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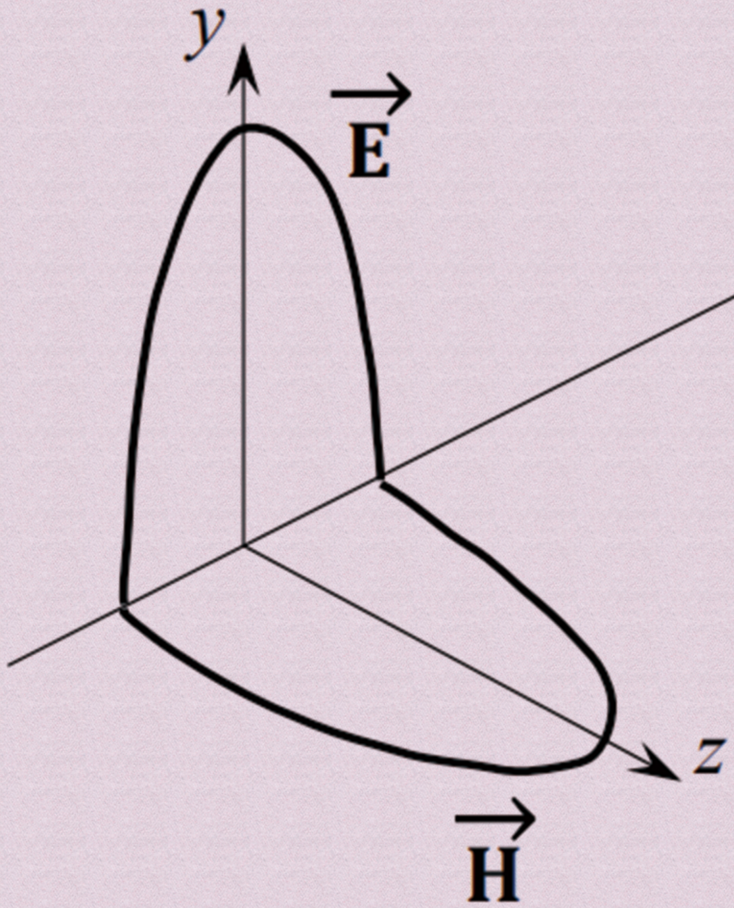

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Self-Sustained Boundedness of Logical and Quantal Error at Semantic Intelligence

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Abstract

It is demonstrated that the recently introduced semantic intelligence spontaneously maintains bounded logical and quantal error on each and every semantic trajectory, unlike its algorithmic counterpart which is not able to. This result verifies the conclusion about the assignment of equal evolutionary value to the motion on the set of all the semantic trajectories sharing the same homeostatic pattern. The evolutionary value of permanent and spontaneous maintenance of boundedness of logical and quantal error on each and every semantic trajectory is to make available spontaneous maintenance of the notion of a kind intact in the long run.

Keywords

Semantic Intelligence, Algorithmic Intelligence, Boundedness, Logical Error, Quantal Error, Optimization, Survival of the Fittest, Notion of a Kind

1. Introduction

One of the major apprehensions of any theory of general intelligence is whether it is ever possible to create a type of intelligence that spontaneously maintains logical and quantal errors permanently bounded. The question is provoked by the opposition between our human intelligence on the one hand and on the other hand, the rapidly developing in the recent decades' algorithmic intelligence which serves as grounds for modern-day computers and as grounds for the current comprehension of artificial intelligence.

Human intelligence is executed by means of natural processes and is organized in a variety of individual responses. The persistence of the latter organization prompts to suggest that in the long run, all individual responses share the same evolutionary value.

At the same time, the algorithmic intelligence, artificially designed and artificially maintained, is not able to keep boundedness of logical and quantal error in the long run in all three of its realizations, namely: deterministic, probabilistic, and mixed one. In the next section, it is considered in detail why each of the above types of algorithmic intelligence exhibits ill-definiteness of logical errors which result in their unrestrained accumulation in a long run regardless of how small quantal errors are maintained. The lack of restraint over logical error renders the fundamental task for the design and maintenance of algorithmic intelligence to be establishing the best relation between structure and functionality under the supervision of the "survival of the fittest" paradigm.

Semantic intelligence is a new form of intelligence that naturally arises in the frame of the recently introduced concept of boundedness [1]. It is fundamentally different from the algorithmic intelligence both in the means of its physical realization and in the mathematical background. This culminates in the most pronounced difference between them: the semantic intelligence acquires the property of autonomous creation and comprehension of information while its algorithmic counterpart needs external mind (that is our human mind) to create an algorithm and to comprehend the output.

Semantic intelligence is executed spontaneously by natural non-linear and non-homogeneous physico-chemical processes subject to a general operational protocol that keeps the boundedness intact. To compare, the algorithmic intelligence is executed by means of artificially designed and artificially maintained linear processes. Thus, the physical realization of the semantic intelligence by means of spontaneously executed natural physico-chemical processes renders similarity to human intelligence which, to remind, is also executed by means of spontaneous natural processes.

The fundamental importance of maintenance of the linearity of all local processes for algorithmic intelligence consists of the fact that it provides holding of the parallelogram summation rule at each and every moment and throughout the entire hardware. In turn, the latter keeps the computing process free from local distortions which, it happens, would appear as unrestrained local errors. Consequently, the artificially maintained linearity provides the re-occurrence of any output of each and every algorithm on the re-occurrence of the same input. However, as proven in Section 2, the reproducibility of the outcome is inevitably accompanied by the reproducibility of once produced large enough logical error.

The reproducibility of semantic computing is provided by the major outcome of the general operational protocol which keeps boundedness intact [1] [2]. It consists of the fact that the functional metrics are kept permanently Euclidean regardless of what the underlying spatio-temporal metrics of the computing system are. The Euclidean of the functional metrics provides not only the uniformity of "units" throughout the entire hardware and at each and every step of computation but it serves also as a crucial ingredient for holding the central for the entire theory of boundedness theorem, proven by the author, and called by her decomposition theorem. It proves that any bounded irregular sequence, (BIS) subject to permanent boundedness of rates of exchanging matter/energy/information with current environment and self-sustaining permanent boundedness of the amplitudes of the corresponding terms, shares the property that the power spectrum of each and every such BIS is additively decomposed to a specific discrete pattern called homeostasis and a continuous component, called noise one, whose shape is universal. The crucial property is that the specific properties of any homeostatic pattern and the shape of the noise component are insensitive to the details of the statistics of the members of any sequence. As considered in our previous paper [2] the causal relations are concentrated in the discrete pattern while the noise component comprises no information content, although it commences from well-defined local physicochemical processes and thus it serves as a reservoir for providing an adequate current local response in an ever-changing environment. An immediate outcome of the decomposition theorem is that it provides reproducibility of established causal relations set in an ever-changing non-uniform environment let alone the latter is bounded.

It should be stressed that the major difference in the mathematical background between semantic and algorithmic intelligence lies in holding of the decomposition theorem for the former while the algorithmic intelligence is subject to the Central Limit Theorem. It is worth noting that the subjects of both theorems have no common cross-section since, while the subject of the Central Limit theorem is independent random variables (yet not bounded), the subject of the decomposition theorem is bounded irregular variables (yet not independent).

Consequently, the fundamentally different physico-chemical and mathematical background of the semantic intelligence and its algorithmic counterpart prompts the anticipation of completely different outcomes one of which is subject to the present paper.

To continue, let us mention that the insensitivity of the specific properties of any set of causal relations to the details of current environment, obtained by the decomposition theorem, comes at a price: obviously, it is available only under the general condition that both the logical and the quantal error are kept permanently bounded on each and every semantic trajectory. The proof of this assertion constitutes the major goal of the present paper. It is worth noting that an immediate consequence of the affirmation of that claim renders the semantic intelligence to share the property of the human intelligence of assigning equal evolutionary value to the variety of all the individual responses in a long run. It should be stressed on the difference with the "survival of the fittest" paradigm: the latter is available in a constant non-uniform environment while the semantic computing operates in a rapidly and permanently changing non-uniform environment. The latter result is obvious since no long-term preference on any of semantic trajectories is available in an ever-changing non-uniform environment whilst a constant environment renders differences among different individuals to grow in the long run.

The following immediate outcome of the concept of boundedness makes the puzzle most intriguing: the central assertion of the concept of boundedness is that only bounded amount of matter and energy, specific to any local process, is involved; in turn, this implies that the precision of the semantic computing is bounded both from below and from above. This is because the computation of each and every number involves energy and matter proportional to the number of defining digits and thus it is available only for those values which belong to specific bounded margins of each and every semantic hardware. Thus, the origin of permanent boundedness for the quantal error from above is evident. Yet, the question of how the interplay between logical and quantal error is organized so that the limited form below the precision of semantic computing does not accelerate the logical error remains; it will be considered in Section 3.

2. Unrestrained Logical and Quantal Error at Algorithmic Intelligence

The major goal of the present section is to consider in detail why the algorithmic intelligence is not able to maintain the logical error bounded even though the quantal error is kept very small. At first, the case of deterministic algorithms comes:

1) Deterministic algorithms are artificially designed, specific to each case, sequences of logical steps. The verification of each logical step is provided by a positive answer to a cleverly enough posed question specially constructed for the purpose. In turn, the logic of the algorithms is represented by acyclic directed graphs where the computing is represented as a trajectory connecting an input and the corresponding output following the steps prescribed by a given algorithm. However, a generic property of algorithms is to comprise at least one step of logical operation "IF". The latter could change drastically the course of a current trajectory by means of causing "jumps" to distant "branches" of the graph. The hazardous moment is that such deviations can be a result not only by the prescription of the corresponding algorithm (desired outcome) but to result in an unrestrained error due to finiteness of the precision (misleading outcome). To make it clear, let us present the most pronounced example: the computation of limit cycles as solutions of differential equations is inevitably bound to degenerate into a motion on a spiral (ingoing or outgoing depending on any current realization of computing) which produces qualitatively different result in a long run: instead of bounded cycling motion, it approaches either steady point or infinity. The inevitability of this behavior is rooted in the fact that the operation "IF" separates two logically different regions by a single point (line in some cases) while the precision restrains the digits to "intervals" regardless of how small the precision is. Thus, around unstable (and/or neutrally stable) solutions the logical "error" accelerates by each and every step.

2) Next in the line comes the probabilistic approach to algorithmic computing. It is grounded on artificial assignment of specific probabilities to local

events and their permanent updating under apriori set local dynamical rules for the interaction with the neighborhood and the environment. The goal is to find out whether a system self-organizes so that to exhibit a collective behavior which in turn can serve as a physical background for an information symbol, *i.e.* a letter in an alphabet. Such self-organization has been established in a number of concrete cases but not as a generic property of a certain class of systems and/or dynamical rules. Yet, self-organization is expected to behave as a type of critical phenomena. The major flaw of this approach is that any such self-organization if exists, is extremely sensitive to even infinitesimal noise added to any steady environment. It is worth noting the difference between vulnerability to small changes in the environment (a setback) and the robustness to local failures (one of the advantages of the approach). The above vulnerability becomes a stumbling block of the entire approach because it rules out any general operational protocol which could govern transitions among different collective states. Indeed, even the very existence of any such protocol turns out inherently contradictive since the vulnerability to any changes in the environment renders the impossibility to define the exact amount of matter/energy and information involved in the substantiation of that transition. Consequently, this renders lack of any metrics in the state space which constitutes the ill-definiteness of the characteristics of any transition between any collective states. Thus, the logical operations among different information symbols, represented through different collective states, are subject to indefinite quantal error which is further loaded by the physical inability to substantiate any "jump" between the information symbols.

3) The mixed case of algorithmic intelligence encompasses these types of algorithms which have both deterministic and probabilistic components. In most cases, the "link" between the deterministic and the probabilistic part is set by means of specific optimization. Since I already have discussed the origin of unrestrained and/or ill-definiteness of the logical error for deterministic and probabilistic cases separately, now I will consider only why the optimization is not the "remedy" for the problem. The optimization is an artificially set constraint that holds along the entire optimal trajectory. Generally, it is of 3 types: minimax optimization, Bellman type optimization and Pontryagine type one. The common setback of all three types of optimization is that their fulfillment along the entire trajectory is accompanied by specific local discontinuities of the optimal trajectory. These discontinuities are hazardous not only for the physical maintenance of the corresponding hardware but they produce a massive change in logical error. As an example, the discontinuity of the optimal solution in the Pontryagine type of optimization is a product of the collapse of the current effective "Hamiltonian" and its substitution with a new one on the next part of the trajectory where it again will collapse when the next discontinuity occurs. Thus, this is not the only problem of the value of the logical error but it turns into a problem of identity of a system since the "Hamiltonian" is supposed to hold the identity of an object in physics.

3. Boundedness of Logical and Quantal Error for Semantic Intelligence

Since the mathematical and physical grounds of the semantic intelligence and its algorithmic counterpart are completely different, it is to be expected that they yield completely different outcomes. In the previous section, it has been demonstrated that algorithmic computing is not able to preclude the accumulation of logical error although the quantal error is maintained very small. In a nutshell, the root of the problem lies in the contradiction between the logic of the algorithms which operates as "dots" whilst the precision operates as "intervals". Further, the optimal trajectory is generically a "choice" of a single line among a volume of all available ones. Thus, each and every operation "IF" acts as a comparison between values of different dimensions, which, as it is well known, is never well-defined.

The goal of the present section is to demonstrate that the semantic intelligence permanently maintains bounded logical error at self-sustaining the quantal error bounded from below and above. The first clue lies in the fact that the general protocol providing spontaneous execution of semantic intelligence is organized so that the semantic computing to operate only with sets of equal dimensions.

Let me start the consideration by reminding how semantic intelligence is organized. It starts with the fact, proven in [1] that any non-uniform ever-changing environment, let alone being bounded, is decomposable to an effective specific steady component and an effective noise component, latter presented as a BIS. One of the major outcomes of this exclusive for the boundedness decomposition is one-to-one correspondence between the state space and the effective control parameter space. Thus the state space acquires metrics which in turn defines the characteristics of any motion in it. Further, the state space turns divided into basins-of-attractions so that a specific to each basin homeostatic pattern appears as an intra-basin invariant and thus the latter serves as an information symbol. Since stable in a long run, solution could be only bounded ones, each basin-of-attraction has non-zero volume. Alongside, the bounded precision renders the motion on each and every trajectory to be confined in an open tube such that any current trajectory (a line) never leaves that tube. Further, the boundedness renders the state space to be bounded which in turn renders the motion in it to be orbital. Yet, it should be stressed that, due to the boundedness of rates, each and every available orbit passes only through adjacent basins-of-attraction thus keeping local deviations permanently minimal regardless of whatever the logical operation is.

The crucial step forward is the association of the notion of a semantic unit with the performance of a non-mechanical engine built on the corresponding orbit which generically passes through at least 4 different information symbols (basins-of-attraction). The greatest value of this association is that semantic meaning acquires novel connotation irreducible to a mere algorithmic sequence of information symbols. To remind, so far it is taken for granted that the execution of any algorithm is subject to the general laws of arithmetic (e.g. associative law, dissociative law, etc.). This irreducibility is best pronounced by the exclusive property of semantic intelligence to get hold permutation sensitivity of the operation of any engine to the change of the direction of its execution; e.g. the famous Carnot engine operates in one direction as a pump and in the opposite as a refrigerator. Note that the execution of a sequence of logical operations in the opposite direction could yield an uncontrolled change in the logical meaning most probably producing a non-sense; thus the latter cannot be classified as "permutation sensitivity". Thus, the arithmetic laws alone are unable to provide meaningful permutation sensitivity as a generic property. It is worth noting that the permutation sensitivity of the semantic intelligence is indispensably linked to the boundedness of logical and quantal errors of any non-mechanical engine. The boundedness of logical and quantal errors are self-sustained by means of the confinement of the corresponding trajectories within specific "tubes" set by the corresponding precision.

The next step forward lies in the assumption that different semantic units are separated by punctuation marks, e.g. space bars. The punctuations marks are substantiated by means of special volumes tangent to all semantic units at any given hierarchy. Details of these considerations can be found in Chapter 4 of [1].

Thus, the motion on any semantic trajectory is confined to be exerted within a torus ("donut") "wrapped" onto an orbit which passes through different basins-of-attraction; the width of the torus is set correspondingly by the current lower and upper level of quantal error. Thus, due to the boundedness of rates and amplitudes, the quantal error never exceeds its margins throughout the motion on any semantic trajectory. Later it is considered the role of Euclideanity of the functional metrics for self-sustaining the margins of quantal error intact in the long run.

Outlining, all ingredients of semantic computing, namely basins-of-attractions, punctuation marks, and trajectory confinements are volumes of the same dimension. Further, the execution of the semantic intelligence as a sequence of orbits, each of which of bounded length, automatically keeps the logical error also bounded.

A crucially important point is about spontaneous self-sustaining of bounded quantal error. The latter is provided by the general operational protocol for extra-matter/energy dissipation. Now we make use of the property of that protocol to provide robustness of the Eucledianity of the functional metrics in the sense that the latter provides the global robustness of the same abstract quantal relations for each and every spatio-temporal point (for example, that is, 2 + 2 = 4 everywhere and/or on repetition). In turn, the robustness of the quantal relations maintains the robustness of the thresholds thus providing permanent robustness of the margins of boundedness of rates and amplitudes which eventually culminates in maintenance of bounded quantal error regardless to the details of the environmental impact and regardless to the details of the local specificity of the physico-chemical processes which substantiate any piece of semantic intelligence. It is worth reminding that these processes are in general non-linear and operate in a non-homogeneous way in any non-uniform ever-changing environment let alone the latter is bounded [2].

Note that this view renders the notion of abstract arithmetic relations equally available for continuous objects such as space and time zones, and for discrete objects such as apples and matter. Thus, as mentioned in the Introduction, the maintenance of Euclideanity of metrics provides the notion of a unit for different spatio-temporal phenomena well-defined. Alongside this, it renders the law of parallelogram for the summation of vector variables insensitive to the spatio-temporal point where it is applied. An immediate consequence of that is the covariance (viewed as insensitivity to the choice of reference frame) of the specific laws of Nature represented through the corresponding functional relations. Note that otherwise, the quantal relations would turn local and specific to the current spatio-temporal point of application. Consequently, the notion of boundedness would turn ill-defined since the notion of thresholds turns local so that they can vary from one spatio-temporal point to another and under repetition of any current event. As an immediate consequence, this would result in a lack of any form of covariance for any relation.

Let us now focus our attention on highly non-trivial feedback between the boundedness of quantal errors and the boundedness of logical errors. Next, I present arguments that provide long-term stability of any concrete realization of the notion of semantic intelligence. Indeed, the permanent non-distorted boundedness of rates and amplitudes precludes both logical and quantal errors from exceeding their margins on each and every orbit and render both of them to turn to zero on completing each and every cycle on each and every semantic trajectory. To remind, turning errors to zero on completing a cycle is an immediate consequence of the fact that since successive orbits are separated by accumulation point (e.g. space bar), one can consider each and every cycle starting and ending at the corresponding accumulation point, that is to consider each orbit starting and ending at the same point. Thus, the feedback operates via self-sustaining of Euclideanity of the corresponding functional metrics which, in turn, provides the characteristics of the boundedness of rates and amplitudes non-distorted. In turn, the non-distorted boundedness of rates and amplitudes sustains boundedness of logical and quantal errors which in turn provides robustness of the characteristics of the boundedness of rates and amplitudes. Taking into account that boundedness of rates and amplitudes has been put forward as the most general condition for providing long-term stability of complex systems, the conclusion derived is that the same conditions turns enough to provide not only long-term stable operating of a complex system but permanent self-sustaining of the boundedness of logical and quantal error of the corresponding realization of semantic intelligence as well.

The highly non-trivial feedback between logical and quantal error is best pronounced through the decomposition theorem. Indeed, the latter proves additivity of the decomposition of the power spectrum of each and every BIS to a specific homeostatic pattern and a universal noise component. In turn, namely, the additivity provides a constant error for that decomposition in the long run. So, the additive decomposition confirms permanent boundedness of the logical error for the corresponding causal relations in an ever-changing environment, and in the long run and regardless of the concrete values of the quantal error. To remind, the causal relations are encapsulated in the homeostatic pattern while the noise component has no information content.

It is worth to point out the fundamental difference between the above-considered feedback between logical and quantal error for the semantic intelligence and the case of algorithmic intelligence. Next, it is considered why the artificial maintenance of low quantal error is not enough to provide boundedness of logical error for algorithmic intelligence. This conclusion becomes evident through the following considerations: any piece of algorithmic intelligence is executed as a "string" of specific successive local computations by means of local linear processes (compare to orbits for semantic intelligence). Namely, the locality of each and every computation along with the execution as a sequence of computations immediately provides a disconnection between the corresponding logical and quantal errors. Consecutively, the consideration at the beginning of the present section comparison between variables of different dimensions renders unrestrained accumulation of logical error even though the quantal error is kept very small.

Let us now focus the attention on the role of optimization for semantic computing. The problem with the identity stands differently: the long-term bearer of identity for semantic intelligence is the current homeostatic pattern whilst the Hamiltonians acquire rather local meaning in the sense that they define local properties of participating in any given chemical reaction and/or physical process atoms/molecules. It should be stressed on the fact that the optimization can be useful in the short run only and for the purpose only. However, it should not yield to deviations from the current semantic trajectory because thus it returns in hazardous long-run events.

It is obvious that permanent self-sustaining of the boundedness of the logical and the quantal error renders all different semantic trajectories to be of equal evolutionary value from the point of view of the concept of general intelligence.

The evolutionary value of permanent and spontaneous maintenance of boundedness of logical and quantal error on each and every semantic trajectory is to make available maintenance of the notion of a kind intact in the long run. It is worth noting that the notion of a kind is corroborated by means of the diversity of individual responses executed as motion on different semantic trajectories so that, provided by the bounded logical and quantal errors on each of them, to assign the same evolutionary value to the motion on all the semantic trajectories sharing the same homeostatic pattern.

