup> can describe any rest-mass in terms of kg, including both subatomic and cosmological objects. The Planck constant is indeed a constant, and so is the speed of light. The only factor that differs between masses of different sizes (weights) is then the Compton wavelength of the mass. The Compton wavelength has only been measured for fundamental particles such as the electron. However, even larger masses that don't have their own Compton wavelengths still consist of a series of subatomic particles that must have Compton wavelengths. The Compton wavelengths of elementary particles are additive based on the following formula:

$$\bar{\lambda} = \sum_{i=1}^n \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \dots + \frac{1}{\lambda_n}} \quad (15)$$

This means that the Formula (14) can be used for composite masses and even astronomical objects like the sun or the moon. But what does the formula truly represent? The Planck constant is linked to the quantization of energy. Some will find it strange that the speed of light is embedded in the mass formula. We are all familiar with  $E = mc^2$ , but few physicists are familiar with the idea that the speed of light is integrated in the mass at a deeper level. This indicates something inside a fundamental particle, a mass, is linked to the speed of light, and also to composite masses, as they consist of fundamental particles. But how? Mass is known at the quantum level to be a wave-particle duality. But what exactly is a wave-particle duality? Newton assumed light consisted of indivisible particles; later, the view that light was a wave evolved from some experiments strongly indicating wave behaviour. Then Einstein introduced his photoelectric effect and again showed that light had particle-like properties, and light was re-defined as having a mystical wave-particle duality; not mystical in the terms of the math, but in terms of the interpretation of the math. Then Louis de Broglie [28] [29] suggested that matter, in addition to having particle-like properties, also likely had wave-like properties, and he suggested that the matter wave was given by the following formula  $\lambda_B = \frac{\hbar}{mv\gamma}$ , where  $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ . Einstein quickly

endorsed the idea, and some years later it was confirmed that masses such as electrons had wave-like properties; see [30] [31]. This was considered almost a

<sup>3</sup>This way of to describe the kilogram mass was possibly first described by Haug [25] [26] [27]. To express the kilogram mass,  $m = \frac{\hbar}{\bar{\lambda}} \frac{1}{c}$  is naturally simply the Compton wavelength formula solved with respect to  $m$ , but even if this is very easy to do, it has not, to our knowledge, actually been done before in these papers.



proof that the de Broglie hypothesis was rooted in reality. Next, in a series of steps, an entire quantum wave theory emerged from this line of thought, based on the important work of Heisenberg [32], Schrödinger [33], Klein Gordon, Pauli, and Dirac, among others. Further, the quantum mechanical theory fit experiments extremely well. And just before this development, gravity theory had evolved into Einstein's [34] general relativity theory. Since then, for more than 100 years, many of the world's most brilliant physicists have tried to unify gravity with quantum mechanics into a quantum gravity theory but without much success.

However, in our rest-mass formula,  $m = \frac{\hbar}{\lambda} \frac{1}{c}$  we do not have the de Broglie wavelength, but the reduced Compton wavelength;  $\bar{\lambda}$ . Compton was more of an experimental researcher than de Broglie and he had measured the wavelength of an electron around the same time that de Broglie had presented his hypothesis of the matter wave. That is, the Compton wavelength has been measured, at least indirectly. There is a very simple mathematical relation between the Compton wavelength and the de Broglie wavelength, namely  $\bar{\lambda} = \bar{\lambda}_B \frac{v}{c}$ . However, if  $v = 0$ , then the de Broglie wavelength is infinite [35] [36], or even mathematically undefined as it is not allowed to divide by zero ( $\lambda_B = \frac{\hbar}{mv\gamma} = \frac{\hbar}{m \times 0}$ ). An infinite

matter wave for a subatomic particle is, to put it mildly, a bizarre prediction. We will claim, as we have done in other papers [21], that the de Broglie wavelength is not a physical wavelength; it should be seen as a mathematical derivative of the true physical Compton wavelength. In short, the de Broglie wavelength is simply a mathematical artifact that is never needed. A theory built around the de Broglie wavelength will, in general, give a series of correct predictions, but the interpretations will often be absurd, as one has not discerned what matter is directly linked to the Compton wavelength and what is linked to the de Broglie wavelength. Why does modern physics have two different types of wavelengths for mass—one being the experimentally-observed Compton wavelength, the other being the hypothetical de Broglie wavelength? Well, this is a topic for another time.

Let's return to our mass definition in kg in terms of the Compton wavelength. The formula can be rewritten as:

$$\frac{\hbar}{\bar{\lambda}} \frac{1}{c} = \frac{\frac{c}{\bar{\lambda}}}{\frac{\hbar}{1 \times c}} \quad (16)$$

We can see that the kg of the mass in question is simply the Compton frequency of the mass in question divided by the Compton frequency of one kg. That is, the kg definition of mass at a deeper quantum level is a frequency ratio. At each Compton time we will claim there is a Planck mass event. Such Planck

mass events consist of two indivisible particles colliding. Such indivisible particles, when not colliding with other particles, move at the speed of light over the reduced Compton length. For example, an electron will then have the following number of Planck mass events per second:

$$f_e = \frac{c}{\lambda_e} \approx 7.76 \times 10^{20} \quad (17)$$

Each Planck mass event is  $10^{-8}$  kg, but the Planck mass event only lasts for one Planck time, so this gives a mass in kg for the electron of:

$$m_e = \frac{c}{\lambda_e} \approx 7.76 \times 10^{20} \times m_p t_p = \frac{\frac{c}{\lambda_e}}{\frac{c}{\hbar}} = 10^{-31} \text{ kg} \quad (18)$$

However, this mass definition that indeed is a collision ratio does not tell anything about how long each collision lasts; it disappears in the equation, as the Planck length will cancel out between the Planck mass in terms of kg and the Planck time. The standard kg definition of mass is a collision ratio, and that is all we need when working with most observable phenomena. An exception to this is gravity. Gravity is not some magical force; all mass is also gravity. That is, gravity is the collisions between the indivisible particles that exist in matter. The collision only lasts for a Planck time, as we can find from gravity observations. This is, however, not embedded in today's mass definition, and it must come from somewhere in the gravity models to make the gravity formulas predict correctly. This is where the gravity constant comes in. The so-called Newton's gravitational constant adds to the formula what is missing in the kg definition of mass. Luckily what is missing is only something that is constant, namely the Planck length, and we need to take something out of the kilogram mass, namely the Planck constant. The Planck constant is the units of energy relative to the collision ratio in a kg. That is, the Planck constant is the amount of energy in an indivisible particle in the form of a collision ratio where the collision ratio is relative to the collisions in one kg per second.

The quantum aspects of this theory and a unified quantum gravity theory are explained in much more detail in [21] [22] [37] [38]. Just as important is the fact that one can find the Planck length (and other Planck units such as the Planck time and the Planck mass) totally independently of any knowledge of  $G$ , see [39] [40] [41]. The Newton gravitational constant that Newton never invented or used is, at a deeper level, a composite constant of the form  $G = \frac{l_p^2 c^3}{\hbar}$  as described by Haug in some of the papers just mentioned, as well as in [25] [27] [42], something we soon will get back to in this paper.

## 5. The Newton Mass from a Quantum Perspective, the True Mass and the Newton God Particle

Let us look closer at what the "mysterious" mass we get out of the original New-

ton formula actually represents from a quantum particle perspective. The reason we use the term “God Particle” is simply because Newton called such particles so:

“... and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces, no ordinary Power being able to divide what God himself made one in the first Creation.” Isaac Newton, see full quote in the appendix.

With this, we think Newton indicated that the indivisible particle was the most fundamental of all particles. We will next show how we can measure important properties of this particle that we now have reasons to think are directly linked to the Planck scale.

The mass of the Earth, for example, we predicted (using Newton’s original formula) to be  $1.52 \times 10^{-11}$  Light-seconds. We believe that we can find the mass of Newton’s indivisible particle from this and claim it must be given by the following formula (a formula we have already shown is directly linked to the Planck length, [43])

$$\tilde{m}_i = \sqrt{\tilde{M} \bar{\lambda}} \quad (19)$$

where  $\bar{\lambda}$  is the reduced Compton wavelength of the Newtonian mass  $\tilde{M}$  of for example the Earth, (e.g. the gravity object of which we have observed the mass). How can we find the Compton wavelength of the Earth? We can measure the Compton wavelength of an electron without knowing the mass of the electron. The reduced Compton wavelength of an electron can be found by Compton scattering and it is about  $3.86 \times 10^{-13}$  m. Also be aware that the Planck constant is not needed for finding this, because we have:

$$\lambda_e = \frac{\lambda_{\gamma,2} - \lambda_{\gamma,1}}{1 - \cos \theta} \quad (20)$$

where  $\lambda_{\gamma,1}$  and  $\lambda_{\gamma,2}$  are the wavelength of the photon before and after it hit the electron, and  $\theta$  is the angle between the incoming and outgoing photon. In light-seconds, the reduced Compton wavelength ( $\bar{\lambda}_e = \lambda_e / (2\pi)$ ) of the electron is about  $1.28 \times 10^{-21}$  light-seconds. This can be measured without knowing the mass of the electron first, see also [44]. Further, the Compton wavelength of a proto can be found by simply checking the cyclotron frequency of a proton relative to an electron. The cyclotron frequency is given by:

$$f = \frac{qB}{2\pi m} \quad (21)$$

where  $q$  is the charge of the particle, and  $B$  is the magnetic field, and  $m$  is the mass of the particle. Since protons and electrons have the same charge, we must have:

$$\frac{f_e}{f_p} = \frac{\frac{qB}{2\pi m_e}}{\frac{qB}{2\pi m_p}} = \frac{m_p}{m_e} = \frac{\bar{\lambda}_e}{\bar{\lambda}_p} \approx 1836.15 \quad (22)$$

The well-known (measured) cyclotron frequency ratio [45] [46] is about 1836.15247, so the reduced Compton wavelength of the proton is simply the measured reduced Compton wavelength of the electron divided by the cyclotron frequency ratio, that is  $\bar{\lambda}_p = \bar{\lambda}_e \frac{f_e}{f_p} \approx \frac{\bar{\lambda}_e}{1836.15247}$ . Interest in the Compton wavelength of the proton goes back to at least 1958 and has recently garnered more interest; see [47] [48]. Now we just need to know the number of protons (assuming neutrons have same mass or do we need to make a slight adjustment for this) in the Earth, which we could count hypothetically, even if this is impossible directly in practice, but we will soon look at indirect methods to do so. In any case, there are about  $3.57 \times 10^{51}$  protons in the Earth (we assume neutrons have approximately the same mass as protons). In addition, there would be a small adjustment for binding energy, the nuclear binding energy, and the bond energy that keeps atoms together, but that is so small compared to the rest-mass energy of the atoms that it will not make much of a difference in the predicted Compton wavelength. The reduced Compton length of the Earth is then given by:

$$\begin{aligned} \bar{\lambda}_E &= \sum_{i=1}^n \frac{1}{\frac{1}{\lambda_1} + \frac{1}{\lambda_2} + \frac{1}{\lambda_3} + \dots + \frac{1}{\lambda_n}} = \frac{1}{3.57 \times 10^{51} \times \frac{1}{\bar{\lambda}_e / 1836.15}} \quad (23) \\ &= 1.96 \times 10^{-76} \text{ light-seconds} \end{aligned}$$

The mass of Newton's indivisible particle we can now calculate by:

$$\tilde{m}_i = \sqrt{\tilde{M} \bar{\lambda}} = \sqrt{1.52 \times 10^{-11} \times 1.96 \times 10^{-76}} \approx 5.46 \times 10^{-44} \text{ light-seconds} \quad (24)$$

Some will recognize this number; it is the Planck time, which is  $5.46 \times 10^{-44}$  seconds. This is the case because we have chosen seconds as our time scale but remember this is also directly linked to our length scale. The ultimate subatomic mass is a collision between two indivisible particles; this collision lasts for approximately  $5.46 \times 10^{-44}$  seconds. Our interpretation is that two indivisible particles spend this amount of time in collision (standing still) during the period in which one non-colliding indivisible particle (moving at the speed of light) travels a distance equal to the Planck length, that has a distance of  $5.46 \times 10^{-44}$  light seconds (or approximately  $1.61 \times 10^{-35}$  m). This is explained in more detail, but from a slightly different perspective, in our two collision space-time unified quantum gravity papers, see [21] [38].

Keep in mind, we never relied on the so-called Newton gravitational constant (that Newton never invented) that was invented to fit the arbitrary kilogram mass, and the mass definition of kg (pounds); as we have said, at a deeper level the kilogram mass is just a collision ratio. Nor do we need the Planck constant to find the Planck time [49]. One can mistakenly think this is only theory as it seems impossible to directly count the number of protons in the Earth. Still, we can do this indirectly. This is when a Cavendish apparatus comes in handy. Here, we can start out by finding the Newton gravitational mass of a small practical

mass like a lead ball, given by:

$$\tilde{M}_c = \frac{2\pi^2 R^2 L \theta}{T^2} \quad (25)$$

where  $\tilde{M}_c$  is the Newton mass of one of the large balls in the Cavendish apparatus and  $L$  is the distance between the smallest balls in the Cavendish apparatus, and  $R$  is the distance from the centre of the small ball to the centre of the larger ball, and  $\theta$  is the angle of deflection (in radians), and  $T$  is the oscillation time. This formula is only valid when  $c = 1$ ; otherwise one must divide it by  $c^3$ .

To find the Compton wavelength of the ball in the Cavendish apparatus, we can count the number of protons in that object; this is also a challenge, but is fully possible; see [50] [51] [52]. When we know the Newton mass (light-seconds) of the ball, we can easily find the Newton mass of the Earth relative to that. Also, if we know the Compton wavelength of the mass in the Cavendish apparatus (by counting atoms in it as described above), then we can find the reduced Compton wavelength of the Earth from the following equation:

$$\bar{\lambda}_E = \frac{\tilde{M}_c}{M_E} \bar{\lambda} \quad (26)$$

where  $\bar{\lambda}$  is the reduced Compton wavelength of the sphere in the Cavendish apparatus. We could also have found the reduced Compton wavelength of the Earth simply by using the Compton formula:

$$\bar{\lambda}_E = \frac{\hbar}{M_E c} \quad (27)$$

but then we need to know the Planck constant, and part of our purpose is to demonstrate we need fewer constants than in standard physics when understanding gravity and physics from a deeper perspective.

We also have that:

$$\tilde{M} = \frac{G}{c^3} M = \frac{l_p^2 c^3}{\hbar c^3} \times \frac{\hbar}{\lambda} \frac{1}{c} = \frac{l_p}{c} \frac{l_p}{\lambda} \quad (28)$$

which is the collision time of that mass over the shortest possible time interval it can be observed, as described by Haug in his unified quantum gravity theory [53]. Again, the collisions between indivisible particles last only for the Planck time; this is given by  $t_p = \frac{l_p}{c}$ , and multiplied by how often these collisions happen  $\frac{l_p}{\lambda}$ . The part  $\frac{l_p}{\lambda}$  can also be seen as a frequency probability if  $l_p < \bar{\lambda}$ , when observed over the shortest possible time interval, which is the Planck time. Be aware that for anything that has been measured in relation to the Newton formula, one of the masses in the derivations for what one wants to predict will always cancel out; we are always operating with just  $GM$  in any observable prediction and never  $Gmm$ . Modern physics appears to have missed the point that the invented  $GM$  is actually identical to the mass in the original Newton formula. That  $GM$  is the Newtonian mass holds when we have linked length and time

through the speed of light, e.g., when  $c = 1$ . When we have units such that  $c \gg 1$ , then the collision time mass is given by  $\tilde{M} = \frac{l_p l_p}{c \lambda} = \frac{GM}{c^3} = \frac{1}{2} \frac{r_s}{c}$ . In the special case  $c = 1$  we naturally get  $\tilde{M} = \frac{1}{2} r_s$ . When  $c \gg 1$  it is interesting to note that we also have<sup>4</sup>  $l_p = \sqrt{\frac{1}{2} r_s \lambda}$ .

Back to the gravity constant  $G$ ; why on earth would the universe invent something that is length cubed divided by time and kg (the output units of  $G$ ). Of course, the universe never invented such a thing. Modern physics invented a gravity constant to fit a misinterpreted mass view of Newton's formula, which was needed to get physicists' ill-specified mass model to fit experiments. Newton never mentioned a gravitational constant himself. He calculated relative masses based on orbital time squared (and adjusted for distance between the gravity objects; that is, the masses.).

## 6. The So-Called Newton's Gravity Constant $G$ Is Just a Composite Constant Needed to Fix the Incomplete Kilogram Mass

In 1984, Cahill [54] already suggested that the Newtonian gravity constant could perhaps be a composite constant of the form  $G = \frac{\hbar c}{m_p^2}$  and that the Planck units could be more fundamental; in other words, simply solving the Planck mass formula  $m_p = \sqrt{\frac{\hbar c}{G}}$  with respect to  $G$ . However, in 1987, Cohen [55] pointed out that if one needs  $G$  to find the Planck units this will simply lead to a circular problem, so it seemed one needed to know  $G$ . This is the main view among most researchers to this day, and has been repeated as late as 2016 in an interesting paper by McCulloch [56]. However, in recent years, we have had a breakthrough in understanding the Planck units. We can now extract the Planck length and Planck time from a series of gravity observations without any prior knowledge of  $G$ ,  $c$ , and  $\hbar$ , see [40] [41] [57]. There also exist other suggestions for how to get the gravity constant from such things as its hypothetical relation to electromagnetic constants to suggestions of how to extract  $G$  from cosmological constants; see, for example, [58] [59] [60]. However, here we will focus on expressing the gravity constant from the Planck units as this seems to lead to a significant step forward in understanding gravity. Some of these approaches are actually closely related when they are compared carefully; see [61]. In 2016 we [42] suggested this to express  $G$  as a composite constant of the form:

$$G = \frac{l_p^2 c^3}{\hbar} \quad (29)$$

This is nothing more than solving the Planck length formula of Max Planck

<sup>4</sup>As first described by [43].

with respect to  $G$ . Back then, I had also not been able to yet solve the circular problem. That is, we had not yet found a way to find  $l_p$  or other Planck units independent of  $G$ . A year later, Haug [39] solved the circular problem for the first time, so  $G$  can indeed be expressed as a composite constant and the Planck length can be found independent of any knowledge of  $G$ . Later on, we showed  $l_p$  can be found independent of any prior knowledge of  $G$ ,  $c$  and  $\hbar$ . See, for example, [62].

Still, it is first when one combines this composite view of  $G$  with the idea that any kilogram mass can be expressed as  $m = \frac{\hbar}{\lambda} \frac{1}{c}$ , one gets a real breakthrough in the understanding of gravity. All observable gravity phenomena rooted in today's standard gravity theory contain  $GM$  and not  $GMm$ . The 1873, modified, Newton gravity formula indeed contains  $GMm$ , but the small mass  $m$  always cancels out in derivations of formulas that can be used to predict observable gravity phenomena, and can thereby be checked with observations. This is for observable gravity phenomena where the small mass  $m$  has insignificant gravitational impact relative to  $M$ ; in other words, when we have  $m \ll M$ . For real two body problems where both masses are significantly large relative to each other to have significant impact the gravity parameter is  $\mu = G(M_1 + M_2) = GM_1 + GM_2$ , so then one multiplies both the kilogram masses with the gravity constant.

One can ask why it is necessary to always multiply the mass with  $G$  when used it for gravity. At a superficial level, this is simply how we have to calibrate the gravity formula for it to be useful for predictions. First, we must find the value of  $G$  from one gravity observational phenomena and then we can use the same  $G$  to predict other types of things related to gravity that we can observe. In other words,  $G$  seems to be a constant; it is an empirically-observed or calibrated constant, not a derived constant, or something understood from a very deep perspective. The physics' community has no idea what  $G$  truly represents, or exactly why it is there. In 1961, Thüring [63] concluded that  $G$  had been inserted quite ad hoc and that it is not clear how it is related to the physical nature. In our view,  $G$  contains something missing in the model. When one introduced the kilogram mass, something was missing in the formula  $F = Mm/R^2$  so one had to multiply it by an unknown constant and get  $F = GMm/R^2$ . The constant  $G$  was unknown and had to be found by calibration to observable data. It then worked, but no one knew exactly why, because they had, and still have, no knowledge of why exactly  $G$  must be included and what it represents at a deeper level. This we can first really understand when we multiply  $G$  in the composite form,  $G = \frac{l_p^2 c^3}{\hbar}$ , with the kilogram mass. This gives:

$$GM = \frac{l_p^2 c^3}{\hbar} \times \frac{\hbar}{\lambda} \frac{1}{c} = c^3 \frac{l_p}{c} \frac{l_p}{\lambda} \quad (30)$$

That is, the Planck constant in the kilogram mass cancels out with the Planck constant embedded in  $G$ , so to calculate  $GM$  we need less information than to

find  $G$  and  $M$  separately. In our view,  $G$  is needed to get  $\hbar$  out of the kilogram mass and  $l_p^2$  into the mass. Further:

$$GM = c^3 \frac{l_p}{c} \frac{l_p}{\lambda}$$

can be seen as a gravity constant  $c^3$ , multiplied by a new mass definition  $\tilde{m} = \frac{l_p}{c} \frac{l_p}{\lambda}$ , which we have called collision-time mass.

This means the Newton gravity force formula can be described as:

$$F = c^3 \frac{\tilde{M}\tilde{m}}{R^2} \quad (31)$$

This force formula does not give the same output units as the 1873 version of the Newton formula, as its output unit is  $\text{m}\cdot\text{s}^{-1}$  versus the 1873 formula's output that gives  $\text{m}\cdot\text{kg}\cdot\text{s}^{-2}$ , so one could mistakenly think there must therefore be something very wrong with our newly-suggested gravity force formula, see also [41]. The thing is that the Newton's gravity force is never observed, and neither is the force coming from the 1873 formula. What is observable is when the small  $m$  has canceled out from the formula through derivations of predictions of observable gravitational phenomena. The new gravity force formula is simple and gives exactly the same predictions and also the same output units as the 1873 formula, and in the special case of setting  $c = 1$ , the new formula is the original Newton formula:  $F = \frac{\tilde{M}\tilde{m}}{R^2}$ .

**Table 1** shows the original Newton formula as well as observations we can derive from it, in addition to the modified Newton version of 1873, which has the gravity constant  $G$ . The two formulas, at a deeper level, predict exactly the same for observable phenomena. However, the Newton formula is simpler, requires fewer constants and is much more intuitive. If two theories are identical in predictions, then the simplest theory should win. In the original Newton formulation, we are totally independent of the value of  $\hbar$ , so this is not simply setting  $G = c = \hbar = 1$ , in the original Newton formula  $F = \frac{\tilde{M}\tilde{m}}{R^2}$  all that is set to 1 is  $c$ .

## 7. The Uncertainty in Measurement of $G$ Is It Still Relevant

We have demonstrated in this paper that Newton never invented nor used the so-called Newton gravitational constant  $G$ . Further, from **Table 1**, it is clear  $G$  is not needed to predict any observable gravitational phenomena. The Newtonian gravitational constant introduced in 1873 is needed when one uses the incomplete kilogram definition of mass, to fix that kilogram mass into a gravitational mass. The kilogram mass is not in line with Newton's thought that matter ultimately consists of indivisible particles, which recent research strongly indicates are linked to the Planck length and Planck time.

It is well known that there is a large uncertainty in the measurement of the Newtonian gravitational constant compared to most other physical constants. See, for example [64]-[69]. However, it would be a misunderstanding to think



**Table 1.** In the first formula column, the table shows what is rooted in the 1873 modified Newton theory, and the second formula column shows the original Newton formula. In addition, we show what both the 1873 framework and the original Newton framework means at the deepest level, where both theories are identical, except in the original Newton theory  $c = 1$ . Further, pay attention to the fact that all observable gravity phenomena are linked to  $GM$  and not  $GMm$  in the 1873 modified Newton gravity theory and only to  $\tilde{M}$  in the original Newton theory, rather than  $\tilde{M}\tilde{m}$ .