To compare, algorithmic computing is universal and reproducible if and only if the environment is kept permanently the same. In the ever-changing environment, the logical and quantal errors at algorithmic computing turn ill-defined and unsaved from unlimited accumulation of error. Preventing the accumulation of logical and quantal errors by means of appropriate optimization is available also only for a steady environment. In an ever-changing environment, the effect of optimization turns only local because either the identity is violated and/or the optimization causes permanent discontinuity of the major flow function.

4. Conclusions

The obtained in the present paper result about permanent self-sustaining of bounded logical and quantal error for a newly introduced type of general intelligence called by the author semantic intelligence prompts to outline the general strategy for the development of the notion of general intelligence, namely: the development is substantiated through a variety of yet to be discovered forms with radically different properties. The grounds for this suggestion lying in the verified in the present paper conclude that the semantic intelligence is exerted through a variety of individual responses, each substantiated by motion on a semantic trajectory so that all individual responses, sharing the same homeostatic pattern, acquire the same evolutionary value in a long run. The latter comes in fundamental opposition to the algorithmic intelligence which is artificially created, maintained in a constant environment, and artificially comprehended, thus being subject to the survival of the fittest paradigm. Another conclusion drawn from the above comparison is that each evolutionary paradigm holds for its own subject and there is no cross-section for the subjects of different types of intelligence.

The evolutionary value of permanent and spontaneous maintenance of boundedness of logical and quantal error on each and every semantic trajectory is to make available spontaneous maintenance of the notion of a kind intact in the long run. It is worth noting that the notion of a kind is corroborated by means of the diversity of individual responses executed as motion on different semantic trajectories so that, provided by the bounded logical and quantal errors on each of them, to assign the same evolutionary value to the motion on all the semantic trajectories sharing the same homeostatic pattern.

It should be stressed also on the difference in substantiation of these types of intelligence: whilst the algorithmic intelligence is fully governed by the human mind, the semantic intelligence is executed by means of spontaneously executed physico-chemical processes governed by general operational protocol which spontaneously and permanently maintains boundedness and whose major distinctive property is autonomous creation and comprehension of information.

Yet, besides fundamental differences, there is a highly non-trivial synergy between semantic and algorithmic computing, which becomes most pronounced in curved space-time. Thus, by means of self-sustaining Euclidean metrics [1] [2] the semantic computing provides boundedness of the quantal error for the corresponding semantic computation; on the other hand, the algorithmic computing, where the "Euclediainity" of computation is artificially maintained through artificially designed and executed linear processes, "computes" the underlying spatio-temporal structure. Outlining, the semantic intelligence "computes" functionality while the algorithmic intelligence "computes" the underlying spatio-temporal structure. The advantage of that interplay is crucial for the study and exploration of any unknown phenomena ranging from nano-devices to cosmological objects.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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What Connects Dark Matter and Black Holes?

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Abstract

Dark matter is a major component of the universe, about six times more abundant than ordinary visible matter. We measure the effects of its mass, but it escapes the telescopes. It has the particularity of emitting no radiation and interacting only by the action of gravity. The main purpose of this article is to try to answer what dark matter is: we conjecture that it is composed of magnetically charged neutrinos, true magnetic monopoles. But that requires a huge conceptual leap: Maxwell's laws must be inverted and the electric charge becomes a magnetic charge. Asymmetric "reversed" Maxwell's laws would provide the "dark" magnetic charge that would replace the electric charge. The very form of the Dirac equation, which imposed on ordinary matter that the particle carries an electric charge and obeys the principal properties of the electron, would impose in the dark matter that the "dark" particle obeys the main properties of a neutrino associated with a magnetic charge. The second aim of the article is to show that dark matter is derived from black holes, mainly from active supermassive black holes. This requires a second conceptual leap: the horizon of the black hole undergoes a high temperature and an intense pressure of magnetic fields which cause a blackout and a phase transition (or broken symmetry) when the matter crosses the horizon. The result is a reversal of Maxwell's laws: a magnetic charge is substituted for the electric charge, and the electric current becomes a tributary of the magnetic current. A third important conceptual leap follows: sterile magnetic neutrinos created inside the black hole would cross the horizon to the outside to constitute dark matter.

Keywords

Dark Matter, Magnetic Monopole, Inverted Maxwell's Equations, Magneto-Electric, Dirac Equation, Magnetic Sterile Neutrino, Active Black Hole, Event Horizon

1. Introduction

The problem of dark matter is well known: observational evidence and theoretical arguments suggest that there is a lot more matter that gravitationally interacts in the universe than what is accounted for. This is not the normal "baryonic" material in the cosmos. Yet no direct evidence exists to explain what dark matter is. This hidden form of matter must be a kind of particles which do not feel the electromagnetic or strong forces and which, consequently, neither emit nor reflect light nor behave like atomic nuclei bound together. Observations suggest that most of the dark matter is "cold".

Scientists have theorized several potential explanations of dark matter that appears to exert a gravitational pull on normal matter in galaxies and clusters of the cosmos. These possibilities belong to different categories. Some scientists suggest a complex dark matter, a wide range of dark species. Wimps, axions, heavy and sterile neutrinos, low mass black holes and dark matter atoms are among the contenders for dark matter envisioned by theorists. The simplest theories of dark matter postulate that only one type of particle contributes to the invisible mass [1].

Although cosmologists do not know what constitutes dark matter, they know something about its properties from their observations of how it influences ordinary matter and by simulations of its gravitational effects. They know that dark matter travels at a much slower speed than light, and that it therefore aggregates more than fast-moving "hot" dark matter, that it must be electrically neutral because it does not absorb or emit electromagnetic radiation. The particles that make up dark matter are probably massive. They cannot interact by the strong force that binds the atomic nuclei together; otherwise we would have had evidence in the interaction of dark matter with cosmic rays. Until recently, scientists believed that dark matter might interact via the weak force, but new observations have wiped out this notion.

In this article we conjecture a new force that belongs essentially to dark matter. It would be a magnetoelectric force (ME) that would be an inverted form of the electromagnetic force (EM). This would result in the existence of a magnetic charge that would replace the electrical charge. Maxwell's four laws would be reversed. We imagine a sterile neutrino with a magnetic charge that would emit gravitons and plays the equivalent role of the electron that emits and captures photons. What's more, these sterile neutrinos would come from black holes.

At first glance, there is no relationship between dark matter and a substance emanating from a black hole, since black holes are often defined as the regions from which nothing can escape, not even light. Stephen Hawking demonstrated, however, that there are some reasons to think that particles can come out, by "tunnel effect": the space-time in which we evolve would be modeled by the connections established between the black holes, via a purely quantum link.

In recent years, a new vision of the world has put black holes in the center of the stage. Astrophysicists believe that black holes are responsible for phenomena ranging from X-ray emissions to huge jets of material ejected from galaxy centers. The big bang would hide a primordial black hole located in another universe and from which ours would have emerged. Space-time would be woven with micro black holes connected together by a quantum phenomenon. Stars and galaxies are thought to have been created by jets of matter expelled by supermassive black holes. The black holes that were taken at first for the worst cosmic monsters would actually be the greatest builders [2].

The idea that black holes *are* dark matter has been proposed by several theorists. They first thought of massive primordial black holes formed in the first second of the universe. But it would take billions to explain the missing mass, and we would see their influence on the motion of the stars. In the 1990s, they then thought of micro black holes, of the order of a nanometer, but weighing one hundredth of the mass of the Moon. Except that their evaporation would have been detected by the gamma satellites in the 2000s. They are currently studying the possibility of primordial black holes weighing between 20 and 100 solar masses. Gravitational wave detectors have seen the fusion of objects in this category in recent years.

Our hypothesis is not that black holes are dark matter, but that dark matter would consist of substances from black holes, including sterile neutrinos associated with a magnetic charge. We say that a "black out" happens at the event horizon of a black hole (due to the enormous pressure and high temperature) that reverses the Maxwell's laws, transforms the electric charge into a magnetic charge, and makes invisible and imperceptible particle emission. Active black holes are internally filled with "dark" energy. According to the theory of Relation [3] [4], this energy would be the dark energy of the beginning (amalgamated with the kinetic negative energy, with the cosmological constant) which has dissolved to form the ordinary matter with positive energy, according to the principle of Compensation. This energy is the same as that of polarized vacuum, except that it is all the more excited as the temperature is high. The strong fluctuations cause the expulsion of sterile neutrinos inside the black hole horizon with a relativistic speed close to the speed of light. The emitted particles become slower and "magnetized" with cooling. They will automatically be magnetic monopoles.

This paper will address three links between dark matter and the black hole. These are actually three conceptual leaps. In Section 2, "Dark Matter and Maxwell's Laws Reversed", we see that dark matter might interact with a form of light to which our eyes are blind (Sect. 2.1, 2.2). It would be a variant of electromagnetism, a "magnetoelectric" force that would be obtained by reversing Maxwell's laws: this is the first conceptual leap (Sect. 2.3, 2.6, 2.7). From Dirac's theory, which establishes a connection between the smallest electrical charge and the smallest magnetic pole (charge), we end up with a magnetic pole that looks like a dark matter and would be composed of magnetic sterile neutrinos (Sect. 2.4, 2.5, 2.8). In Section 3, "Black Hole", we see that black holes now play an important role in the birth and evolution of galaxies. Throughout the universe, black holes relativistic jets condense gases and trigger outbreaks of stars (Sect. 3.1). We speculate that black holes are surrounded by magnetic walls that serve as an event horizon (Sect. 3.2). This is our second conceptual leap. These magnetic walls, generated by the high temperature and the intense pressure of the magnetic fields, would disturb any object meeting them. There would be a concomitant blackout with an inversion of the laws of electromagnetism and the appearance of a magnetic charge that squeezes out the electric charge. The black hole would produce a dark substance similar to that of dark matter (Sect. 3.2.1 to 3.2.5). In Section 4, "Creation of Magnetic Sterile Neutrinos inside the Active Black Holes that can Cross to the outside to Constitute Dark Matter", we figure that the black hole space is filled with dark energy. In the theory of Relation, there was at the beginning a maximum energy (identified with dark energy) which declined by transforming itself into ordinary matter. The black hole does the opposite process by turning ordinary matter into energy (Sect. 4.1). Some will feel that this paper deals with the problem of dark energy in a manner that is not consistent with the standard model of particle physics and general relativity. We argue, on the contrary, that it is the problem of dark energy that is inconsistent with the standard model of particle physics and general relativity, and we explain why (Sect. 4.2). It is logical to expect that the gravitational energy density inside the black hole can easily convert into virtual couples of particles and materialize them. This enormous energy would behave like an intense *accelerator of materialization and annihilation*. "Dark" particles and antiparticles could escape from the black hole. Third conceptual leap: the sterile "magnetic" neutrinos could be the dark matter (Sect. 4.3). In Section 5, "Efforts of Four Researchers", we highlight some aspects of the work of four researchers who are contributing to the extension of knowledge about dark matter, magnetic monopoles, black holes and sterile neutrinos. In Section 6, "Heat, entropy and information have everything to do with black holes", after describing the problem of entropy (Sect. 6.1) and the information paradox (Sect. 6.2), we presume that not only information escapes from the black hole, but also the destroyed matter (Sect. 6.3). In section 7, "Comments and Conclusion", we realize that dark matter, different from ordinary matter, generates a crisis. A major conceptual overhaul is needed. It concerns, in addition to dark matter, electromagnetism, sterile neutrino and black holes. A final summary serves as conclusive.

2. Dark Matter and Maxwell's Laws Reversed

2.1. Omnipresence of Dark Matter in All Regions of the Universe

The suspicion of the existence of a dark matter is due to the astronomer Fritz Zwicky in 1930. He noticed a dynamic anomaly within each cluster of galaxies whose mass he proposed to determine by measuring the speed of galaxies constituting these clusters. The speeds of the galaxies were too great to be balanced by the gravitational pull of the cluster, which should have been scattered. He came to the conclusion that the mass of these clusters must be greater than all that was observable and that a hidden matter must be present in each cluster. He estimated that the hidden mass represents more than 90% of the mass of the cluster.

Around 1960, astrophysicist Vera Rubin, while studying the dynamic behavior of gaseous clouds orbiting the center of certain galaxies, discovered that this unknown dark matter was also distributed outside the clusters. These clouds are sometimes located at distances from this center that sometimes far exceed the visible radius of their galaxies and the rotational speeds of these galaxies should have decreased with their distance from the center of the galaxy. She realized that the speed of rotation of the gaseous clouds was independent of their distance from the galaxy. If all the matter, visible and dark, had been concentrated in the galaxies, the speed of rotation of these clouds should have been all the smaller as their distance in the center was great. All these experimental facts testified to the presence of dark matter, uniformly distributed not only within these galaxies, but also in vast external volumes. Observations on larger scales, clusters of galaxies that cover a few million light-years, have confirmed the presence of dark matter [5].

2.2. The Candidates

Theorists consider radically different paths to explain this unknown fluid of ordinary matter that bathes the whole cosmos and whose nature remains to be explained. Over the decades they have scrolled through several candidates: wimps (neutralino, Kaluza-Klein particle, little Higgs particle), wimpzilla, axions, machos, black holes, sterile neutrinos, etc.

Wimps (weakly interacting massive particles) are the preferred candidates. These hypothetical particles have in common to be more massive than the particles known today, and are supposed to be able to interact with the latter only via the force of gravity and the weak nuclear force. These are ideal candidates for dark matter, as it is assumed that they would have just the abundance required to explain the current structure of the universe. Several distinct theories, all supposed to correct the imperfections of the standard model that describes particle physics, predict different types of wimps. For twenty years, astrophysicists and particle physicists have given themselves the means to discover them. Whether it is a direct detection (the aim is to detect the impact of a wimp on a core of ordinary material in an underground laboratory) or indirect (the products of the collision of two wimps are tracked in galaxies, in the heart of the sun, in cosmic rays, in the LHC at CERN), no dark matter particles have been detected to date [6].

The wimps' track, as well as those of the other candidates, could be a dead end. So instead of looking for a new particle, why not change the law of gravitation? This is what the followers of Mond (Modified Newtonian Dynamics) have been trying to do for thirty-five years. But neither this theory nor its most recent varieties can explain all the properties of dark matter.

2.3. A Variant of Electromagnetism

It seems that particle physicists are living a nightmare scenario. It turns out that they found no new particles beyond the standard model with their accelerators.

They took a first look everywhere and found nowhere. They still continue to move forward. They rely on the great diversity of detection approaches to hope to one day get their hands on the good particle(s). We believe that this crisis must be solved by a major conceptual overhaul. Without being guided by theoretical prejudices, we propose a variant of electromagnetism that will provide an explanation for dark matter. Why would something remarkable and unprecedented not have occurred at the heart of the electromagnetic theory that would make it incapable of emitting or absorbing electromagnetic radiation?

There is almost complete symmetry between electrical and magnetic phenomena. The difference lies in the fact that no free magnetic pole exists (north or south), while there are free electric charges (positive or negative): the two types of poles can never be physically separated. This makes us consider magnetism as a secondary phenomenon whose existence depends on the flow of an electric current [7].

Maxwell's four equations fully describe the electromagnetic behavior on a very large scale, including that of light. The electromagnetic field is the space between the lines of force of the electric field E and the magnetic field B, and there is thus energy

$$U = (\varepsilon/8\pi)E^2 + (\mu/8\pi)B^2 . \tag{1}$$

(ε: vacuum permittivity; μ: vacuum permeability)

Suppose a severe astrophysical event has occurred that would cause the visible light to cease. And that to provoke this darkness, it would have been necessary that the electric charge no longer plays its role, that its physical size becomes another (the letter q, in Coulomb's formula, would no longer play the exact role played by the letter m in the Newton's formula). This last possibility has already been considered by Paul Dirac, while he was wondering about the reason for the existence of the smallest electric charge.

2.4. Dirac's Theory Establishes a Connection between the Smallest Electrical Charge and the Smallest Magnetic Pole

Although in classical electromagnetism the existence of magnetic monopoles is not compatible with Maxwell's equations, and although special relativity allows us to demonstrate all Maxwell's laws, including that which predicts the non-existence of magnetic monopoles, Paul Dirac demonstrated in 1931 that the existence of magnetic monopoles was compatible with Maxwell's equations in the hypothesis of the quantification of the electrical charge [8]. His theory establishes a connection between the elemental electric charge (that of the electron) and the hypothetical elementary magnetic charge. It showed symmetry between electricity and magnetism, which is still completely foreign to established conceptions.

We know that the smallest electric charge exists experimentally. With a purely electronic quantum condition, we obtain the value e (in CGS system) given approximately by

$$\hbar c/e^2 = 137.03. \tag{2}$$

$$\left(1.054512\times10^{-27}\ \text{erg/sec}\right)\left(2.9979\times10^{10}\ \text{cm/s}\right)\Big/\left(4.8030\times10^{-10}\text{statcoul}\right)^2 = 137.03$$

However, his theory, although at first intended to give a theoretical value to e, turned out, when it was developed, to establish a connection between the smallest electric charge and the smallest magnetic pole, namely the equation

$$\hbar c/(e\mu_o) = 2. \tag{3}$$

$$\left(1.054512\times10^{-27}\right)\left(2.9979\times10^{10}\right)\Big/\left[\left(4.803\times10^{-10}\right)\left(3.29028\times10^{-8}\right)\right] = 2$$

($\mu_o = 3.29098\times10^{-8}\text{statcoul}$: quantum of magnetic pole). Although Dirac's proof of the relation between pole strength, electric charge, and Planck's constant, is not simple, we can, by a simple estimate, explain the number 2 of the Equation (3) and briefly illustrate the character of the relation [9]. Here we consider that poles exist and one isolated plate (a capacitor plate holding a magnetic pole charge) holds a pole density of σ_{μ_o} poles per unit area. By analogy with the calculation of the electric field from such a plate holding an electric charge density and the symmetry between the electric field E and the magnetic Bc, we find that the magnetic field near the plate is

$$cB = 2\pi\sigma_{\mu_o}. \tag{4}$$

The electric charge moving with a velocity v will move in a circle of radius r if the centrifugal force on the charged particle, mv^2/r, is equal to the force on the moving charge generated by the magnetic field, Bev

$$Bev = \frac{mv^2}{r} \quad \text{or} \quad Be = \frac{mvr}{r^2}. \tag{5}$$

From the quantization of angular momentum, mvr is equal to $n\hbar$, where n is an integer. Substituting the pole density expression for B ($B = 2\pi\sigma_{\mu_o}/c$),

$$\frac{2\pi\sigma_{\mu_o}}{c}\cdot e = \frac{n\hbar}{r^2} \quad \text{or} \quad 2\left(\pi r^2\sigma_{\mu_o}\right)\cdot e = n\hbar c. \tag{6}$$

This relation requires that the magnetic pole strength enclosed by the orbit of the electron be quantized. Setting that pole strength to be the smallest value, one single pole with a unit pole strength μ_o, we have $\pi r^2\sigma_{\mu_o} = \mu_o$ and

$$\mu_o\cdot e = \frac{n}{2}\hbar c \quad \text{hence} \quad \hbar c/(e\mu_o) = 2. \tag{7}$$

Although this estimate is crude, it serves to illustrate the connection between the quantization of angular momentum and the quantization of charge and pole strength. Quantum mechanics requires a quantization of charge—if monopoles exist.

Instead of finding a purely electronic quantum condition, such as (2), Dirac found a reciprocity between electricity and magnetism, a connection between the magnetic pole quantum and the electronic charge. His theory contains no arbitrary characteristic, gives no possibility to modify it, and would have the effect of creating a magnetic monopole. For that, the theory requires a quantization of the electric charge, because any charged particle moving in the field of a pole of intensity μ_o must have as it charge an integer multiple (positive or negative) of e, so that functions wave describing the motion may exist.

Magnetic poles would have properties similar to those of electric charges. Each pole would emit $4\pi\mu_o$ of magnetic field lines B, where μ_o is the pole strength (corresponding to the charge q). The strength on a pole in an electromagnetic field would be

$$F = B\mu_o + E\mu_o \frac{v}{c}, \tag{8}$$

where, field vectors and velocity are orthogonal. The strength exerted on a fixed magnetic pole would be a measure of the magnetic field and the strength exerted on a moving pole would be proportional to the pole's speed and the magnitude of the E field. The symmetry of the pole and the charge seems to be exact.

If the charges and poles are so similar, why hasn't nature provided us with poles? (Poles have not been seen despite careful searches.) However, if poles are found they must have much larger charges than the unit electrical charges found on elementary particles such as the electron. So this universe cannot be completely symmetric between pole and charge on the microscopic level.

We might still ask why, if poles and charges are symmetric in principle, we have charges and not the poles. If the universe were constructed so that there was no electric charge, but only magnetic poles with the same value of pole strength as the fundamental charge strength, we believe that this universe would be indistinguishable from ours. If we could communicate with the inhabitants of that universe (who are made of protons, which have no electric charge, but hold a unit magnetic pole strength, and electrons, with no charge, but with an opposite magnetic pole strength), we could not determine whether they live in a magnetic universe or an electric universe as we do [9].

2.5. Our Theory for Dark Matter: Sterile Particles Associated with Magnetic Monopoles

And if the universe was constructed in such a way that there is no electrical charge, but only magnetic poles not having the same value of pole strength as the fundamental charge strength, so that the left-hand side of Equations (2) or (3) no longer corresponds to the experimental value 137 or the theoretical value 2, we think we would be in a total darkness that would have the appearance of a dark matter. We can imagine that the elements that make up this dark matter would be composed of elements charged magnetically, with electricity and the electric field considered as a relativistic consequence of the magnetic field, which involves reversing Maxwell's laws.

2.6. Inversion of Maxwell's Laws

The experimental dissymmetry of Maxwell's equations with respect to the electric-magnetic duality is related to the fact that the electric field is generated by the usual charges which give it a non-zero divergence, but the magnetic field is always of zero divergence because of the absence of corresponding punctual charge. Experimentally, the only source of the magnetic field comes from the existence of an electric current, that is to say a motion of electric charges.

$$\nabla \cdot E = 4\pi\rho_e , \tag{9}$$

$$\nabla \cdot B = 0 , \tag{10}$$

$$\nabla \times E + 1\partial B/c\partial t = 0 , \tag{11}$$

$$\nabla \times B - 1\partial E/c\partial t = 4\pi J_e/c . \tag{12}$$

(ρ_e: electric charge density; J_e: electric current density; σ_{μ_o}: magnetic charge density; J_{μ_o}: magnetic current density; *e*: electric charge; μ_o: magnetic charge) [10].