Non observable (contains $GMm$ or $\tilde{M}\tilde{m}$ )		
	1873 modified Newton and forward:	“Original” Newton:
Gravity force	$F = G \frac{Mm}{R^2} (\text{kg} \cdot \text{m} \cdot \text{s}^{-2})$	$F = c^3 \frac{\tilde{M}\tilde{m}}{R^2} = \frac{\tilde{M}\tilde{m}}{R^2} (\text{m} \cdot \text{s}^{-1})$ when $c = 1$
Mass must be	$M = \frac{\hbar}{\lambda_M} \frac{1}{c} (\text{kg})$	$\tilde{M} = l_p \frac{l_p}{\lambda_M}$ (collision time, see [21])
Gravitational constant	$G, \left( G = \frac{l_p^2 c^3}{\hbar} \right)$	$c = 1$
<b>Observable predictions, identical for the two methods: (contains only <math>GM</math>)</b>		
Gravity acceleration	$g = \frac{GM}{R^2} = \frac{c^2}{R^2} \frac{l_p^2}{\lambda_M}$	$g = \frac{\tilde{M}}{R^2} = \frac{1}{R^2} \frac{l_p^2}{\lambda_M}$
Orbital velocity	$v_o = \sqrt{\frac{GM}{R}} = cl_p \sqrt{\frac{1}{R\lambda_M}}$	$v_o = \sqrt{\frac{\tilde{M}}{R}} = l_p \sqrt{\frac{l_p}{R\lambda_M}}$
Orbital time	$T = \frac{2\pi R}{\sqrt{\frac{GM}{R}}} = \frac{2\pi\sqrt{\lambda} R^3}{cl_p}$	$T = \frac{2\pi R}{\sqrt{\frac{\tilde{M}}{R}}} = \frac{2\pi\sqrt{\lambda} R^3}{l_p}$
Velocity ball Newton cradle	$v_{out} = \sqrt{2 \frac{GM}{R^2} H} = \frac{cl_p}{R} \sqrt{\frac{H}{\lambda}}$	$v_{out} = \sqrt{2 \frac{\tilde{M}}{R^2} H} = \frac{l_p}{R} \sqrt{\frac{H}{\lambda}}$
Periodicity Pendulum (clock)	$T = 2\pi \sqrt{\frac{L}{g}} = 2\pi R \sqrt{\frac{L}{GM}} = \frac{2\pi R}{cl_p} \sqrt{L\lambda}$	$T = 2\pi \sqrt{\frac{L}{g}} = T = 2\pi R \sqrt{\frac{L}{\tilde{M}}} = \frac{2\pi R}{l_p} \sqrt{L\lambda}$
Frequency Newton spring	$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} = \frac{1}{2\pi R} \sqrt{\frac{GM}{x}} = \frac{cl_p}{2\pi R} \sqrt{\frac{1}{\lambda x}}$	$f = \frac{1}{2\pi} \sqrt{\frac{k}{\tilde{m}}} = \frac{1}{2\pi R} \sqrt{\frac{\tilde{M}}{x}} = \frac{l_p}{2\pi R} \sqrt{\frac{1}{\lambda x}}$
Gravitational red-shift	$z = \frac{\sqrt{1 - \frac{2GM}{R_1 c^2}}}{\sqrt{1 - \frac{2GM}{R_2 c^2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p^2}{R_1 \lambda_M}}}{\sqrt{1 - \frac{2l_p^2}{R_2 \lambda_M}}} - 1$	$z = \frac{\sqrt{1 - \frac{2\tilde{M}}{R_1}}}{\sqrt{1 - \frac{2\tilde{M}}{R_2}}} - 1 = \frac{\sqrt{1 - \frac{2l_p^2}{R_1 \lambda_M}}}{\sqrt{1 - \frac{2l_p^2}{R_2 \lambda_M}}} - 1$
<b>Observable predictions (from GR): (contains only <math>GM</math> or only <math>\tilde{M}</math>)</b>		
Time dilation	$T_R = T_f \sqrt{1 - \frac{2GM}{R}} / c^2 = T_f \sqrt{1 - \frac{2l_p^2}{R\lambda_M}}$	$T_R = T_f \sqrt{1 - \frac{2\tilde{M}}{R}} / c^2 = T_f \sqrt{1 - \frac{2l_p^2}{R\lambda_M}}$
Gravitational deflection (GR)	$\delta = \frac{4GM}{c^2 R} = \frac{4}{R} \frac{l_p^2}{\lambda_M}$	$\delta = \frac{4c^3 \tilde{M}}{c^2 R} = \frac{4}{R} \frac{l_p^2}{\lambda_M}$

**Continued**

Advance of perihelion	$\sigma = \frac{6\pi GM}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)} \frac{l_p^2}{\lambda_M}$	$\sigma = \frac{6\pi\tilde{M}}{a(1-e^2)c^2} = \frac{6\pi}{a(1-e^2)} \frac{l_p^2}{\lambda_M}$
<b>Quantum analysis:</b>		
Constants needed	$G, \hbar$ , and $c$ or $l_p, \hbar$ , and $c$	$l_p$ and indirectly $c$ , but $c=1$
Variable needed	one for mass size	one for mass size

we are getting away from this uncertainty after we have got rid of  $G$ . So, this paper is not about improving or getting rid of this uncertainty. From a deeper perspective, the uncertainty in  $G$  ultimately comes from uncertainty in measurements of the Planck length. This also explains why the uncertainty in  $G$  is so large compared to in what has been found in most other physical constants. The reason is that the Planck length is the shortest possible observable length, and it is therefore not so strange that it is hard to measure it accurately when it is the smallest of all things there are.

The standard uncertainty in the gravity constant  $G$  is exactly twice that of the standard uncertainty in the Planck length. Just as an illustrative example, assume the measured standard uncertainty in the Planck length is 1%, then relative uncertainty in the gravitational constant must be:

$$\frac{\partial G}{\partial l_p} \frac{l_p}{G} = 2\% \tag{32}$$

NIST 2018 CODATA states the one standard deviation uncertainty in the gravity constant is given by  $2.2 \times 10^{-5}$ , and for the Planck length the one standard deviation uncertainty is given as  $1.1 \times 10^{-5}$ . This perfectly matches our view that the standard uncertainty in the Newton gravity constant is exactly twice of that of the Planck length. But since where we have  $G$  in the 1873 Newtonian framework we have  $l_p^2$  embedded, this since  $G = \frac{l_p^2 c^3}{\hbar}$  as understood from a deeper level, then the standard uncertainty in the gravity observations are the same as before. Bear in mind that  $c$  and  $\hbar$  are defined as exact constants so they do not add to any uncertainty in  $G$  or in gravitational observations, because  $\hbar$  also cancels out for any observable gravity phenomena. All these studies, which try as accurately as possible to find the value of the gravitational constant, can be seen as simply methods to find an accurate value of the Planck length, even if the researchers looking into measuring  $G$  are not aware of this. They are of the view that the Planck length only can be found after one has found  $G, c$  and  $\hbar$  through dimensional analysis. In recent years, we have demonstrated how to find the Planck length and Planck time independent on any knowledge of  $G$  and  $\hbar$  and even of  $c$ . Still, these experiments, trying to accurately measure  $G$ , are just as relevant as before, but it is the uncertainty in  $l_p^2$  the experimenters are looking at,

without knowing so. It could be that when this becomes widely known, one could devise even more accurate ways to measure  $l_p^2$ , but this only time can tell. We do not claim to know any new ways to measure  $G$  more accurately than before.

From **Table 1**, we see that both the standard 1873 Newtonian formalization, as well as the original Newtonian formulation that is without  $G$ , when understood from a deeper perspective, contain the same two constants for prediction of all observable phenomena; that is,  $l_p$  and  $c$ .

Still, our insight that we do not need  $G$  to make gravity predictions is not an argument to reduce the uncertainty in gravity measurements. It is an argument for the possibility to understand gravity through deeper and simpler principles. It is also an argument to reduce the number of universal constants from  $G$ ,  $h$  and  $c$  to just  $l_p$  and  $c$ .

## 8. Conclusion

As we have seen, it is by using Newton's original formula that we obtain the correct unit measure of mass. The kg definition of mass is a manmade, arbitrary unit of mass that has caused great confusion in modern physics. The kg definition and similar manmade arbitrary units (such as the pound) are why the gravitational constant had to be invented. Nature does not work in kg; it has its own, more fundamental units. Arbitrary incomplete units have added an unnecessary layer of complexity to modern physics, and Newton's original theory is superior in many ways. Naturally, the theory was not complete in terms of quantum mechanics and relativity theory. However, if the field of physics had stayed with Newton's original formula, it is possible that a full understanding of mass and a unified quantum theory might have been developed much earlier.

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## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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## Appendix: Some Quotations from Newton

Below are some quotations from Newton on gravity

*If there be several bodies consisting of equal particles whose forces are as the distances of the places from each, the force compounded of all the forces by which any corpuscle is attracted will tend to the common centre of gravity of the attracting bodies, and will be the same as if those attracting bodies, preserving their common centre of gravity, should unite there, and be formed into a globe.*  
p 236

*I say, that the whole force with which one of these spheres attracts the other will be reciprocally proportional to the square of the distance of the centres. The force with which one of these attracts the other will be still, by the former reasoning, in the same ratio of the square of the distance inversely. Cor. 3. The motive attractions, or the weights of the spheres towards one another, will be at equal distances of the centres as the attracting and attracted spheres conjunctly; that is, as the products arising from multiplying the spheres into each other. p. 223.*

*Cor.2 The force of gravity towards several equal particles of any body is reciprocally as the square of the distance of the places of the particles. p. 393.*

*Cor.2 The force of gravity which tends to any one planet is reciprocally as the square of the distance of places of that planet's center. p. 393.*

*That all bodies gravitate towards every planet, and that the weights of bodies towards any the same planet, at equal distances from the centre of the planet, are proportional to the quantities of matter which they severally contain. p. 394, book 3.*

*If two spheres mutually gravitating each towards the other, if the matter in places on all sides round about and equidistant from the centres is similar, the weight of either sphere towards the other will be reciprocally as the square of the distance between their centres.*

*Wherefore the absolute force of every globe is as the quantity of matter which the globe contains, but the motive force by which every globe is attracted towards another, and which, in terrestrial bodies, we commonly call their weight, is as the content under the quantities of matter in both globes applied to the square of the distance between their centres (by Cor. IV, Prop. LXXVI), to which force the quantity of motion, by which each globe in a given time will be carried towards the other, is proportional. And the accelerative force, by which every globe according to its quantity of matter is attracted towards another, is as the quantity of matter in that other globe applied to the square of the distance between the centres of the two (by Cor. II, Prop. LXXVI): to which force, the velocity by which the attracted globe will, in a given time, be carried towards the other is proportional.*

*That there is a power of gravity tending to all bodies, proportional to the several quantities of matter which they contain. p. 397.*

Newton only uses the word "mass" once in his book:

*The quantity of matter is the measure of the same, arising from its density and bulk conjunctly. It is this quantity that I mean hereafter everywhere under the name of body or mass.*

In other words, mass is the quantity of matter.

In the Principia, Newton is also clear on the idea that the smallest particles of all bodies have spatial extension and are hard (indivisible) and can move. And he follows up with the comment, “And this is the foundation of all philosophy.”

*Since every particle of space is always, and every indivisible moment of duration is everywhere, certainly the Maker and Lord of all things cannot be never and nowhere.* p. 505.

*And thence we conclude the least particles of all bodies to be also extended, and hard and movable, and endowed with their proper vires inertia. And this is the foundation of all philosophy.*

In his book Optica, Newton is even clearer that he think matter consists of fully-hard forever-lasting particles; that is, indivisible particles:

*All these things being consider'd it seems probable to me, that God in the Beginning form'd Matter in solid, massy, hard, impenetrable, movable Particles, of such Sizes and Figures, and in such Proportion to Space, as most conduce to the End for which he form'd them; and that these primitive Particles being Solids, are incomparably harder than any porous Bodies compounded of them; even so very hard, as never to wear or break in pieces, no ordinary Power being able to divide what God himself made one in the first Creation. While the Particles continue entire, they may compose bodies of one and the same Nature and Texture in all Ages, But should they wear away, or break in pieces, the Nature of Things depending on them, would be changed. Those minute rondures, swimming in space, from the stuff of the world: the solid, coloured table I write on, no, less than the thin invisible air I breathe, is constructed out of small colourless corpuscles; the world at close quarters looks like the night sky — a few dots of stuff, scattered sporadically through and empty vastness. Such is modern corpuscularianism.*

There are many more references showing that Newton believed that the smallest particles were indivisible, even though he also said it would be hard to prove. This seems to be a view he held from the time of his unpublished notebook, to his published works Opticks and Principia. He wrote more about this in unpublished draft versions than he did in published versions. Keep in mind that even to talk about atomism had been forbidden in most of Europe for hundreds of years. Giordano Bruno was burnt at the stake in 1600 mainly for talking openly about atomism. As another example of the suppression and persecution taking place in that era, in 1624 the Paris Parliament decreed that a person maintaining or teaching atomism would be liable for the death penalty. Lancelot Law Whyte, who claimed to have worked with Albert Einstein on the unified field theory, noted,

*The aggressive rise of physical atomism as an adequate explanation of the universe ... provoked a crusade (1660-1700) against it.*



In addition, recent research has shown that the Galileo affair may have been related to the fact that he openly talked about atomism; see [70]. For example, in the late 1680s, the Holy Office ordered local inquisitors to refuse to licence books which stated that: “*substantial composites are not made by matter and form but by atoms or corpuscles*” [71].

In England, the climate for discussing atomism was slightly more relaxed than in continental Europe, but even Newton probably had to be careful, especially if he was considering the possibility of visiting France, for example.

# The Concept of Time: A Grand Unified Reaction Platform

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## Abstract

The universe is things which change and called events. The events are matter and field. A boundary divides a system to things and environment. The things which belong to the environment have no significant effect on the things which belong to the system. The physical observables are the variations of things and it is always assumed that the conscious thing is placed in environment because the science cannot explain consciousness. There is not only an obligated minimum boundary between things (space) but also between past and future (present). The gravitational field has significant effect on these obligated minimums, especially at Planck scale. By using the above concept we introduce a grand unified reaction platform for categorizing the current physical paradigms and possible future explanation of the universe as a whole.

## Keywords

Spacetime, Planck Scale, Gravity, Universe, Conscious

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## 1. Introduction

From many years ago, people have been familiar with the concepts of matter and field. Einstein have explored the relativistic mechanics and shown that matter can convert to energy and vice versa. By exploring the quantum field theory, people had attributed specific quanta to each field. During the last century, people had tried to unify the existed different physical theories. Standard model unifies the matter and all known different fields, except gravitational field. Some scientists believe that abortiveness in adding the gravitational field to standard model is referred to the concept of time [1] [2]. Nowadays, there are two different answers to the question about the reality of time. We chose some important

statements of each answer and rewrite them in **Table 1**.

However, Marchesini have evaluated the Barbour's and Smolin's answers through the lens of Henri Bergson's metaphysics of time [3]. He has explained how both ideas encounter with some paradoxes and in consequence run into dead ends [3]. The some chosen statements of Bergson's idea are rewritten in **Table 2** [3] [4] [5].

In this paper, we intend to show how to define certain boundaries between physics in different dimensions, through which a conceptual shift paradigm occurs. Following this explanation, we intend to introduce a reference platform based on which the grand unified theory may be described and explained. Therefore, we review some current different concepts of time, firstly. Then, we explain how a conceptual shift paradigm happens when we consider the universe as a whole instead of a part of the universe. Also, we explain how some specific concepts, such as the level of awareness (LoA), obligated minimum boundary between things and obligated minimum boundary between past and future can help us in planning a grand unified reaction platform (GURP) for explaining the different concepts of time and thinking about the different physical paradigms. The structure of article is as follows: in Section 2, the different current concepts of time are reviewed. The concept of the grand unified reaction platform is explained in Section 3. The summary is provided in Section 4.

**Table 1.** Two different answers to the question about the reality of time.

Professor Barbour [1]	Professor Smolin [2]
The change of things is time.	Time is the most real aspect of our perception of the world.
Time is simply a complex of rules that govern the change.	Non-causal children time is the most real aspect of our perception of the world.
Time is inferred from things.	Space is emergent and approximate.
Time is in the instant.	Laws of nature evolve in time.

**Table 2.** Marchesini's evaluation of Barbour's and Smolin's answers [3] [4] [5].

Marchesini's evaluation through the lens of Henri Bergson's metaphysics of time [3]
Things precede nothingness. Creative order precedes disorder.
Movement and change precede inertia and immobility.
Time must not be confused with space; to pass from one to the other one had only to change a single word: juxtaposition was replaced by succession.
As time passes, existence is merely added to what was already possible.
Our states of consciousness are continuous, indivisible, and interpenetrate one another.
The whole of the universe moves and changes much like our conscious.

## 2. Different Concepts of Time

There are different explanations about the concept of time. In below, we choose and review the most important ones.

### 2.1. Classical Concept of Time

If we assume that the world contains things and time ( $t$ ) flows in the world then we can conclude that things change in time, have volume and occupy the space ( $\bar{x}$ ). Therefore, time and space are different concepts. Since, the time is considered as a flux, we should divide the time axis to three different sections, called past, present (now), and future. Now, if  $\Psi(\bar{x})$  stands for the state of a thing; its variation in time (its evolution) *i.e.*,  $\Delta\Psi(\bar{x})/\Delta t$  will provide us the future state of the thing. If we know the past of a thing (called boundary conditions or history in general) we will be able to find its future. It means that we receive the information from the past of things and in consequence we called  $\Psi(\bar{x})$  the retarded state of things. It means that we receive the information with some delays. Therefore, the world is like a big mechanical machine which works deterministically, based on the classical laws. It should be noted that if one knows the future state ( $\Psi(\bar{x})$ ); he/she will be able to find its past by reversing the axis of time and solving the deterministic classical equations (of course, if dissipative terms do not exist). It means that the future information is built by the past information. In this case, we called  $\Psi(\bar{x})$  the advanced state of things. It means that the future information comes from the past. In the point of view, the evolution of things is absolute and is not relativistic.

But, we can rewrite the above scenario in a new style. We can assume that the world contains things and they move along their world line,  $\gamma$ , in the world. The world lines are made by the existence of the thing, their interaction with gravitational field and their relative evolution. They fill the entire world. Therefore the state of thing ( $\Psi(\bar{x}, t)$ ) is confined to change on its world line. Here, the time ( $t$ ) has no physical meaning and can be changed, freely but things occupy the spacetime  $(\bar{x}, t)$ . For calculating the time one should study the state of a clock (thing) when it moves along its world line. In the point of view, the evolution of things is relativistic and is not absolute. It means that there is no difference between present, past, and future [2]. It should be noted that since the time ( $t$ ) can be changed freely, one is able to disappear it from the classical Hamiltonian-Jacobi formulation and find a time independent equation such as Wheeler-Dewitt (WdW) equation. Simply speaking, the WdW equation says  $\hat{H}\Psi = 0$ , where  $\hat{H}$  is the Hamiltonian constraint in quantized general relativity and  $\Psi$  stands for the wave function of the universe. Unlike ordinary quantum field theory or quantum mechanics, the Hamiltonian is a first class constraint on physical states. We also have an independent constraint for each point in space. Therefore, WdW equation does not describe a frozen world [6].

Finally it can be concluded that, all other observable quantities change in space by passing the time or change in spacetime. Basically in the above both

point of views, the world contains things and they change absolutely or relative to each other. It means that the other physical concepts are defined (confined) by the concept of time and space.

## 2.2. Quantum Concept of Time

Although, the Schrödinger equation is a quantum mechanical equation but it evolves in classical space when time is considered as a flux. Dirac and Klein-Gordon equations have been found by using the relativistic laws but both evolve in spacetime. On the right-hand side of the Heisenberg uncertainty relation between space coordinate and momentum of a particle, the time parameter ( $t$ ) does not appear. It means that the priority is not important. Therefore, we encounter the classical concept of time or spacetime (called background dependent). By considering the Schrodinger, Dirac, Klein-Gordon, Yang-Mills and so on as a field function ( $\Psi(\vec{x}, t)$ ) and quantizing them by using the second quantization methods, the term  $\delta(t-t')$  appears on the right-hand side of the Heisenberg uncertainty relation [7]. Therefore, the priority is important. But similar to the previous case, we encounter the background dependent case, again.

Also, we know that there is an uncertainty relation between energy and time in quantum mechanics which is  $\Delta E \Delta t \geq \hbar/2$  and  $(\Delta E(t))^2 = \langle E(t)^2 \rangle - \langle E(t) \rangle^2$ . It means that the change in time is connected to the change in energy [2]. For understanding the relation let us consider an experiment. An electron, which is in its ground state, is excited to a higher energy level and after passing sometimes it comes back to the ground state. It can be shown that the staying time in the excited state ( $\Delta t$ ) is proportional to the inverse of the energy difference between the excited and ground states ( $\Delta E$ ) [8]. But, we are in the classical atmosphere of time else. Also, we can consider a particle with mass  $M$  and energy  $E = Mc^2$ , where  $c$  is the velocity of light. We can localize the particle in a sphere with radius  $R \sim MG/c^2$  where  $G$  is the gravitational constant. Using the Heisenberg uncertainty principle, it can be shown that it is not possible to localize anything with a precision better than the Planck length which is equal to  $10^{-33}$  cm [9] [10]. It means that anything smaller than the Planck length is hidden inside its on mini-black hole [10]. This is called the length (Planck) scale. By using the Planck length, the minimum time can be calculated and called the Planck time which is equal to  $10^{-44}$  seconds. These are the granules of quantum gravity [11]. Since, the quantum spacetime is a physical object and fluctuates; it can be in a superposition of different configurations (spacetime) [11]. Of course, due to the relational aspect of quantum physical variables, the quantum gravitational field does not have determined values until it interacts with something else [11]. By ignoring the microscopic details of the world, a blurring is seen *i.e.*, it is produced by the intrinsic quantum indeterminacy of things [11]. The time of physics is the expression of our ignorance of the world *i.e.*, time is ignorance [11].

Based on the quantum methodology, for finding the quants of space and area we should define the volume and area operators [9] [10]. It can be shown that

there is a kinematical Hilbert space  $\mathcal{K}_{Diff}$  which admits a basis of states in which certain area and volume operators are diagonal. It means that there is a spin network state which describes a quantized three-geometry [9] [10]. It should be noted that a spin network state is not in space but it is space [9] [10]. Therefore, it introduces a background independent physics and in consequence we are in quantum atmosphere of space. It is well known that, the quantum dynamics of a particle is entirely described by the transition probability amplitudes  $W(x, t, x', t') = \langle x | e^{-\frac{i}{\hbar} H_0(t-t')} | x' \rangle = \langle x, t | x', t' \rangle$  which depend on two events  $(x, t)$  and  $(x', t')$  that bound a finite portion of a classical trajectory [9]. It should be noted that the argument of  $W$  is the eigenstate of the corresponding Hamiltonian operator,  $H_0$ . Similarly, we can consider a spin network as the argument of  $W$  which represents the possible outcome of a measurement of the gravitational field (or the geometry) on a closed three dimensional surface [9]. By defining a suitable scalar product, it can be shown that the transition amplitude between two states is simply their scalar product [9]. Here,  $H$  acts on the node of spin network and in consequence spinfoam is constructed [9] [10]. The transition amplitudes  $W(s, s')$  do not depend explicitly on time and in consequence introduces a new concept called the physic without time. It means that the theory allows us to calculate the relation between observables which is what we see and does not give us their evolution in terms of an unobservable quantity which is called time in classical physics. In the other words, there are no good clocks at the Planck scale [9]. The world is a network of events and is not things [11]. In classical physics time exists with many determinations and here the main concept is: things happen [11]. The difference between things and events is that things persist in time but events have a limited duration [11].