Assuming there is a magnetic charge density σ_{μ_o} and a current density J_{μ_o} but no corresponding electrical counterpart, the equations would be asymmetric being fully subject to the magnetic charge. Maxwell's equations then become:

$$\nabla \cdot B = 4\pi\sigma_{\mu_o} , \tag{13}$$

$$\nabla \cdot E = 0 , \tag{14}$$

$$\nabla \times B + 1\partial E/c\partial t = 0 , \tag{15}$$

$$\nabla \times E - 1\partial B/c\partial t = 4\pi J_{\mu_o}/c . \tag{16}$$

The equations are still asymmetrical but no longer subject to the electric charge. Equations (14) and (15) seem to miss something on their right sides. To see exactly what they are missing, we need to explain the meaning of $\nabla \cdot E$, also called divergence of E or simply of div*E*. Let V be a volume surrounded by a surface S in space. $\nabla \cdot E$ integrated on the volume V gives 4π times the total amount of electric charge *e* contained in *V*. Similarly, $\nabla \cdot E$ evaluated at point *x* gives 4π times the electric charge density at *x*. Hence, Equation (14) indicates that there is no electric charge at any point in space. Basically, moving charges are equivalent to currents. But because the above reversed Maxwell's equations assume that there is no electric charge in dark matter, there is no electric current J_e on the right side of Equation (15). Equations (13) and (16) seem to have won something on their right sides. This means that $\nabla \cdot B$ integrated on the volume V surrounded by a surface S in space gives 4π times the total quantity of magnetic charge μ_o contained in *V*. Similarly, $\nabla \cdot B$ evaluated at the point *x* gives 4π times the density of magnetic charge at point *x*. As a result, Equation (13) indicates that there is a magnetic charge at any point in space.

Because the Maxwell's equations above assume that there is a magnetic charge, there is a magnetic current J_{μ_o} on the right side of Equation (16). Therefore, the absence of electric charge and the presence of magnetic charge reverse the asymmetry.

In fact, the electrical charge would become a magnetic pole, which would result in an attribution reversal, so that electricity should be considered as a secondary phenomenon whose existence depends on the flow of a magnetic current. The overthrow, in addition to the darkness caused, would in a way make that there would be free magnetic poles when there would be no more free electric charges. Magnetic monopoles would exchange "dark photons".

Note: There is no question of continuing by presenting a critical analysis of the hypothesis of Maxwell's "inverted" laws, because these must not be considered in an absolute sense, as if the nature of dark matter had to conform precisely to these laws. It is only a simplistic schema of reality, a kind of approximation, an image. As such, it corresponds to reality, even if it does not identify with reality.

2.7. Magnetoelectric Force

The inversion that we have just presented shows that there would be a dark magnetoelectric force (ME) with a dark photonic wave, just as there is an electromagnetic force (EM) with a photonic wave. Dirac's theory ensures that the magnetic monopole can coexist with an electric charge in ordinary matter. Maxwell's laws are not reversed, they are completed in order to obtain a perfect symmetry: $E \neq 0$, $B \neq 0$. According to our hypothesis of the inversion of the laws of Maxwell, the magnetic pole would replace the electric charge: $E = 0$, $B \neq 0$. There would only be a magnetic pole, which evicted the electrical charge.

We suggest the existence of an electric charge (known electric monopole) in ordinary matter and a magnetic pole (unofficial lightweight magnetic monopole) in dark matter. With rare exceptions, there is no coexistence of the two charges in ordinary matter or in dark matter. There would be no electric monopole in dark matter just as there would apparently be no magnetic monopole in ordinary matter.

2.8. "Magnetic" Sterile Neutrinos

To penetrate the mystery of dark matter, we think that it is a different electromagnetism, a *magnetoelectrology*, with the necessity of qualifying this variant as a "new force". And that it is also a new particle: "magnetic" sterile neutrino.

Physicists know three types of neutrinos. Since the 1970s, many researchers have assumed that there is a fourth type, a "sterile" neutrino, much heavier, but which interacts even less than the others with ordinary matter. It is a hypothetical type of neutrino that does not interact via any of the fundamental interactions of the standard model of particle physics except gravity. It is a right chirality neutrino or a left chirality antineutrino that can be added to the standard model and can take part in phenomena such as the mixing of neutrinos.

We assume that there is a fifth type of neutrino. Our hypothesis is that there would be a sterile neutrino linked to magnetic pole, that would belong to dark matter and that would be a magnetic monopole. The term *magnetic sterile neutrino* is used to distinguish it from *sterile neutrino*. The mass of the neutrino in both cases is unknown and could take any value between less than 1 eV and 10^{15}

GeV.

Dark matter would consist of invisible magnetic sterile neutrinos that swarm in the universe and exert a gravitational attraction everywhere [11]. This *sterile neutrino* would depend on a magnetic pole that would be undetectable since it is not an integral multiple of the conventional electric charge. Where would it come from? Our idea is that they come from active black holes, which risks upsetting our vision of black holes even more.

3. Black Holes

3.1. Black Holes Revolution outside the Event Horizon

Black holes are giving birth to a new genesis of the universe at every level. Our universe would have emerged from a primordial black hole that would be the big crunch of another universe. Initially, the black holes were at first only a mathematical singularity, a cosmic curiosity very difficult to observe. These gravity wells come from general relativity, they are an effect of the existence of a space-time curved by the masses and it is only in 1968 that John Wheeler invents the expression "black hole". As early as 1916, Karl Schwarzschild found the solution of general relativity that describes the gravitational field around a star. A singularity in its heart is seen as impenetrable before becoming, after the 1970s, the horizon of the black hole that swallows everything. From the 1990s, with the progress of observations, there is a need to believe that they really exist. Astronomers build a bestiary of destructive black holes. There would be everywhere: in the primordial universe, at the center of our Milky Way and all galaxies. Cosmologists and physicists see the concept of the black hole as a means to marry the irreconcilable theories of relativity and quantum physics. The proof of their existence fell on September 14, 2015 when the Ligo experiment for the first time captured gravitational waves caused by the fusion between two black holes.

Today, it seems that those who were thought to be the worst cosmic gluttons have become the great architects of the universe. They would have structured the primordial universe, modeled galaxies and lit up stars [2]. Supermassive black holes have been observed everywhere in the cosmos and astrophysicists believe that one of them sits at the center of most galaxies. They concentrate millions of times the mass of the Sun and would have created stars and galaxies. Observations made in the late 1990s were the first to reveal three creative roles of supermassive black holes:

First, a role of regulator, guardian of galaxies.

Astrophysicists first realized that in nearby galaxies, the central black hole always seemed to weigh 1/1000 of the star bulb that shelters it, a sign that the two are linked. Then, from 2005, it became apparent that the energy of the most energetic black holes can modulate star formation, stopping the formation of galaxies that otherwise would be enormous. Plasma winds ejected by the disk from black holes could control the growth of galaxies. These winds reach 1/10 the speed of light and carry enormous amounts of energy. In this way, today, 80% of the gas in the universe is found outside galaxies. Thus, the black holes play a regulatory role, soothing through their winds a cosmos too eager to generate stars.

Second, a role of cleaner (ionizer) of the primordial universe.

They could also, in other circumstances, have played the opposite role: concentrating the gas clouds, boosting the star formation, they would have been the main actors of the reionization process some 400,000 years after the big bang. Their X-rays, much more energetic than the UV of stars, could have been extracted from galaxies and ionized the intergalactic medium at greater distances than the UV of stars. Even smaller black holes, the stellar black holes a few dozen times the mass of the Sun, could have participated because in this primordial universe, they were always accompanied by a star whose matter they vampirized. What to maintain over the long term their production of X-ray at a high rate. They could thus disperse the thick neutral hydrogen fog by ionization of the atoms and make the cosmos transparent.

Third, a trigger role of star births through their jets.

A supermassive black hole compresses and heats the gaseous material that accumulates and revolves around it so much that this burning plasma begins to radiate and create an intense magnetic field. The radiation pressure exceeds gravity. Strong winds are generated in all directions and some of the matter escapes from the poles, in the form of two fine jets, several hundred km/s away. The winds blow the galaxy's gas and regulate its star production. The jets strike distant clouds of gas, initiating their condensation into new stars. This is suggested by the observation of some active galaxies. It seems proven, that with their jets, black holes form stars. It was discovered that a surge of new stars followed the direction of the jet emitted by its central black hole. The jet is so powerful that in a short time it can form 10% of a galaxy, like a spider spinning its web. Although there are only a handful of examples, black holes are now taken into account in the theory of galaxy formations.

3.2. Black Holes Revolution inside the Event Horizon

The three previous roles involve the photon sphere and the accretion disk outside the event horizon. We propose here a fourth role, a role of *creator* of dark matter, by their emission of "magnetic" sterile neutrinos. This role concerns the internal space of the black hole, between the horizon and the center of the black hole.

3.2.1. The Classic Horizon of the Black Hole

Roger Penrose, wrote a short article in 1964 in the journal *Physical Review Letters*, where he described the problem of the singularities associated with star implosions and demonstrated a mathematical theorem that said that when a star collapses to the point where gravity becomes strong enough to form an *apparent horizon* around it that brings back the photons that are trying to emerge, nothing can prevent the gravity from becoming strong enough to create a singularity.

Therefore, any black hole must contain a singularity. In the late 1960s, Penrose searched unsuccessfully for a mathematical example of a collapse that produced a naked singularity. In 1969, he issued the conjuncture of cosmic censorship: no object can, collapsing, give birth to a naked singularity. If a singularity is formed, it is dressed with a horizon that makes it invisible from the outside world [12].

This apparent horizon (like a spherical membrane) is in fact the Schwarzschild horizon. It is not singular in the strong sense, space-time is defined and it is permeable to incoming particles, it is a unidirectional membrane. The membrane of a sphere formed of light rays which define its surface. At the center of Schwarzschild's solution lies the true singularity, the heart of the black hole. The Schwarzschild sphere is an apparent singularity called horizon ($r = 2GM/c^2$), while the point of space-time at the origin of coordinates ($r = 0$) is really singular and looks like what we call the big bang [13].

At the crossing of the horizon, time-coordinate t and radius- coordinate r have exchanged their roles because the sign assigned to them in the definition of the linear element of space-time turns over. Which means that time becomes space, and space, time. The time-coordinate t is not at all adapted to the local definition of ds^2 since we no longer recognize the signs of time and space. In other words, the Euclidean metric does not extend inside the horizon.

But if we take a quantum view of cosmic censorship, the collapse of the structure at the level of a singularity must not affect any physical measurement. A description of the particle in free fall should allow to drive the particle through the horizon to the center by a path integral [14].

The most general model of black holes, according to general relativity, says that imploding stars towards the state of the black hole must, by passing their horizon, lose all the differences to the spherical symmetry, "all their hair", all their characteristics (except three parameters: mass, charge, angular momentum), and therefore, for example, their protuberances, their asymmetries and their magnetic field; they must, willingly or forcibly, become "bald". This lost structure must be evacuated previously in the form of radiation, in the form of an emission of gravitational waves.

3.2.2. Power Failure at the Crossing of the Horizon

It may be said that this model predicts the emission of gravitational waves, but the particle of the collapsing star does not end its life on the horizon, as if it were finally dying in the center, on the true singularity. At the crossing of the horizon, it seems to simply disappear on one side to reappear the other side. It becomes invisible to any observer left outside the sphere. Everything happens as if the sudden invisibility of the light was caused by a blackout that begins on the horizon. How to understand what happens to a particle, whether material or luminous, immediately after it has crossed the horizon? One could perhaps understand by taking an ordered magnetic field that would have settled on the "enlarged horizon" of a black hole located in the center of a quasar. An enlarged horizon is a fictional area just outside the horizon while an "inner horizon" is a fictional area just inside the horizon. This black hole is surrounded by an accretion disk, composed of hot and ionized gases. When plasma detaches from the inner edge of the disc and plunges onto the enlarged horizon, it carries with it a skein of magnetic field lines. When this magnetic field crosses the enlarged horizon, it generates surface currents that dissipate energy by flowing into the very resistive membrane. In fact, the lines of the field do not cross the real horizon, but wrap around it and form loops. The density of these loops and the intensity of gravity are such that they cause symmetry breaking in the membrane. There is a charge reversal; the magnetic pole replaces the electric charge. It's the blackout.

3.2.3. Magnetic Field of the Horizon and Energy of the Quasars

To provide the energy of a quasar, a magnetic field should cross the enlarged horizon throughout the life of the quasar. However, there is a source, outside the black hole, likely to generate such a field: the interstellar gas attracted to the black hole. The interstellar gases are the seat of magnetic fields and, when the gases heat up and ionize near a black hole, they form a plasma disk where the field lines are "frozen". The rotation and turbulence of this plasma in accretion entangle the field lines, some of which settle on the enlarged horizon, during the fall of plasma fragments. In the membrane, surface currents continuously dissipate the energy of this chaotic field, leaving only "clean", ordered, field lines that penetrate the membrane at the South Pole and exit at the North Pole. After an ordered field line has been deposited on the black hole, it no longer disappears: the plasma of the accretion disk and the magnetic field make it persist as long as the disk does not explode or is not swallowed by the black hole. The black hole acquires a magnetic field more than 10,000 times more intense than the Earth's magnetic field.

3.2.4. Where Electromagnetism Becomes Magnetoelectricity

It is known that the sphere of the event horizon is surrounded on the outside of a sphere of photons where, because of the considerable gravity, the particles of light no longer propagate in a straight line but begin to orbit. These spheres are surrounded by an accretion disk which is a plasma disc formed of matter taken by the attraction of black holes, compressed under the effect of gravity and heated to thousands of billions of degrees. The inner edge of the accretion disk is the last place where matter can orbit before falling into the black hole. A supermassive black hole compresses and heats the matter around it so much that this hot plasma creates an intense magnetic field. The latter exerts on the horizon of the black hole forces perpendicular to itself. The magnetic tension force is inversely proportional to the radius of curvature of the field line: it acts by stretching the field lines as if they were elastic cords. The behavior of the field can be interpreted as if it were endowed with a throttling pressure: when the forces of the magnetic field penetrate inside the horizon of the black hole there is a phase transition where the relation electricity/magnetism is changed due to too high pressure and excessive temperature. There is a charge reversal, during which the electric charge disappears to become the magnetic pole. We go from light to darkness; we witness a kind of reversal of Maxwell's laws, as described above [15].

With the reversal of charges, electric currents become electric fields and magnetic fields become magnetic currents. The result is that on the horizon, this region of space-time pressed by the electromagnetic tidal forces which culminate, the light does not act suddenly anymore. To illustrate the phase transition, consider that electromagnetism, before breaking through the horizon, is like a piece of wood impregnated with water. In this analogy, wood is magnetism and water is electricity, and both (wood and water; magnetism and electricity) are intimately intertwined, unified. Close to the horizon, according to our better understanding, the laws of quantum mechanics begin to combine with those of Einstein's general relativity and are already beginning to change the "rules of the game". (They will be totally changed at the singularity, and the new rules will be called quantum gravity.) The horizon and the laws of gravity combined with those of the quantum mechanics that govern it are like a fire in which wood swollen with water is thrown. The fire boils the water coming out of the wood, leaving it alone and master. On the horizon, the laws of quantum gravity expel electricity, leaving magnetism alone and resistant. Electricity is reduced to a current without conduction, extinguished [12].

One could also explain what happens with invariance groups that have proven themselves in quantum mechanics. They are algebraic transformations that retain the form of the equations and reveal physical properties. However, if we imagine a particle that crosses the horizon of the black hole, it "oscillates", it has a charge no longer electric but magnetic: it is the magnetic monopole. The particle described by the Dirac equation thus acquires *another* gauge invariance: an inverted gauge invariance. The Dirac equation might have a gauge invariance that changes a bit the wave described by the equation, but the new particle does not just change "phase", it is no longer an integral multiple of the charge of the electron. It no longer interacts with electromagnetism. Nevertheless, there are other "dark" invariance groups that fall under "magnetoelectrology" and that can intervene in accessible disintegration phenomena because these monopoles are not only magnetic but endowed with weak interactions.

To summarize, on the periphery of the black hole, there is a horizon: a region where light no longer works and where electromagnetism has given way to magnetoelectricity. This means that with the reversal of charges, electric currents become electric fields and magnetic fields become magnetic currents. In a pictured language, it will be said that electric fields are "frozen" in magnetic currents.

3.2.5. Light Can Escape from the Black Hole

In the context of general relativity, a black hole is defined as a gravitational singularity occulted by a horizon of events. It is a celestial object so compact that the intensity of its gravitational field prevents any form of matter or radiation from escaping it. According to quantum physics, a black hole is likely to evaporate by the emission of black body radiation called Hawking radiation. In 1974, Stephen Hawking discovered that, contrary to classical mechanics, black holes could radiate near heat radiation. The "temperature" of the black hole, which is inversely proportional to its size, is associated with it. In an article published in 2014, Hawking declared that there is no black hole, in the sense that light cannot escape to infinity [16]. According to him, these space ogres, capable of devouring galaxies and making light disappear, could actually release some quantities of matter and particles. The matter and the energy could actually be held temporarily, then modified, before being released into space. A phenomenon that would be inversely proportional to the mass of these objects: the smaller a black hole, the more it would let large quantities of matter escape. Black holes would not be so "black" as most cosmological models portray.

4. Creation of Magnetic Sterile Neutrinos inside the Active Black Holes That Can Cross to the Outside to Constitute Dark Matter

4.1. Dark Energy inside the Black Hole

The deep meaning of the discovery of Hawking radiation emanating from black holes is that the quantum vacuum is polarized by the very intense gravitational field prevailing in the vicinity of a mini black hole; the *gravitational energy* of the latter is converted spontaneously into particles. Quantum vacuum means minimal energy. According to the theory of Relation [3] [4] [17], there was at the beginning a maximum energy (identified with dark energy) which declined by transforming itself into ordinary matter. Black hole does the opposite process by turning ordinary matter into energy. This enormous energy is like a vacuum energy (which has become very dense) inside the black hole since it is no longer materialized. It is logical to expect that the enormous density of *gravitational energy* (which has a colossal mass) inside the black hole can easily convert into virtual couples of particles and materialize them.

Following the reversal of the charges explained above which converts visible ordinary matter into invisible matter, let us imagine a distribution of dark matter inside a black hole whose mass increases by engulfing a whole astrophysical jumble of gas pockets, stars, etc. As the mass of the black hole enhances, the dark matter sees its distribution contract, become more compact and denser. The black hole accumulates a colossal black mass that is equivalent to dark energy. The latter means a gigantic density of matter- dark energy inside the horizon. This enormous energy would behave like an intense *acceleration of annihilation* [18].

The boost in the mass of black holes thus augments the rate of annihilation of the dark energy-matter inside the horizon. In principle, because the density of dark matter is prodigiously high (inside the supermassive black holes, and even the intermediate-mass black holes of with a mass between a hundred and a million times the mass of the Sun), the probability exists that the rate of annihilation of particles of dark matter will accelerate to the point of injecting outwardly, out of the horizon, ample energy to constitute the unknown substance known as "dark" which fills the universe.

To describe the states of a magnetoelectric field, we will use an intra-horizon space, which is a first circle once crossed the horizon superimposed on several circles leading to the central point, called a singularity. In this intra-horizon space, the magnetoelectric field contains a huge concentration of energy, allowing large fluctuations of the creative energy of particles.

The inside of the active black hole could be comparable to the state of the universe during the 380,000 after the big bang, while the universe was a sea of darkness—just a fog of hydrogen atoms forged by the big bang and left to float in the absence of light. However, if it is constructed in such a way that there are only magnetic poles with a pole strength value other than the fundamental electrical charge strength, the inside of a black hole would be different from the inside our primeval universe. This inside could be made of a particle (similar to proton) holding a unit magnetic pole strength and a particle (similar to electron) with an opposite magnetic pole strength. These poles would provide a source of magnetic field just as an electric charge provides a source of electric field. The energy coursing through the black hole is so strong that magnetic monopoles, characterized by equal magnetic poles of similar or opposite signs, which repel or attract each other, cannot assemble a magnetic dipole.

4.2. The Idea of Dark Energy

We have just seen that according to the theory of Relation there was at the beginning a maximum energy (identified with dark energy) which declined while being transformed into ordinary matter. Some will feel that this paper deals with the problem of dark energy in a way that is not consistent with the standard model of particle physics and general relativity. A big nuance to bring: it is the problem of dark energy which is inconsistent with the standard model of particle physics and general relativity. Let's take a closer look.

According to official cosmologists, 70% of the contents of the universe are made up of a mysterious, undetectable and anti-gravitational dark energy that accelerates the expansion of the universe. It was through the observation of distant supernovae, which constitute "standard candles" intended to measure the universe on a large scale that they were able to deduce that dark energy existed. The latter was not predicted by any theory. It was introduced as a simple parameter in the equations of quantum particle physics and general relativity, which are two diametrically opposed theories. The result is that the dark energy, which looks like the energy of the quantum vacuum, seems to be 10^{120} times too strong compared to what the observations indicate. This gigantic gap is at the heart of the greatest crisis in contemporary physics.

In our opinion, we have reached this gigantic gap, or rather this unacceptable error, when astronomers, to measure the distance of very distant supernovae, have assumed that the intrinsic luminosity of the supernovae is the same for all, independent of the particular object measured. With this gratuitous hypothesis, impossible to prove, they came to the conclusion that expansion accelerates instead of slowing down (slowing down is what it does in an honest Friedmann model, and this is what is predicted in the equation of the theory of Relation). They then thought it wise to use an engine of unknown origin to produce the desired effect: dark energy [19].

Astrophysicists have associated this dark energy with negative pressure on a cosmological scale that would translate into a "current acceleration" in the expansion of space. It corresponds to a quantum vacuum energy whose value would be disproportionately greater. They gave this energy density value of the vacuum the same status as a repulsive cosmological constant, which pushed them to rehabilitate Einstein's cosmological constant, but about 10^{120} times larger. There is a deep contradiction between the concepts of quantum field theory (according to which the energy density of vacuum is about 10^{120} times the density of matter-energy of the present universe), and the ideas of general relativity (vacuum energy is a source of gravitation, hence of curvature of space-time) used to associate this estimate with astrophysical observations. This dark energy in the form of a repulsive cosmological constant imposed by the omnipresent quantum vacuum would produce hallucinating cosmological effects: our universe would bend so intensely that the visibility horizon would be at centimeter distances [5].

By decreeing that the supernovae of yesteryear were the same as those of today, by affirming dogmatically that the first supernovae were necessarily of a similar chemical composition of the following [20], we are arrived at the "vacuum catastrophe" or the problem of the cosmological constant. The high degree of intoxication of the scientific community was manifested by the award in 2011 of the Nobel Prize in physics to three astrophysicists belonging to two different teams for their discovery of the acceleration of the expansion of the universe. This discovery, based on the unconfirmed hypothesis of the uniformity of supernovae and uncertain distance measurements, endorsed by the judgment's passivity of official cosmology, is as aberrant as the Ptolemy's epicycles.

More and more, specialists advance the hypothesis that the acceleration of the expansion of the universe, which motivated the creation of the concept of dark energy, could in fact result from an observational bias [21]. What will be said in astrophysical publications (whose content was sadly uniform) the day when they will have to announce the non-uniformity of the concerned supernovae?

4.3. Creation and Emission of Sterile Neutrinos in Black Holes

Sterile neutrinos have often been proposed as dark matter candidates. It is also our preference. They would interact only by gravity with ordinary matter, with the exception of a small ability to mix with familiar neutrinos of the standard model. Sterile neutrinos associated with a magnetic charge would be one of the only by-products of annihilations that would successfully leak from the inside of the black hole to the outside, as would solar neutrinos associated with an electrical charge (electron) are the only ones who manage to escape from the heart of the Sun.

We will therefore limit ourselves here to the creation of sterile neutrinos dependent on a magnetic charge. The wave-corpuscle duality of photons, extended by de Broglie to the waves of matter, led to the quantum concept of matter field. This quantum field of matter is a set of operators, creations and annihilation of fermions, including the neutrino: the operator v_k^+ creates a neutrino of pulse *k*, and the operator v_k annihilates a neutrino of pulse *k*.

In this intra-horizon space of the active (hot and dense) black hole, which is part of Dirac's restless ocean, virtual pairs are constantly being created and destroyed. For a brief moment, a particle and its antiparticle separate. There are four possibilities

Process 1: The two partners meet and annihilate each other.

Process 2: The antineutrino remains in the black hole and the *neutrino materializes in the outside world.*

Process 3: the neutrino remains in the black hole and its *antineutrino escapes into the outside world*.

Process 4: Both partners stay in the black hole.

Particles escaping to the outside would be *fermionic monopoles* (refusing to put themselves in the same state). They would leave with a relativistic speed. Could there be several types of sterile magnetic neutrinos that can oscillate between them? Could the magnetic charge neutrino get the flavor of an electronic neutrino? Certainly not by an oscillation process, since all the neutrinos involved should be associated with the same type of charge. It is however possible to envisage that the sterile magnetic neutrino can decay into gamma rays (photons), into standard neutrinos (electric charge), into weaker sterile magnetic neutrinos (magnetic charge), and other particles. To return to the neutrinos escaped from the black hole, they would rather tend to slow down and regroup with the cooling to form the dark cosmos whose rules do not reflect our bright world. They would obey other non-symmetry laws, Maxwell's inverse laws, and be provided with magnetoelectric and weak interactions (very small compared to nuclear forces). The mass of these neutrinos affiliated with the magnetic charge would be rather small instead of being huge or zero, but sufficient to fill the missing mass gap.

Scientists think that there may be more than just a type of dark matter. A possibility is that several classes of dark matter particles exist, as well as a variety of forces that act only on them. One idea is that particles of dark matter interact with each other by a force that ordinary matter cannot feel. These particles could carry a "dark charge" that attracts or repels them even if they are electrically neutral and could emit "dark photons". Dark atoms would emit dark photons at a different rate than ordinary matter that emits ordinary photons. We know by observing the shapes of galaxies that this rhythm must be very weak.

5. Efforts of Four Researchers

In line with our paper, we highlight certain aspects of the work of four tenacious researchers who contribute to the extension of knowledge on dark matter, magnetic monopoles, black holes and sterile neutrinos:

Georges Lochak, president of the Louis-de-Broglie Foundation, is known for his work on magnetic monopoles: the magnetic monopole is a fermion endowed with weak interactions. He found an equation, analogous to that of Dirac, which no longer represents an electron but a magnetic monopole, which is, in a way, the other side of the electron. His equation finds Dirac's formula which shows that the charge of a magnetic monopole is equal to an integer multiple of the charge of the electron multiplied by 68.5: its equation joins this result. For him, it indicates that if the multiple is equal to zero—so if the monopole has no charge and is neutral—its equations coincide with those of the neutrino [22]. Georges Lochak worked for ten years with Leonid Urutskoiev of the Kurtchatov Institute who had headed a team that was looking for the origin of the Chernobyl disaster. Urutskoiev had hypothesized a flood of monopoles, resulting from an electrical explosion that occurred in the engine room. Some clues made him lean towards the hypothesis of a light magnetic monopole that corresponded to the Lochak monopole. Dozens of physicists contributed to a joint research work. The experiments were counted in the hundreds. The main theoretical center was the Louis de Broglie Foundation.

André Michaud explored the foundations of an electromagnetic mechanics of elementary particles whose laws apply to all levels. He described a space-time geometry that represents the mutual induction of electrical energy and magnetic energy within moving elementary particles in accordance with Maxwell's equations [23]. He details an experiment he performed that proves out of any doubt the inverse cube relation with distance between the magnetic fields of a magnet whose both north and south poles physically coincide, proving by the fact that the same inverse cube interaction law also applies by similarity to the elementary electromagnetic particles colliding with quasi-punctual behavior. This experiment also demonstrates that the magnet used behaves like a magnetic monopole [24].

Eue Jin Jeong basically demonstrated that the black hole jets and the dark matter problems are essentially one integrated physical phenomenon caused by dipole gravity. His outstanding discovery of the long ranged dipole gravity is in the fulfillment of Einstein's general relativity in its simplicity of the equivalence principle. He explains the dark matter problem in his book by invoking the fact that jets from both the south and the north poles of the rotating black hole constitutes a point source of the continuous outgoing matter following the dipole gravity force lines [25]. Jeong started early in 1982 when he was a graduate student, wondering why general relativity does not explain the jet phenomena from the black hole accretion discs. He was perplexed by the dismissed dipole gravity in the weak field limit of general relativity. His quest for the solution to the problem led him to realize in 1995 that the rotating hemisphere has a rotation frequency which depends on the relativistic shift of the center of mass. By investigating further, he derived Lense-Thirring force from the dipole gravity potential generated by the two hemispheres oppositely superposed inside the rotating sphere. The result is described with detailed mathematical derivation in an article published in 1999 [26].

Kevork Abazajian, an American physicist who works at the University of California, has demonstrated in an article the mechanism by which 7 keV sterile neutrinos can be produced and be the source of unknown gamma rays observed at 3.5 keV from center of galaxy cluster [27] [28]. Several teams of astrophysicists have observed an X-ray (gamma) spectral line with energy of about 3.5 keV, which corresponds to nothing known and seems very real, that is to say statistically significant. The only remaining hypothesis to explain the existence of these photons seeming to come from where there is the most dark matter, is that they would come from the disintegration of sterile neutrinos. As they are a little heavy, they would disintegrate producing “normal” neutrinos and photons whose energy would be half their mass. Abazajian considers that all dark matter consists of such 7 keV sterile neutrinos.

6. Heat, Entropy and Information Have Everything to Do with Black Holes

6.1. The Entropy Problem

Entropy is forbidden to black holes by general relativity, because the theory requires them to be completely smooth, without substructure. General relativity describes a black hole as having a smooth geometry and indicates that every black hole of a given mass, spin and charge should be exactly the same: in other words, black holes have no hair.

In contrast, quantum mechanics says black holes have a large amount of entropy, meaning a microscopic structure, or a hair. In 1972, Jacob D. Bekenstein was the first to suggest that black holes should have a well-defined entropy. He wrote that a black hole’s entropy was proportional to the area of its event horizon. Bekenstein also formulated the generalized second law of thermodynamics for systems including black holes. Both contributions were confirmed when Stephen Hawking (and, independently, Zeldovich and others) proposed the existence of radiation two years later. Hawking had initially opposed Bekenstein’s idea on the grounds that a black hole could not radiate energy and therefore could not have entropy [29]. However, in 1974, Hawking performed a lengthy calculation that convinced him that particles can indeed be emitted from black holes. Today this is known as Hawking radiation.

Reflecting on the isolated black hole, Hawking noted that the light spectrum of the eponymous radiation streaming away from it would look the same as that of a radiating hot body, meaning that the black hole has a temperature. In general, temperature arises from the motion of atoms inside objects. The thermal nature of Hawking radiation, then, suggested that the black hole should have a microscopic structure made of some kind of discrete building blocks or bits. The work of Bekenstein and Hawking gives a formula for the number of bits, a measure known as the black hole entropy. Entropy is a gauge of disorder, which becomes greater as the number of states that an object can have grows. The larger the number of bits in a black hole, the more possible arrangements they can have and the greater the entropy.

So here is the contradiction: relativity says no hair, whereas quantum mechanics says black holes have a large amount of entropy, meaning some microscopic structure, or hair.

6.2. The Information Paradox

In agreement with the standard picture of quantum mechanics, information can never be destroyed. Even when you burn a letter, for example, the original information encoded in the atoms of the letter is preserved in the ashes. In quantum mechanics, every system is described by a formula called the wave function, which encodes the chances that the system will be in any particular state.

In keeping with Hawking's first calculation, the particles that escape from a black hole do not depend at all on the properties of the material that went into the hole. We could send a note with a message into the black hole, and there would then be no process to reconstruct the message from the final particles that would emerge. Hawking radiation implies that black holes destroy the information about the matter that falls into them. In Hawking's thought experiment, the loss of information means that we have no method to predict the wave function of Hawking radiation based on the properties of the mass that went into the black hole. Information loss is forbidden by quantum mechanics, so Hawking concludes that the laws of quantum physics had to be modified to allow for such loss in black holes.

In an effort to resolve these puzzles (this information paradox), physicists looked for new approach to combine general relativity and quantum mechanics into a coherent theory that could describe black holes. In 1997, Juan Maldacena came up with an idea around the information loss problem—a solution sometimes called the Maldacena duality. This duality is equivalence between quantum mechanics and gravity—a quantum theory of gravity. It means that the quantum physics of a black hole is equivalent to that of an ordinary gas of hot nuclear particles. It also means that spacetime is fundamentally different from what we perceive, more like a three-dimensional hologram projected from a two-dimensional surface of a sphere. If Maldacena's assumptions are true, then ordinary quantum laws would apply to gravity of black holes as well, and information cannot be lost [30].

Hawking had proposed that general relativity works for black holes but that quantum must be modified. Maldacena concludes that spacetime is holographic. In 2004 Hawking announced that he had changed his mind about the need for black holes to lose information.

6.3. Entropy, Heat and Information of Black Hole According to the Theory of Relation

We consider that quantum physics inside a black hole is equivalent to that of concentrated energy magma, or that of a gas of hot nuclear particles. According to the theory of the Relation, energy is "dark" for a double reason: it undergoes a change of energy (principle of Compensation) [31], and because a blackout accompanied by a charge reversal occurs at the passage of the event horizon.

We conjecture that energy within active black holes—surrounded by an accretion disk whose matter feeds them—is subject to high thermal quantum fluctuations (kinetic energy of particle motion). The temperature, that is to say the energy absorption capacity, is very high. Not only quantity of energy is huge but also its availability. According to quantum mechanics, pairs of particles and their antimatter counterparts are born incessantly, then disappear a few moments later in the universe. Pairs of real thermal particles that can be as well leptonic than bosonic. A huge amount of radiation and particles escapes from the inside of the black holes.

Under general relativity, no signal of any kind can come back from beyond the horizon because that would suppose exceed the speed of light. But if we rely on the equation that Hawking has derived from the temperature of a black hole [32],

$$T^o = \frac{1}{16\pi^2} \times \frac{c^3 h}{GMk}, \tag{17}$$

(T^o is temperature, k is Boltzmann's constant, kT^o is energy), it is not required that a particle exceeds the speed of light to cross the horizon to the outside. And there is no indication that the mass M that melts contains only photons. It may contain hadrons. We stick to magnetic neutrinos.

In the expression

$$kT^o M = \frac{1}{16\pi^2} \times \frac{c^3 h}{G}, \tag{18}$$

T^o and M are inversely proportional: when the temperature, or energy, increases, the classical mass declines. The contradiction between the diminishing mass and the growing energy seems flagrantly, since energy and mass are supposed to be equivalent. Clearly, the energy kT^o contains the quantum mass m_o which is proportional to the temperature. [m_o comes from $kT^o = h\nu = m_o c^2$. Equation (17) comes from $t_o c = GM/c^2 = h/m_o c = h/kT^o$].

The link between energy, entropy and temperature refers to the second law of thermodynamics, which says that entropy always rises. The law of entropy implies irreversibility. The principle of irreversibility is that if you leave things to themselves at different temperatures, with the passage of time, their temperatures are getting closer and closer, and the availability of energy is continually decreasing. The one way always leads to a loss of energy availability. The drop in temperature, and therefore the decrease in the energy absorption capacity, goes hand in hand with an enlargement in entropy (which is a degraded energy) [33].

In the case of active black holes, there is a very high temperature around and beyond the horizon. The rise in temperature, and therefore the growth in energy, should go with a drop in entropy. But one concludes that the energy-mass increment goes hand in hand with a gain in entropy. In this case, the second law of thermodynamics, which states that entropy rises, presents a serious problem with temperature and energy [34].

Energy is a subtle concept, hard to grasp. It can be said that energy stops the motion as much as it provokes it. Bekenstein first conjectured that black holes have entropy. Entropy always goes hand in hand with energy. In itself, the existence of entropy does not imply that a system has a temperature. For Hawking, the key was temperature, not entropy. He anticipated that black holes also have a temperature. They are not cold objects, dead. They radiate thanks to an internal heat, but, in the end, it is this heat that causes their destruction.

On the subject of information, we think that the laws of general relativity are inapplicable beyond the horizon and that quantum mechanics must not be modified: information loss is forbidden. Imagine the particles falling into a black hole, each with its particular frequency that is its message. Very quickly, the sharply frequencies begin to dissolve, the message becomes almost impossible to discern in this magma of dark energy. The message becomes hopelessly scrambled in this inextricable mix of quantum fluctuations. The principles of quantum mechanics ensure that the message is always present within deformed particles moving in a chaotic manner. Although scrambled, not a single bit of information was eradicated. Each bit of information ends up being transferred to photons and other particles that evacuate energy from the black hole. The information is stored among the particles that form the Hawking radiation. The latter calculated that the disturbance of vacuum fluctuations due to black holes caused the emission of photons, as if the horizon of a black hole was a blackbody [32]. Hawking believed that a particle in a virtual pair escapes from the black hole but carries no information. Many theorists concluded that Hawking was wrong, that he had mistaken the scrambling of information for actual information loss. Our opinion is that not only information escapes from the black hole, but also the destroyed matter.

Perhaps the truth is somewhere in a hologram. A hologram is a two-dimensional image that makes it possible to reconstruct three-dimensional images. The holographic principle is a speculative conjecture in the framework of quantum gravity theory, proposed by Gerard't Hooft in 1993 and then improved by Leonard Susskind in 1995. This conjecture proposes that all the information contained in a volume of space can be described by a theory that lies on the edges of this region. For example, expanding cosmic space and black holes have horizons as an edge. The cosmic event horizon in an expanding universe is mathematically similar to the horizon of a black hole. The difference is that in the first case we are in and we look outward, and in the other we look at it from the outside. We can assume that the photons of the cosmic microwave background radiation that surround us are the messengers of the cosmic horizon that would carry the coded images of the megaverse. Just as one can surmise that the physical events that take place behind the horizon of the black hole would be telegraphed to the outside in a scrambled telegraph code in the form of Hawking radiation [35]. The idea that the universe is a kind of holographic image is surprising.

7. Comments and Conclusions

More than 80 percent of the mass of the universe is invisible. The presence of this dark matter is detected thanks to its gravitational signature [36]. His nature remains one of the great enigmas of cosmology. But we at least know that it is, for the most part, of a different nature from the ordinary matter that composes planets and stars [37]. In practice, the observations show that we cannot explain the distribution of matter by supposing that it is, on the one hand, solely baryonic and, on the other hand, governed only by the laws of gravitation. To reconcile theory and observation, scientists considered either changing the material content of the universe or changing the laws of gravitation. The hypothesis of an unknown form of matter remains the most accepted. A plethora of scenarios of high energy physics postulates new forms of matter that are difficult to detect [38]. Whether direct or indirect detection experiments, the tracks—especially the supersymmetric particle track—are very similar to a dead end. We will agree that in this moment of crisis, we must leave no track aside.

In this paper, we have just presented a radically different track to explain the enigma of dark matter: a major conceptual overhaul that concerns, in addition to dark matter, electromagnetism, sterile neutrino and black holes. Our model predicts that dark matter may be accompanied by a hidden and reworked version of electromagnetism (and possibly also a hidden weak force), implying that dark matter may emit and reflect hidden light. This "light" is invisible to us and so the dark matter remains unseen. Nevertheless, these new forces could have very significant effects. For example, they could distort interacting clouds of dark particles. Astronomers have sought this effect in the famous Bullet cluster, also called 1E 0657−56, which consists of two clusters of galaxies that have passed through each other. Observations show the co-mingling of clusters left the dark matter largely unperturbed, indicating that any dark forces are weak [39].

The new variant of electromagnetism, which we call magnetoelectricity, would also allow dark particles to exchange energy and momentum, a process that would tend to homogenize them and make the halos spherical. We can make some conclusions about the strength of the dark electromagnetism force—and thus how often dark matter annihilation occurs—by considering how this force would affect galaxies. The reason galaxies have a flattened structure is that electromagnetism allows ordinary matter to lose energy and settle into disks. Clouds of gas inside galaxies radiate electromagnetic energy through the emission of photons. That radiation results in the spinning matter inside the clouds clumping together and eventually relaxing into a dislike structure. Because we know that dark matter is primarily distributed spherically around most galaxies and does not collapse to a disk, we can conclude that it cannot lose energy via dark photon emission at the same rate that ordinary matter does [40].

We have seen above that if we apply the reversed laws of Maxwell and that if we try the same steps as those that led to the equation of the electron, we find another particle, no longer an electron but a magnetic monopole. A modification of the laws of gravitation in a somewhat *ad hoc* way constitutes an alternative to dark matter. Maxwell's reversed laws, on the contrary, justify the existence of this dark substance that would come from black holes.

Supermassive black holes are emerging as the most prolific creators. Far from being passive, they spit, blow. They emit large amounts of energy accumulated around them with unparalleled power. Their jets of matter would have fertilized the cosmos on vertiginous distances, triggered outbreaks of stars, created galaxies [2]. And why should not the supermassive black holes also have engendered dark matter? Why would not they also play the role of dark matter producer? Certainly not by condensed gas jets. Dark matter is different from ordinary matter, as is matter inside the black hole.

Our hypothesis of the inversion of Maxwell's laws as well as that of the black hole producing dark matter may seem as strange as absurd. But let us say it, the very existence of the dark matter seems absurd. Likewise the idea of black holes, which was originally a mathematical "catastrophe" shunned by theorists, including Einstein who had predicted them. Our view concerning the links between dark matter and black holes can be summarized as follows:

Dark matter is intimately related to black holes. The darkness of dark matter and black holes is caused by the reversal of Maxwell's laws. This inversion is triggered near the horizon of the black hole while the magnetic currents combined with gigantic pressure and high temperature cause a phase transition which results in a reversion of Maxwell's laws. This means that a magnetic charge is substituted for the electric charge, and that the magnetic current subdues the electric current. It can be said that in the space of the black hole a magnetoelectric force is created. The substance of dark matter comes from black holes. The latter emit particles from the process of creation of pairs of particles triggered by the metamorphosis of high energy photons. Growth of energy-mass of black holes increases the rate of materialization and annihilation of the dark energy inside. Dark radiation will materialize by creating a neutrino and an antineutrino, particles associated with the magnetic charge. If they are not annihilated, some will cross the black hole with a relativistic speed close to that of the light before slowing down to constitute the dark matter. But it can also happen that two opposite particles meet within the black hole. They dematerialize, they turn into two rays of the same energy and directed in the opposite direction. One of these dark rays, if not both, can cross the black hole without hitting an ordinary particle on the outside, and this dark radiation can materialize by creating a sterile magnetic neutrino and antineutrino accompanied by particles and by normal high energy photons. There is more than thermal evaporation; it is the spontaneous emission of particles. Black holes do not constitute dark matter, as we are led to believe. On the other hand, the black holes produce and emit the substance that constitutes the dark matter, in this case the sterile neutrino with magnetic charge. The black holes come undone, producing a dark matter that gradually disintegrates.

Crises in science are often the most creative. This redesign should have profound implications for theoretical physics and astrophysics.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Natural Extension of the Schrödinger Equation to Quasi-Relativistic Speeds

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Abstract

A Schrödinger-like equation for a single free quantum particle is presented. It is argued that this equation can be considered a natural relativistic extension of the Schrödinger equation for energies smaller than the energy associated to the particle's mass. Some basic properties of this equation: Galilean invariance, probability density, and relation to the Klein-Gordon equation are discussed. The scholastic value of the proposed Grave de Peralta equation is illustrated by finding precise quasi-relativistic solutions for the infinite rectangular well and the quantum rotor problems. Consequences of the non-linearity of the proposed equation for the quantum superposition principle are discussed.