Finally it can be concluded that, the spin network is space, and its evolution by acting the Hamiltonian, constructs the spinfoam. But, it is not clear that how other observable quantities change in the spin network. For example, how can we study the quantum transport of a particle in a spin network and find its spinfoam? What may be the general laws and rules? Whether, the above explained theory can be only used for studying the some specific problems of quantum gravity and the structure of spacetime at Planck scale [12]? It means that we do not know how the other physical concepts are defined (confined) by the concept of the spin network.

### 2.3. Biological Concept of Time

Three types of time can be defined in biology as follow:

- 1) The number of complete cycles per unit time such as heart rate and metabolic rate;
- 2) Aging due to the cell dividing (splicing) process;
- 3) Internal clock of body.

Many Biological variables can be defined as the product of three power function as below

$$[Y] = [M^\alpha \cdot L^\beta \cdot T^\gamma] \quad (1)$$

where,  $M$ ,  $L$ , and  $T$  stand for mass, length, and time, respectively [13]. For comparing the empirical findings with calculated values, the biological variable  $Y$  can be expressed as

$$Y = \log a + b \cdot \log M \quad (2)$$

which is called Huxley's allometric equation [13] [14]. Using the dimensional analysis, one can investigate the influence of earth's gravity on heart rate and metabolic rate. It can be shown that there are two solutions for allometric coefficient  $b$  as below [13] [15]

$$b_M = \alpha + \frac{1}{3}\beta + \frac{1}{4}\gamma \quad (3)$$

and

$$b_W = \frac{5}{6}\alpha + \frac{1}{3}\beta + \frac{1}{4}\gamma \quad (4)$$

where,  $b_M$  and  $b_W$  are body mass and body weight coefficients, respectively [13] [15]. As an example, for metabolic rate  $b_M = 0.91$  and  $b_W = 0.75$  the difference  $\Delta b = 0.17$  which is referred to the influence of the gravity [13]. It should be noted that the predominance of the cyclic nature of almost all functions leads to the granulation of time. It appears as flicker-fusion-frequency in neurophysiological realm, heart rate, respiratory rate, and specific metabolic rate in organ physiology [13].

One of the strange natural phenomena is frozen wood frogs which have been seen in Alaska [16]:

*“The hearts of the frogs stop beating and their blood no longer flows. The freezing patterns help the frogs convert more of the glycogen stored in their liver into glucose. It is the high levels of glucose in the cell of frogs that keep them alive through the long, cold winter. The main function of the glucose is to keep water inside the cell. By making the cells super sweet with glucose, the frogs keep the water from leaving their cells. It should be noted that, they do not freeze totally solid, but they do freeze mostly solid. Two-thirds of their body water turns to ice.”*

Therefore, under very special and critical thermo dynamical conditions, the cyclical character of the biological time can be changed such as the heart beating of wood frogs in Alaska and in consequence can be set in laboratory.

There are two different ideas about whether ageing is a disease or not [17]. Aging can be considered as a natural process and consequently it is not a disease [17] [18] [19]. They believe that aging constitute a natural and universal process, while diseases are seen as deviation from normal state. Some other scientists believe that aging has specific causes, each of which can be reduced to a cellular and molecular level, and has recognizable signs and symptoms [17] [20]. However it is well known that, certain genes and pathways that regulate splicing factors play a key role in the aging process [21]:

*“The ERK and AKT pathways are repeatedly activated throughout life, through aspects of aging including stockticker DNA damage and the chronic inflammation of aging. By using specific inhibitors which are already used as cancer drugs in clinics, it is possible the activity of the ERK and stockticker ATK pathways are stopped and in consequence an increment in splicing factor is seen. It means that a better communication happen between protein and genes. It can cause a reduction in the number of senescent cells and reverse many of their features which have been linked to the aging process and in consequence leading to a rejuvenation of cells.”*

Some scientists plan to rejuvenate dogs using gene therapy. They plan to make animals younger by adding new stockticker DNA instructions to their bodies [22]. Also, the eroding effects of aging can be controlled by making a complex and protective shield which is made by combination of stem cell with anti-aging gene [23]. Therefore, the ageing character of the biological time can be changed and set in laboratory.

Another biological concept of time is referred to the body clock [24]:

*“People belong to each time zone of the earth have a specific rhythms which is in sync with the day-night cycle of the zone. The rhythms dictate times for eating, sleeping, hormone regulation, body temperature variations and other functions. The rhythms have been changed by rapid long-distance trans-meridian (east-west or west-east) travel. The adjustment speed of jet lagged body depends on the individual as well as the direction of travel. It has been shown that the jet lag is a chronobiological problem.”*

Therefore, it can be concluded that the body clock is some kind of complete cycles per day which is adjusted by some specific duration and depends on individual and travel direction.

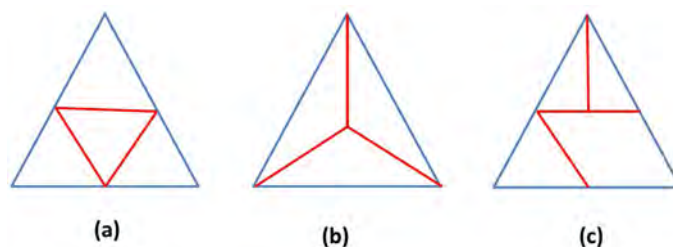
## **2.4. Cosmological Concept of Time**

If the universe (world) is the variable things, a thing divides the world to two main parts which are called system (thing) and environment (here, background). Of course one can consider a set of things as a system and calls the remained part of the universe as its environment. Therefore, there is a boundary between system and its environment. By adding more things to the system, the system inflates and its environment condensates (respect to its previous condition). The boundary can be defined as: *“things which are placed outside the boundary have negligible effect on the system and the effect of things which are placed inside the boundary should be considered when we want to study the specific characteristics of the system. Outside a boundary contains environment i.e., other things”*. For example, a confined gas in a region (box) can be considered as a system. By attaching a thermometer on the outside boundary of the system (walls of the box), one can measure the temperature (kinetic energy of gas molecules) of the box. By setting a clock outside the box, it is possible to study the time variation of the temperature when the boundary of system (the volume of the box) in-



creases or decreases. Also we can calculate the entropy of the system. Conlon has defined the entropy of an object as a measure of its number of degrees of freedom *i.e.*, “the total number of ways to rearrange its internals while keeping its external unaltered”. For example, “the entropy of gas is a measure of the total number of way the gas molecules can arrange themselves within box” [12]. It means that by increasing the number of rearrangement the entropy increases and in consequence the system is more stable than before. Smolin has defined the entropy as: “how many microstates could give the same macrostate. The entropy of a building is the measure of the number of different ways to put the parts together to realize the drawing of the architect” [2]. Of course, the entropy is proportional to information, inversely. For example, one can use four small equilateral triangles (case No.1) or three small isosceles triangles (case No.2) and make a bigger equilateral triangle by using them. Also, it is possible one make an equilateral triangle by using three triangles and one parallelogram (case No.3) (Figure 1). It is obvious that for manufacturing the case No.3 we need more information respect to the case No.2 and for manufacturing case No.1 we need less information respect to the case No.2. Also, the case No.3 is more orderless than case No.2 and the case No.1 is more order than case No.2. However, we can set the small triangles of Figure 1(a) in many different ways and find the big triangle. But the number of setting the small triangles of Figure 1(b) for finding the big triangle is less than Figure 1(a) (*i.e.*, we need more information). Similarly, the number of setting the small triangles of Figure 1(b) for finding the big triangle is more than Figure 1(c) (*i.e.*, we need less information). Therefore, the information is proportional to the orderless (entropy), inversely. But it should be noted that in both Conlon’s and Smolin’s point of view, there is a boundary which separates the system from its environment.

The biggest problem about the universe as whole is: how can we define the boundary of the universe? If there is not an environment (the existence of other things which have not serious effects on the universe) how can we define a clock outside the universe and measure the time? How can we put a thermometer outside the universe and measure its temperature? How can we define the order or orderless, entropy and information when we are not able to define the environment (outside) of the universe? How can we define the inflation or condensation? It should be noted that, we cannot put a clock outside the universe and in



**Figure 1.** (Color online) A equilateral triangle composed by (a) four equilateral triangles, (b) three isosceles triangles, and (c) three triangles and one parallelogram.

consequence its time is zero and the energy of the universe is equal to zero too, based on the Heisenberg uncertainty principle. Therefore, it can be concluded that the universe is frozen [2] [11]. Is it a correct conclusion when we cannot put a clock outside the universe?

When we consider the universe as a whole, we encounter a conceptual paradigm shift. Time (using a clock outside the system), temperature (using a thermometer outside the system), entropy (the total number of ways to rearrange the internals of the system while keeping its external unaltered) and information (the degree of order) has been defined for a system which has boundary with the remained part of the universe.

When, we consider the universe as whole, a conceptual paradigm shift happen. Here, we should define new concepts for time, temperature, entropy, order (orderless), information and so on and pay enough attention to using the concepts which belong to a system and its environment. If we use these old concepts and laws which have satisfied for system and its environment and not the universe as a whole, we will find that the universe is frozen and we need a Big Bang or Big Bounce for describing the current measured experimental data [2] [11]. Of course in next section, we will explain how we need the Big Bang and or Big Bounce based on our definition of obligated minimum boundary between things and obligated minimum boundary between past and future.

### 3. Ground Unified Reaction Platform

Using the explanations in section II, we consider the below principles (axioms):

- 1) The world is things which change *i.e.*, it is variable things (events).
- 2) Things interact with each other and their interactions and variations make the observables of the universe.
- 3) A system is composed by variable things and boundaries.
- 4) A boundary is the border of system which separates system from its environment (background).
- 5) All things which are placed outside the boundary have negligible effects on the observables of the system while all things inside the boundary have mutual significant effects on each other.
- 6) A system composed by things is completely and conceptually different from the universe as a whole.
- 7) When we shift from a system to universe, a conceptual paradigm shift happen. During the usage of the concepts (such as time, temperature, entropy, information, ) and laws (second law of thermodynamic, relativity, quantum physics ) belong to a system when we deal with the universe as a whole, we should be very careful and pay enough attention to the conceptual paradigm shift. It may be necessary; we redefine the concepts (e.g. time) and extract the laws (e.g. quantum gravity) again with compatibility with the definition of the universe.
- 8) There are two main interactions between things belong to a system. One is based on the biological conscious and other is based on quantum field theory in-

cluding gravity or not.

9) The entangled interaction is a kind of conscious which can be called pre-conscious because the organism aware that a specific relation exist between other parts of the system before doing any experiment. Here, the different outcomes of the experiment can be determined with equal probability before doing the experiment (such as, half and half occurrence probability between entangled spin up and spin down electrons).

10) If the occurring probability (based on the quantum physics) is very high (such as classical physics limit) due to any reason (such as interactions between things of a system or interaction between system and its environment) and an organism is a part of the system, the conscious is the past-conscious since before doing the experiment one can guess the final results, deterministically (such as falling a ball from top of a tower).

In continuing, we try to introduce a new concept which is called grand unified reaction platform (GURP) by considering the above axioms. Before doing that, we should review the concept of consciousness. The variable things of the universe can be categorized in two different branches which are organism and non-organism things. Organism things have the consciousness capability while the non-organism things have not. Here, the capability is composed by two concepts which are capable and ability. For example, a health two years old child has a hidden inherent learning aptitude for learning the science but he/she should passes the different scientific courses for learning the science. It means that the emergence of each capability needs two main elements which are inherent aptitude and programing for maturing and improving the aptitude. Organism has the experience capability. Organism is experimenting every day through his/her five main senses which are seeing, hearing, touching, tasting, olfaction and emotion. It should be noted that [25] [26]:

*“We usually explain what a thing does, how it changes and how it is put together, in science. But, an explanation of consciousness i.e., answering the question: why is it conscious (awake)?; goes beyond the method of science”.*

Sometimes, our experience does not coincide with external reality. Let us to do an experiment by using two light emitting diodes (LED) in green and red colors. An observer who is not aware of these LEDs sees the flash of red LED for only 20 milliseconds (ms) followed immediately by the flash of green LED for 20 ms duration. What, he/she will report, is seeing a flash of yellow light [13]. It means that his/her central nervous system (SNC) integrates the two successive processes and in consequence he/she reports seeing the yellow color because there is a period of time during which he/she is aware of an event or an entity. It is called the duration of the present [13] [27]. In biology, time has physiological periodicities where future passes to past though the present (an infinitely thin boundary) [13].

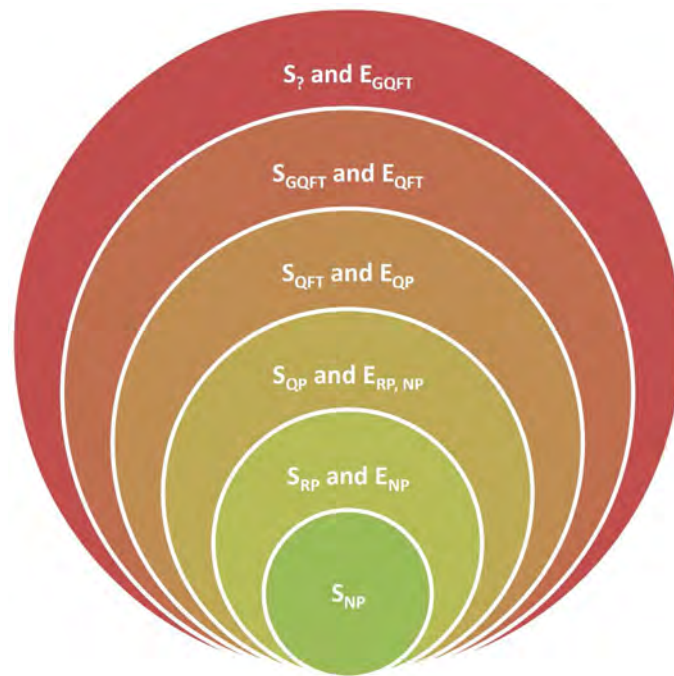
For understanding and explaining the behavior of a system (not universe as whole) composed by non-organism variable things; an organism should interact with it. There are three cases as below:

1) The organism is placed outside the boundary and its consciousness, which is emerged through its interaction with the system, coincides with the reality. Under the condition, he/she is able to explore the physical laws which govern over the system and inside its boundary. Here, his/her level of awareness (LoA) is complete. The observer is able to specify (guess) the output of his/her experiment deterministically (probabilistically). We prefer to call the LoA as past-awareness (pre-awareness and/or non-deterministic).

2) The organism is placed outside the boundary and its consciousness does not coincide with the reality. Since, he/she is not aware about his/her mistake the explored laws will be considered as govern physical laws over the system and inside its boundary. Here, his/he level of awareness (LoA) is not complete but the probability of his/her awareness about own mistake is not always zero and someday he/she will be aware and try to find the correct physical law. From the point of view, sometimes it seems that the physical laws have been changed by increasing his/her LoA.

3) Sometime the system and organism are not separable and in consequence the organism is placed inside the boundary of the system. Since, the explaining the consciousness goes beyond the method of science [25] [26], he/she is not able to explore the physical laws which govern over the system and inside its boundary. The situation is out of the scope of science.

Now, let us to review the credibility border of current physical paradigms based on the above categorizations. By putting the organism outside the boundary of a system (not universe as a whole), we assume that the velocity of object is shown by  $v$  and the velocity of light ( $c$ ), the Planck constant ( $\hbar$ ) and the gravity constant ( $G$ ) stand for the special relativity effects, quantum physic effects, and gravity effects, respectively. Of course, the symbols only use for showing the paradigm shift. For both  $(v/c)$  and  $\hbar \rightarrow 0$ , the Newtonian physics, for only  $\hbar \rightarrow 0$ , the relativistic physics and for only  $(v/c) \rightarrow 0$ , the quantum physics satisfies inside the boundary of the system. Also, for both  $(v/c)$  and  $\hbar \rightarrow 0$ , the quantum field theory satisfies inside the boundary. Of course in the above four cases, it is assumed that the gravity has no significant effect on the system (*i.e.*,  $G \rightarrow 0$ ) and in consequence it is placed outside their boundaries. Also in Newtonian and relativistic physics, the LoA is past-awareness and the observables follow the deterministic laws but in quantum physics and quantum field theory the LoA is pre-awareness and/or non-deterministic. In consequence, the observables do not follow the deterministic laws. It should be noted that, the pre-awareness about the observables causes the entanglement between them but does not remove their probabilistic behavior before doing the measurement, although by measuring one observable, the organism is able to specify its entangled observable with certainty *i.e.*, deterministically (Figure 2). When the system is not the universe of a whole, each physical law satisfies inside own boundary and credibility border. Therefore, in a part of the universe (not a universe as a whole) by changing the credibility border of laws, the laws change due to the



**Figure 2.** (Color online) The creditability border of different physical theories. Here,  $S_{NP}$ : Both  $v/c$  &  $\hbar \rightarrow 0$  and  $G \rightarrow 0$ ,  $S_{RP}$ : Both  $G$  &  $\hbar \rightarrow 0$ ,  $S_{QP}$ : Both  $G$  &  $v/c \rightarrow 0$ ,  $S_{QFT}$ : Both  $v/c$  &  $\hbar \rightarrow 0$  and  $G \rightarrow 0$ , and  $S_{GQFT}$ : Both  $v/c$  &  $\hbar \rightarrow 0$  and  $G \rightarrow 0$ . Here, S and E stand for system and environment, respectively. Also, NP, RP, QP, QFT, and GQFT are abbreviations of Newtonian physics, relativistic physics, quantum physics, quantum field theory, and gravitational quantum field theory, respectively.

change of LoA. Smolin has claimed that by considering the universe as a whole it seems that the laws have changed by passing the time [2].

But, what's about the universe as a whole? Before discussing about the subject, let us review the biological immortality. Wikipedia says [28]:

*“Biological immortality is a state in which the rate of mortality from senescence is stable or decreasing, thus decoupling it from chronological age. Various unicellular and multicellular species, including some vertebrates, achieve this state either through their existence or after living long enough. A biologically immortal living being can still die from means other than senescence, such as through injury, disease, or lack of available resources. Biologists chose the word immortal to designate cells that are not subject to the Hayflick limit, the point at which cells can no longer divide due to DNA damage or shortened telomeres.”*

And also it says [29]:

*“Turrítopsis nutricula is a small hydrozoan that once reaching adulthood, can transfer its cells back to childhood. Hydrozoans have two distinct stages in their life, a polyp stage and a medusa stage. Generally in hydrozoa the medusa develops from the asexual budding of the polyp and the polyp results from sexual reproduction of medusa Turrítopsis nutricula in any point of the medusa stage has the ability to transfer back into its polyp stage.”*

Of course, some scientists [30] [31] have discussed about the Turrítopsis nu-

trricula and reversed the life cycle, respectively.

Now let us to assume that the universe is a *Turritopsis nutricula* (thing) plus a conscious observer (organism) whose presence has no significant effect on the thing. What will be the observer's report about the changes in the thing? Depends on the observation conditions, he/she will report a cyclic change between polyp stage and medusa stage of the thing or will report that the thing is always juvenile. In the other word, when the measurement is always done at polyp stage *i.e.*, at times,  $= nT_0$ , where  $T_0$  is the cyclic time and  $n = 1, 2, 3, \dots$ , the observer see a frozen universe while when the measurement is always done at medusa stage *i.e.*, at times,  $(n-1)T_0 < t < nT_0$ , the observer see a universe that changes. When the measurement is done at infinitely thin boundary between medusa stage (future) and polyp stage (past) the observer see a cyclic change. The simple example has the below lessons for us:

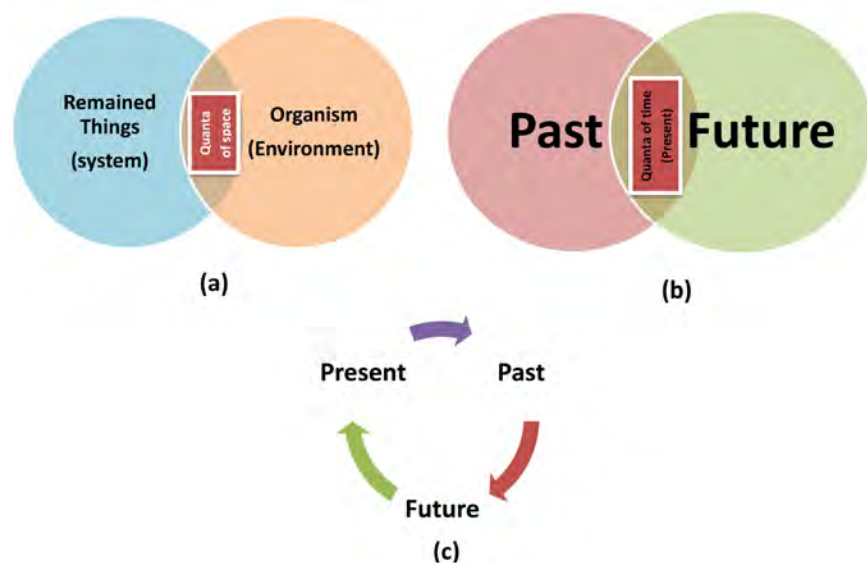
1) The remained things are separated from other things and conscious observer (organism) by a specific boundary which divides the universe to system (remained things) and its environment (other things and organism). The obligated smallest dimension of boundary (OSDB) between things under the observation conditions is called the “quanta of space” (Figure 3(a)).

2) The changes of things are observable. If no change is observed the thing is frozen.

3) The observed changes of things depend on the observation conditions.

4) Based on the observation conditions, the changes of things can be classified in three categories called past, present, and future.

5) If an observation is always done at “*polyp state*” of the thing (past), the observer sees no changes and reports a frozen state of thing. But, if an observation is always done at “*medusa state*” of thing (future), the observer sees changes and reports a variable (non-frozen) thing.



**Figure 3.** (Color online) (a) The quanta of space; (b) the quanta of time; and (c) a cyclic evolution.

6) There is always an infinitely thin boundary between future and past which is called present. The duration of present (*i.e.*, the width of the thin boundary) depends on the obligated smallest time duration (OSTD) under the observation conditions. The smallest time duration is called the “quanta of time” (**Figure 3(b)**).

7) For better understanding the concept of the present stage we should pay much attention to a cyclic change. In a cyclic change, the future passes to the past though the present (**Figure 3(c)**).

Now let us to mix the concepts of **Figure 2** with the concepts of **Figure 3**. In Newtonian universe since  $(v/c) \rightarrow 0$ , there is an obligated broad boundary between past and future. In consequence, the assumption of continues time (a stream of time) is meaningful. But, by increasing the ratio  $(v/c)$ , the width of the obligated boundary becomes thinner. When the ratio is equal or greater than a critical value, the width of the boundary approached to zero and past touches the future at a point which can be called the Dirac point. It is a singular point and in consequence we are not able to define an abstract time. After touching past with future the correct physical theory is relativistic theory. However, not only in Newtonian physics but also in relativistic physics, we can recognize A-thing from B-thing with very high accuracy and without perturbing them and in consequence we can imagine that the universe contains things. It means that there is an obligated broad boundary not only between things but also between observer and observable in these theories. But by decreasing the width of the boundary, which can be shown mathematically as  $\hbar \rightarrow 0$ , the separation between them and also their non-perturbing interaction disappears gradually and we reach to the atomic scale. Here, the quantum physics is true theory which can be non-relativistic or relativistic. Therefore, the Planck constant  $\hbar$  can be proportional to the bench mark of the obligated smallest dimension of the boundary between things (minimum space).