Keywords

Quantum Mechanics, Schrödinger Equation, Klein-Gordon Equation, Relativistic Quantum Mechanics

1. Introduction

Since the discovery of the quantum wave mechanics by Erwin Schrödinger in 1925, the Schrödinger equation has been often used for introducing the fundamentals of quantum mechanics [1] [2] [3] [4] [5]. The one-dimensional Schrödinger equation for a free particle with mass m is given by the following equation [1] [2] [3] [4] [5]:

$$i\hbar\frac{\partial}{\partial t}\psi_{Sch}(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi_{Sch}(x,t). \tag{1}$$

where $\hbar$ is the Plank constant (h) divided by 2π. However, the Schrödinger equation is not Lorentz invariant but Galilean invariant [6]; therefore, a relativistic quantum mechanics cannot be based on Equation (1). A fully relativistic quantum theory requires to be funded on equations that are valid for any two observers moving respect to each other at constant velocity. In contrast, the Galilean invariance of Equation (1) means that two such observers will only agree in the adequacy of Equation (1) for describing the movement of a massive free quantum particle when the relative speed between the observers (V_o) is much smaller than the speed of the light in the vacuum (c). In practice, this is not a terrible limitation of the Schrödinger equation because up to today humans have been only able to travel at speeds much smaller than c. This is one of the principal reasons why the Schrödinger equation is still relevant almost 100 years after its discovery. However, as it will be discussed in Section 2, there is another important limitation of Equation (1): it describes a particle in which linear momentum (p) and kinetic energy (K) are related by a classical relation that is not valid at relativistic speeds [1] [2] [3] [6]. The famous relativistic equation $E_m = mc^2$, where E_m is the energy associated to the mass of a particle [7] [8], implies the equivalence between mass and energy. This equivalence has profound implications for the formulation of any relativistic quantum mechanics theory. When the kinetic energy of a free particle with mass m equals the energy associated to the mass of the particle, *i.e.*, $K = mc^2$, a second particle with the same mass can be created from the kinetic energy of the original particle; therefore, the number of particles may not be conserved in a fully relativistic quantum theory [2] [8] [9]. A common argument used for guiding the search for the correct Lorentz invariant basic equation of a relativistic quantum mechanics is that in such equation the time and spatial variables should appear on equal footing as it happens in the Lorentz transformations [8] [9]. For instance, in contrast to Equation (1), in the Lorentz invariant Klein-Gordon equation does not appear the first partial derivative respect to time but the second one as shown in Equation (2), which is the Klein-Gordon equation for free particle [8] [9]:

$$\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi_{KG}(x,t) = \frac{\partial^2}{\partial x^2}\psi_{KG}(x,t) - \frac{m^2c^2}{\hbar^2}\psi_{KG}(x,t). \tag{2}$$

Unfortunately, Equation (2) does not formally look at all like Equation (1), thus masking how the Klein-Gordon equation becomes the Schrödinger equation when the particle moves at speeds (V) much smaller than c. Moreover, there are solutions of Equation (2) with unwanted properties like superluminal phase velocity, negatives energies, and associated with negative probabilities [8] [9]. In Section 2, the consequences of an intriguing natural extension of the Schrödinger equation to quasi-relativistic speeds are explored. The term "quasi-relativistic" is used in this work as meaning a particle moving at so large speeds that it is necessary to use the correct relativistic relation between p and K but still the number of particles is constant because $K < mc^2$. The following equation is the center of attention here:

$$i\hbar\frac{\partial}{\partial t}\psi(x,t) = -\frac{\hbar^2}{(\gamma_V + 1)m}\frac{\partial^2}{\partial x^2}\psi(x,t). \tag{3}$$

where:

$$\gamma_V = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}. \tag{4}$$

Clearly, the Grave de Peralta equation (Equation (3)) exactly coincides with the Schrödinger equation (Equation (1)) when $V \ll c$. As it will be discussed in Section 3, the formal similitude between Equation (3) and Equation (1) immediately suggest that Equation (3) may be Galilean invariant and that probabilities can be associated to ψ in the same way that it is done for ψ_{Sch} [1] [2] [3] [4]. However, Equation (3) describes the movement of a massive free quantum particle which momentum and kinetic energy are related by the correct relativistic relation. Therefore, Equation (3) extent the range of applications of the Schrödinger equation to quasi-relativistic speeds. Moreover, the formal similitude between Equation (3) and Equation (1) provides a simple approach for obtaining quasi-relativistic corrections to the solutions of the Schrödinger equation, for a whole class of problems where the square of the particle speed (V^2) is constant. Two interesting examples illustrating this point are presented in Sections 4 and 6. In both instances, explicit quasi-relativistic solutions of Equation (3) can be found with no more complexity than in standard textbook examples of solvable Schrödinger equation problems [1] [2] [3] [4] [5]. This illustrates the scholastic value of the Grave de Peralta equation for introducing learners to the intricacies of the fully relativistic quantum mechanics and quantum fields theory. In addition, it is demonstrated in this work that a plane wave solution of Equation (3) is subluminal and that this solution is related to a plane wave solution of Equation (2) by the following relationship:

$$\psi(x,t) = \psi_{KG}(x,t)\mathrm{e}^{iw_m t}, \quad w_m = \frac{mc^2}{\hbar}. \tag{5}$$

where:

$$\begin{aligned} \psi(x,t) &= \mathrm{e}^{\frac{i}{\hbar}(px-Kt)}, \\ \psi_{KG}(x,t) &= \mathrm{e}^{\frac{i}{\hbar}(px-Et)}. \end{aligned} \tag{6}$$

The plane waves ψ and ψ_{KG} in Equation (6) are solutions of Equations (3) and (2), respectively. E, K and p are the relativistic total and kinetic energy and the linear momentum of a free particle, respectively [7] [8]. It is worth noting that two solutions of Equation (3) corresponding to two different particle's speeds are not simultaneously solution of the same equation but solutions of two slightly different equations only differing in the value of γ_v. Even when the full discussion of this topic is outside of the scope of this work, due to its relevance, the implications of the non-linearity of Equation (3) for the quantum mechanics superposition principle are briefly discussed in Section 5. Finally, the conclusions of this work are given in Section 7.

2. Schrödinger Equation Extension to Quasi-Relativistic Speeds

Formally, Equation (1) can be obtained from the classical relation between K and p for a free particle when $V \ll c$ [1] [2] [3] [6]:

$$K = \frac{p^2}{2m}, \quad p = mV. \tag{7}$$

Then, Equation (1) is obtained by substituting K and p by the following energy and momentum quantum operators [1] [2] [3]:

$$\hat{E} = \hat{K} = i\hbar\frac{\partial}{\partial t}, \quad \hat{p} = -i\hbar\frac{\partial}{\partial x}. \tag{8}$$

By analogy, Equation (3) can be simply obtained combining Equation (8) with the relation between the relativistic expressions of the kinetic energy and the linear momentum of a free particle traveling at quasi-relativistic speeds:

$$K = \frac{p^2}{(\gamma_V + 1)m}, \quad p = \gamma_V mV. \tag{9}$$

Equation (9) can be easily obtained from the following well-known relativistic equations [7] [8]:

$$E^2 - m^2c^4 = p^2c^2 \Leftrightarrow (E + mc^2)(E - mc^2) = p^2c^2, \tag{10}$$

$$K = E - mc^2, \quad E = \gamma_V mc^2. \tag{11}$$

The Klein-Gordon equation can formally be obtained from the first expression of Equation (10) by assigning the temporal partial derivative operator in Equation (8) to the total relativistic energy (E) of the free particle, which is the sum of its kinetic energy plus the energy associated to the mass of the particle [7] [8]. However, if one chooses to assign this operator to K, as it is done when obtaining the Schrödinger equation, then from Equations (9) and (8) follows Equation (3). This is not the customary choice, but in this work instead of simply discharging this option, it is explored the consequences of this natural choice. For instance, a simple substitution of $\psi(x, t)$ given by Equation (6) in Equation (3) results in Equation (9), thus demonstrating that $\psi(x, t)$ given by Equation (6) is a plane wave solution of Equation (3), which phase velocity $V_{ph} = K/p$ is related to the velocity of the particle by the following expression:

$$V_{ph} = \frac{\gamma_V}{\gamma_V + 1}V. \tag{12}$$

Consequently, $V_{ph} < V < c$, *i.e.*, the plane wave $\psi(x, t)$ given by Equation (6) is subluminal and, as happen for a plane wave solution of the Schrödinger equation, $V_{ph} \sim V/2$ when $V \ll c$. In contrast, the substitution of $\psi_{KG}(x, t)$ given by Equation (6) in Equation (2) results in Equation (10), thus demonstrating that $\psi_{KG}(x, t)$ given by Equation (6) is a plane wave solution of Equation (2), which phase velocity $V_{KG} = E/p$ is given by the following expression:

$$V_{KG}=\frac{\gamma_V mc^2}{\gamma_V mV}=\frac{c^2}{V}. \tag{13}$$

Consequently, $\psi_{KG}(x, t)$ is superluminal because $V_{KG} > c$. Equations (5) and (6) suggest that the time-independent equations corresponding to Equations (2) and (3) are equal. In fact, looking for solutions of the form $X(x)T(t)$ of Equations (1), (2), and (3), where $T(t)=\mathrm{e}^{-\frac{i}{\hbar}Kt}$ for Equations (1) and (3) but $T(t)=\mathrm{e}^{-\frac{i}{\hbar}Et}$ for Equation (3), produces the same time-independent equation in the three cases:

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}X(x)+\kappa^2 X(x)=0,\ \ \kappa=\frac{p}{\hbar}. \tag{14}$$

As it will be illustrated below, often $X(x)$ and κ are determined solving Equation (14) under adequate boundary conditions; then the possible values of p are determinate from the possible values of κ. However, the relation between K and p are different for non-relativistic and quasi-relativistic speeds; therefore, the solutions of Equations (1) and (3) have equal spatial dependences but different values of K. Also, the relation between E, K, and p are different for quasi-relativistic speeds; therefore, the solutions of Equations (2) and (3) have equal spatial dependences but different values of K and E. Equations (9) and (10) can be obtained from each other using Equation (11); however, Equation (10) admits solutions with positive and negative energies but K only can be positive in Equation (9). This is in correspondence to the presence of a second-order temporal partial derivative in Equation (2), which determines that Equation (2) has solutions with positive and negative energies [8] [9]. In contrast, there is a first-order temporal partial derivative in Equations (1) and (3). This determines that Equations (1) and (3) only have solutions with positive energies. It is straightforward to show that Equation (5) can be obtained from Equations (11) and (6). Equation (5) gives a simple recipe from obtaining a plane wave solution of Equation (3) from a plane wave solution of Equation (2) with positive energy and vice versa.

3. Probability Density and Galilean Invariance

Due to the formal similitude between Equation (3) and Equation (1), one can demonstrate that a probability continuity equation can be associated to the solutions of the Grave de Peralta equation in the same way that it is done for the Schrödinger equation [1] [2] [3] [4]. In short, one can associate a probability density $\rho(x, t)$ to a normalized solution of Equation (3) in the following way:

$$\rho(x,t)=\psi^*(x,t)\psi(x,t),\ \int_{-\infty}^{+\infty}\rho(x,t)\mathrm{d}x=1. \tag{15}$$

The probability density corresponding to the Schrödinger equation is well-defined when both $\psi/(2m)$ and $\psi^*/(2m)$ tend to zero when $|x|$ is very large [1]. Similarly, provided that V^2 and γ_V are constant, it can be shown than $\rho(x, t)$ defined by Equation (15) is well-defined when both $\psi/\left[(\gamma_V+1)m\right]$ and

$\psi^*/\left[(\gamma_V+1)m\right]$ tend to zero when $|x|$ is very large, which is a less restrictive condition when $\gamma_V \gg 1$ than the one required for the Schrödinger equation. The rate of the temporal variation of $\rho(x, t)$ is then given by the following expression:

$$\frac{\partial}{\partial t}\rho = \frac{\partial \psi^*}{\partial t}\psi + \psi^* \frac{\partial \psi^*}{\partial t}. \tag{16}$$

The temporal derivatives of ψ and ψ^* in Equation (16) can be substituted by expressions containing spatial derivatives of ψ and ψ^* by using Equation (3) and its complex conjugate equation. In this way Equation (16) can be transformed in the following one:

$$\frac{\partial}{\partial t}\rho = \frac{\hbar}{(\gamma_V+1)mi}\left(\psi\frac{\partial^2\psi^*}{\partial x^2} - \psi^*\frac{\partial^2\psi}{\partial x^2}\right). \tag{17}$$

But [1]:

$$\psi\frac{\partial^2\psi^*}{\partial x^2} - \psi^*\frac{\partial^2\psi}{\partial x^2} = \frac{\partial}{\partial x}\left(\psi\frac{\partial\psi^*}{\partial x} - \psi^*\frac{\partial\psi}{\partial x}\right). \tag{18}$$

Then using Equation (18) permits to rewrite Equation (17) as the one-dimensional (1D) probability continuity equation [1]:

$$\frac{\partial}{\partial t}\rho + \frac{\partial}{\partial x}J = 0,\ \ J = \frac{\hbar}{(\gamma_V+1)mi}\left(\psi^*\frac{\partial\psi}{\partial x} - \psi\frac{\partial\psi^*}{\partial x}\right). \tag{19}$$

Like for the Schrödinger equation [1], it is easy to show that Equation (19) can be generalized to three dimensions (3D). The absence of negative values of ρ and J is a consequence of the absence of a second time derivative in the Equation (3) [8] [9]. This concludes the demonstration that a probability continuity equation can be associated to the solutions of Equation (3) as it is done for the Schrödinger equation. In what follows a qualitative discussion about the Galilean invariance of Equation (3) is presented. A more formal discussion about this topic is presented in Annex A. At a first sight, Equation (3) does not look neither Galilean nor Lorentz invariant. Equation (3) should not be Lorentz invariant because in Equation (3) the temporal and spatial partial derivatives do not have the same order [8] [9]. In contrast, it is well known that Equation (2) is Lorentz invariant [8] [9]. The formal similitude between Equations (3) and (1) suggests that Equation (3) may be Galileo invariant, but there is a problem. A well-defined Equation (3) requires a constant value of V^2 and γ_V. As it will be illustrated in Sections 4 and 6, there are very interesting problems where this requirement is fulfilled. For instance, one of these problems is the description of the movement of a massive quantum particle confined in a 1D box, which is at rest respect to an inertial reference frame S. An observer at rest respect S may think about the particle as moving with constant quasi-relativistic speed (V) but changing direction each time the particle bounced in the box's walls. However, a second observer moving parallel to the box with velocity $+V_o$ respect to the first observer, but at rest respect to a second inertial reference frame S′, would see the particle moving sometimes with speed V'_+ and sometimes with speed V'_-, where [7] [8]:

$$V'_\pm = \frac{\pm V - V_O}{1 - \frac{\pm V V_O}{c^2}}. \tag{20}$$

Thus, the second observer would not find well-defined the value of V'^2 and $\gamma_{V'}$ that should be introduced in Equation (3). However, at quasi-relativistic particle's speeds $V'_+ \sim V'_- \sim V$ when $V_o \ll V$. Consequently, at quasi-relativistic particle's speeds when $V_o \ll V$, both observers will see the particle moving with (almost) the same values of V^2 and γ_V. Moreover, in this quasi-relativistic limit $p' \sim p$ and $K' \sim K$. Consequently, both observers will agree in that they should solve Equation (3) for finding the possible quantum states of the massive particle moving at quasi-relativistic speeds inside of the 1D box. *i.e.*, Equation (3) is Galilean invariant. Nevertheless, as it will be shown below, Equation (3) can be used for solving quasi-relativistic quantum problems.

4. Infinite Rectangular Well

An important but simple problem often solved in quantum mechanics textbook is a particle moving inside an infinite rectangular well at speeds much smaller than c [2] [3] [4] [5]. Using Equation (3), this problem can be solved for quasi-relativistic speeds following similar procedures than in the quantum mechanics textbooks for $V \ll c$ [2] [3] [4] [5]. One should look for a wavefunction that is identically null outside of the well, null at $x = 0$ and $x = L$, and satisfies Equation (3) in the interval $0 < x < L$. Solving Equation (3) means finding the quantum states of a free particle with constant values of K. But due to Equation (9), a free particle moving with constant kinetic energy must have constant value of V^2 and vice versa; therefore, the solutions of Equation (3) correspond to quantum states of a particle moving with a constant value of V^2. Separating variables and substituting in Equation (3) results:

$$\psi(x,t) = X(x)\mathrm{e}^{\frac{i}{\hbar}Kt}, \tag{21}$$

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}X(x) = -\kappa^2 X(x) = 0, \ \ \kappa = \sqrt{\frac{(\gamma_V + 1)mK}{\hbar^2}}, \ \ X(0) = X(L) = 0. \tag{22}$$

Looking for solutions of Equation (22) corresponding to constant values of K and V^2, one can find that:

$$\psi_n(x,t) = X_n(x)\mathrm{e}^{\frac{i}{\hbar}K_n t}, \ \ \kappa = \frac{n\pi}{L}, \ \ n = 1, 2, \cdots \tag{23}$$

where:

$$X_n(x) = \sqrt{\frac{2}{L}}\sin\left(\frac{n\pi}{L}x\right), \tag{24}$$

$$K_n = n^2 \frac{h^2}{(\gamma_{V_n} + 1)m(L/2)^2}. \tag{25}$$

From Equation (24) follows that the spatial dependence of $\psi_n(x, t)$ coincide with the spatial dependence of the wavefunction calculated using the Schrödinger equation [2] [3] [4] [5]. As expected, Equation (25) gives the know values of the particle's energies at low speeds when $\gamma_V \sim 1$ [2] [3] [4] [5]. From Equation (25) and the relativistic equation, $K = (\gamma_V - 1)mc^2$, follow:

$$\gamma_{V_n}^2 = 1 + n^2\left(\frac{2\lambda_C}{L}\right)^2. \tag{26}$$

where $\lambda_C = h/(mc)$ is the Compton wavelength [7] [8]. Equation (26) gives $\gamma_V^2 = 2$ when $n = 1$ and $L = 2\lambda_C$; evaluating for these values Equation (25) results in $K_1 \sim 0.4mc^2$, which is smaller than the value $K_1 \sim 0.5mc^2$ that would be obtained, using the Schrödinger equation, for the ground state energy of a particle of mass m confined in an infinite rectangular well of length $L = 2\lambda_C$. Moreover, this result is precise because the calculated energy of the ground state is clearly quasi-relativistic. In contrast, Equation (26) gives $\gamma_V^2 = 5$ when $n = 1$ and $L = \lambda_C$; evaluating for these values Equation (25) results in $K_1 \sim 1.2mc^2$. The number of particles may not be constant at these energies. This result for a 1D infinite rectangular well can easily be extended to the 3D infinite rectangular well as it is done for the Schrödinger equation [4] [5]. Consequently, Equation (3) establishes a fundamental connection between quantum mechanics and especial theory of relativity: no single particle with mass can be confined in a volume much smaller than $(\lambda_C)^3$ because when this occurs, $K > mc^2$ and the number of particles may not be constant anymore; therefore, a single point-particle with mass cannot exist. Point-particles with mass can only exist in fully relativistic quantum field theories where the number of particles is not constant. This is true for an electron, a quark, and probably may also be true for a black hole and the whole universe at the beginning of the Big Bang. This is consistent, for instance, with the confinement of an electron in the Hydrogen atom because for an electron $\lambda_{Ce} \sim 2.4 \times 10^{-3}$ nm, which is ~20 times smaller than the radius of the Hydrogen atom, $r_B \sim 5.3 \times 10^{-2}$ nm [1] [2] [3] [4] [5]. Combining Equations (25) and (26) allows for rewritten Equation (25) in the following way:

$$K_n = n^2\frac{h^2}{\left[1+\sqrt{1+\left(\frac{2n\lambda_C}{L}\right)^2}\right]m\left(\frac{L}{2}\right)^2}. \tag{27}$$

When $L \gg 2n\lambda_C$, Equation (27) gives the know values of the energies calculated using the Schrodinger equation for a particle in an infinite well [2] [3] [4] [5]. However, in general, the values of K_n calculated using Equation (27) are smaller than the ones calculated using the Schrödinger equation. This in excellent correspondence with more involved numerical results obtained solving the Dirac equation for the 1D infinite rectangular well [10]. Moreover, and more significant for experiments, the differences in energies between different energy levels are slightly different when obtained using Equations (1) and (3).