But, some questions can be asked. What are the nature of the obligated minimum boundary between things (minimum space) and the obligated minimum boundary between past and future (present)? Whether they are things if things are matter and fields? Whether the boundaries are made by the gravitational field? Based on the general relativity theory, the spacetime is gravitational field and the standard model, which is a non-gravitational quantum field theory, has unified mater with fields except gravitational field. Therefore, for all mentioned cases in the previous paragraph, the bench mark of gravitational field approaches to zero *i.e.*,  $G \rightarrow 0$ . But, if we accept (or assume) the boundaries are made by the gravitational field what is about the case  $G \rightarrow 0$ ? At the beginning of the section, we assume that the universe is variable things as a whole, and in consequence it is not possible we consider other things (such as clock and, thermometer) outside it. It means that the state of things change and in consequence a set of data is generated. The content of the set depends on the existence of other things, the interaction between them, and their interaction with the gravitational

field, if  $G \rightarrow 0$ . Generally, based on the quantum physics, we first prepare the necessary conditions for doing a specific measurement and then measure the observables. Therefore, there are two different states which are state-prepared and state-measured [9] [10]. Now, if the system is not frozen and the observer (organism) has no significant on the system, each pair of events include state-prepared and state-measured. The state of the pairs *i.e.*, the space of data set is shown by  $\mathfrak{H}$ . It should be noted that, the space  $\mathfrak{H}$  includes not only the data of observable variation but also the variations of gravitational field. In The other word, when  $G \rightarrow 0$ , we should not only measure the distance of the parts of the measurement apparatus and the time lapsed between them but also the variation of the physical observable [9] [10]. But for  $G \neq 0$ , it is not possible we separate the change of spacetime from the change of observables. In the other words, for  $G \neq 0$  ( $G \rightarrow 0$ ) we deal with one (two) measurement(s) because the mentioned two measurements are on the same ground [9] [10].

Also based on the quantum physics, the transition probability amplitude can be shown as  $W(s_{out}, s_{in})$  where  $s_{out}$  ( $s_{in}$ ) stands for state-measured (prepared) [9] [10]. However, if we can define a suitable inner scalar product then  $W(s_{out}, s_{in})$  will be equal to the inner scalar product between  $s_{out}$  and  $s_{in}$  *i.e.*, their correlations,  $W(s_{out}, s_{in}) = \langle s_{out} | s_{in} \rangle$ . Of course, for calculating the dynamic of the system, we should find (define) the Hamiltonian as a function on  $H$ . For example, if operator  $P$  stands for projection on the space including the solution of the WdW-equation, the transition amplitude will be  $W(s_{out}, s_{in}) = \langle s_{out} | P | s_{in} \rangle$  and the dynamic of the system is explained by a function on  $\mathfrak{H}$  which is called Hamilton (not Hamiltonian) [9] [10]. Therefore, we can summarize the differences between physical paradigms as what is shown in **Table 3**.

Based on our best current knowledge, the universe is things which are matter, fields of standard model plus gravitational field. Some important subjects about our current knowledge are:

1) We measure the observables of the universe by using the equipment and tools which have been made by using the current physical theories and they work based on them. The theories are satisfied over a part of the universe and on the specific conditions. In the other words, they satisfy over a region of universe which is separated from the remained part of the universe by some boundaries.

2) Since the gravitational field is a part of the universe, the data set include state-prepared and state-measured. Therefore, there is not only an obligated minimums boundary between things but also an obligated minimum boundary between past and future (future does not touch past at Dirac point) when the effect of the gravitational field is significant on the boundaries.

3) We are a part of the universe and it seems that we stand on the future when we measure the cosmological radiations which belong to our past, *i.e.*, they have traveled a long distance in the cosmos. Therefore, the universe as a whole is at its “*medusa stage*” now and we see the universe as variable things. It should have a beginning point (Big Bang). As we see, the govern laws on Big Bang conditions



**Table 3.** The differences between physical paradigms. GQFT, QFT, RQP, QP, RM, and NM are abbreviations of gravitational quantum field theory, quantum field theory, relativistic quantum physics, quantum physics, relativistic mechanics, and Newtonian mechanics, respectively.

Paradigm	Data space ( $\mathfrak{S}$ )	Transition amplitude	Hamiltonian	OSDB	OSTD
<b>GQFT</b>	Collection of both state-prepared and state-measured	$W(s_{out}, s_{in}) = \langle s_{out}   s_{in} \rangle$	Hamilton function is defined on $\mathfrak{S}$ [9] [10]	There is a minimum gap between things	There is a minimum gap between past and future
<b>QFT (RQP)</b>	Collection of state-measured at specific points of spacetime	$W(s_{out}, s_{in}) = \langle s_{out}   P   s_{in} \rangle$ $P$ stands for projection on the space including the solution of the special equation [9] [11]	Hamiltonian function is defined on $\mathfrak{S}$	Gravitational field has no significant effect	Future touches the past at Dirac point
<b>QP</b>	Collection of state-measured at specific points for specific times	Same as QFT	Same as QFT	A continuous space is considered.	A stream of time is considered
<b>RM</b>	Same as QFT	Things change deterministically	Can be defined [9] [10]	A continuous spacetime is considered	Future touches the past at Dirac point
<b>NM</b>	Same as QP	Things change deterministically	Can be defined [9] [10]	The world contain things	A stream of time is considered

differ from govern laws on sometimes after Big Bang (such as our current conditions) since things, the interactions between them and with gravitational field had changed. The interaction between things and between things and gravitational field create new things (mater and or fields) in the world and in consequence it is expected that the govern laws on the universe change by changing the universe (set of data *i.e.* state-prepared plus state-measured). Then it seems that a flow of time exist in the universe which separates the future from the past. Under our current situation we are trying to repeat the Big Bang condition (e.g., Long Hadronic Collider (LHC) set up) and find the suitable gravitational quantum field theories.

4) If universe as a whole is at its “polyp stage” it means that it is always in past *i.e.*, it is frozen. The frozen universe may be explained by a correct developed version of WdW-equation probably.

5) If universe has a cyclic evolution, the future passes through the past and in consequence we should have a Big Bounce instead of Big Bang. If it is true there is thin boundary between past and future and the experimentalists should find some cyclic phenomena and their rhythms at the scale of whole universe.

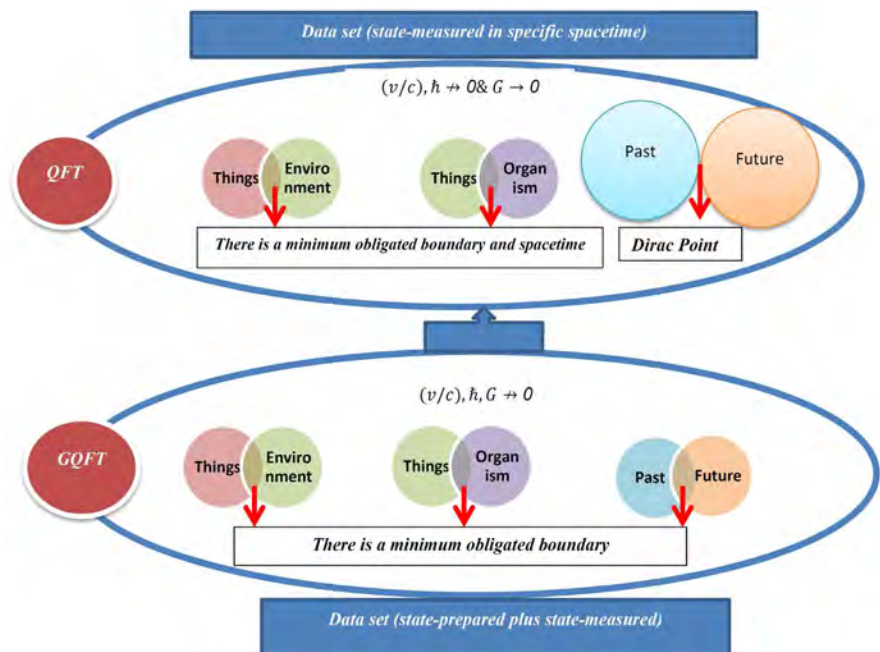
Anyhow, it seems that a reaction platform can be made by using the below materials:

1) Events (variable and interacting things) which are mater, fields of standard model and gravitational field.

- 2) A boundary separates system (things inside the boundary) from its environment (means things outside the boundary). The outside things have no significant effect on the inside things.
- 3) An obligated boundary between things exists and its minimum can be effected by the gravitational field (specially at Planck scale).
- 4) An obligated boundary between past and future exists and its minimum can be effected by the gravitational field (specially at Planck scale).
- 5) A conscious organism (observable) which has no significant effect on data set (state-prepared plus state-measured) is placed outside the system.
- 6) The observables are variation of things and their interactions.
- 7) If a conscious organism is placed inside the boundary the science cannot explain the system due to its inability to explain the consciousness.

How the above materials are used and what are their preconditions will specify the type of physical theory under the used condition. When the gravitational field has significant effect the reaction platform is called Grand Unified Reaction Platform (GURP). **Figure 4** shows the GURP and its different branches.

The grand unified reaction platform suggest us to think about a new theory based on the above mentioned principles which can categorize different physical paradigms based on their differences in the obligated minimum boundary between things, between past and future and between things and conscious organism. We prefer to call the new theory as “Boundaries Theory”. In boundaries theory we should show how the different physical paradigms can be extracted, the gravitational field acts on the boundaries and how the different boundaries can be transferred to each other using a suitable transformation.



**Figure 4.** (Color online) The grand unified reaction platform for building a physical theory for universe.

## 4. Summary

Universe is variable things. The observables are variation of things and their interactions. Organism is a conscious thing. Science can explain conscious but cannot explain the consciousness. A boundary divides a system to two parts called things and environment. The things which are placed outside the boundary have no significant effect on the inside things. If organism is placed inside the boundary the science cannot explain the system due to its inability to explaining the consciousness. The boundary between things has an obligated minimum value which is called minimum space. The obligated minimum boundary between past and future is called present. The gravitational field has significant effect on the present and the obligated minimum boundary between things in Planck scale. They are the necessary material for manufacturing a reaction platform for explaining the current situation of different physical paradigms respect to each other. We hope that the article can motivate the mathematical physicists to work on the subject and provide the necessary infrastructure for next physical followers.

## Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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# Possible Modular Structure of Matter Based on the “YY Model” Approach

—An Overview of the Construction Rules for Quarks and Atomic Nuclei with Configurative Examples for Neutron, Proton, Deuteron and Dineutron

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## Abstract

The newly developed YY model contains a set of constitutive rules to describe the structures of atomic nuclei and subatomic particles, by using two elementary sub-quark particles, the Yin and Yang fermions of charge  $1/3$  forming all the particles of the Standard Model. This model suggests a modular structure of the universe, in which two elementary constituents recursively form all the matter. The advantage of this hypothesis is that it provides a total symmetry and a noticeably clear conceptual understanding. Moreover, it justifies the cosmological formation of a limited number of atoms, e.g., H and Li with their isotopes, considering that matter can be produced as a free agglomerate of semi-stable neutrons, which would lead to the feeding of baryonic matter in the universe. In this current article, some further theoretical aspects are proposed as an evolution of the YY model. They cover correlation paths between interacting quarks, the considerations of color forces between yin-yang elementary elements. Moreover, an agreement of the YY model with the Teplov approach based on harmonic quarks and oscillators is established, and the mass of Yin and Yang is considered. Two example nuclei are used for the analysis: a radioactively stable deuteron (containing a neutron and a proton) and a possible semi-stable dineutron (roughly “consisting of two neutrons”), which is purely theoretical, represent a very natural and legal nuclear state within YY model. Based on the results obtained here, some indications are given for a possible simple experimental verification providing proof for the stability or instability of the dineutron.

## Keywords

YY Model Approach, Color Confinement Aggregate State CCAS, Quark Correlation Path, Color Forces, Constituent Quarks, Harmonic Oscillator,

## 1. Introduction

This is not the first time that a sub-quark structure has been used to understand matter. Starting from the YY model, a structural new method for building matter is proposed here, quantitatively augmented by Teplov's harmonic oscillating quarks to form a consistent model of the universe. Since the YY model for atomic nuclei is relatively new and uses the Yin and Yang, two hypothetical elementary symmetric particles, we first give a summary of the basic concept already published with examples (Section 2), in particular for its constituents for building subatomic particles (quarks, electron and positron) and atomic nuclei (neutron, proton and deuteron). This also includes main construction rules to ensure that nuclear aggregate states are logically and physically consistent (in the construction of units for electrical and color charges) and remain compliant with knowledge artifacts from the standard model and standard experimental physics.

Beyond the structural nature of a nucleus, the manifestation of quantum colors on its Yins and Yangs play an important role. Based on previously published results on the "color confined aggregate state" (CCAS), which can be considered as a color confined snapshot reflecting invariant symmetry with the fundamental energy level of the aggregate, and using deuteron as an example, correlation paths between the constituent quarks are identified. They are invariant units in transformations between different CCAS (Section 3). They are interpreted here as the linearized axes of the interactions for the strong forces. This is also another theoretical result in the development of the YY model.

Unresolved so far is the "triple charge binding" of the YY model: why three Yins (arithmetically all with  $-1/3$  electrical charge unit) bind in one vertex to a whole negatively charged node, just as three Yangs (arithmetically all with  $+1/3$  electrical charge unit) bind in one vertex to a whole positively charged node. "Color forces" (not necessarily in the sense of existing formulations) are treated between a pair of two Yins, of two Yangs, or of Yin-Yang depending on colors (Section 4). This is another approach to linearizing the strong forces, which is simple but can be combined with a particular correlation path of two quarks for a quantum field treatment. Probably two or three specific parts relating to these color forces need to be introduced into the Hamiltonian and Lagrangian setup. We believe that this would lead to simpler and more precise quantum field solutions than previously done.

A conceptual overview of harmonic quarks, harmonic oscillators and their masses is given by O. A. Teplov (Section 5)—in particular, the inherent mechanism of their recursive composition (Section 6). Using this approach and our extension by "down-exciting" the harmonic quarks, the masses of the particles in the Standard Model can be accurately determined (Section 7). While the Teplov

approach provides a universal, precise mass composition, the YY model, by corresponding Yin and Yang with harmonic quarks, complements this composition with a structural view that includes the interactions of color and electrical charges.

A nucleus model for dineutron is given (Section 8), derived purely by following the constituent rules, initially without reference to existing theories and experiments on dineutron. In its constitution, a dineutron and a deuteron are very similar. In preparation for a future quantitative calculation, all quark correlation paths for the dineutron are given, allowing a detailed comparison with the deuteron. An experimental scenario involving transmutation of deuterons (bombed with neutrons) to dineutrons is considered (Section 9), rather than the usual assumption of neutron absorption. In addition to the collision of neutrons with deuteron, a collision scenario of neutrons with tritons (tritium nucleus) is also considered.

Before the last part with conclusion and outlook, we give the conformal nuclear models for possible trineutron, tetra-neutron and more (Section 10) without performing any investigation.

## 2. Constitution of Atomic Nuclei with the YY Model and Conservation Rules

As an architectural approach to the modelling of the interior structure of atomic nuclei, the YY model was first published in the summer of 2020 (Ref. [1]). It has mainly descriptive character and allows rule-based modelling of atomic nuclei and their isotopes with detailed internal structures, which is lacking in the standard model. In this section, we roughly summarize some constituent aspects of the YY model to provide a knowledge base for rapid understanding.

Starting from an up quark, which according to the standard model has an electricity of  $+2/3$  charge unit and a mass of  $1/3$  of the atomic unit (Ref. [2]), is considered in the YY model as an aggregate state shown on the left panel of **Figure 1**. In contrast, a down quark, which has an electricity of  $-1/3$  charge unit and a mass of  $1/3$  of the atomic unit, is an aggregate state, as shown on the right panel of **Figure 1**.



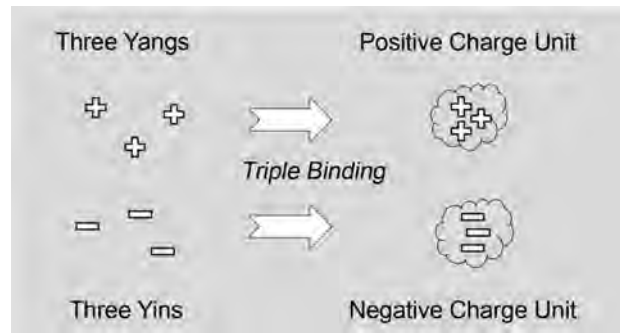
**Figure 1.** Up quark and down quark state model.

According to the smallest electrical charge unit of one third, Yin with the symbol “-” and Yang with the symbol “+” were introduced, which serve as more elementary constituents for particles than used in the standard model. The shading symbolizes the energetic and materialized states and stands for a certain

amount of mass (e.g., one third of an atomic mass unit). The handle for the up quark, consisting of a Yin-Yang pair, is called “Pairing Space Link” (PSL), which is a construct for the exchange of gluons with quarks. The construction rules make use of the following formalism, corresponding **Figure 1**:

Up quark:  $u$  or  $(++<+->)$ ; Down quark:  $d$  or  $(-)$

The origin of electrical charges in the YY model is based on “triple charging”: three Yangs bind to form a charged node with a positive electrical unit, while three Yins bind to form a charged node with a negative electrical unit, **Figure 2**.



**Figure 2.** Triple bonds to electrical charges.

The mechanism for the triple bond is discussed as an effect of color forces in Section 4. A triple charged node is the base for an electron or a positron, this is expressed as follows:

Electron ( $e^-$ )  $\Rightarrow (-)(-)(-)$  or  $(- - -)$ ;

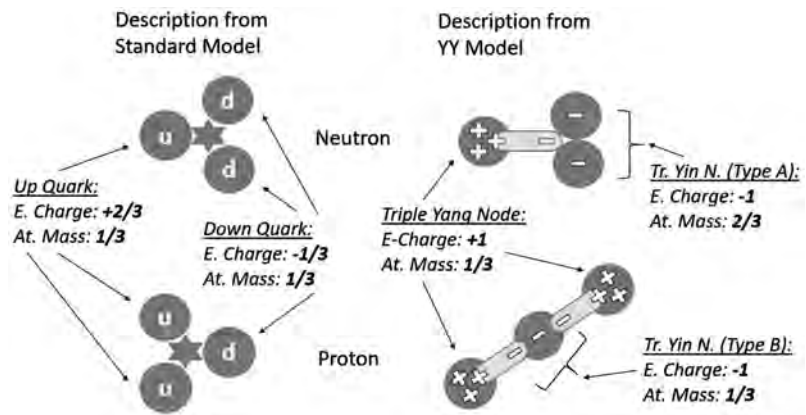
Positron ( $e^+$ )  $\Rightarrow (+)(+)(+)$  or  $(+ + +)$ .

While the positively charged node is considered a permanent bond, the negatively charged node is decomposable and can recombine (asymmetric behavior in transmutations).

In all subatomic transmutation processes, the total number of Yins and Yangs and thus the charges of each Yin and each Yang must be universally conserved. Yin and Yang are elementary entities of the architectural model. They are permanent carriers of charges, but spontaneous carriers of quantum colors. The YY model attempts to build a universe in which all matter and particles emerge from them. As we will see later, Yin and Yang find their counterpart in the harmonic quarks in the sense of Teplov. They are recursively involved in the construction of the mass and structure and obey the clearly defined rules of behavior for the harmonic oscillators as well as for the color and electrical charge interactions

According to the known artifacts of the standard model, a neutron consists of one up and two down quarks and a proton consists of two up and one down quarks (Ref. [3]), **Figure 3**, left panel. The strong forces holding the individual quarks together are symbolized in each case by the star in the center of the figure parts. In contrast, the description of the YY model uses the tubular PSLs to connect the integer charged triple nodes, **Figure 3**, right panel.





**Figure 3.** Description models for neutron and proton.

The total electrical charge within a neutron or a proton resulting from the YY model corresponds to the quantity in the standard model, the difference to the description in the standard model is the use of integer (triple) charged nodes by Yin and Yang entities. The construction makes use of the following formalism:

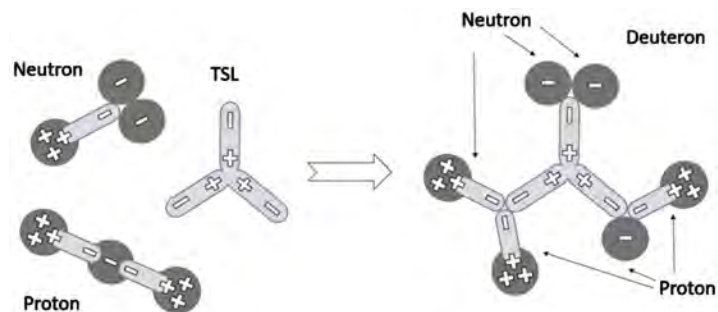
Neutron (udd):  $(++<+->)(-)(-)$ ; Proton (udu):  $(++<+->)(-)(<-+>++)$ .

The pairing-space link PSL and the triple-space link TSL, which is the “Y particle” predicted by the YY model, form fundamental gluon- and pion-conformal constructs (Ref. [4] [5] [6]) that serve as exchange particles within the nucleus, **Figure 4.**



**Figure 4.** Pairing Space Link (PSL) and Triple Space Link (TSL).

TSLs are involved in the constitution of complex atomic nuclei, e.g., in the building of deuteron (deuterium nucleus) by fusion of a neutron and a proton, **Figure 5.** In this paper, deuteron is used as the reference model.

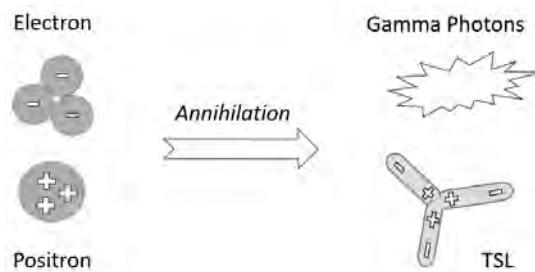


**Figure 5.** Deuterium nucleus as a result of fusion of neutron, proton and TSL.

Combining the standard symbols (u—up quark, d—down quark) with the TSL ( $\Rightarrow Y$ ), we obtain the following charming symbolic for a deuteron—which is an equivalent expression to the right part of **Figure 5**, but using quarks and their interaction linkage TSL:



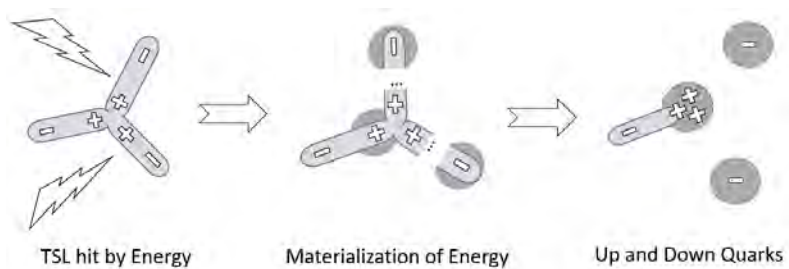
Many fusion processes from simple nuclei to a complex require TSLs from the environment, lead to recombination of their constituent quarks following the basic rules of the YY model and yield one or more resulting aggregates. All the original up and down quarks are “stretched” in their distances from each other, but still indirectly connected to form neutrons and protons. TSLs are themselves a byproduct of the electron-positron annihilation during nuclear fusion – this is also a prediction of the YY model, as an extension of the familiar annihilation description from the standard model: in addition to producing gamma photons, the YY model restores a TSL as the original constitutive state, taking into account the Yang and Yin conservation rule, **Figure 6** (The resulting TSL as a particle is the difference from the standard model description).