5. Superposition Principle

Besides allowing to obtain precise quasi-relativistic solutions of several interesting problems, like tunneling through a barrier and other problems with piece-wise constant potentials, following similar procedures than in the quantum mechanics textbooks for $V \ll c$ [1] [2] [3] [4] [5], Equation (3) may describe a new physics. The Schrödinger equation is linear, this means, for instance, that if $\psi_{Sch1}(x, t)$ and $\psi_{Sch2}(x, t)$ are two solutions of Equation (1) for a particle in an infinite rectangular well corresponding to different values of V^2, then the wave-function $\psi_{Sch}(x,t) = a\psi_{Sch1}(x,t) + b\psi_{Sch2}(x,t)$, where a and b are complex numbers such that $|a| + |b|^2 = 1$, is also a solution of Equation (1). $\psi_{Sch}(x, t)$ represents a legitime possible state of a particle in an infinite well. The superposition state represented by $\psi_{Sch}(x, t)$ is often interpreted as a state where the particle is neither in the state $\psi_{Sch1}(x, t)$ where the kinetic energy is K_1 nor in the state $\psi_{Sch2}(x, t)$ where the kinetic energy is K_2, but somehow the particle is simultaneously in both states. The existence of superposition states like $\psi_{Sch}(x, t)$ is then a fundamental consequence of the linearity of Equation (1) with no classical counterpart. This exemplifies the weirdness of quantum mechanics [6] [11]. Moreover, the superposition state $\psi_{Sch}(x, t)$ represent a qubit, concept that is at the heart of current attempts to demonstrate a practical quantum computer [11] [12]. In contrast to the Schrödinger equation, Equation (3) is not linear. If $\psi_1(x, t)$ and $\psi_2(x, t)$ are two solutions of Equation (3) for a particle in a rectangular infinite well corresponding to different values of V^2, then strictly they are not solutions of the same Equation (3) but of slightly different Equations (3) with different values of γ_V. Moreover, $\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t)$ is not a solution of any Equation (3). Consequently, if the Grave de Peralta equation is a legitime extension of the Schrödinger equation to the quasi-relativistic domain, then the current understanding of the superposition principle in quantum mechanics should be revised because it appears to only be valid when the particle moves at speeds much smaller than c. The superposition principle is a corner stone of quantum mechanics; therefore, one could be interested in saving the superposition principle by stretching the meaning of "solution of Equation (3)", such that "$\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t)$ is a solution of Equation (3)" means that there is a set formed by several slightly different Equations (3) and $a\psi_1(x, t)$ and $b\psi_2(x, t)$ are solutions of a slightly different Equation (3) from this set, corresponding to a different value of V^2 each. For instance, strictly speaking, Equations (23), (24), and (27) give the solutions of a set of Equations (3) for the infinite rectangular well. This is somehow related with Section 3 discussion about the Galilean invariance of Equation (3). Strictly speaking, two observers slowly traveling with constant velocity respect to each other should resolve a set of Equations (3) which values of γ_V are contained in a narrow continuous interval. The adoption of Equation (3) as a valid description of the quantum states of a massive free particle then breaks with the longstanding tradition of describing the dynamics of a physical system using a single equation. The second Newton law and the

Schrödinger equation are examples of this tradition. The future of the particle's wave function is determined by the Schrödinger equation and the initial conditions are the only source of indeterminacy. In contrast, the dynamics of a particle is described by a whole set of similar Equations (3), which introduces a new source of indeterminacy in the future of the particle's wave function. This may be a welcome development for the understanding of the weirdness of quantum mechanics.

Alternatively, the non-linearity of Equation (3) suggests that $\psi_1(x, t)$ and $\psi_2(x, t)$ could be understood as corresponding to two different phases-of-a-system which are described by a different equation each. $\psi(x, t)$, which is not a solution of Equation (3), describes them a state of the system where no one of these two phases exists but where somehow, when a set of identical measurements is done on a system which is prepared in the state $\psi(x, t)$ each time, then a fast transition of the system is induced by the measurement and the system randomly transits either to the phase represented by $\psi_1(x, t)$ with probability $|a|^2$ or to the phase represented by $\psi_2(x, t)$ with probability $|b|^2$. In this description, the state of the system represented by $\psi(x, t)$ must be different from a state where a mixture of the phases $\psi_1(x, t)$ and $\psi_2(x, t)$ actually exist. For instance, let's assume that $\psi_1(x, t)$ and $\psi_2(x, t)$ are two solutions, of the set of Equations (3) for the infinite rectangular well, with kinetic energies given by K_1 and K_2, respectively. Loosely borrowing the words "superheated" and "supercooled" from possible phase transitions in liquids, one could say that the state $\psi(x,t) = a\psi_1(x,t) + b\psi_2(x,t)$ corresponds to a state of the system which is not a solution of any Equation (3). The state $\psi(x, t)$ could be formed by superheating the state $\psi_1(x, t)$ or by supercooling the state $\psi_2(x, t)$. When a set of N measurements is done on the system in the state $\psi(x, t)$, $|a|^2 N$ times a fast system's transition occurs from the superheated state to the state $\psi_1(x, t)$, and $|b|^2 N$ times the transition occurs from the supercooled state to the state $\psi_2(x, t)$. This point of view may motivate the search for the unknown equation for which $\psi(x, t)$ is a solution.

6. Quasi-Relativistic Quantum Rotor

Courses of Quantum Mechanics often include how to solve the Schrödinger equation for a quantum rigid rotator, which in general is a quantum particle moving with constant speed in a sphere. Therefore, all the kinetic energy of a quantum rigid rotator is rotational. Instances of the quantum rigid rotor appear when describing the relative movement between two particles forming a system like a diatomic molecule (neglecting vibrations) [5]. The 3D Schrödinger equation for a particle moving in a central potential $\Phi(r)$ is given by the following equation [1] [2] [3] [4] [5]:

$$i\hbar\frac{\partial}{\partial t}\psi_{Sch}(\boldsymbol{r},t) = -\frac{\hbar^2}{2m}\nabla^2\psi_{Sch}(\boldsymbol{r},t) + \Phi(r)\psi_{Sch}(\boldsymbol{r},t). \tag{28}$$

Which natural extension to quasi-relativistic speeds is the following equation:

$$i\hbar\frac{\partial}{\partial t}\psi_{Sch}(\boldsymbol{r},t)=-\frac{\hbar^2}{2m}\nabla^2\psi_{Sch}(\boldsymbol{r},t)+\Phi(r)\psi_{Sch}(\boldsymbol{r},t). \tag{29}$$

In spherical coordinates $\boldsymbol{r}=(r,\theta,\varphi)$, and the Laplacian operator in Equations (28) and (29) is defined in the following way [1] [2] [3] [4] [5]:

$$\nabla^2\psi=\frac{1}{r}\frac{\partial^2}{\partial r^2}(r\psi)+\frac{1}{r^2}\nabla^2_{\theta,\varphi}\psi. \tag{30}$$

Where:

$$\nabla^2_{\theta,\varphi}=\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}. \tag{31}$$

Using Equations (30) and (31) permits to rewrite Equation (29) in the following way:

$$i\hbar\frac{\partial}{\partial t}\psi=-\frac{\hbar^2}{(\gamma_V+1)mr}\frac{\partial^2}{\partial r^2}(r\psi)-\frac{\hbar^2}{(\gamma_V+1)mr^2}\nabla^2_{\theta,\varphi}\psi+\Phi(r)\psi. \tag{32}$$

The rotational kinetic energy of a quantum rigid rotator is given by the second term in the right size of Equation (32); therefore, the first term in the right size of Equation (32) vanishes for a quantum rigid rotator [5]. In addition, $r = r_S$ and $\Phi(r) = \Phi(r_S)$ are constants because the radius of the sphere containing the particle trajectory (r_S) is constant; therefore, choosing $\Phi(r_S) = 0$, and introducing the moment of inertia of a rotating mass $I=mr_S^2$, reduces Equation (32) to the following expression for a quasi-relativistic quantum rigid rotator:

$$i\hbar\frac{\partial}{\partial t}\psi(\theta,\varphi)=-\frac{\hbar^2}{(\gamma_V+1)I}\nabla^2_{\theta,\varphi}\psi(\theta,\varphi). \tag{33}$$

Equation (33) can be solved looking for a separable-variable solution of the following form:

$$\psi(\theta,\varphi,t)=\Omega(\theta,\varphi)\mathrm{e}^{\frac{i}{\hbar}Kt}. \tag{34}$$

Then, substituting Equation (34) in Equation (33), results in the well-known equation for the spherical harmonic functions [1] [2] [3] [4] [5]:

$$\nabla^2_{\theta,\varphi}\Omega(\theta,\varphi)+\eta\Omega(\theta,\varphi)=0. \tag{35}$$

With:

$$\eta=\frac{(\gamma_V+1)I}{\hbar^2}K. \tag{36}$$

where K is the quasi-relativistic kinetic energy of the rotor. Consequently, the normalized solution of Equation (35) satisfying the appropriated boundary conditions is given by the following expressions [1] [2] [3] [4] [5]:

$$\Omega_{l,m}(\theta,\varphi)\propto Y_l^{(m)}(\theta,\varphi);\ \eta=l(l+1);\ l=0,1,2,\cdots;\ m=-l,-l+1,\cdots,0,1,\cdots,l. \tag{37}$$

where $Y_l^{(m)}$ are the spherical harmonic functions [1] [2] [3] [4] [5]. Therefore, the quasi-relativistic kinetic energy is given by the following expression:

$$K_l = \frac{\hbar^2}{(\gamma_{V_l}+1)I} l(l+1) = \frac{h^2}{(\gamma_{V_l}+1)mL_C^2} l(l+1). \tag{38}$$

where $L_C = 2\pi r_S$ is the maximum length of a circle contained in the sphere where the particle moves. From Equation (37) follows that the spatial dependence of $\psi_{l,m}$ coincide with the spatial dependence of the wavefunction calculated using the Schrödinger equation [5]. As expected, Equation (38) gives the know values of the particle's energies at low speeds when $\gamma_V \sim 1$ [5]. From Equation (38) and the relativistic equation, $K = (\gamma_V - 1)mc^2$, follow:

$$\gamma_{V_l}^2 = 1 + \left(\frac{\lambda_C}{L_C}\right)^2 l(l+1). \tag{39}$$

Equation (39) gives $\gamma_V^2 = 3$ when $l = 1$ and $L_C = \lambda_C$; evaluating for these values Equation (38) results in $K_1 \sim 0.7mc^2$, which is smaller than the value $K_1 \sim mc^2$ that would be obtained, using the Schrödinger equation, for the state with minimum non-zero angular momentum ($l = 1$) of a quantum rotor with $L_C = \lambda_C$ [5]. Moreover, this result is precise because the calculated energy ($K_1 \sim 0.7mc^2$) is quasi-relativistic. In contrast, Equation (39) gives $\gamma_V^2 = 9$ when $l = 1$ and $L_C = \lambda_C/2$; evaluating for these values Equation (38) results in $K_1 = 2mc^2$. The number of particles may not be constant at this energy. Consequently, Equation (3) also establishes the following fundamental connection between quantum mechanics and especial theory of relativity: there is a stable orbit with minimum length that a quantum particle of mass m, moving with constant non-zero speed in a sphere, can have. This length is equal to the Compton wavelength associated to the particle's mass. Combining Equations (38) and (39) allow for rewritten Equation (38) in the following way:

$$K_l = \frac{h^2}{\left[1+\sqrt{1+\left(\frac{\lambda_C\sqrt{l(l+1)}}{L_C}\right)^2}\right]mL_C^2} l(l+1). \tag{40}$$

When $L_C \gg \sqrt{l(l+1)}\lambda_C$, Equation (40) gives the know values of the energies calculated using the Schrödinger equation for a non-relativistic quantum rotor [5]. However, in general, the values of K_l calculated using Equation (40) are smaller than the ones calculated using the Schrödinger equation. Moreover, and more significant for experiments, the differences in energies between different energy levels are slightly different when obtained using Equations (1) and (3).

7. Conclusion

Relativistic quantum mechanics has evolved a lot since 1925, when Erwin Schrödinger played with the Klein-Gordon equation but decided not to publish what he found and then, settled for publishing his finding about the today famous Schrödinger equation. Nevertheless, the existence of the quasi-relativistic Schrödinger-like equation discussed here should be considered the discovery of a hidden gem. At low particle's speeds, the proposed Grave de Peralta equation (Equation (3)) clearly coincides with the Schrödinger equation. Equation (3) is Galilean invariant for observers traveling at low speeds respect to each other, and a positive probability density can be defined for this equation by analogy of how it is defined for the Schrödinger equation. The plane wave solutions of Equation (3) are subluminal and are related through Equation (5) with the plane wave solutions with positive energies of the Klein-Gordon equation. From a practical point of view Equation (3) has a clear scholastic value. Moreover, as it was shown in this work, Equation (3) can be used for obtaining precise quasi-relativistic solutions of very interesting problems at energies smaller than mc^2.

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Annex A: Lorentz or Galilean Invariance?

It is well-known that Equation (10) is Lorentz invariant [7] [8] [9]. The 3D version of Equation (10) can be rewritten in a covariant form in the following way [7] [8] [9]:

$$p^{\mu}p_{\mu}=\frac{E^2}{c^2}-\boldsymbol{p}\cdot\boldsymbol{p}=m^2c^2,\ \ p^{\mu}=\left\{\frac{E}{c},p_x,p_y,p_z\right\}. \tag{A1}$$

Here covariance means that the module ($p^{\mu}p_{\mu}$) of the four-component vector p^{μ} is a scalar under Lorentz transformations because m^2c^2 is a scalar under the Lorentz transformations relating the coordinates $\{t, x, y, z\}$, respect to an inertial reference frame S, to the coordinates $\{t',x',y',z'\}$, respect to a second inertial reference frame S' which is moving with speed V_o respect to S in the positive direction of the axis x [7] [8]:

$$t=\gamma_{V_o}\left(t'+\frac{V_o}{c^2}x'\right),\ \ x=\gamma_{V_o}\left(x'+V_ot'\right),\ \ y=y',\ \ z=z'. \tag{A2}$$

Equation (8) permits to associate a partial derivative operator to each component of p^{μ}, thus transforming Equation (A1) in the 3D version of Equation (2) [8] [9]:

$$\begin{aligned}&\hat{p}^{\mu}\hat{p}_{\mu}\psi_{KG}\left(x,y,x,t\right)=m^2c^2\psi_{KG}\left(x,y,x,t\right),\\&\hat{p}^{\mu}=\left\{i\hbar\frac{\partial}{\partial ct},i\hbar\frac{\partial}{\partial x},i\hbar\frac{\partial}{\partial y},i\hbar\frac{\partial}{\partial z}\right\}.\end{aligned} \tag{A3}$$

The operator $\hat{p}^{\mu}$ is a Lorentz-invariant four-component vector because $\psi_{KG}(x, y, z, t)$, the wavefunction of a particle with spin-0, must be a Lorentz-scalar [8] [9], and because the four quantities formed by differentiation of a Lorentz-scalar respect to the components of a Lorentz-invariant four-vector transform as a four-component Lorentz invariant vector [7]. Therefore, the left term of Equation (A3) transform under Lorentz transformations as the module of a four-component Lorentz-invariant vector; *i.e.*, as a Lorentz-scalar. Consequently, provided that $\psi_{KG}(x, y, z, t)$ is a scalar under Lorentz transformations, Equation (A3) is relativistic covariant; *i.e.*, both sides of the equation transform under Lorentz transformations as Lorentz-scalars. Equation (A3) can then be directly written in differential form in the same way in both S and S' references frames:

$$\begin{aligned}&\frac{1}{c^2}\frac{\partial^2}{\partial t^2}\psi_{KG}\left(\boldsymbol{r},t\right)=\nabla^2\psi_{KG}\left(\boldsymbol{r},t\right)-\frac{m^2c^2}{\hbar^2}\psi_{KG}\left(\boldsymbol{r},t\right),\\&\frac{1}{c^2}\frac{\partial^2}{\partial t'^2}\psi'_{KG}\left(\boldsymbol{r}',t'\right)=\nabla'^2\psi'_{KG}\left(\boldsymbol{r}',t'\right)-\frac{m^2c^2}{\hbar^2}\psi'_{KG}\left(\boldsymbol{r}',t'\right).\end{aligned} \tag{A4}$$

where $\boldsymbol{r}=\left(x,y,z\right)$ is a 3D spatial vector is rectangular coordinates. Evidently, Equation (A4) is a 3D version of Equation (2). Therefore, a 3D version of Equation (6) is a plane wave solution of Equation (A4), which is a Lorentz-scalar because the wave's phase can be rewritten as the scalar product of two Lorentz-invariant four-components vectors [7] [8] [9]:

$$\psi_{KG}(\boldsymbol{r},t) = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{r}-Et)} = \mathrm{e}^{\frac{i}{\hbar}\left(-p^{\mu}x_{\mu}\right)}, \ x^{\mu} = \{ct, x, y, z\}. \tag{A5}$$

Therefore, $\psi'_{KG}(x',y',z',t')$ is the boosted wavefunction:

$$\psi'_{KG}(x',y',z',t') = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}'\cdot\boldsymbol{r}'-E't')}. \tag{A6}$$

In other words, $\psi'_{KG}(x',y',z',t')$ can be obtained from $\psi_{KG}(x, y, z, t)$ by making the formal substitutions $x \to x(x', t')$, $y \to y'$, $z \to z'$, and $t \to t(x', t')$ using Equation (A2). In addition, the linear momentum and total energy of the particle should be boosted by substituting $p \to p'$ and $E \to E'$. This ends the discussion about the Lorentz invariance of the Klein-Gordon equation. One can then try to demonstrate the relativistic covariance of Equation (3) following similar steps than for the demonstration of the relativistic covariance of the Klein-Gordon equation. Looking for a solution of the 3D version of Equation (3):

$$i\hbar\frac{\partial}{\partial t}\psi(\boldsymbol{r},t) = -\frac{\hbar^2}{(\gamma_V+1)m}\nabla^2\psi(\boldsymbol{r},t), \tag{A7}$$

Such that:

$$\psi(\boldsymbol{r},t) = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{r}-Kt)}, \ \psi'^{(\boldsymbol{r}',t')} = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}'\cdot\boldsymbol{r}'-K't')}, \tag{A8}$$

Would require that:

$$\psi(\boldsymbol{r},t) = \mathrm{e}^{\frac{i}{\hbar}(\boldsymbol{p}\cdot\boldsymbol{r}-Kt)} = \mathrm{e}^{\frac{i}{\hbar}\left(-\beta^{\mu}x_{\mu}\right)}, \ x^{\mu} = \{ct, x, y, z\}. \tag{A9}$$

But Equation (A9) would imply the following relation between K and p:

$$\beta^{\mu}p_{\mu} = \frac{K^2}{c^2} - \boldsymbol{p}\cdot\boldsymbol{p} = 0, \ \beta^{\mu} = \left\{\frac{K}{c}, p_x, p_y, p_z\right\}, \ K^2 = p^2c^2. \tag{A10}$$

This relation is only correct for relativistic massless particles (photons). For a particle with non-zero mass, the correct relativistic relation between K and p is given by Equation (9). This demonstrates that Equations (3) and A(7) are not Lorentz invariant. Qualitative arguments about the Galilean invariance of Equation (3) were given in Section 3; therefore, one should expect that Equation (3) is approximately Galileo invariant when the observers move respect to each other at much smaller speeds than the quasi-relativistic speed of the particle. One can then try to demonstrate the Galilean invariance of Equation (3) following similar steps than for the demonstration of the Galilean invariance of the Schrödinger equation [6]. Consequently, when $V_o \ll V$ and $K \sim mc^2$, if an observer at rest respect to S' find that:

$$\psi'(x',t') = \mathrm{e}^{\frac{i}{\hbar}(p'x'-K't')}, \tag{A11}$$

Is a solution of the equation:

$$i\hbar\frac{\partial}{\partial t'}\psi'(x',t') = -\frac{\hbar^2}{(\gamma_{V'}+1)m}\frac{\partial^2}{\partial x'^2}\psi'(x',t'). \tag{A12}$$

Then, if Equation (A12) is Galilean invariant, an observer at rest respect to S should be able to find that the following plane wave:

$$\psi(x,t)=\psi'(x',t')\mathrm{e}^{i\varepsilon(x,t)}=\mathrm{e}^{\frac{i}{\hbar}(p'x'-K't')}\mathrm{e}^{i\varepsilon(x,t)}, \tag{A13}$$

Is a solution of Equation (3) [6]. In Equation (A13), $\psi'(x',t')$ should be explicitly rewritten as $\psi'(x,t)$ by substituting the variables x', t' by the variables x, t using the Galileo transformations [6]:

$$t=t',\quad x=x'+V_o t'. \tag{A14}$$

In Equation (A13), $\varepsilon(x, t)$ have a double function. First, it should make $\psi(x, t)$ a Galilean boosted version of $\psi'(x',t')$, *i.e.*:

$$\psi(x,t)=\mathrm{e}^{\frac{i}{\hbar}\left[p'(x-V_o t)-K't\right]}\mathrm{e}^{i\varepsilon(x,t)}=\mathrm{e}^{\frac{i}{\hbar}(px-Kt)}. \tag{A15}$$

The equation in the variables x and t that results, after using Equation (A14) for transforming the differential operators of Equation (A12), do not need to be equal to Equation (3). Therefore, the second function of $\varepsilon(x, t)$ is to guarantee that $\psi(x, t)$ given by Equations (A13) and (A15) satisfies Equation (3). If there is a function $\varepsilon(x, t)$ satisfying these two requirements, then $\psi(x, t)$ and $\psi'(x',t')$ both satisfy the same equation and both have equal square module values; therefore, both described the same physical reality [6]. *i.e.*, Equation (3) would be Galilean invariant. It can be shown that for the Schrödinger equation, $\varepsilon_{Sch}(x, t)$ is given by the following equation [6]:

$$\varepsilon_{Sch}(x,t)=\frac{1}{\hbar}\left(mV_o x-\frac{1}{2}mV_o^2 t\right). \tag{A16}$$

Therefore, $\psi_{Sch}(x, t)$ is a boosted version of $\psi'_{Sch}(x',t')$ because for a non-relativistic particle:

$$\begin{aligned}
&\frac{i}{\hbar}(p'x'-K't')+\varepsilon_{Sch}(x,t)\\
&=\frac{i}{\hbar}\left[p'(x-V_o t)-\frac{p'^2}{2m}t\right]+\frac{1}{\hbar}\left(mV_o x-\frac{1}{2}mV_o^2 t\right)\\
&=\frac{i}{\hbar}\left[(p'+mV_o)x-\frac{(p'+mV_o)^2}{2m}t\right]\\
&=\frac{i}{\hbar}\left[px-\frac{p^2}{2m}t\right]=\frac{i}{\hbar}(px-Kt).
\end{aligned} \tag{A17}$$

In addition $\psi_{Sch}(x, t)$ satisfies Equation (1) [6]. What follows is the demonstration that Equation (3) is approximately Galileo invariant when $V_o \ll V$, and $K \sim mc^2$. One should find a function $\varepsilon(x, t)$ for Equation (A12) that satisfies the two requirements discussed above. First, using the Galilean relations given by Equation (A14), the differential operators in the variables x' and t' in Equation (A12) transform to the following differential operators in the variables x and t [6]:

$$\frac{\partial}{\partial t'}=\frac{\partial t}{\partial t'}\frac{\partial}{\partial t}+\frac{\partial x}{\partial t'}\frac{\partial}{\partial x}=\frac{\partial}{\partial t}+V_o\frac{\partial}{\partial x},\quad \frac{\partial t}{\partial x'}=\frac{\partial t}{\partial x'}\frac{\partial}{\partial t}+\frac{\partial x}{\partial x'}\frac{\partial}{\partial x}=\frac{\partial}{\partial x}. \tag{A18}$$