**Figure 6.** Annihilation of electron-positron pair described by the YY model.

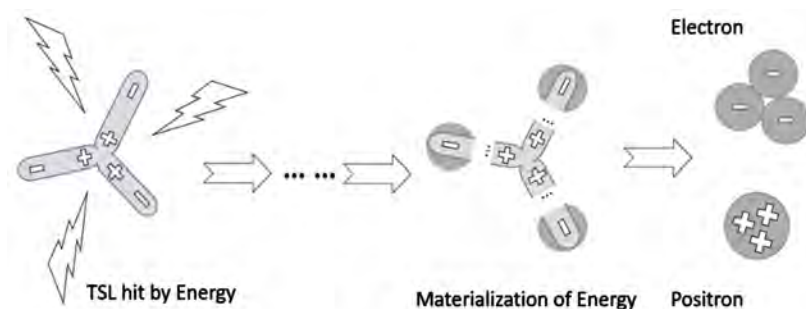
The transmutation is thus expressed as:  $e^- + e^+ \rightarrow 2\gamma + \text{TSL}$ .

In addition to bonding, TSL itself serves as a universal particle with a very basic physical state from which various other fundamental particles emerge under the conservation of yang and yin. Under high energy density, TSLs can be transformed into states for one up quark and two down quarks, **Figure 7** ( $\Rightarrow$  origin of quarks).





**Figure 7.** Conversion of a TSL into an up quark and two down quarks.

Moreover, under certain energy conditions and in a chain of transmutations, a TSL can also be split into an electron-positron pair, as shown in **Figure 8**.

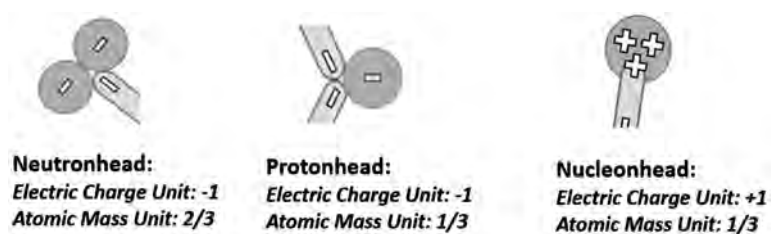


**Figure 8.** Conversion of a TSL into a positron-electron pair according to the YY model.

Moreover, as a bonding structure for quarks, the TSL itself can change state into a pair of up and anti-up quarks or a pair of down and anti-down quarks, taking into account color aspects.

Not least for the theoretical basis, the “internal charge balance (ICB) rule” states that within a nuclear aggregate, the constituent TSLs (electrically positively charged triple nodes ) must be numerically balanced by electrically negatively charged triple nodes (). ICB is important for the stability of aggregate structures, and the underlying mechanism remains to be investigated mathematically and physically. In the example of deuterium (**Figure 3**, right panel), a TSL in the center is balanced by the one triple node on the left, which is negatively charged.

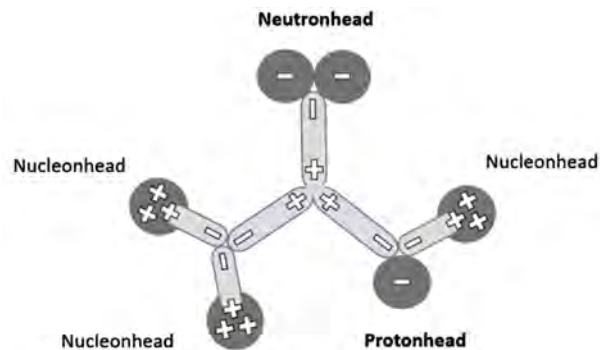
The “vertex element rule” requires that all surrounding vertices of a nuclear aggregate must be one of the so-called “neutronhead”, “protonhead” or “nucleonhead” (called “protonid” in our early publication), see also **Figure 9**.



**Figure 9.** Neutronhead, protonhead and nucleonhead; Their charges and masses.

Each neutronhead requires a corresponding nucleonhead to build a neutron within a nuclear aggregate. Each protonhead requires two corresponding nucleonheads to form a proton within the same nuclear aggregate, refer also **Figure 5**, right part. The implication is that the occurrence of down quarks (double or single) determines the number of neutrons and protons within a nucleus, while the occurrence of nucleonheads serves only to satisfy the requirements from the occurrence of protonheads and neutronheads.

Thus, the net electrical charges of an atomic nucleus result from the summation of all built outer vertices—all up and down quarks combinations. **Figure 10** gives the vertex notations for the deuterium nucleus.

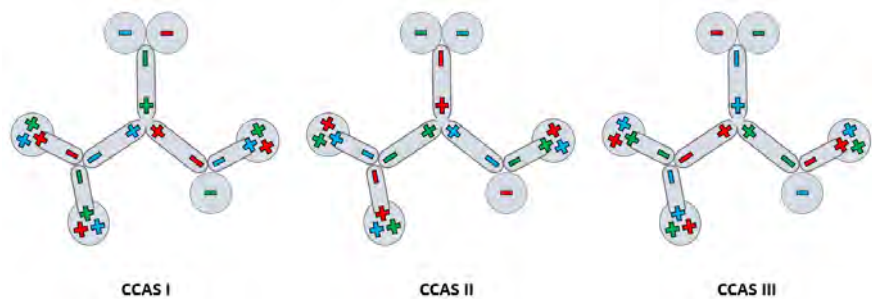


**Figure 10.** Notation of neutronhead, protonhead and nucleonhead for a deuterium.

### 3. Quantum Color Manifestation and Quark Correlation Path

The manifestation of quantum colors (Ref. [7]) was published last fall, based on the description of the YY model. We first give a rough summary about it and report here on a more developed aspect called “quark correlation path”.

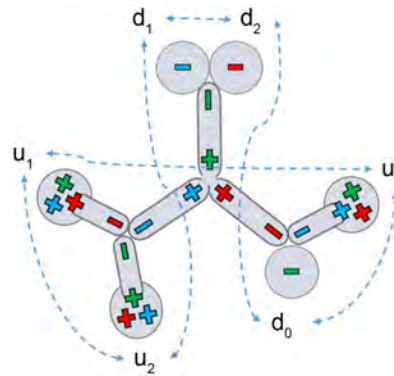
A consideration of quantum color dynamics QCD in the structural descriptions of YY model revealed a particular aspect as “Color Confined Aggregate State” (CCAS): a nuclear aggregate occupies a set of CCAS. Each CCAS means that all triple nodes are colored white (from the colors red, green and blue) and all PSLs are color-balanced (the two poles Yang and Yin are evenly colored, either red, green, or blue). **Figure 11** shows three examples of CCAS for the deuterium nucleus (The YY model considers a quantum color—e.g., “red”—as manifested on a Yang node, whereas its anti-color—anti-red—is just the application of red on a Yin node).



**Figure 11.** Three examples of CCAS for the deuterium nucleus.

The determining mechanism for the CCAS is not yet fully understood. But it seems that CCAS is closely related to chiral symmetry (Ref. [8] and [9]) and is an important stability factor for the atomic nucleus. Due to vertex-based separations of the constituent quarks for neutrons and protons, we need to reconsider the spins and the implication of the Pauli exclusion principle within a nucleus

built according to the YY model. But for now, very clear and deterministic “quark correlation paths” can be derived from each constituent quark to its corresponding partner quark. They are invariant with respect to CCAS. To do this, consider an arbitrarily chosen CCAS I from **Figure 11** and redraw it in **Figure 12** by labeling the individual quarks with indices.



**Figure 12.** Selected CCAS I from **Figure 11** with all quarks and correlation paths denoted.

Starting from the blue colored down quark d1 ( $\Rightarrow$  anti-blue), entering the triple node TSL via the green colored PSL, and following a blue/green alternating path (“tumbling”), the correlation path ends on the green colored up quark u2 ( $\Rightarrow$  anti-green). This correlation path can be denoted as (with r—red,  $\check{r}$ —anti-red, g—green,  $\hat{g}$ —antigreen, b—blue,  $\check{b}$ —antiblue):

$$d1 - \text{blue/green} - u2 \Rightarrow \mathbf{d1}(\check{b}) \langle \hat{g} \rangle \langle \check{b} \check{b} \rangle \mathbf{u2}(\hat{g}).$$

The up quark u2 can take the same path back to d1 (symmetric bidirectional path). In addition, it has a second, alternative correlation path (green/red), which leads to the red colored up quark u1:

$$u2 - \text{green/red} - u1 \Rightarrow \mathbf{u2}(\hat{g}) \mathbf{u1}(\check{r}).$$

Furthermore, if u1 follows its alternative path back (red/blue path), it ends up on the blue colored up quark u0:

$$u1 - \text{red/blue} - u0 \Rightarrow \mathbf{u1}(\check{r}) \langle \check{b} \check{b} \rangle \langle r \check{r} \rangle \mathbf{u0}(\check{b}).$$

Continuing this scheme, the tracing yields the other three correlation paths:

$$u0 - \text{blue/green} - d0 \Rightarrow \mathbf{u0}(\check{b}) \mathbf{d0}(\hat{g});$$

$$d0 - \text{green/red} - d2 \Rightarrow \mathbf{d0}(\hat{g}) \langle \check{r} r \rangle \langle g \hat{g} \rangle \mathbf{d2}(\check{r});$$

$$d2 - \text{red/blue} - d1 \Rightarrow \mathbf{d2}(\check{r}) \mathbf{d1}(\check{b}).$$

The entire correlation path closes on the start quark, regardless of which start quark is chosen as the starting point and the direction of the tracking. All vertex-quark pairs have a short correlation path within themselves. “Remotely connected” quark pairs always take a path through a TSL node.

By using the Yin-Yang symbols for quarks (u)  $\Rightarrow$  (+ + < + - >) and (d)  $\Rightarrow$  (-), by assigning them with colors, we obtain the equivalent, more accurate expressions for all correlation paths (symbol “Y” stands for positively charged triple-yang node—TSL, and  $\check{Y}$  for negatively charged triple-yin node) in **Table 1**.

**Table 1.** All quark correlation paths of deuteron in different expressions.

$d1 - b/g - u2$	$(b) \hat{Y} < \hat{g} g > Y < b b > \hat{Y} (< \hat{g} g > b   r)$	$(-) \hat{Y} < - + > Y < + - > \hat{Y} (< - + > +   +)$
$u2 - g/r - u1$	$(b   r < g \hat{g} >) \hat{Y} (< r \hat{r} > g   b)$	$(+   + < + - >) \hat{Y} (< + - > +   +)$
$u1 - r/b - u0$	$(g   b < r \hat{r} >) \hat{Y} < b b > Y < r \hat{r} > \hat{Y} (< b b > r   g)$	$(< +   + < + - > \hat{Y} < - + > Y < + - > \hat{Y} (< - + > +   +)$
$u0 - b/g - d0$	$(r   g < b \hat{b} >) \hat{Y} (\hat{g})$	$(+   + < + - >) \hat{Y} (-)$
$d0 - g/r - d2$	$(\hat{g}) \hat{Y} < \hat{r} r > Y < g \hat{g} > \hat{Y} (\hat{r})$	$(-) \hat{Y} < - + > Y < + - > \hat{Y} (-)$
$d2 - r/b - d1$	$(\hat{r}) \hat{Y} (b)$	$(-) \hat{Y} (-)$

Correlation paths as linearized force axes allow to form simplified Hamiltonians for analysis—further investigations must be carried out in the future. In a sense, the quantum fields become a superposition of all these correlation paths. Each PSL is traversed twice (back and forth) and each TSL is touched three times (as a paired input-output combination).

As will be seen, the dineutron as a bound state has a very similar set of properties as a deuteron. Even the transmutations between these two nuclei are easily possible.

### 4. Acting Forces

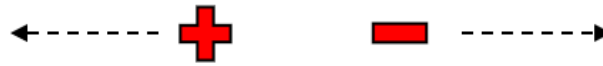
So far, the answer to the question of what the holding forces for the triple bond of three Yins are and for that of three Yangs has been omitted, so that an electron or a positron can be formed on the basis of this bond (Figure 2 and Figure 8, right part). There is also the question of how a PSL can take the “tubular form”, a path-like state. In this section, two simple assumptions—attractive and repulsive color forces—are made, which become effective immediately when the colors are assigned to the Yin-Yang elements:

- **Attractive Color Force:** A pair of adjacent Yins with different colors exerts an attractive force on each other; a pair of adjacent Yangs with different colors also exerts an attractive force on each other. Thus, the triple bond is the result of attractive color forces between three adjacent Yin’s or between three adjacent Yang’s, Figure 13, left. These bonding states also correspond to the “white” states because three different colors of red-green-blue (or of anti-red-antigreen-antiblue) must be involved for this to occur.



**Figure 13.** Attractive color forces.

- **Repulsive Color Force:** A Yin-Yang pair of the same color ( $\Rightarrow$  color-balanced PSL) repels from each other, **Figure 14**.

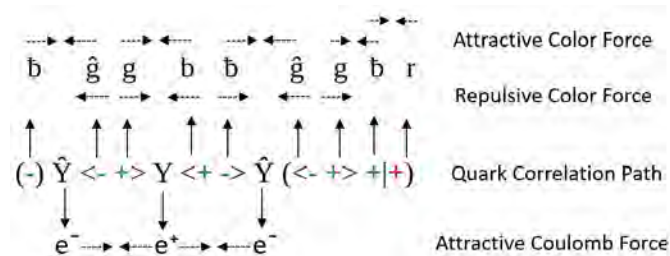


**Figure 14.** Repulsive color force.

- **Coulomb Forces:** In contrast to the color forces, a Yin (electrically charged in the fraction  $-1/3$ ) and a Yang (electrically charged in the fraction  $+1/3$ ) exert an attractive electrically conditioned force on each other. Two adjacent Yins repel each other electrically. Similarly, two neighboring Yangs also repel each other electrically.

**Overall Force Balance:** Within a certain differential space region, the color forces and the electric forces are balanced, so that in the case of **Figure 13**, the triple nodes (TSL nodes) do not collapse, and in the case of **Figure 15**, a Yin-Yang pair can form a stable tubular structure corresponding to a Pairing Space Link PSL. We will show later (Section 5) that the physical background is the “quasi-annihilating quark and anti-quark pair”, which has its own harmonic oscillation mass in the sense of Teplov theory.

Considering the whole nuclear aggregate, an overall equilibrium of all forces must be reached on each correlation path between two quarks to achieve a stable path state: These include the color forces and the Coulomb forces resulted from the charges of Yin’s and Yang’s, as shown in **Figure 15** based on the first correlation path of **Table 1**.



**Figure 15.** Overall force balance on a quark correlation path.

Although the terminology “strong forces” of quantum field theory is not directly used here, the YY model considers the color forces described above as an interpretation of the strong forces. The advantage of the acting forces discussed here lies in their clarity and simplicity: The overall effect of the forces is always a balance of color forces and electric force.

### 5. Harmonic Quarks According to O. A. Teplov, Coincidence of the YY Model with the Teplov Approach and Further Extensions

Between 2002 and 2005, O. A. Teplov introduced the concept of harmonic quark

oscillators based on a quark-antiquark pair and developed the formalism for calculating the exact masses of harmonic quark oscillators (Ref. [10] [11] [12] [13]). According to this approach, the quark mass is understood as the physical rest mass of the single particle state of an interacting quantum field. The flavor quantum number (reflecting quark production) is essentially a reflection of the quark's internal energy—its physical mass. The quark mass model with a multiplicative pattern in the mass transformation between quark flavors focuses on the quark-antiquark interaction and its outcome: either a meson (e.g., a vector boson) or complete annihilation of the pair with the birth of photons or lower mass quarks or other particles is produced.

Consider flavor changes in the weak fundamental interaction of quarks as expressed in the following terms ( $n$  is the quark generation number,  $\nu$  is the neutrino):

$$Q_{(n)} + W_{+/-} \sim Q_{(n+1)} \quad (1)$$

$$Q_{(n)} + e_{+/-} \sim Q_{(n+1)} + \nu \quad (2)$$

Teplov derived the formula for calculating the mass of harmonic quarks based on a multiplicative pattern:

$$m_{(n+1)} = \frac{\pi}{4 - \pi} \times m_{(n)} \quad (3)$$

The mass of the quark oscillator of generation  $n+1$  can be determined exactly by the mass of its lower generation  $n$ , starting from a hypothetical initial mass of the generation 0 quark. Moreover, for a given quark, its two neighbors can be considered as having an upward excitation (the quark with the larger mass) and a downward excitation (the quark with the smaller mass), with the electric charges of the two excitations being the same. According to Teplov, such harmonic oscillators form a series of quarks, starting with the lightest down quark (with a harmonic oscillator mass of 28.815 MeV), which are considered below as successive up excitations, see **Table 2**.

Using these harmonic quark masses, Teplov gave a mass composition model for some leptons and baryons—numerically very accurate. The research of O. A. Teplov reveals a deep fact of the quark generation model, which essentially states that a quark generation of  $n + 1$  results from the quark generation  $n$  by binding an electron or positron of its own flavor (its own generation), as expressed in Formula (2). In particular, the masses (harmonic quark masses) can be accurately calculated between these two generations according to a simple Formula (3).

Teplov treated the down quark with a harmonic oscillator mass of 28.815 MeV as the “lower limit” of his series described in **Table 2**. However, he also mentioned a possible “down excitation” quark with a harmonic oscillator mass of about 7.87 MeV (Ref. [11]), calculated from  $28.815/\pi(4 - \pi)$ . But he did not pursue this idea further. This is made up for in our current work, as calculated in the following **Table 3**.



**Table 2.** The masses of harmonic quarks after Teplov.

Harmonic quark defined by Teplov	down	up	strange	charm	bottom	top	b'
Harmonic quark mass (MeV)	28.815	105.456	385.95	1412.5	5169.4	18919	69239
Notation with generation index	Q1	Q2	Q3	Q4	Q5	Q6	Q7

**Table 3.** The masses of (pseudo) harmonic quarks after “down excitation” using Formula (3).

Harmonic quark defined by Teplov	-	-	-	-	-	-	-	?
Harmonic quark mass (MeV)	0.00089	0.00328	0.0120	0.0439	0.161	0.588	2.151	7.87
Notation with generation index	Q-7	Q-6	Q-5	Q-4	Q-3	Q-2	Q-1	Q 0

Teplov treated in his series of articles the structure formation of subatomic particles and atomic nuclei from the quarks. But his focus was essentially on a decomposition into harmonic pairs. We will show in the next two sections that a decomposition must take into account not only the harmonic pairing, but also the asymmetric “remote” pairing of the quarks themselves. We will show by examples that all these harmonic quarks from **Table 2** and **Table 3** are together “primary” and “discrete” building blocks of any particle in mass and electric charge.

It should also be mentioned that these “Teplov quarks” or synonymously “harmonic quarks” are theoretical in nature, unlike the “physical quarks” of the Standard Model, which have already been well treated in both fundamental theories and experiments.

First of all, we point out that a Yin or Yang in the YY model so far mainly represents a “summed up” charge part (one third charge unit): Yin:  $-1/3$  and Yang  $+1/3$ . We only claim that they have a certain mass quantum. We will show in this section how Yin and Yang obtain their masses. We will also show that there is a very good correspondence of Yin (Yang) with Teplov harmonic quarks as well as with their grouping states in the conservation of charge quantum. Moreover, there is a very simple explanation of PSL (Pairing Space Link) when considering the harmonic quark pairs. This is also true for TSL (Triple Space Link).

To this end, we first give a numerical approach to the Teplov mass Formula (3), using a very close approximation of  $\pi \sim 355/113 (=3.14159292, \text{Ref. [14]})$ , as follows:

$$m_{n+1} = \frac{\pi}{4 - \pi} \times m_n = \frac{355}{97} \times m_{(n)}$$

which expresses the ratio between the Teplov quark mass of generation  $n + 1$  and generation  $n$ . By applying the Teplov algorithm for successive down calculations:

$$m_{(n)} = \frac{355}{97} \times m_{(n-1)}$$

$$m_{(n-1)} = \frac{355}{97} \times m_{(n-2)}$$

$$m_{(n-2)} = \frac{355}{97} \times m_{(n-3)}$$

We obtain the following numerical mass series of harmonic quarks over five generations:

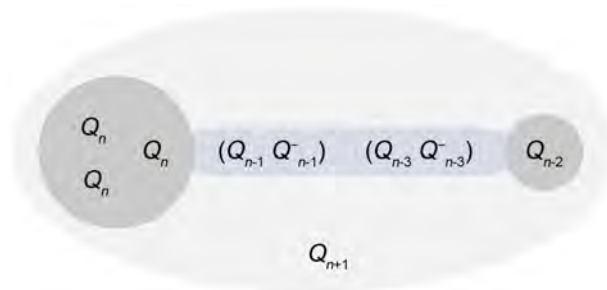
$$m_{(n+1)} = 3m_{(n)} + 2m_{(n-1)} + m_{(n-2)} + (167750635/88529281) \times m_{(n-3)}$$

Considering the coefficient in the last term ( $\approx 1.89$ ) as an approximate factor of 2—also because the mass of  $m_{(n-3)}$  is smaller than that of  $m_{(n-2)}$  by a factor of 3.66—we obtain the following expression, with an inaccuracy of minus  $0.11 \times m_{(n-3)}$ :

$$m_{(n+1)} = 3m_{(n)} + 2m_{(n-1)} + m_{(n-2)} + 2m_{(n-3)} \tag{4}$$

The physical interpretation of factor of 3 in Formula (4) is the “triple binding” of Yins or Yangs (Figure 2 and Figure 14) to give a “Teplov electron/positron” with a whole electric charge unit ( $-1$  or  $+1$ ). The factor of 2 in Formula (4) represents a charge-neutral “quasi-annihilating harmonic quark pair”, while the factor of 1 represents an up- or down-harmonic quark that compensates against the Teplov electron/positron so that the total state has a total electric charge unit of  $-1/3$  or  $+2/3$ —becoming a next “higher” down-or “higher” up-harmonic quark.

This leads to a modular structure of matter: it states that the mass of a Teplov quark is composed of three Teplov quarks excited downward  $3 \times m_{(n)}$ , plus two harmonic oscillators  $2 \times m_{(n-1)}$  and  $2 \times m_{(n-3)}$ , and another Teplov quark excited downward by three generations  $m_{(n-2)}$ . Figuratively speaking, this corresponds to the following construction (Q stands for a Teplov quark and the index  $n$  stands for a certain generation) (Figure 16):



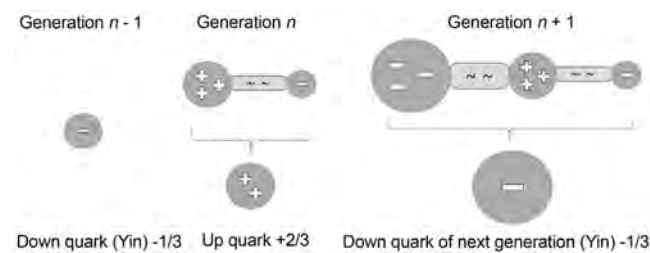
**Figure 16.** Teplov harmonic quark composition (generation  $n + 1$  of  $n$ ,  $n - 1$ ,  $n - 2$  and  $n - 3$ ).

According to the YY model, Yin (−) and Yang (+) would correspond exactly to a harmonic Teplov quark, namely Yin to a down quark and Yang to an anti-down quark, both in a certain generation. This in turn results in a more precise form for the YY modeling—namely the cascading up excitations from a down quark (Yin) to an up quark and further to a down quark of the next generation (Yin), see **Figure 17**.