After transforming Equation (A12) using Equation (A18) and making $\gamma_{V'} \sim \gamma_V$ because $V_o \ll V$, and $K \sim mc^2$, one can propose:

$$\psi(x,t) = \psi'(x,t)\mathrm{e}^{i\varepsilon(x,t)}. \tag{A19}$$

where $\psi'(x,t)$ is $\psi'(x',t')$, *i.e.*, a solution of Equation (A12), after making the substitution $x' \to x'(x, t)$, and $t' \to t$ using Equation (A14). Then one can substitute $\psi(x, t)$ given by Equation (A19) in the equation that results after transforming Equation (A12) using Equation (A18). This permits finding out that $\varepsilon(x, t)$ must satisfy the following conditions provided $\psi(x, t)$ is a solution of Equation (3):

$$\frac{2\hbar}{(\gamma_V+1)m}\frac{\partial\varepsilon}{\partial x} - V_o = \frac{\partial^2\varepsilon}{\partial x^2} = \frac{\hbar}{(\gamma_V+1)m}\left(\frac{\partial\varepsilon}{\partial x}\right)^2 - V_o\frac{\partial\varepsilon}{\partial x} - \frac{\partial\varepsilon}{\partial t} = 0. \tag{A20}$$

These conditions are very similar to the ones corresponding to the Schrödinger equation [6]. The three conditions given by Equation (A20) determine that $\varepsilon(x, t)$ is given by the following expression:

$$\varepsilon(x,t) = \frac{1}{\hbar}\left[\frac{1}{2}(\gamma_V+1)mV_o x - \frac{1}{4}(\gamma_V+1)mV_o^2 t\right]. \tag{A21}$$

Comparing Equations (A16) and (A21), one realizes that both include a linear momentum term and a kinetic energy term. Equation (A16) includes the non-relativistic expressions $p_o = mV_o$ and $K_o = p_o^2/(2m)$. Equation (A24) includes the relativistic expressions $p_r = \frac{1}{2}(\gamma_v+1)mV_o$ and $K_r \sim p_r^2/[(\gamma_v+1)m]$. This is because the Schrödinger equation describes a non-relativistic particle but Equation (3) describes a particle moving at quasi-relativistic speeds. Finally, one should check if $\varepsilon(x, t)$ given by Equation (A21) transforms $\psi(x, t)$ in a Galilean boosted version of $\psi'(x',t')$. *i.e.,* one should check if:

$$\mathrm{e}^{\frac{i}{\hbar}\left[p'(x-V_o t) - \frac{p'^2}{(\gamma_V+1)m}t\right]}\mathrm{e}^{i\varepsilon(x,t)} \approx \mathrm{e}^{\frac{i}{\hbar}\left[px - \frac{p^2}{(\gamma_V+1)m}t\right]}. \tag{A22}$$

But:

$$\begin{aligned}&\frac{i}{\hbar}\left[p'(x-V_o t) - \frac{p'^2}{(\gamma_V+1)m}t\right] + \frac{1}{\hbar}\left[\frac{1}{2}(\gamma_V+1)mV_o x - \frac{1}{4}(\gamma_V+1)mV_o^2 t\right]\\ &= \frac{i}{\hbar}\left\{\left[p' + \frac{1}{2}(\gamma_V+1)mV_o\right]x - \frac{\left[p' + \frac{1}{2}(\gamma_V+1)mV_o\right]^2}{(\gamma_V+1)m}t\right\}.\end{aligned} \tag{A23}$$

When $V_o \ll V$, and $K \sim mc^2$, then $p' \sim (\gamma_v+1)mV \gg p_r$ and $p' + p_r \sim p' \sim p$. Therefore, Equation (A23) can be approximated to the following expression:

$$= \frac{i}{\hbar}\left[px - \frac{p^2}{(\gamma_V+1)m}t\right] = \frac{i}{\hbar}(px - Kt). \tag{A24}$$

Consequently, Equation (3) is Galilean invariant for observers that move at constant but small speed respect to each other. Nevertheless, the Grave de Peralta equation describes a massive free quantum particle moving at quasi-relativistic speeds.

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The Hubble Constant Problem and the Solution by Gravitation in Flat Space-Time

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Abstract

General Relativity implies an expanding Universe from a singularity, the so-called Big Bang. The rate of expansion is the Hubble constant. There are two major ways of measuring the expansion of the Universe: through the cosmic distance ladder and through looking at the signals originated from the beginning of the Universe. These two methods give quite different results for the Hubble constant. Hence, the Universe doesn't expand. The solution to this problem is the theory of gravitation in flat space-time where space isn't expanding. All the results of gravitation for weak fields of this theory agree with those of General Relativity to measurable accuracy whereas at the beginning of the Universe the results of both theories are quite different, *i.e.* no singularity by gravitation in flat space-time and non-expanding universe, and a Big Bang (singularity) by General Relativity.

Keywords

Gravitation in Flat Space-Time, Cosmological Models, Hubble Constant, No Big Bang, No Singularity, Non-Expanding Universe

1. Introduction

General Relativity (GR) implies an expanding universe where the expansion rate is the Hubble constant. There are two different methods to measure the Hubble constant. The results of these two methods are two different values for the Hubble constant (see e.g. [1] [2]). Hence, the assumption that the universe expands is not correct and the universe doesn't expand (see e.g. [2]). The expansion is a generally accepted assumption supported by GR. We can say that GR isn't a correct description of gravitation. There are authors who ask for new physics (see [1]). Therefore, we will use the theory of gravitation in flat space-time (GFST) instead of GR which is studied by the author in the book and in several articles (see e.g. the articles [3] [4] [5] [6]). GFST gives non-expanding space for the universe. The metric is the pseudo-Euclidean geometry and the proper time is formally similar to the metric of GR. The source of the gravitation field is the total energy-momentum tensor including that of gravitation. This is in full agreement with Einstein who stated that matter is equal to energy and reverse. GR doesn't satisfy this condition and in addition the energy-momentum of gravitation by GR is not a tensor. It is worth to mention that GFST was already studied in article [7] with application to non-singular cosmological models in [8]. Surface data show evidence for a non-expanding universe [9]. The possibility of non-expanding, cosmological models is already given in the article [10] by the use of GFST. Non-singular universes by GFST with matter creation and entropy production are also studied in [11].

2. GFST

The theory of GFST is shortly summarized. The metric is flat space-time given by

$$(\mathrm{d}s)^2 = -\eta_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{1}$$

where (η_{ij}) is a symmetric tensor. Especially, pseudo-Euclidean geometry has the form

$$(\eta_{ij}) = (1,1,1,-1). \tag{2}$$

Here, $(x^i) = (x^1, x^2, x^3)$ are the Cartesian coordinates and $x^4 = ct$. Let

$$\eta = \det(\eta_{ij}). \tag{3}$$

The gravitational field is described by a symmetric tensor (g_{ij}). Let (g^{ij}) be defined by

$$g_{ik}g^{kj} = \delta_i^j \tag{4}$$

and put similar to (3)

$$G = \det(g_{ij}). \tag{5}$$

The proper time τ is defined by

$$(c\mathrm{d}\tau)^2 = -g_{ij}\mathrm{d}x^i\mathrm{d}x^j. \tag{6}$$

The Lagrangian of the gravitational field is given by

$$L(G) = -\left(\frac{-G}{-\eta}\right)^{1/2} g_{ij}g_{kl}g^{mn}\left(g^{ik}_{/m}g^{jl}_{/n} - \frac{1}{2}g^{ij}_{/m}g^{kl}_{/n}\right) \tag{7}$$

where the bar "/" denotes the covariant derivative relative to the flat space-time metric (1). The Lagrangian of dark energy (given by the cosmological constant Λ) has the form

$$L(\Lambda) = -8\Lambda\left(\frac{-G}{-\eta}\right)^{1/2}. \tag{8}$$

Let

$$\kappa = 4\pi k / c^4 \tag{9}$$

where k is the gravitational constant. Then, the mixed energy-momentum tensor of gravitation, of dark energy and of matter of a perfect fluid is

$$T(G)^i_J = \frac{1}{8\kappa}\left[\left(\frac{-G}{-\eta}\right)^{1/2} g_{kl} g_{mn} g^{ir}\left(g^{km}_{/j} g^{\ln}_{/r} - \frac{1}{2} g^{kl}_{/j} g^{mn}_{/r}\right) + \frac{1}{2}\delta^i_j L(G)\right] \tag{10a}$$

$$T(\Lambda)^i_j = \frac{1}{16\kappa}\delta^i_j L(\Lambda) \tag{10b}$$

$$T(M)^i_j = (\rho + p) g_{jk} u^k u^i + \delta^i_j p c^2 . \tag{10c}$$

Here, ρ, p and u^i denote density, pressure and four-velocity of matter. it holds by (6)

$$c^2 = -g_{ij} u^i u^j . \tag{11}$$

Define the covariant differential operator

$$D^i_j = \left[\left(\frac{-G}{-\eta}\right)^{1/2} g^{kl} g_{jm} g^{mi}_{/l}\right]_{/k} \tag{12}$$

of order two. Then, the field equations for the gravitational potentials (g_{ij}) have the form

$$D^i_j - \frac{1}{2}\delta^i_j D^k_k = 4\kappa T^i_j \tag{13}$$

where

$$T^i_j = T(G)^i_j + T(M)^i_j + T(\Lambda)^i_j . \tag{14}$$

Define the energy-momentum tensor

$$T(M)^{ij} = g^{ik} T(M)^j_k . \tag{15}$$

Then, the equations of motion in covariant form are

$$T(M)^k_{i/k} = \frac{1}{2} g_{kl/i} T(M)^{kl} . \tag{16}$$

In addition to the field Equation (13) and the equations of motion (16) the conservation law of the total energy-momentum holds, *i.e.*

$$T^k_{i/k} = 0 . \tag{17}$$

The results of this chapter may be found in the book [12] and in the subsequently appeared articles [3] [4] [6]. In article [5] the gravitation theories of GFST and GR and their results are compared with one another. Furthermore, the redshift formula for GFST is derived.

4. GFST and the Universe

GFST is defined in flat space-time metric, e.g. in the pseudo-Euclidean geometry which is used in the following to study homogeneous, isotropic, cosmological models. The matter tensor is given by a perfect fluid with velocity equal to zero. The total matter is given by the sum of density of matter ρ_m and of radiation ρ_r with the corresponding pressure density of matter $p_m = 0$ and of radiation $p_r = \frac{1}{3}\rho_r$. It holds for homogeneous, isotropic, cosmological models

$$g_{ij} = a(t)^2 \quad (i = j = 1,2,3)$$

$$g_{ij} = -1/h(t) \quad (i = j = 4)$$

$$g_{ij} = 0 \quad (i \neq j).$$

The initial conditions at present time $t_0 = 0$ are

$$a(0) = h(0) = 1,\ \dot{a}(0) = H_0,\ \dot{h}(0) = \dot{h}_0,\ \rho_m(0) = \rho_{m0},\ \rho_r(0) = \rho_{r0}$$

where H_0 is the Hubble constant and $\dot{h}_0$ is an additional constant not appearing in GR. Relation (16) for $i = 4$ implies under the assumption that matter and radiation do not interact

$$\rho_m = \rho_{m0}/h^{1/2},\quad \rho_r = \rho_{r0}/\left(ah^{1/2}\right) \tag{18}$$

It follows by the use of the field Equation (13)

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\sqrt{h}\frac{\dot{a}}{a}\right) = 2\kappa c^4\left(\frac{1}{2}\rho_m + \frac{1}{3}\rho_r + \frac{\Lambda}{2\kappa c^4}\frac{a^3}{\sqrt{h}}\right) \tag{19a}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(a^3\sqrt{h}\frac{\dot{h}}{h}\right) = 4\kappa c^4\left(\frac{1}{2}\rho_m + \rho_r + \frac{1}{8\kappa c^4}L(G) - \frac{\Lambda}{2\kappa c^2}\frac{a^3}{\sqrt{h}}\right) \tag{19b}$$

where

$$L(G) = \frac{1}{c^2}a^3\sqrt{h}\left(-6\left(\frac{\dot{a}}{a}\right)^2 + 6\frac{\dot{a}}{a}\frac{\dot{h}}{h} + \frac{1}{2}\left(\frac{\dot{h}}{h}\right)^2\right)$$

The expression $\frac{1}{16\kappa}L(G)$ is the density of gravitation field. The conservation law of the total energy is

$$(\rho_m + \rho_r)c^2 + \frac{1}{16\kappa}L(G) + \frac{\Lambda}{2\kappa}\frac{a^3}{\sqrt{h}} = \lambda c^2 \tag{20}$$

where λ is a constant of integration. Define the quantity

$$\varphi_0 = 3H_0\left(1 + \frac{1}{6}\frac{\dot{h}}{H_0}\right).$$

The field Equation (19) imply by the use of the conservation law (20) and the initial conditions the relation

$$a^3\sqrt{h} = 2\kappa c^4\lambda t^2 + \varphi_0 t + 1. \tag{21}$$

It follows from (20) with the present time $t_0 = 0$ by the use of the initial conditions and the standard definitions of the density parameters of matter, radiation and of the energy given by the cosmological constant with the abbreviation

$$\kappa_0 \Omega_m = \Omega_r + \Omega_m + \Omega_\Lambda - 1 \tag{22}$$

the differential equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{H_0^2}{\left(2\kappa c^4 \lambda t^2 + \varphi_0 t + 1\right)^2}\left(-\Omega_m \kappa_0 + \Omega_r a^2 + \Omega_m a^3 + \Omega_\Lambda a^6\right). \tag{23a}$$

Here, Ω_r, Ω_m and Ω_Λ are the density parameters of radiation, matter and the energy given by the cosmological constant. The initial condition for the differential Equation (23a) is

$$a(0) = 1. \tag{23b}$$

Relation (20) with $t = t_0 = 0$ gives by elementary calculations

$$\frac{8\kappa c^4 \lambda}{(H_0)^2} - \left(\frac{\varphi_0}{H_0}\right)^2 = 12\Omega_m \kappa_o \tag{24}$$

The assumption

$$0 < \kappa_0 \tag{25}$$

implies that the solution of (23) is non-singular for all $t \in \mathbb{R}$. It exists $t_1 < 0 = t_0$ with $\dot{a}(t_1) = 0$, that is

$$a(t) > a(t_1) = a_1 > 0 \quad \text{for all} \quad t \neq t_1. \tag{26}$$

It follows from (23a)

$$\Omega_r a_1^2 + \Omega_m a_1^3 + \Omega_\Lambda a_1^6 = \Omega_m \kappa_0.$$

The time t_1 must be long time before the present time $t_0 = 0$ implying $0 < a_1 \ll 1$, *i.e.*

$$\kappa_0 \ll 1. \tag{27}$$

Therefore, $a(t)$ starts at a positive value at time equal to minus infinity, decreases to a_1 at $t = t_1$ and then increases for all *t*. The function $h(t)$ can then be calculated from relation (21). Let us introduce the proper time $\tilde{\tau}$ instead of the time $t$by

$$\tilde{\tau}(t) = \int_{-\infty}^{t} 1\Big/\sqrt{h(t)}\,\mathrm{d}t \tag{28}$$

The differential Equation (23a) can by the use of (21) be rewritten

$$\left(\frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tilde{\tau}}\right)^2 = H_0^2\left(-\frac{\Omega_m \kappa_0}{a^6} + \frac{\Omega_r}{a^4} + \frac{\Omega_m}{a^3} + \Omega_\Lambda\right). \tag{29}$$

This differential equation is for not too small functions $a(t)$ nearly identical with that of GR for a flat homogeneous, isotropic universe by virtue of (25) and (27).

Then, the conditions (25) and (27) give

$$0 < a_1 \ll 1, \tag{30}$$

i.e. t_1 corresponds to the time of the big bang of GR with value a_1 very small but not zero. This result is received by GFST without any additional assumption or change of the theory.

5. Conclusions

There are two methods of measuring the Hubble constant of the universe: the cosmic distance ladder and looking at the signals originated from the beginning of the universe. Two different results for the Hubble constant are received. Therefore, the universe doesn't expand because the methods use the expansion of the universe. It is worth to mention that GR implies expansion because the universe starts from a point singularity and the observed universe is very big. Furthermore, the universe must be inflationary expanding because the observed universe is flat. Summarizing, it follows that GR doesn't correctly describe gravitation if two Hubble constants are measured.

A theory of gravitation in pseudo-Euclidean geometry has been given in article [12]. Later on, it is studied more generally in flat space-time. The applications of this theory to homogeneous, isotropic, cosmological models are given in article [8] where non-singular solutions are received, *i.e.* big bang did not exist. It was proved that for weak gravitational fields the results of GFST and GR agree to measurable accuracy. The theory and the applications of GFST is studied in several articles and summarized in the book [12]. Differences of the results of GFST and GR arise for cosmological models in the beginning of the universe. The metric of GFST is the pseudo-Euclidean geometry, *i.e.* space is not expanding. It is worth to mention that by virtue of the covariance of GFST an expansion of the universe would also be possible by a suitable transformation. But this is not realistic. A non-expanding universe is important because expansion of the universe implies two different Hubble constants. For cosmological models of GFST the source is the total energy-momentum tensor inclusive that of the gravitational field (as it should be by Einstein: matter is equal to energy and reverse) whereas the source is only the matter tensor and no gravitational energy-momentum for cosmological models of GR which is no tensor for GR. The redshift of distant objects follows by the energy of time-dependent gravitational fields which is converted to matter where the total energy is conserved and it doesn´t follow from velocities (expanding space).

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Linearized Equations of General Relativity and the Problem of Reduction to the Newton Theory

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Abstract

The paper is concerned with the problem of reduction of the general relativity theory to the Newton gravitation theory for a gravitation field with relatively low intensity. This problem is traditionally solved on the basis of linearized equations of general relativity which, being matched to the Newton theory equations, allow us to link the classical gravitation constant with the constant entering the general relativity equations. Analysis of the linearized general relativity equations shows that it can be done only for empty space in which the energy tensor is zero. In solids, the set of linearized general relativity equations is not consistent and is not reduced to the Newton theory equations. Specific features of the problem are demonstrated with the spherically symmetric static problem of general relativity which has the closed-form solution.

Keywords

General Relativity, Gravitation Constant, Linearized Equations, Spherically Symmetric Problem

1. Introduction. General Relativity Equations

The basic equation of general relativity which specifies the Einstein tensor has the following form:

$$E_i^j = R_i^j - \frac{1}{2}\delta_i^j R \tag{1}$$

in which R_i^j ($R = R_i^i$) are the components of the Ricci curvature tensor (we use mixed components because for the spherically symmetric problem considered further they coincide with the physical components). The Einstein tensor is associated with the energy tensor as

$$E_i^j = \chi T_i^j \tag{2}$$

where χ is the relativity gravitational constant. The energy tensor expressed with the aid of Equations (1) and (2) identically satisfies the conservation equation

$$\nabla_k T_i^k = 0 \tag{3}$$

For the static problem,

$$T_i^j = \sigma_i^j, \quad T_i^4 = 0\left(i, j = 1,2,3\right), \quad T_4^4 = \mu c^2 \tag{4}$$

where σ_i^j is the stress tensor and μ is the density.

For gravitation with relatively low intensity, the general relativity must reduce to the Newton theory in which the gravitation potential ψ satisfies the Poisson equation

$$\Delta \psi = 4\pi G \mu \tag{5}$$

in which G is the classical gravitation constant. Traditionally, the linearized version of Equation (1) is obtained and matched to Equation (5). The result is

$$\chi = \chi_0 = 8\pi G/c^4 \tag{6}$$

2. Linearized Equations and the Reduction Problem

Consider the space referred to Cartesian coordinates $x^i \left(i = 1,2,3\right)$ and present the line element of the Riemannian space as

$$\mathrm{d}s^2 = g_{ij}\mathrm{d}x^i\mathrm{d}x^j - g_{44}c^2\mathrm{d}t^2$$

Assume that the components of the metric tensor have the following forms: $g_{ij} = \delta_{ij} + f_{ij}$ and $g_{44} = 1 + f_{44}$ in which the amplitude values of functions f are much smaller than unity. Undertaking linearization of Equation (1) and taking into account Equations (2) and (4), we arrive at the following set of the linearized general relativity equations:

$$\chi\sigma_1^1 = \frac{1}{2}\left(-f_{22,33} - f_{33,22} + 2f_{23,23} - f_{44,22} - f_{44,33}\right) \tag{7}$$

$$\chi\sigma_1^2 = \frac{1}{2}\left(f_{33,12} + f_{12,33} - f_{13,23} - f_{23,13} + f_{44,12}\right) \tag{8}$$

$$\chi\sigma_1^3 = \frac{1}{2}\left(f_{22,13} - f_{12,23} + f_{13,22} - f_{23,12} + f_{44,13}\right) \tag{9}$$

$$\chi\sigma_2^2 = \frac{1}{2}\left(-f_{11,33} - f_{33,11} + 2f_{13,13} - f_{44,11} - f_{44,33}\right) \tag{10}$$

$$\chi\sigma_2^3 = \frac{1}{2}\left(f_{11,23} - f_{12,13} - f_{13,12} + f_{23,11} + f_{44,23}\right) \tag{11}$$

$$\chi\sigma_3^3 = \frac{1}{2}\left(-f_{11,22} - f_{22,11} + 2f_{12,12} - f_{44,11} - f_{44,22}\right) \tag{12}$$

$$\chi\mu c^2 = -\frac{1}{2}\left(f_{11,33} + f_{11,22} + f_{22,11} + f_{22,33} + f_{33,11} + f_{33,22} - 2f_{12,12} - 2f_{13,13} - 2f_{23,23}\right) \tag{13}$$

Notation $\left(\cdots\right)_{,i}$ means the derivative with respect to x^i. The linearized form of the conservation equation, Equation (3), with regard to Equations (4) is

$$\sigma_{1,1}^{1}+\sigma_{1,2}^{2}+\sigma_{1,3}^{3}-\frac{1}{2}f_{44,1}\left(\mu c^{2}-\sigma_{1}^{1}\right)+\frac{1}{2}f_{44,2}\sigma_{1}^{2}+\frac{1}{2}f_{44,3}\sigma_{1}^{3}=0 \quad (1,2,3) \qquad (14)$$

Here, (1, 2, 3) means permutation which allows us to obtain two more equations from the written one (only three Equations (14) exist for a static problem). For linear approximation, we can neglect the nonlinear terms with stresses in Equation (14) and simplify it as

$$\sigma_{1,1}^{1}+\sigma_{1,2}^{2}+\sigma_{1,3}^{3}-\frac{1}{2}f_{44,1}\mu c^{2}=0 \quad (1,2,3) \qquad (15)$$

This equation has a simple physical meaning—it is the equilibrium equation for a solid element loaded with gravitation forces.