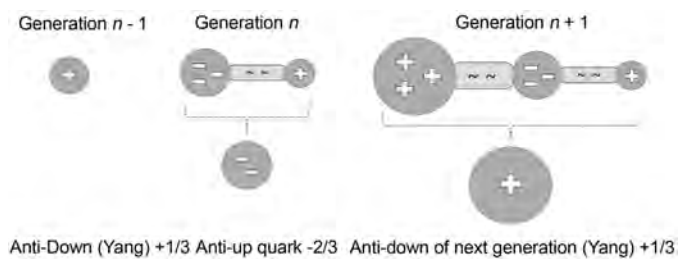
The harmonic oscillating quark pairs are symbolized by  $\sim$ . According to Teplov, they are “almost” or “quasi” annihilating, but cannot because the binding states exist with their neighbors.

Similarly, cascading up excitations from an anti-down quark (Yang) to an anti-up quark and further to an anti-down quark of the next generation (Yang), see **Figure 18**.

Note that **Figure 17** represents a set of harmonic quarks, while **Figure 18** represents a set of harmonic anti-quarks. The triple-bonded Yins and Yangs in **Figure 17** and **Figure 18** correspond to the “Teplov electrons” and “Teplov positrons” of different generations, respectively, which are also expressed in Formula (2).



**Figure 17.** Two up excitations from a Yin turns a Yin of next generation.



**Figure 18.** Two up excitations from a Yang turns a Yang of next generation.

To generalize the correspondence of Yin Yang with Teplov quarks, we claim that any Teplov quark representing a particular harmonic mass (in **Table 2** and **Table 3**)—for example, 28.81 MeV—can be charged with both  $-1/3$  and  $+1/3$  to serve as a down or anti-down quark. The same Teplov quark can also be charged with both  $+2/3$  and  $-2/3$  to serve as an up and anti-up quark. Consequently, the composite (up-excited) Teplov quark can also be charged with  $-1/3$ ,  $+1/3$ ,  $-2/3$ , or  $+2/3$ . For example, the combination of one aggregate  $m_{(n-2)}$  charged with  $-1/3$  and three  $m_{(n)}$  each charged with  $+1/3$  will result in an aggregate  $m_{(n+1)}$  with charge  $+2/3$ . From the aggregate of  $m_{(n-2)}$  charged with  $+2/3$  and from three  $m_{(n)}$

each charged with  $-1/3$ , an aggregate  $m_{(n+1)}$  with a charge of  $-1/3$  is formed.

We also see that harmonic oscillating quark pairs play an important role in the link between a Teplov quark and a Teplov electron or positron. In the YY model, this corresponds to the PSL (Pairing Space Link). In other words, the harmonic oscillating quark pairs are a kind of “load-bearing assembly” of the tubular structure of the PSL in the YY model, they constitute a large part of the mass, as described in Formula (4). We will show that the CCAS and the color forces described in Section 4, which balance the electric forces, are the determining factor for the tubular structure of PSL.

Based on these results, we can compose an arbitrary particle described by the Standard Model from both harmonic Teplov quarks and the YY model. We will see that the YY model has the advantage here of describing the deep binding mechanism based on color confinement (CCAS).

It should be mentioned that an alternative derivation of Formula (4) and thus an alternative interpretation of the modular structure of matter is also interesting:

$$m_{(n+1)} = \frac{\pi}{4 - \pi} m_{(n)} = (355/97) m_{(n)} = 4m_{(n)} - (33/97)m_{(n)}$$

With an inaccuracy of 0.007 factor of  $m_{(n)}$ , the following formula expresses the quantitative relationship for the masses of two neighbor generations:

$$m_{(n+1)} = 4m_{(n)} - \frac{1}{3}m_{(n)} \quad (4a)$$

The structural setup resulting from 4a is comparable to **Figures 16-18**. The physical explanation for this could be: the mass of a Teplov harmonic quark results from the sum of the four constituent harmonic quarks of the next lower generation minus one third of this mass. This minus part of the mass will be interesting for future research to find out its relationship to the “binding energy” of a harmonic quark.

## 6. Examples of Harmonic Quark Construction by Down-Excited Harmonic Quarks

Let us first consider how a heavy Teplov harmonic up quark (Q2, **Table 2**) is itself composed of other down-excited Teplov harmonic quarks, in particular in terms of numerical mass. A mass composition for a harmonic quark can be easily derived purely numerically. Based on the numerical composition, a suitable structural composition is obtained as follows (the following calculation is rounded to two decimal places).

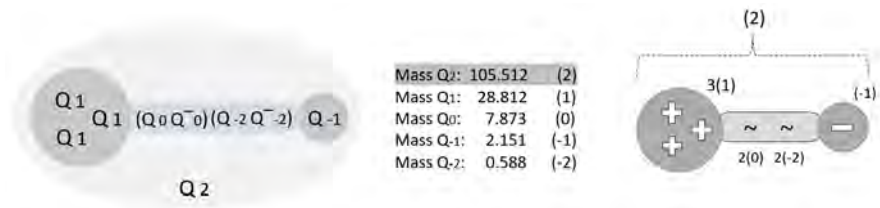
**Harmonic up quark** (charge  $+2/3$ , theoretical mass 105.456 MeV and calculated 105.512 MeV below):

mass composition:  $3 \times 28.815 + 2 \times 7.87 + 2.151 + 2 \times 0.588$ ;

quark composition: (Q1 Q1 Q1) (Q0<sup>-</sup> Q0) Q-1 (Q-2<sup>-</sup> Q-2).

The following configuration (**Figure 19**) reflects the correspondence between the description of the YY model and the mass decomposition from the point of

view of harmonic Teplov quarks:



**Figure 19.** A heavy harmonic up quark, its composition of down-excited harmonic quarks.

In the Figure above, the notation 3(1), for example, represents three Teplov quarks of generation 1. The up quark consists of a Teplov positron of generation 1 and a down quark of generation -1 (right part), between which there are two harmonic quark pairs:

Triple Yangs (+++) ⇔ (Q<sub>1</sub> Q<sub>1</sub> Q<sub>1</sub>), Ending Yin (-) ⇔ Q<sub>-1</sub>;

PSL (tuples) ⇔ (Q<sub>0</sub><sup>-</sup> Q<sub>0</sub>) (Q<sub>-2</sub><sup>-</sup> Q<sub>-2</sub>), composed of two harmonic quark pairs.

It should be mentioned that all calculations here and in the following are rough, because the compositions are done in a rough way. Nevertheless, the power of harmonic quarks is obvious.

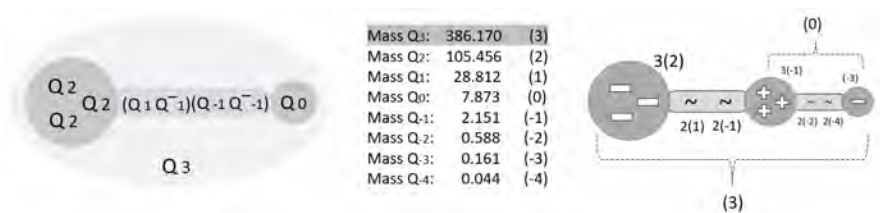
Next, we consider how a heavy Teplov harmonic down quark (Q<sub>3</sub>, Table 2) is composed of other down-excited Teplov harmonic quarks, especially in terms of masses:

**Harmonic down quark** (charge -1/3, theoretical mass 385.95 MeV and calculated 386.17 MeV below).

mass composition: 3 × 105.456 + 2 × 28.815 + 7.87 + 2 × 2.151;

quark composition: (Q<sub>2</sub> Q<sub>2</sub> Q<sub>2</sub>) (Q<sub>1</sub><sup>-</sup> Q<sub>1</sub>) Q<sub>0</sub> (Q<sub>-1</sub><sup>-</sup> Q<sub>-1</sub>);

This results in the following configuration (Figure 20):



**Figure 20.** A heavy harmonic down quark, its composition of down-excited harmonic quarks.

In the Figure above, the notation 3(2), for example, represents three quarks of generation 2. Thus, the down quark is composed of a generation 2 Teplov electron and a generation 0 up quark, which in turn is composed of a generation -1 Teplov positron and a generation -3 down quark (right part):

Triple Yins (---) ⇔ (Q<sub>2</sub> Q<sub>2</sub> Q<sub>2</sub>), Ending up quark (++) ⇔ Q<sub>0</sub>;

PSL (tuples) ⇔ (Q<sub>1</sub><sup>-</sup> Q<sub>1</sub>) (Q<sub>-1</sub><sup>-</sup> Q<sub>-1</sub>), composed of only harmonic quark pairs.

Third, we consider how a harmonic down quark of intermediate weight (Q<sub>1</sub>,

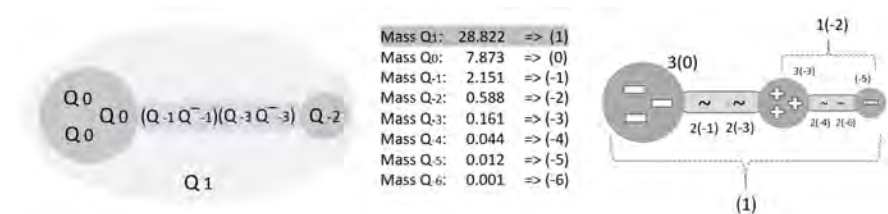
**Table 2)** is composed of downward excited harmonic quarks, in particular at masses:

**Harmonic down quark** (charge  $-1/3$ , theoretical mass 28.815 MeV and calculated 28.822 MeV below).

mass composition:  $3 \times 7.87 + 2 \times 2.151 + 0.588 + 2 \times 0.161$ ;

quark composition:  $(Q_0 Q_0 Q_0) (Q_{-1}^- Q_{-1}) Q_{-2} (Q_{-3}^- Q_{-3})$ ;

The resulting configuration is shown in **Figure 21**:



**Figure 21.** A harmonic down quark of middle weight, its composition of harmonic quarks.

In the Figure above, the notation 3(0), for example, represents three quarks of generation 0. Thus, the down quark is composed of a generation 0 Teplov electron and a generation  $-2$  up quark, which in turn is composed of a generation  $-3$  Teplov positron and a generation  $-5$  down quark (right part):

Triple Yins (---)  $\Leftrightarrow (Q_0 Q_0 Q_0)$ , Ending Yang (++)  $\Leftrightarrow Q_{-2}$ ;

PSL (tuples)  $\Leftrightarrow (Q_{-1}^- Q_{-1}) (Q_{-3}^- Q_{-3})$ , composed of only harmonic quark pairs.

From a structural point of view, the generation 0 down quark at 28.812 MeV (**Figure 21**) has the same structure or decomposition as the generation 2 down quark at 386.95 MeV (**Figure 20**)—an inherent recursion that also applies to up quarks.

In the same way, each harmonic quark in the Teplov series can be considered as a composition of its downward excited harmonic quarks. They all follow the same rules for mass and composition (Formula (4) and **Figure 16**). A very important aspect of this matter building rule is “recursive” or “fractal”: Contrary to the understanding of the Standard Model, there are no “final particles” that compose everything.

### 7. Examples of Particles of the Standard Model Decomposed into Harmonic Quarks

As distinguished from the Section above (harmonic Teplov quarks), in this section we consider the building of the Standard Model particles—quarks, leptons and baryons (neutron and proton)—through the composition of harmonic Teplov quarks by mass and charge. The YY model plays an important role in structural considerations. The deuteron nucleus and a (hypothetical) dineutron are also considered. The conceptual universality of our architectural model becomes clearer.

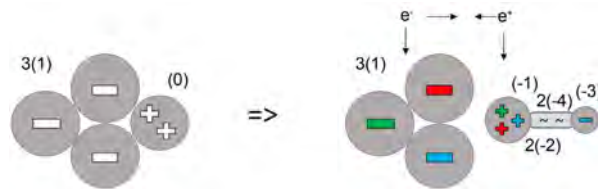
Let us start here with a simple structure case, namely the strange quark, which is described in the Standard Model as an elementary particle (The mass calculation here is accurate and the structural configuration is shown in **Figure 22**):

**Strange quark** (electrical charge  $-1/3 e$ , bare mass 95.00 MeV and calculated mass 94.315 MeV below).

harmonic mass composition:  $3 \times 28.815 + 7.87$ ;

harmonic quark composition: (Q1 Q1 Q1) Q0;

structural and charge composition:



**Figure 22.** Strange quark of standard model, its compositing harmonic quarks and their colors.

The relevant spectrum of harmonic quark generations is spanned between  $(-4)$  and  $(1)$ . The binding of the aggregate is based on the generation 1 Teplov electron and the generation  $-1$  Teplov positron (both color-confined) and two harmonic quark pairs ( $\sim \sim$ ) terminating at a down quark of generation  $-3$ . As a strange quark, the total aggregate takes on an (anti-)blue color—note that this strange quark does not exist “alone”, as it is embedded in a higher-level context aggregate by being adjacent to other neighboring down or up quarks.

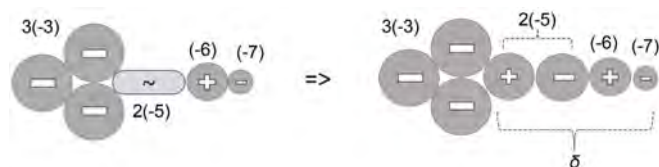
The next case concerns the electron with its harmonic quarks all in the downward excited region from **Table 3**, with exact mass calculation.

**Electron** (electrical charge  $-1e$ , bare mass 0.511 MeV and calculated mass 0.511 MeV below).

mass composition:  $3 \times 0.161 + 2 \times 0.012 + 0.0033 + 0.001$ ;

quark composition: (Q-3 Q-3 Q-3) (Q-5<sup>-</sup> Q-5) Q-6 Q-7;

structural and charge (**Figure 23**):



**Figure 23.** Electron, its composition of harmonic quarks.

This is a more accurate description of the electron composite structure shown in **Figure 2**. The  $\delta$ -part (right part of **Figure 23**) is an “appendage” to the Teplov electron of generation  $-3$ . Its role is not explained here. However, Teplov has made an interpretation for a muon (see below), whose approach can be considered for the electron in the future.

A further sample is given for the muon, an elementary particle of the standard model with a short lifetime.

**Muon** (electrical charge  $-1e$ , bare mass 105.658 MeV and calculated mass 105.661 MeV below).

mass composition:  $105.456 + 0.161 + 0.044$ ;

quark composition: Q2 Q-2 Q-4;

structural and charge (**Figure 24**):



**Figure 24.** Muon and its harmonic quark composition.

Remark: The further down-exciting gives the following more detailed compositions for muon ( $\Rightarrow 105.656$  MeV):

mass composition:  $3 \times 28.815 + 2 \times 7.87 + 2.151 + 2 \times 0.588 + 3 \times 0.044 + 0.012$ ;

quark composition: (Q1 Q1 Q1) (Q0<sup>-</sup> Q0) Q-1 (Q-2<sup>-</sup> Q-2) (Q-4 Q-4 Q-4) Q-5.

In explaining the small difference in mass between a harmonic u quark and a lepton muon, Teplov pointed out that: "... muon is a successful attempt of Nature to explicitly fix the single u-quark mass state as a lepton suppressing color and fractional charge."

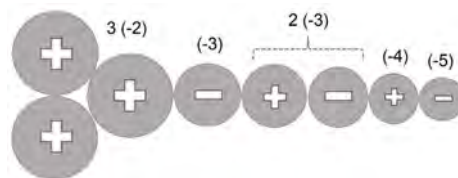
Now we turn to the consideration of the "well-known" up- and down-quarks of the standard model and the decay process  $d \rightarrow u + e^- + \nu e^-$ .

**Up quark** (electrical charge  $+2/3 e$ , bare mass 2.3 MeV and calculated mass 2.303 MeV below).

mass composition:  $3 \times 0.588 + 2 \times 0.161 + 0.161 + 0.044 + 0.012$ ;

quark composition: (Q-2 Q-2 Q-2) (Q-3<sup>-</sup> Q-3) Q-3 Q-4<sup>-</sup> Q-5;

structural and charge (**Figure 25**):



**Figure 25.** Standard model up quark, its composition of Teplov harmonic quarks.

The mass of the up quark is close to the mass of the harmonic quark of generation  $-1$  (Q-1 in **Table 3**).

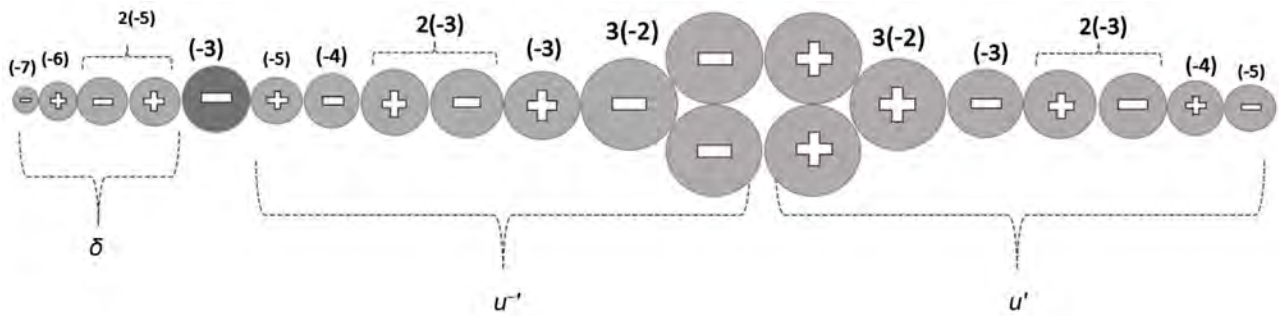
**Down quark** (electrical charge  $-1/3 e$ , bare mass 4.8 MeV and calculated mass 4.795 MeV below).

Mass composition:  $2 \times 2.303 + 0.161 + 2 \times 0.012 + 0.0033 + 0.001$ ;

Quark composition: (u<sup>-1</sup> u') Q-3 (Q-5<sup>-</sup> Q-5) Q-6 Q-7;

Structural and charge (**Figure 26**):

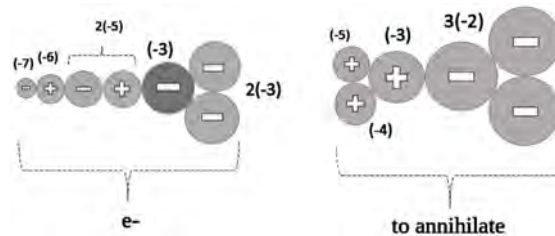




**Figure 26.** Standard model down quark, its composition of Teplov harmonic quarks.

From the structural point of view, the down quark can be considered as a compound of three parts: an up-quark  $u$  (see also **Figure 25**), an anti-up quark  $u^{-}$ , a down quark of generation  $-3$ , and a charge-neutral aggregate  $\delta$ . The quark-antiquark pair ( $u u^{-}$ ) is “almost” annihilating but cannot because of the binding state with the neighboring down quark ( $-3$ ) and the  $\delta$ -part, which is identical to the  $\delta$ -part in an electron (**Figure 23**).

Thus, the decay process ( $d \rightarrow u + e^{-} + \nu e^{-}$ ) is simply a separation of the up quark  $u$  and a transmutation of the anti-up quark  $u^{-}$ , the down quark ( $-3$ ), and  $\delta$  into an electron and an electron antineutrino (annihilation), as shown in the following **Figure 27**.



**Figure 27.** Transmutation of  $\delta$ ,  $(-3)$  and  $u^{-}$  into an electron and electron antineutrino.

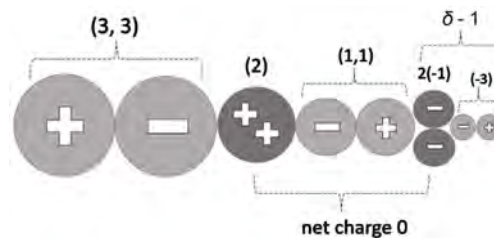
In the following, further examples of particles from the Standard Model are given neutron, proton, deuteron and finally a possible dineutron.

**Neutron**, electrical charge 0, bare mass 939.565 MeV and calculated mass 939.61 MeV below:

mass composition:  $105.456 + 2 \times 385.95 + 2 \times 28.815 + 2 \times 2.151 + 2 \times 0.161$ ;

quark composition:  $Q2 (Q3^{-} Q3) (Q1^{-} Q1) Q-1 Q-1 (Q-3^{-} Q-3)$ ;

structural and charge (**Figure 28**):



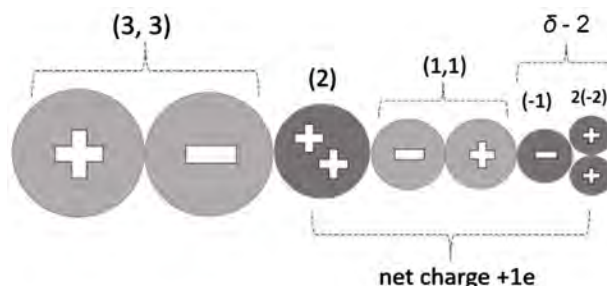
**Figure 28.** Neutron and its composition of harmonic quarks.

**Proton**, electrical charge  $+1e$ , bare mass 938.272 MeV and calculated mass 938.313 MeV below:

Mass composition:  $105.456 + 2 \times 385.95 + 2 \times 28.815 + 2.151 + 2 \times 0.588$ ;

Quark composition:  $Q_2 (Q_3^- Q_3) (Q_1^- Q_1) Q_1 (Q_2^- Q_2)$ ;

Structural and charge (**Figure 29**):

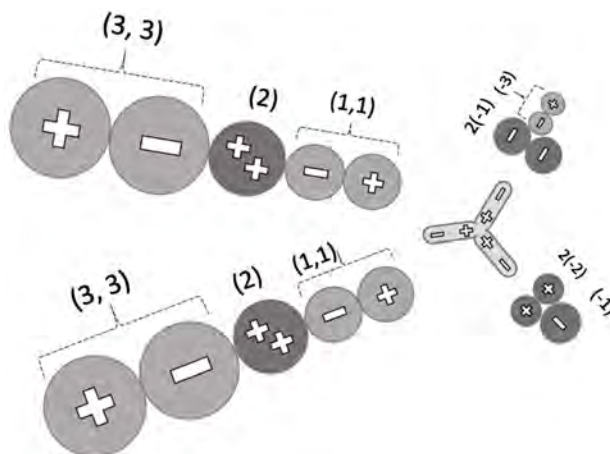


**Figure 29.** Proton and its composition of harmonic quarks.

The decay of a free neutron into a proton ( $N \rightarrow P + e^- + \nu e^-$ ) is considered here as a transmutation of the small right part  $\delta-1$  in **Figure 28** into the small right part  $\delta-2$  in **Figure 29**. It is mainly created by the decay of one harmonic quark  $Q_1$  with 2.151 MeV into its downward excited harmonic quarks, recombining with the harmonic quark pair  $(Q_3^- Q_3)$  and finally emitting an electron with 0.511 MeV and an electron antineutrino with an energy of about 0.769 MeV.

Experiments (CLAS Collaboration) have shown the existence of strange and anti-strange quark pairs in the proton’s mass structure by shooting electron beams into liquid hydrogen, scattering  $K^+$ -mesons and  $\Lambda$ -hyperons (refs. [15] and [16]). At least part of the products, the  $K^+$ -meson with a bare mass of 493.667 MeV, can be calculated here almost directly by adding the harmonic quark masses ( $105.456 + 385.95 + 2.151 = 493.557$  MeV, splitting two harmonic pairs of generation 3 and 1, respectively, in **Figure 29**).

**Deuteron**, electrical charge  $+1e$ , composed of a neutron and a proton as follows, using a TSL with vanishingly small mass (**Figure 30**):

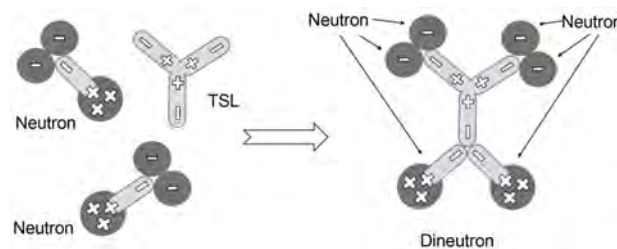


**Figure 30.** Deuteron and its composition of harmonic quarks.

In contrast to a single neutron, a bound neutron in a deuteron does not have the tendency to decay, since this would change the charge of the total system from +1e to +2e, which would imply an excess of positive charges.

### 8. A Possible Bounded Aggregate Model for Dineutron

In the past, the dineutron has been studied in some theoretical and experimental works (Ref. [17]-[24]). In particular, the existence of dineutron has been observed experimentally [19]. In general, it is considered as a semi-stable construction—in the sense of being short-lived and, above all, conditional occurrence. The YY model gives the aggregate state for it very simply, like deuteron, as nuclear fusion of two single neutrons and by consumption of a free TSL, which is a product of electron-positron annihilation in the same fusion environment, **Figure 31**.

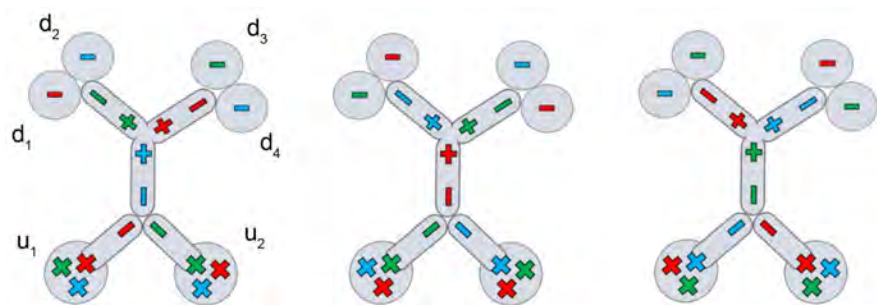


**Figure 31.** Dineutron nucleus fused from two neutrons by consuming one TSL.

By using the standard symbols (u—up quark, d—down quark) with the TSL ( $\Rightarrow Y$ ) we get the expression for a dineutron:

$$\begin{matrix} dd & dd \\ Y \\ uu \end{matrix}$$

For a QCD consideration for the dineutron, **Figure 32** below shows three selected CCAS (the left panel gets its all-constituent quarks are noted in the left panel):



**Figure 32.** Three CCAS of a dineutron.

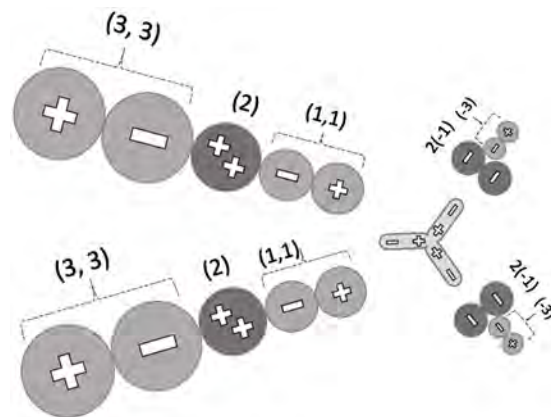
Selecting the first CCAS on the left panel of **Figure 33**, all correlation paths

are given in **Table 4** below:

**Table 4.** All quark correlation paths of dineutron in different expressions.

d1 - r/g - d3	$(\check{r}) \hat{Y} \langle \hat{g} g \rangle Y \langle r \check{r} \rangle \hat{Y} (\hat{g})$	$(-) \hat{Y} \langle - + \rangle Y \langle + - \rangle \hat{Y} (-)$
d3 - g/b - d4	$(\hat{g}) \hat{Y} (b)$	$(-) \hat{Y} (-)$
d4 - b/r - u1	$(b) \hat{Y} \langle \check{r} r \rangle Y \langle b b \rangle \hat{Y} \langle \check{r} r \rangle b   g \rangle$	$(-) \hat{Y} \langle - + \rangle Y \langle + - \rangle \hat{Y} \langle - + \rangle   + \rangle$
u1 - r/g - u2	$(b   g \langle r \check{r} \rangle) \hat{Y} \langle \hat{g} g \rangle r   b \rangle$	$(+   + \langle + - \rangle) \hat{Y} \langle - + \rangle   + \rangle$
u2 - g/b - d2	$(r   b \langle g \hat{g} \rangle) \hat{Y} \langle b b \rangle Y \langle g \hat{g} \rangle \hat{Y} (b)$	$(+   + \langle + - \rangle) \hat{Y} \langle - + \rangle Y \langle + - \rangle \hat{Y} (-)$
d2 - b/r - d1	$(b) \hat{Y} (\check{r})$	$(-) \hat{Y} (-)$

For a more detailed description including the mass consideration, the composition model for dineutron is presented as follows, using a TSL with vanishingly small mass:



**Figure 33.** Possible dineutron and its composition of harmonic quarks.

The structure is symmetric—it is slightly different from the detailed model for the deuteron in **Figure 30**. There can easily be a decay of a neutron into a proton, which converts the aggregate from a dineutron to a deuteron by emitting an electron and an electron antineutrino.

This aggregate for dineutron is a legal atomic nucleus state which obeys all constitutional rules described by the YY model, for example the rule “Internal Charge Balance”. It is electrically neutral as a whole because the surrounding parts are also electrically charge balanced. There is no electron orbiting around it. In this sense, a dineutron is also an atom in its own right that does not tend to bond with other atoms due to the lack of chemical valence. The mass of a dineutron has two atomic mass units. As for collisions with matter, their behavior can be related to that of the single neutron and is closely related to deuteron.

From the point of view of the transmutation process, there is no difference between the formation of a dineutron (**Figure 33**) and the formation of a deuteron (**Figure 30**). The physical conditions for them may differ, but they may also exist simultaneously. The mere existence of neutrons within a stellar fusion state

(Ref. [25] and [26]) is sufficient to assert, that dineutrons will be formed there (e. g., in the interior of the Sun) as byproducts – the TSLs are produced there, in the electron-positron annihilations, and are thus available.

Moreover, from the point of view of the holding forces for the nucleons, the mechanism is the same in the case of a dineutron and in the case of a deuteron: they are strong forces described by the standard model or as reinterpreted by the YY model, the superposition of all correlation paths.

The Pauli exclusion principle does not allow a legal bound state of two neutrons. However, a dineutron according to the YY model is not this constellation it is a bound state of two pairs of down quarks and one pair of up quarks. This constellation must lead to a reconsideration of the spin states of all the sub-particles involved without violating the Pauli exclusion principle (In this paper, we will not investigate this further). The question how stable a free dineutron is can be related to the stability of a deuteron: The two nuclear aggregates have an internal charge balance (ICB) of “one-positive to one-negative”. Compared to the deuteron’s external charge balance of “three-positives to two-negatives”, the dineutron must have a strong structural bond because its external charge balance has a ratio of “two-positives to two-negatives”, resulting in little repulsions.

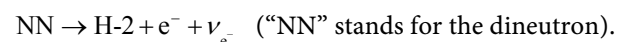
The Bethe-Weizsäcker formula and its refined variants (Ref. [27] [28] [29]) for calculating nuclear binding energies are generally not suitable for treating small nuclei such as a deuteron or a dineutron. F. C. Hoh has made insightful considerations of the binding energies of a twin nucleon system (Ref. [30]), which included the aspects of spin orientation, electrostatic confinement, and the distance range of the quarks involved. Based on plausible model configurations, he calculated the binding energy for a deuteron (closely matching the measured 2.23 MeV) and for a dineutron (1.78 MeV). Although the binding energy of a dineutron is weaker than that of a deuteron, he concluded that a triplet dineutron is electromagnetically bound and is a stable nucleus, similar to a triplet deuteron, which is also electromagnetically bound. F. C. Hoh pointed out that a dineutron can energetically decay into a deuteron by neutron beta decay—this is no surprise. He predicted a decay time to be half of the neutron decay time or 440 sec.

We therefore prefer the term “semi-stable” for the stability of a dineutron.

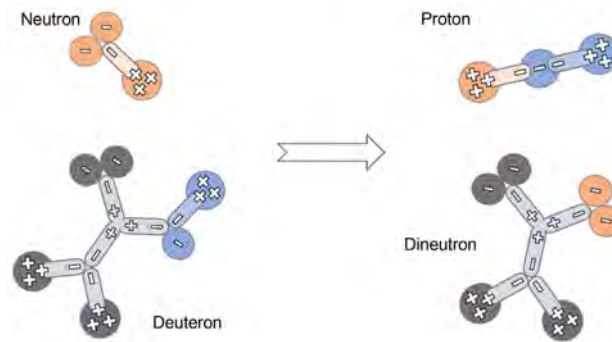
## 9. Dineutron and Possible Transmutations

Furthermore, the close relationship between a dineutron and a deuteron is also evident in their transmutations into each other, possibly under the condition of collisions. We describe such scenarios to provide clues for experimental evidence that can support the finding of dineutrons with light nuclei.

A natural decay of a dineutron (**Figure 33**) to a deuteron (**Figure 30**) emitting an electron and an electron anti-neutrino is mostly possible:



A collision of neutron and deuteron can lead to a transmutation into a proton and a dineutron, **Figure 34**, formulated as:  $N + H-2 \rightarrow p + NN$ .

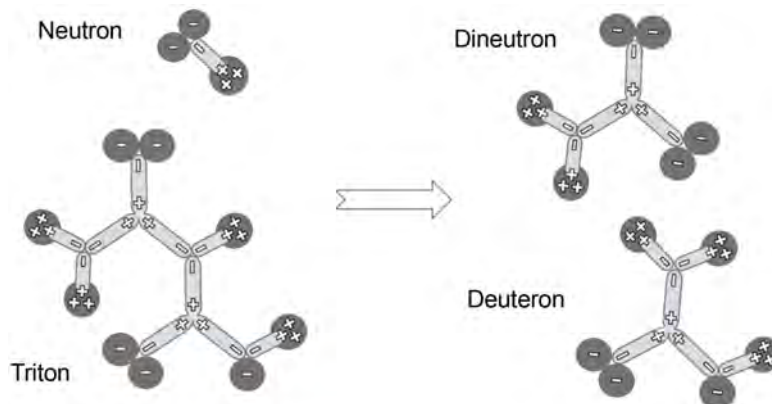


**Figure 34.** Collision of a neutron with a dineutron and their outcomes.

Theoretically, a fusion of neutron and deuteron can also produce a triton – tritium nucleus—(Ref. [31] [32] [33]). This would require a nuclear fusion condition and is not considered here. The products of a neutron-deuteron collision are usually considered to be a scattered proton plus two individual neutrons, each of which subsequently decays into a proton and an electron plus an electron antineutrino (beta decay). Assuming a dineutron does not decay, unlike a single neutron, a significant shortage of protons and electrons (and electron-antineutrinos) can be detected experimentally by observing the total particle balance: absorbed neutrons—actually part of them—are bounded to truncated deuterons—to dineutrons. Only the transmutation from deuteron to hydrogen nucleus will be dominant. Detection by such an experiment would mean detection of stably bound dineutrons—without participation of large nuclei.

The dineutrons produced in the above transmutation can also be subsequently hit by other firing neutrons, producing single neutrons ( $N + NN \rightarrow 3N$ ). In this case, much more nuclear binding energies will be released because the early fused hadron state for the deuteron is dissolved—corresponding to a nuclear fission reaction.

Similarly, a collision of neutron and tritium nucleus can lead to a transmutation into a deuteron and a dineutron, **Figure 35** (The tritium nucleus has already been described in our first paper on the YY model), formulated as:  
 $N + H-3 \rightarrow H-2 + NN$ .

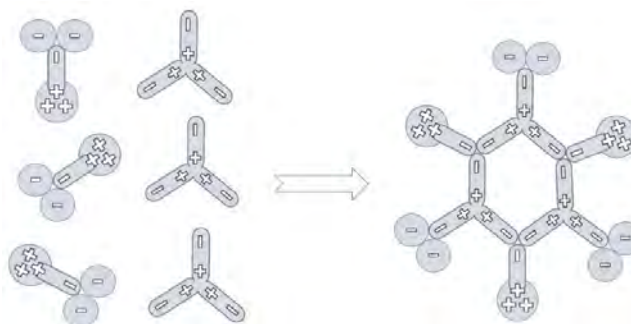


**Figure 35.** Collision of a neutron with a tritium nucleus and their outcomes.

If the experiment is designed as bombardment of a tritium medium with neutrons, the subsequent hit of the previously split deuteron by a following neutron must also be taken into account.

## 10. Possible Bounded State for Trineutron, Tetraneutron and More

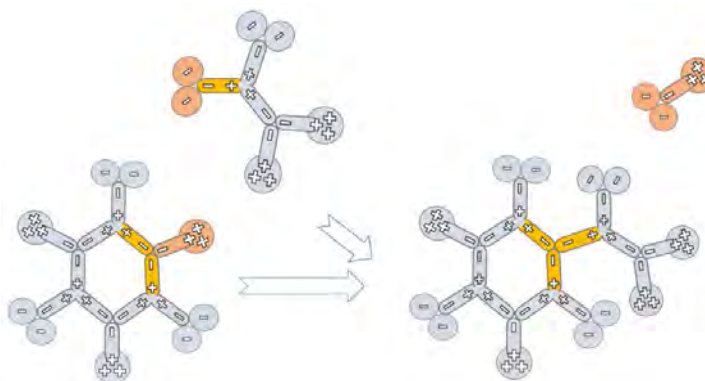
When considering atomic aggregates containing multiple neutrons with zero electrical charge (Ref. [34]), more complex legal aggregate states can be obtained. For example, a model state representing a trineutron (three neutrons aggregated together, **Figure 36**). It is fused from three single neutrons by consuming three TSLs that hold the resulting structure together.



**Figure 36.** Trineutron core fused from three neutrons and three TSLs.

A trineutron has three atomic mass units, is electrically charge neutral. It can also be a fusion byproduct during the stellar fusion process, if there are enough “ingredients”. Here no conclusion can be drawn about the stability. If it would be and participates in a collision with another atomic nucleus, it is mostly decomposed into its composite neutrons.

A tetraneutron (consisting of four neutrons) can be fused from a dineutron and a trineutron. This transmutation releases a single neutron (**Figure 37**).

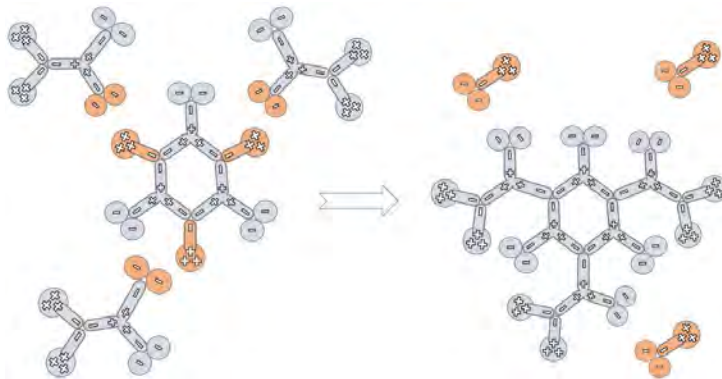


**Figure 37.** Tetraneutron plus a single neutron transmuted from a dineutron and a trineutron.

Theoretically, the fusion of di- and trineutrons can occur at a single docking

site of the involved aggregates, as colored in Figure. At the docking site of trineutron, the up quark is released. At the docking site of dineutron, two down quarks are released. The three “open” PSLs bind together to form a negatively charged node, which links the two (reduced) participating aggregates together. The released double down quarks and the released-up quark combine to form a free neutron.

Due to the symmetric structure of the trineutron, the fusion described in **Figure 38** can take place at three different docking sites, resulting a sixfold neutron aggregate by fusion with three dineutrons together. This process will release three single neutrons, **Figure 38**.



**Figure 38.** Fusion of a trineutron with three dineutrons, releases a sixfold neutron and three single neutrons.

On the one hand, if we consider the color aspect, the resulting sixfold neutron also has a valid set of CCAS. On the other hand, in this case the question of structural stability becomes more complex. We cannot give an answer to this question.

## 11. Conclusions and Outlook

Quark correlation path, color forces and mass composition concept are considered theoretically in this article and represent three new aspects as a further development of the YY model approach. They have been examined using examples of deuteron and dineutron. Future research should combine more of the recent results from the field of QCD (Ref. [35] and [36]). Despite the structural refinement of the constituent quarks—in recursive or fractal form of the lower generations—it is still useful to describe the overall structure initially with only Yins and Yangs of the higher generations, without including details. The constituent masses of low generations play an important role for subsequent investigation approaches. In the future, the distribution of them must be more in the focus of the investigation. Relevant approaches in research, for example [37] may play a role.

Early work by others on the dineutron had quite different starting points than here—for example, in the use of large elementary constituents in theory and



large atomic nuclei in experiment. Thus, early conclusions about the dineutron, especially about the stability of bound states, could be reconsidered by performing the proposed collision of neutrons with deuterons and by analyzing the scattering products: The emergence of net protons in the absence of beta decay suggests the formation of dineutrons, rather than the usual neutron absorption theory. This happens only when the dineutron state is stable enough for their occurrence to be determined.

Along the quark correlation path, the further future work can simplify the mathematical basis based on quantum field theory. For this purpose, some parameters for the attractive and repulsive color forces have to be defined and determined based on empirical values. This quantitative calculation would possibly provide more closed-form solution formulas or more accurate calculation results than with the standard approach. However, the really interesting part is a better theoretical foundation of fundamental artifacts in the standard model, such as spin states in terms of Yin-Yang constituents. The manifestation of energy and matter in constituted quarks and anti-quarks (in the sense of Teplov harmonic oscillators) is another interesting aspect of development for the YY model. The structural constitution of a quark or an anti-quark enforces the “center and distribution” of the expected mass mounts. For example, an anti-up quark consists of two Yins and an energetic bond between them. The center of mass must be near the energetic region. Further work is needed for a more sophisticated view.

A close relationship between the universal TSL (Y particle) in the YY model and photons, neutrinos, and anti-neutrinos forms another interesting research topic for the future. These particles have significant wave-particle duality and cannot be constituted as easily as is possible for particle with short-range interactions. Nevertheless, one might consider participation of the TSL, in transformed form, in the formation of the photon and neutrinos. Beta decay and inverse beta decay have some points that support this consideration.

## Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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# Remarkable Findings in Fundamental Theory of Quantum Mechanics

## —Matter Wave and Discrete Time in Physics

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### Abstract

The relation of matter wave, which is well-known as a hypothesis proposed by de Broglie in 1923, gave basis for establishing the quantum mechanics. After that, experimental results revealed that a micro particle has a wave nature. However, the theoretical validity of the relation itself has never been revealed since his proposal. Theoretical basis that a micro particle has a wave nature has been thus disregarded in the unsolved state. The diffusion equation having been accepted as Fick's second law was derived from the theory of Markov process in mathematics. It was then revealed that the diffusivity  $D$  depends on an angular momentum of a micro particle in a local space. The fact being unable to discriminate between micro particles in a local space resulted in having to accept the existence of minimum time  $t_0 (> 0)$  in the quantum mechanics. Based on  $t_0$  and  $D$  obtained here, the theoretical validity of relation of matter wave was confirmed. Denying the density theorem in mathematics for time in physics indicates that the probabilistic interpretation is essentially indispensable for understanding the quantum mechanics. The logical necessity of quantum theory itself is thus understandable through introducing  $t_0$  into the Newton mechanics. It is remarkable that the value of  $t_0$  between  $1.14 \times 10^{-17} \text{ s} \leq t_0 \leq 1.76 \times 10^{-14} \text{ s}$  obtained here is extremely larger than that of the well-known Planck time  $t_p = 5.39 \times 10^{-44} \text{ s}$ .

### Keywords

Quantum, Diffusion Particle, Matter Wave, Planck Time, Minimum Time

## 1. Introduction

There are sometimes reproducible phenomena expressible by a relation under

the given conditions in physics. When we cannot theoretically reveal the validity of its relation, it has been accepted as a law or a principle. Further, such hypotheses as de Broglie's hypothesis relevant to the matter wave, Planck's hypothesis relevant to the photon energy, Bohr's hypothesis relevant to the atomic model and so on, have been also often accepted in the history of physics. Physics has developed in the theoretical frame based on such laws, principles or hypotheses. For example, Newton's laws are valid in the following preconditions.

Precondition [A]: the absolute time of  $t' = t$  is accepted between the different coordinate systems of  $(t, x, y, z)$  and  $(t', x', y', z')$ .

Precondition [B]: the mathematical density theorem is valid in arbitrary variables of coordinate system, that is,  $\lim_{t_1 \rightarrow t_2} (t_1 - t_2) = 0$ ,  $\lim_{x_1 \rightarrow x_2} (x_1 - x_2) = 0$ , and so on.

Here, when we found a new fact contradictory to the existing laws, principles or hypotheses, themselves or their preconditions should be examined again. For example, Einstein's relativity, which is one of the modern physics, was established by denying the above precondition [A], accepting the constant principle of light speed in contradiction to Newton's law. On the other hand, the quantum theory of another modern physics was established by accepting the hypothesis of de Broglie [1], which had never been understandable in the Newton mechanics until recently [2].

In 1926, Schrödinger [3] derived the wave equation of a micro particle from the hypothesis proposed by de Broglie in 1923. The so-called Schrödinger equation has been in conformity with each behavior of micro particles. Judging from the theoretical frame of physics, however, the quantum theory has been still essentially incomplete without revealing the causality for the Newton mechanics, even if it is justifiable. In fact, we have the unsolved "proposition" having to verify the theoretical basis for wave nature of a micro particle.

To solve the proposition in those days, it seems that Einstein, Bohm, and others tried to transform the diffusion equation of micro particles into the wave equation of Schrödinger. However, their projects ended in failure. In actuality, the above proposition has been disregarded and the quantum theory has developed as an afterthought in the matter of fundamental problems. Incidentally, the diffusion equation has been accepted as a law proposed by Fick in 1855. As far as we thus accept it as a law, the diffusivity is only a mathematical operator in the partial differential equation and we cannot grasp its physical meaning then. Here, Okino [4] thought that their failures are caused by accepting the diffusion equation as a law. To grasp the essential meaning of diffusivity in physics, therefore, deriving the diffusion equation from the theory of Markov process in mathematics was first considered then. As a result, it was revealed that the diffusivity  $D$  depends on an angular moment of a micro particle in a closed local space and  $D = \hbar/2m$  is valid then, where  $\hbar$  and  $m$  are  $\hbar = h/2\pi$  for the Planck constant  $h$  and a mass of micro particle.

The photon energy indicates that the discrimination between two micro particles in a local space is essentially impossible. Here, we accept the matter as an

impossible principle of discrimination between micro particles. In that case, the impossible principle of discrimination between micro particles results in the fact that there is a minimum time  $t_0$  as a real time in physics in contradiction to the density theorem of real time in mathematics [5].

As a result, the wave equation of Schrödinger is reasonably derived from the diffusion equation for micro particles by using the impossible principle of discrimination between micro particles and the diffusivity  $D = \hbar/2m$ . Here, the wave nature of a micro particle was theoretically revealed. Further, the validity of the relation itself of matter wave was reasonably revealed [6]. In addition, such theoretical basis that the probabilistic interpretation is indispensable for the quantum theory is also reasonably revealed.

Judging from the theoretical frame of physics, it is essentially important to understand the logical necessity reaching from the Newton mechanics to the quantum mechanics. Nevertheless, the elucidation of logical necessity has been disregarded for a long time in the unsolved state. Thus, the elucidation is a main purpose in the present work.

As a result, such theoretical bases that a micro particle has wave nature and that the probabilistic interpretation is indispensable for the quantum theory were reasonably revealed in introducing the conception of  $t_0$  into the Newton mechanics. In other words, we will notice that the quantum theory is established by denying  $\lim_{t_1 \rightarrow t_2} (t_1 - t_2) = 0$  in the precondition [B] mentioned above.

## 2. Verification of Matter Wave

For a micro particle of mass  $m$  moving with a speed  $v$  in space-time  $(t, x, y, z)$ , the partial differential equation of wave function  $\Psi = \Psi(t, x, y, z)$  yielding

$$i\hbar \frac{\partial}{\partial t} \Psi = -\frac{\hbar^2}{2m} \langle \tilde{\nabla} | \nabla \rangle \Psi \quad (1)$$

was derived by Schrödinger [3] from the hypothesis of de Broglie [1] of

$$\lambda = h/p, \quad (2)$$

where  $i, \hbar, \lambda$  and  $p$  are a unit imaginary number,  $\hbar = h/2\pi$  for the Planck constant  $h$ , a wave length of matter wave and a momentum  $p = mv$ . In addition, the nabla vector  $\nabla$  is expressed by the Dirac bracket and the notation  $\langle \tilde{\nabla} | = -|\nabla \rangle^\dagger$  is then defined because of the Hermite conjugate.

For the concentration  $C = C(t, x, y, z)$  of diffusion particles, the nonlinear diffusion equation of moving coordinate system given by

$$\frac{\partial C}{\partial t} = D \langle \tilde{\nabla} | \nabla \rangle C \quad \text{for} \quad D = \frac{(\Delta r)^2}{2\Delta t}, \quad r = \sqrt{x^2 + y^2 + z^2} \quad (3)$$

was derived from the theory of Markov process in mathematics [7]. For a diffusion particle in the closed local space, Equation (3) shows that the diffusivity  $D$  is rewritten as  $D = \Delta r p / 2m$  relevant to an angular momentum of the diffusion particle, because of  $\Delta r p = \sqrt{\langle \Delta r \times \tilde{p} | \Delta r \times p \rangle}$  in the present case. This means that the diffusion particle in a local space makes a circuit around the center point of

local space.

On the other hand, the quantum condition  $r_n p = n\hbar$  ( $n = 1, 2, \dots$ ) in the atomic model of Bohr is also able to rewrite as  $\Delta r p = \hbar$  for an orbital electron because of  $\Delta r = r_n - r_{n-1}$  ( $r_0 = 0$ ). After confirming that the relation  $\Delta r p = \hbar$  is even valid in an arbitrary motion of electron because of  $\Delta r p = \langle \Delta r | p \rangle$  in the present case, applying the equipartition law to a free electron in such material as metal revealed that the relation of

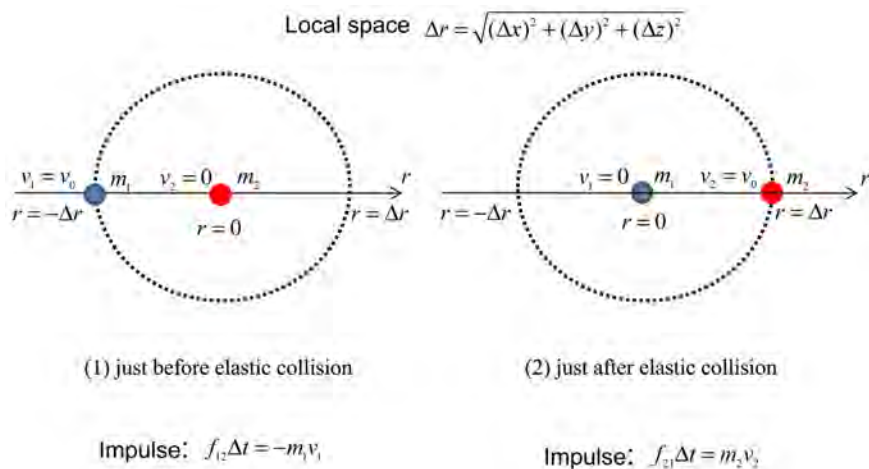
$$\Delta r p = \hbar \tag{4}$$

is also valid for an arbitrary micro particle [6]. Therefore, substituting Equation (4) into  $D = \Delta r p / 2m$  yields

$$D = \frac{\hbar}{2m} \tag{5}$$

for a micro particle in local space [5].

Accepting the impossibility of discriminating between micro particles in a local space, the investigation of an elastic collision process between two micro particles of the same kind revealed that there is a minimum time  $t_0$  as a real time in physics [8]. As can be seen from Figure 1, although we cannot understand behavior of the particle 1 between  $-\Delta r < r < \Delta r$ , it seems then that the particle



**Figure 1.** Elastic collision between two micro particles of the same kind. The figure shows an elastic collision between a particle 1 having a mass  $m_1 = m$  and a velocity  $v_1 = v_0$  at  $r = -\Delta r$  and a particle 2 of the same kind having the mass  $m_2 = m$  and the velocity  $v_2 = 0$  at  $r = 0$  in the initial state. In the Newton mechanics, those impulses are rewritten as  $f_{12} = -m_1 \Delta r / (\Delta t)^2$  and  $f_{21} = m_2 \Delta r / (\Delta t)^2$ . If we cannot discriminate them, however, the relations  $\bar{f}_{12} = m_1 \Delta r / (\Delta t)^2$  and  $\bar{f}_{21} = -m_2 \Delta r / (\Delta t)^2$  obtained by replacement of each suffix 1 and suffix 2 should be then equivalent to the original expressions  $f_{12}$  and  $f_{21}$ , respectively. Therefore, the relations  $\bar{f}_{12} \rightarrow f_{12}$  and  $\bar{f}_{21} \rightarrow f_{21}$  resulting from the impossibility of discrimination between those particles correspond to rewriting  $\Delta t \rightarrow \pm i \Delta t$  in each equation of  $\bar{f}_{12} = m_1 \Delta r / (\Delta t)^2$  and  $\bar{f}_{21} = -m_2 \Delta r / (\Delta t)^2$ . The density theorem in mathematics is thus not valid for the time in physics, but it is still valid for the space. In that meaning, the conception of time is different from that of space.

1 moved from  $r = -\Delta r$  to  $r = \Delta r$  without incident through the impossibility of discriminating between the particle 1 and the particle 2. In other words, consequently we seem as if the particle 2 was nonexistent from the beginning. As mentioned in the caption of **Figure 1**, we must accept the imaginary time  $\pm i\Delta t$  in physics for  $0 \leq \Delta t < t_0$  in mathematics then, denying the mathematical density theorem. It was thus revealed that the minimum time  $t_0$  is existent in the quantum theory and an arbitrary time  $t_j$  is expressed as a discrete time yielding  $t_j = jt_0$  for  $j = -\infty, \dots, -2, -1, 0, 1, 2, \dots, \infty$ .

In accordance with the limit theory, the existence of the minimum time  $t_0$  reveals that the differential operators  $\partial/\partial t$  and  $\partial/\partial x$  become

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \left( \frac{\partial}{\partial t} \right) x \rightarrow \left( \mp i \frac{\partial}{\partial t} \right) x = \mp i \lim_{\Delta t \rightarrow 0} \frac{\Delta x}{\Delta t} = \lim_{\Delta t \rightarrow 0} 1 / \left( \frac{\Delta x}{\pm i \Delta t} \right)$$

and

$$\lim_{\Delta x \rightarrow 0} \frac{\Delta t}{\Delta x} = \left( \frac{\partial}{\partial x} \right) t \rightarrow \left( \pm i \frac{\partial}{\partial x} \right) t = \pm i \lim_{\Delta x \rightarrow 0} \frac{\Delta t}{\Delta x} = \lim_{\Delta x \rightarrow 0} 1 / \left( \frac{\Delta x}{\pm i \Delta t} \right)$$

in the differential equation for a micro particle. Judging from eigenvalues of these operators, therefore, the differential operators  $\partial/\partial t$  and  $|\nabla\rangle$  in the Newton mechanics should be rewritten as

$$\partial/\partial t \rightarrow i\partial/\partial t, \quad |\nabla\rangle \rightarrow -i|\nabla\rangle \quad (6)$$

in the quantum mechanics [6].

Here, substituting Equations (5) and (6) into Equation (3) and rewriting  $\hbar C \rightarrow \mathcal{P}$  give the Schrödinger Equation (1). At this point, the wave nature of an arbitrary micro particle was theoretically verified in accordance with the causality for the Newton mechanics because of the reasonable transformation from the equation of micro particle into the wave equation. At the same time, the wave length  $\lambda$  of matter wave for an arbitrary micro particle is expressed as

$$\lambda = 2\pi\Delta r \quad (7)$$

from the wave characteristic. Here, the above proposition having been disregarded for a long time was thus theoretically solved. Further, Equation (2) was theoretically derived for the first time in the history of quantum theory by eliminating  $\Delta r$  from Equations (4) and (7). Thus, the relation of matter wave is now not a hypothesis but a basic equation in physics judging from the theoretical frame of physics.

In addition, another relation of matter wave was also obtained as

$$\lambda = h / \sqrt{\alpha_n m (k_B T + \varepsilon)} \quad (8)$$

in the analytical process, where  $k_B, T, \varepsilon$  and  $\alpha_n$  are the Boltzmann constant, an absolute temperature in material, a correction term at  $T = 0$  in relation to the uncertain principle and a degree of freedom of micro particle composed of  $n$  atoms [6]. Eliminating  $m$  from Equations (2) and (8), a period  $T_p$  of matter wave is obtained as



$$T_p = \frac{\lambda}{v} = \frac{h}{\alpha_n (k_B T + \varepsilon)}. \quad (9)$$

When a micro particle passes through a local space of the size  $l = 2\Delta r$ , Equations (7) and (9) show that the taken time  $t_{\alpha_n}$  is expressed as

$$t_{\alpha_n} = \frac{l}{v} = \frac{h}{\pi \alpha_n (k_B T + \varepsilon)}. \quad (10)$$

It is thus remarkable that the time  $T_p$  and  $t_{\alpha_n}$  depend only on  $\alpha_n$  and  $T$ .

The mathematical solution  $\Psi = \Psi(t, x, y, z)$ , which is obtained in accordance with the theorem of unique solution for the differential Equation (1), corresponds to either  $\Psi = \Psi(t_j, x, y, z)$  or  $\Psi = \Psi(t_{j+1}, x, y, z)$  between  $t_j \leq t \leq t_{j+1}$  with a certain probability for every  $j$  value, because of the fluctuation caused by the existence of discrete time  $t_j = jt_0$  in the quantum theory. This means that we cannot principally apply the theorem of unique solution for a differential equation in mathematics to analyzing differential equations in the quantum mechanics. At the same time, this indicates that we must accept the probabilistic interpretation as a basic conception in the quantum theory. However, the behavior of a micro particle corresponds to the mathematical solution  $\Psi = \Psi(t, x, y, z)$ , as far as we do not determine the functional value.

Using a probability factor  $A_j$  ( $0 \leq A_j \leq 1$ ) for the solution  $\Psi(t_j, x, y, z)$ , the physical solution  $\Psi_p(t, x, y, z)$  corresponding to  $\Psi(t, x, y, z)$  is expressed as

$$\Psi_p(t, x, y, z) = A_j \Psi(t_j, x, y, z) + A_{j+1} \Psi(t_{j+1}, x, y, z) \text{ for } A_j + A_{j+1} = 1, \quad (11)$$

using a superposition of wave functions for every  $j$  value. In that case, it seems as if  $\Psi_p(t, x, y, z)$  interferes with itself because of the interference between  $\Psi(t_j, x, y, z)$  and  $\Psi(t_{j+1}, x, y, z)$ , resulting from accepting the discrete time  $t_j = jt_0$  in the present theory. Here, we can now understand the theoretical evidence that a wave function interferes with itself in the quantum theory.

In addition, it seems that Einstein did not accept the probabilistic interpretation in the quantum theory in relation to the theorem of unique solution for a differential equation in mathematics. However, we now suppose that he would accept it in those days if he noticed the correlation between  $\Psi(t, x, y, z)$  and  $\Psi_p(t, x, y, z)$  mentioned above. It is, therefore, essentially important that the minimum time  $t_0$  is existent in the quantum theory.

### 3. Revision of Diffusion Theory

In general, we have no such a conception that the space itself moves in physics. However, it is considered that the space within a diffusion region moves relatively with respect to the surface of diffusion region because of the following reason. The expansion or shrinkage of diffusion region is caused by a thermal influence. In other words, an observer on the surface of diffusion region seems that the space within the diffusion region moves then. This means that the coordinate system setting a coordinate origin at a point of space within the closed diffusion region is a moving coordinate system with respect to the outside of

diffusion region.

It was confirmed that the nonlinear diffusion Equation (3) is reasonably transformed into the usual expression of the fixed coordinate system given by

$$\frac{\partial C}{\partial t} = \langle \tilde{\nabla} | D \nabla \rangle C, \quad (12)$$

which has been accepted as a law of Fick for a long time [7]. The diffusion Equation (3) has not been recognized as a nonlinear partial differential equation of a moving coordinate system, in spite of the indispensable one for understanding the diffusion theory. In addition, the universal diffusivity expression of

$$D = \frac{\hbar}{2m} \exp \left[ \frac{U - Q}{k_B T + \varepsilon} \right] \quad (13)$$

applicable to an arbitrary micro particle in a material with an activation energy  $Q$  was also reasonably obtained, where  $U$  is a potential energy between a micro particle in local space and micro particles around the local space.

Judging from the theoretical frame of physics, the diffusion Equation (12) is now not a law but a basic equation in physics. Thus, the finding obtained here gives us a lesson that we should sometimes try to reexamine the relation having been accepted without the demonstration, even if it has been accepted as a law for a long time in physics. It is essentially indispensable for analyzing diffusion problems to discuss the coordinate systems used for the diffusion equation. This means that the existing fundamental theory of diffusion should be revised in accordance with the discussion between the coordinate systems used for the diffusion equation [4]. For example, although the conception of intrinsic diffusion has been widely accepted for a long time, we will notice that such a conception, which was empirically assumed in relation to the Kirkendall effect in those days, is nonexistent from the beginning as if it has been an illusion [7].

Even the general solutions of the concentration  $C = C(t, x)$  and the diffusivity  $D = D(t, x)$  in case of one dimension space for Equation (12) were not obtained. Then, Boltzmann [9] in 1894 transformed Equation (12) in case of the coordinate system  $(t, x)$  into the nonlinear ordinary differential equation of

$$-\frac{\xi}{2} \frac{dC}{d\xi} = \frac{d}{d\xi} \left\{ D \frac{dC}{d\xi} \right\} \quad (14)$$

in the parabolic space  $\xi = x/\sqrt{t}$ . Nevertheless, the general solutions of  $C = C(\xi)$  and  $D = D(\xi)$  of Equation (14) had not been also obtained for a long time. In that situation, recently the general solutions of Equation (14) were first obtained [4]. Using them for the diffusion problems of many elements system, the reasonable analytical method has been thus established [7].

#### 4. Minimum Time in Physics

Oriental people have been used to the word “setsuna” [刹那] defined as a minimum time in the world, resulting from the Sanskrit word in ancient India. The existence of minimum time was also theoretically clarified in physics. The impossibility of discriminating between micro particles in a local space revealed the

existence of minimum time  $t_0$  in the quantum theory then. In comparison with Einstein's relativity established by denying the above precondition [A], the quantum theory developed here is established by denying the precondition [B], *i.e.*, by accepting  $\lim_{t_1 \rightarrow t_2} (t_1 - t_2) = \pm t_0$  in contradiction to the Newton mechanics [6].

The finding obtained here reveals the theoretical evidence that the chronon (quantum-time) proposed by Levi [10] in 1927 as a hypothesis is existent in the quantum mechanics. After that, Caldirola [11] in 1980 reported the time  $\theta_0 = e^2 / 6\pi\epsilon_0 mc^3 (= 6.27 \times 10^{-24} \text{ s})$  as a value of chronon in the electron theory, where  $e, \epsilon_0, c$  and  $m$  are the elementary charge, the dielectric constant, the light speed and the mass of electron. Further, the Planck time  $t_p = \sqrt{\hbar G / c^5} (= 5.39 \times 10^{-44} \text{ s})$  expressed by  $\hbar, c$  and the gravitational constant  $G$  is also well-known as a minimum time in physics.

In relation to Equation (10), the minimum time  $t_0$  is estimated in the following. In general, the temperature effect is not considered in analyzing Equation (1). Using the room temperature  $T_R (\cong 290 \text{ K})$  for Equation (10), therefore, Equation (10) is rewritten as

$$t_{\alpha_n} = \frac{2\hbar}{\alpha_n k_B T_R}, \quad (15)$$

where  $\varepsilon \cong 0$  is acceptable in the present case. If it is possible that Equation (15) corresponds to the minimum time  $t_0$ , the relation  $t_0 \leq t_{\alpha_1} (= 1.76 \times 10^{-14} \text{ s})$  is valid then because of using  $\alpha_n = 3$  for a monatomic molecule ( $n = 1$ ).

The reasonable transformation from the diffusion Equation (3) or (12) into the wave Equation (1) of Schrödinger indicates that the random movement of micro particles is closely relevant to each wave nature of them and further that the parabolic law shown in the concentration profile corresponds to the matter wave. The correlation between the diffusion theory and the quantum theory is thus close with each other. In other words, when the self-diffusion phenomena are observed in a material, a micro particle constituting the material has the wave nature then.

When a vacant local space is generated by a thermal fluctuation in the gas state, a molecule in a neighboring local space jumps to the vacant local space in accordance with the elementary process of diffusion. The random movement of molecules occurs through such iteration. The behavior of gas molecules can be investigated by using not only the diffusion equation but also the state equation for ideal gas.

In the following, the present minimum time  $t_0$  is roughly estimated by using the state equation for ideal gas. In the gas state, when a molecule in local space jumps to the neighboring vacant one in relation to the diffusion phenomena, we think for the present that the jumping time corresponds to a minimum time  $t_0$  resulting from a collision process mentioned above.

Here, Avogadro's law shows that gas molecules of  $N_A (= 6.02 \times 10^{23})$  numbers coexist in the volume  $V_0 (= 2.24 \times 10^{-2} \text{ m}^3)$  at the temperature  $T_0 (= 273 \text{ K})$

and the pressure 1013 hPa. For a size of local space occupied by a molecule at the same pressure, it is roughly considered as  $l_0 = (V_0 T_R / N_A T_0)^{1/3} (= 3.41 \times 10^{-9} \text{ m})$  at the room temperature  $T = T_R$ . Therefore, the minimum time is roughly obtained as  $t_0 \geq 1.14 \times 10^{-17} \text{ s}$  because of  $t_0 > l_0/c$ .

In the present study, it was found that minimum time  $t_0$  depends on a physical system of a micro particle concerned. As a result, the relation of

$$1.14 \times 10^{-17} \text{ s} \leq t_0 \leq 1.76 \times 10^{-14} \text{ s} \quad (16)$$

is thus obtained. In addition, we can roughly discuss a size of micro particle to take account of the quantum effect from Equations (15) and (16), as discussed in the following.

Substituting  $t_{\alpha_n} = t_0 (= 1.14 \times 10^{-17} \text{ s})$  into Equation (15), the value of  $\alpha_n = 4.62 \times 10^3$  is obtained as a degree of freedom of a micro particle at the room temperature  $T = T_R$ . If we can then determine atom numbers  $n$  corresponding to the degree of freedom  $\alpha_n$ , it is considered that a micro particle composed of atoms fewer than  $n$  atoms has a wave nature. On the other hand, it is also considered that the size of micro particle should be smaller than  $l_0 (= 3.41 \times 10^{-9} \text{ m})$  in relation to the size of local space. In addition, the various material structures are possible for a micro particle, for example, a giant molecule, a nanoparticle of metal, and so on. In that situation, since the degree of freedom  $\alpha_n$  depends on the complicated structure of each micro particle concerned, a matter for the correlation between  $\alpha_n$  and  $n$  will be accepted as a subject in the future, but  $n = 1.54 \times 10^3$  is possible if  $\alpha_n = 3n$  is simply acceptable.

For the difference between  $t_p$  and  $t_0$ , the author thinks that the time  $t_p$  is not actual judging from the theoretical frame of physics, because we cannot suppose matters like the physical quantities  $c$  and  $\hbar$  resulting from denying preconditions [A] and [B] in the Newton mechanics are simultaneously used with the gravitational constant  $G$ . The expression of  $\theta_0$  corresponds only to the charged particle, but  $t_0$  is valid for an arbitrary micro particle.

Here, it is remarkable that a minimum time of order  $10^{-17} \text{ s}$  satisfying Equation (16) has been reported in relation to the uncertainty principle [12]. In that situation, Equation (10) indicates that the time  $t_{\alpha_n}$  does not depend on each mass  $m$  of micro particles but  $\alpha_n$  and  $T$ . On the other hand, the diffusivity having the close correlation with the quantum theory depends on a mass of micro particle. Therefore, the detailed estimation of the discrete time  $t_0$  pointed out here should be widely investigated in the future, judging from the importance of fundamental theory in the quantum mechanics. In any case, if the time  $t_0$  obtained here is just valid in the quantum mechanics, some fundamental theories in physics may be unexpectedly influenced by a fluctuation resulting from the minimum time  $t_0$ .

## 5. Discussion and Conclusions

Judging from the theoretical frame of physics, the essential equation in physics

for micro particles is just considered to be Equation (3) itself, which is theoretically derived from the theory of Markov process in mathematics. The reason is as follows. Equation (3) is transformable into Equation (12) using Equation (13) applicable to an arbitrary diffusion field under the condition of  $t_0 = 0$ . On the other hand, Equation (3) corresponds to Equation (1) through substituting Equations (5) and (6) into Equation (3) and rewriting  $\hbar C \rightarrow \Psi$  under the condition of  $t_0 > 0$ . Here, the determination of either  $t_0 > 0$  or  $t_0 = 0$  depends only on whether we investigate behavior of a single micro particle or that of its collective motion.

Since the establishment of quantum theory, some basic problems have been disregarded in the unsolved state. We have been thus unable to understand the theoretical basis that a micro particle has a wave nature. In other words, the theoretical evidence that the hypothesis of de Broglie is valid has never been revealed. In addition, the theoretical bases that the probabilistic interpretation is indispensable for the quantum theory and the matter wave interferes with itself have not been also essentially understood even if it has been plausibly explained in textbooks using a slit.

In that situation, recently those bases were theoretically proved by obtaining the essential diffusivity expression relevant to an angular momentum of micro particle from the theoretical derivation of diffusion equation having been accepted as a law for a long time, and at the same time by revealing the existence of discrete time  $t_j$  in the world of a micro particle. Thus, we could first theoretically solve the problems having been disregarded for a long time in the basic theory of quantum mechanics.

Including the fact that Equation (2) is now not a hypothesis but a basic equation, the new fundamental theory of quantum mechanics resulting from the causality for the Newton mechanics should be discussed in elementary textbooks of physics from a viewpoint of the education for younger people. Further, misunderstanding problems shown in the existing textbooks for the fundamental theory of diffusion should be suitably revised as soon as possible. The diffusion theory should be thus developed in such a way as to start not from accepting the diffusion equation as Fick's law but from deriving itself from the mathematical theory.

Judging from the discussion developed in the present work, there is no doubt that the conception of minimum time is indispensable for understanding behavior of a micro particle. It will be thus no exaggeration to say that the quantum theory is established by incorporating the conception of discrete time  $t_j$  into the Newton mechanics. In conclusion, the author hopes that the values of a minimum time  $t_0$  are highly discussed from various viewpoints in physics.

### Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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