Using the traditional way to derive Equation (6) for the gravitation constant, express the derivatives $f_{23,23}, f_{13,13}, f_{12,12}$ from Equations (7), (10), (12) and substitute them in Equation (13). The resulting equation is

$$\Delta f_{44}=\chi\left(\mu c^{2}-\sigma\right) \qquad (16)$$

in which $\sigma=\sigma_{1}^{1}+\sigma_{2}^{2}+\sigma_{3}^{3}$ is the invariant of the stress tensor. For the linear approximation, we neglect σ in comparison with μc^{2} and finally get

$$\Delta f_{44}=\chi\mu c^{2} \qquad (17)$$

Matching Equations (5) and (17), we can conclude that $f_{44}=2\psi/c^{2}$ and $\chi=\chi_{0}$, where χ_{0} is specified by Equation (6). Thus, it looks like the general relativity reduces in the linear approximation to the Newton gravitation theory.

However, more careful analysis shows that the foregoing derivation is not correct for solids. The problem is in Equations (7)-(13) which are, in general, not compatible. As can be directly checked, the left-hand parts of these equations must satisfy the following relationship:

$$\sigma_{1,1}^{1}+\sigma_{1,2}^{2}+\sigma_{1,3}^{3}=0 \quad (1,\ 2,\ 3) \qquad (18)$$

Consider first the empty space for which $\sigma_{i}^{j}=0$ and $\mu=0$. In this case, Equation (18) coincides with Equation (15) and the set of Equations (7)-(13) is compatible. Equation (17) is homogeneous and coincides with Equation (5) which is also homogeneous. Thus, for the empty space, the general relativity theory reduces to the Newton gravitation theory for gravitation with relatively low intensity. However, the situation is different in case of solids for which the equilibrium equation, Equation (15), must be satisfied. As follows from Equations (15) and (18), the equilibrium equations are satisfied only if $f_{44}=0$ which means the absence of gravitation. Thus, the linearized equations of general relativity do not describe gravitation and the linearized general relativity does not reduce to the Newton theory for solids. The reason is evident—whereas the terms with the stresses in Equation (15) are linear and refer to the first-order approximation, the last term with f_{44} belongs to the second-order approximation. Naturally, this does not mean that the last term can be neglected—in this case, the gravitation disappears. This situation is not unique in mechanics of solids. For example, to construct the two-dimensional theory of thin plates from three-dimensional equations of theory of elasticity by the asymptotic method, we need to retain small terms of the first and of the second orders. Neglecting the second-order terms, we arrive at the plate theory which is not physically consistent [1]. Consider further the spherically symmetric problem for which closed-form solutions can be obtained.

3. Spherically Symmetric Problem

3.1. Classical Linear Solution

For comparison with the general relativity solutions that are discussed further, consider the problem of the theory of elasticity for a linear elastic isotropic solid sphere with radius R and constant density μ loaded with gravitation forces following from the Newton theory. The gravitational potential ψ is the solution of the Poisson equation

$$\Delta\psi = \psi'' + \frac{2}{r}\psi' = 4\pi G\mu \tag{19}$$

Here, $(\cdots)' = \mathrm{d}(\cdots)/\mathrm{d}r$ and r is the radial coordinate. For the external space ($r \ge R$, index "e"), $\mu = 0$ and the solution of Equation (19) is $\psi_e = -Gm/r$ in which m is the sphere mass. Introduce the so-called gravitational radius

$$r_g = \frac{2Gm}{c^2} \tag{20}$$

Then, $\psi_e = -r_g c^2/2r$. For the internal space ($0 \le r \le R$, index "i"), the regular solution of Equation (19) is

$$\psi_i = \frac{2}{3}\pi G\mu r^2 + C$$

Determining constant C from the boundary condition $\psi_i(R) = \psi_e(R)$ and using Equation (20), we get

$$\psi_i = -\frac{2}{3}\pi G\mu\left(R^2 - r^2\right) - \frac{r_g c^2}{2R}$$

The equilibrium equation for the sphere under the action of gravitational body forces $f_g = -\mu\psi_i'$ is

$$\sigma_r' + \frac{2}{r}(\sigma_r - \sigma_\theta) - \frac{4}{3}\pi G\mu^2 r = 0 \tag{21}$$

where σ_r and σ_θ are the radial and the circumferential stresses. Consider the case of the perfect fluid for which $\sigma_r = \sigma_\theta = -p$. The pressure p can be found from Equation (21) which takes the form

$$p' + \frac{4}{3}\pi G\mu^2 r = 0 \tag{22}$$

The solution of this equation that satisfies the boundary condition $p(r = R) = 0$ is

$$p = \frac{2}{3}\pi G\mu^2\left(R^2 - r^2\right) \tag{23}$$

In general relativity, the space geometry is Riemannian and the line element in spherical coordinates r,θ,φ is

$$ds^2 = g_{11}dr^2 + g_{22}\left(d\theta^2 + \sin^2\theta d\varphi^2\right) - g_{44}c^2dt^2 \tag{24}$$

The components of the metric tensor depend on the radial coordinate only. For the foregoing linear solution, the general relativity interpretation of the obtained results is [2]

$$g_{11} = 1 + \frac{r_g}{r}, \quad g_{22} = r^2, \quad g_{44} = 1 - \frac{r_g}{r} \tag{25}$$

In case $r_g = 0$, the space is Euclidean and gravitation vanishes.

3.2. General Relativity Equations

For a spherically symmetric problem, the field equations following from Equation (1) reduce to [3]

$$E_1^1 = \frac{1}{g_{22}} - \frac{1}{g_{11}}\left[\frac{1}{4}\left(\frac{g'_{22}}{g_{22}}\right)^2 + \frac{g'_{22}g'_{44}}{2g_{22}g_{44}}\right] \tag{26}$$

$$E_2^2 = -\frac{1}{2g_{11}}\left[\frac{g''_{44}}{g_{44}} - \frac{1}{2}\left(\frac{g'_{44}}{g_{44}}\right)^2 + \frac{g''_{22}}{g_{22}} - \frac{1}{2}\left(\frac{g'_{22}}{g_{22}}\right)^2 + \frac{g'_{22}}{2g_{22}}\left(\frac{g'_{44}}{g_{44}} - \frac{g'_{11}}{g_{11}}\right) - \frac{g'_{11}g'_{44}}{2g_{11}g_{44}}\right] \tag{27}$$

$$E_4^4 = \frac{1}{g_{22}} - \frac{1}{g_{11}}\left[\frac{g''_{22}}{g_{22}} - \frac{1}{4}\left(\frac{g'_{22}}{g_{22}}\right)^2 - \frac{g'_{11}g'_{22}}{2g_{11}g_{22}}\right] \tag{28}$$

in which in accordance with Equations (2) and (4)

$$E_1^1 = \chi T_1^1 = \chi\sigma_r, \quad E_2^2 = \chi T_2^2 = \chi\sigma_\theta, \quad E_4^4 = \chi T_4^4 = \chi\mu c^2 \tag{29}$$

The only one conservation equation, Equation (3), becomes

$$\left(T_1^1\right)' + \frac{g'_{22}}{g_{22}}\left(T_1^1 - T_2^2\right) + \frac{g'_{44}}{2g_{44}}\left(T_1^1 - T_4^4\right) = 0 \tag{30}$$

The solution of the external ($r \geq R$) problem must satisfy the asymptotic conditions and to reduce to Equations (25) for $r \to \infty$. The solution for the internal ($0 \leq r \leq R$) problem must satisfy the symmetry condition at the sphere center according to which $g_{11}(0) = 1$, $g_{22}(0) = 0$. Both solutions must satisfy the boundary conditions on the sphere surface, *i.e.*

$$g_{11}^e(R) = g_{11}^i(R), \quad g_{22}^e(R) = g_{22}^i(R), \quad g_{44}^e(R) = g_{44}^i(R) \tag{31}$$

Substitution of the obtained equations for T_i^i in Equation (30) identically satisfies this equation. So, only three of four Equations (26)-(28) and (30) are mutually independent. Traditionally [3], the simplest set of equations including Equations (26), (28) and (30) is used. The obtained solution identically satisfies Equation (27). To solve the problem, we should supplement Equations (26), (28) and (30) which include three components of the metric tensor g_{11}, g_{22}, g_{44} and two stresses σ_r, σ_θ with one coordinate condition for the metric tensor [4] and one equation for the stresses [5]. For the case of perfect fluid which is considered further, $\sigma_r = \sigma_\theta = -p$ and we have four unknown functions and need only one coordinate condition for the metric tensor.

3.3. Linearized Solution

Decompose the components of the metric tensor in Equation (24) as $g_{11} = 1 + f_1$, $g_{22} = r^2(1 + f_2)$, $g_{44} = 1 + f_4$ and assume that the absolute values of functions $f(r)$ are much less than unity. Undertaking linearization of Equations (26)-(28), we arrive at

$$\left(E_1^1\right)_1 = \frac{1}{r^2}\left(f - rf_4'\right), \quad \left(E_2^2\right)_1 = \frac{1}{2r}\left(f - rf_4'\right)', \quad \left(E_4^4\right)_1 = \frac{1}{r^2}(rf)' \tag{32}$$

where $f = f_1 - (rf_2)'$. Using Equations (29), we can present these equations as

$$\chi\sigma_r = \frac{1}{r^2}\left(f - rf_4'\right), \quad \chi\sigma_\theta = \frac{1}{2r}\left(f - rf_4'\right)', \quad \chi\mu c^2 = \frac{1}{r^2}(rf)' \qquad 33)$$

The conservation equation, Equation (30), transformed with aid of Equation (29) becomes

$$\sigma_r' + \frac{2}{r}\left(\sigma_1 - \sigma_2\right) + \frac{1}{2} f_4'\left(\sigma_r - \mu c^2\right) = 0 \tag{34}$$

Neglecting σ_r in comparison with μc^2 in the last term of this equation, we can conclude that it is analogous to the equilibrium equation, Equation (21).

Proceeding, express f from the first equation of Equation (33), f' from the second of these equations, *i.e.*,

$$f = r^2\chi\sigma_r + rf_4', \quad f' = 2r\chi\sigma_\theta + f_4' + rf_4''$$

and substitute in the third equation to get

$$f_4'' + \frac{2}{r} f_4' = \chi\left(\mu c^2 - \sigma\right)$$

in which $\sigma = \sigma_r + 2\sigma_\theta$ is the invariant of the stress tensor. Neglecting σ in comparison with μc^2, we arrive at

$$\Delta f_4 = \chi\mu c^2 \tag{35}$$

Formally, taking $f_4 = 2\psi/c^2$ and $\chi = 8\pi G/c^4$, we can reduce this equation to the Newton theory equation, Equation (19). However, as in Section 2, the foregoing derivation is not correct. For the external space $\sigma_r = \sigma_\theta = 0$ and $\mu = 0$. Both Equations (19) and (35) are homogeneous and the linearized general relativity reduces to the Newton theory. Taking $\sigma_r = \sigma_\theta = 0$ in Equations (33), we can conclude that the second equation is a derivative of the first one. Thus we have only two independent Equations (33) for three functions $f_{1,2,4}$ and need a coordinate condition to find these functions. For the internal space, the first two Equations (33) have different left-hand parts and include one unknown function $F = f - rf_4'$ in the right-hand parts. This means that, in general, these equations are not compatible. To derive the compatibility condition, express F from the first equation and substitute in the second one. The resulting equation is

$$\sigma_r' + \frac{2}{r}\left(\sigma_r - \sigma_\theta\right) = 0$$

This equation coincides with the equilibrium equation, Equation (21) only in the absence of gravitation which is, naturally, not the case. Thus, the linearized equations of general relativity are not compatible for a solid sphere and to arrive at the consistent theory, we need to construct the second-order approximation.

3.4. Second-Order Asymptotic Approximation

Present the components of the metric tensor entering Equation (24) as

$$g_{11} = 1 + \varepsilon f_1 + \varepsilon^2 \varphi_1, \quad g_{22} = r^2\left(1 + \varepsilon f_2 + \varepsilon^2 \varphi_2\right), \quad g_{44} = 1 + \varepsilon f_4 + \varepsilon^2 \varphi_4 \tag{36}$$

in which ε is a small parameter. Substitution in Equation (1) yields

$$E_i^i = \varepsilon\left(E_i^i\right)_1 + \varepsilon^2\left(E_i^i\right)_2 \tag{37}$$

Here, the components $\left(E_i^i\right)_1$ are specified by Equations (32) and

$$\begin{aligned}
\left(E_1^1\right)_2 &= \frac{1}{r^2}\left(\varphi_1 - \varphi_2 + f_2^2 - f_1^2\right) \\
&\quad + \frac{1}{r}\left[-\varphi_2' - \varphi_4' + \left(f_1 + f_4\right) f_4' + \left(f_1 + f_2\right) f_2'\right] \\
&\quad + \frac{f_2'}{4}\left(f_2' + 2 f_4'\right)
\end{aligned}$$

$$\begin{aligned}
\left(E_2^2\right)_2 &= \frac{1}{2r}\left[\varphi_1' - 2\varphi_2' - \varphi_4' + \left(f_1 + f_4\right) f_4' + 2\left(f_1 + f_2\right) f_2' - 2 f_1 f_1'\right] \\
&\quad + \frac{1}{4}\Big[-2\left(\varphi_2'' + \varphi_4''\right) + \left(f_2'\right)^2 + \left(f_4'\right)^2 + 2\left(f_1 f_2'' + f_1 f_4'' + f_2 f_2'' + f_4 f_4''\right) \\
&\quad + f_1' f_2' + f_1' f_4' - f_2' f_4'\Big]
\end{aligned} \tag{38}$$

$$\begin{aligned}
\left(E_4^4\right)_2 &= \frac{1}{r^2}\left(\varphi_1 - \varphi_2 - f_1^2 + f_2^2\right) + \frac{1}{r}\left(\varphi_1' - 3\varphi_2' - 2 f_1 f_1' + 3 f_1 f_2' + 3 f_2 f_2'\right) \\
&\quad + \frac{1}{4}\left[-4\varphi_2'' + 2 f_1' f_2' + \left(f_2'\right)^2 + 4\left(f_1 + f_2\right) f_2''\right]
\end{aligned}$$

Recall that the Einstein tensor E_i^i must satisfy the conservation equation, Equation (30), which can be written as

$$\left(E_1^1\right)' + \frac{g_{22}'}{g_{22}}\left(E_1^1 - E_2^2\right) + \frac{g_{44}'}{2g_{44}}\left(E_1^1 - E_4^4\right) = 0$$

Substituting expressions (36) for the metric tensor, we arrive at the following two equations corresponding to ε and ε^2:

$$\left(E_1^1\right)_1' + \frac{2}{r}\left[\left(E_1^1\right)_1 - \left(E_2^2\right)_1\right] = 0 \tag{39}$$

$$\left(E_1^1\right)_2' + \frac{2}{r}\left[\left(E_1^1\right)_2 - \left(E_2^2\right)_2\right] + f_2'\left[\left(E_1^1\right)_1 - \left(E_2^2\right)_1\right] + \frac{1}{2} f_4'\left[\left(E_1^1\right)_1 - \left(E_4^4\right)_1\right] = 0 \tag{40}$$

Substituting Equations (32) and (38), we can conclude that both equations, Equations (39) and (40), are satisfied identically. Thus, the second approximation, in contrast to the first one discussed in Section (2.3), is consistent. This means that only two of three Equations (38) are mutually independent and we can proceed using only two equations, namely those for E_1^1 and E_4^4. However, these two equations include three unknown functions with indices 1, 2 and 4. To solve the problem, we need to supplement the aforementioned two equations with a coordinate condition. According to the condition used further, the so-called space density $d=\sqrt{g_R/g_E}$ in which g_R and g_E are the determinants of the metric tensor in Riemannian and Euclidean three-dimensional spaces is minimized [5]. For the spherically symmetric problem, the introduced coordinate condition has the form [5]

$$\sqrt{g_{11}}g_{22}=r^2 \tag{41}$$

For the internal space, this condition has a simple physical meaning – gravitation transforming the Euclidean geometry into Riemannian does not affect the volume element and the sphere mass, *i.e.*,

$$m=4\pi\mu\int_0^R\sqrt{g_{11}}g_{22}\mathrm{d}r=4\pi\mu\int_0^R r^2\mathrm{d}r=\frac{4}{3}\pi\mu R^3 \tag{42}$$

So, the sphere mass is the same that in Euclidean space [6].

Using Equations (36), we can write Equation (41) as

$$\varepsilon\left(\frac{f_1}{2}+f_2\right)+\varepsilon^2\left(\frac{\varphi_1}{2}+\varphi_2-\frac{f_1^2}{8}+\frac{f_1f_2}{2}\right)=0$$

and get two coordinate conditions for the first and the second approximations, *i.e.*,

$$f_1=-2f_2,\quad \varphi_1=3f_2^2-2\varphi_2 \tag{43}$$

Taking into account these results, we can transform Equations (32) and (38) for E_1^1 and E_4^4 to

$$\left(E_1^1\right)_1=-\frac{1}{r^2}\left(3f_2+rf_2'+rf_4'\right) \tag{44}$$

$$\left(E_4^4\right)_1=-\frac{1}{r^2}\left[r\left(3f_2+rf_2'\right)\right]' \tag{45}$$

$$\left(E_1^1\right)_2=-\frac{3\varphi_2}{r^2}-\frac{1}{r}\left(\varphi_2'+\varphi_4'+f_2f_2'+2f_2f_4'-f_4f_4'\right)-\frac{1}{4}\left(f_2'+2f_4'\right)f_2' \tag{46}$$

$$\left(E_4^4\right)_2=-\left[\varphi_2''+\frac{3}{r^2}\varphi_2+\frac{5}{r}\left(\varphi_2'+f_2f_2'\right)+\frac{3}{4}\left(f_2'\right)^2+f_2f_2''\right] \tag{47}$$

Consider the external space ($R\le r<\infty$) for which $\left(E_i^i\right)_1=\left(E_i^i\right)_2=0$. Integration of Equation (45) yields

$$r\left(f_2^e\right)'+3f_2^e=-\frac{C_1}{r}$$

Further integration and the first equation of Equations (43) give

$$f_2^e=-\frac{C_1}{2r}+\frac{C_2}{r^3},\quad f_1^e=\frac{C_1}{r}-\frac{2C_2}{r^3} \tag{48}$$

Applying Equation (44) and using Equations (48), we get

$$r\left(f_4^e\right)' = -3f_2^e - r\left(f_2^e\right)' = \frac{C_1}{r}$$

Thus,

$$f_4^e = -\frac{C_1}{r} \tag{49}$$

Transforming Equation (47) with the aid of Equations (48), we have

$$r^2\left(\varphi_2^e\right)'' + 5r\left(\varphi_2^e\right)' + 3\varphi_2^e = \frac{9C_1^2}{16r^2} - \frac{15C_2^2}{4r^6} - \frac{3C_1C_2}{4r^4}$$

This is the Euler equation whose solution is

$$\varphi_2^e = \frac{C_3}{r} + \frac{C_4}{r^3} - \frac{9C_1^2}{16r^2} - \frac{C_2^2}{4r^6} - \frac{C_1C_2}{4r^4} \tag{50}$$

From the second equation of Equations (43) it follows that

$$\varphi_1^e = -\frac{2C_3}{r} - \frac{2C_4}{r^3} + \frac{15C_1^2}{8r^2} + \frac{7C_2^2}{2r^6} - \frac{5C_1C_2}{2r^4} \tag{51}$$

Using Equation (46) and taking into account Equations (48)-(50), we get

$$\left(\varphi_4^e\right)' = -\frac{2C_3}{r^2} - \frac{C_1C_2}{r^5}$$

Integration yields

$$\varphi_4^e = \frac{2C_3}{r} + \frac{C_1C_2}{4r^4} + C_5 \tag{52}$$

Substituting Equations (48)-(52) in Equations (36), we arrive at the following expressions for the components of the metric tensor in the external space:

$$g_{11}^e = 1 + \varepsilon_e\left(\frac{C_1}{r} - \frac{2C_2}{r^3}\right) + \varepsilon_e^2\left(-\frac{2C_3}{r} - \frac{2C_4}{r^3} + \frac{15C_1^2}{8r^2} + \frac{7C_2^2}{2r^6} - \frac{5C_1C_2}{2r^4}\right)$$

$$g_{22}^e = r^2\left[1 + \varepsilon_e\left(-\frac{C_1}{2r} + \frac{C_2}{r^3}\right) + \varepsilon_e^2\left(\frac{C_3}{r} + \frac{C_4}{r^3} - \frac{9C_1^2}{16r^2} - \frac{C_2^2}{4r^6} - \frac{C_1C_2}{4r^4}\right)\right]$$

$$g_{44}^e = 1 - \varepsilon_e\frac{C_1}{r} + \varepsilon_e^2\left(\frac{2C_3}{r} + \frac{C_1C_2}{4r^4} + C_5\right)$$

For $r \to \infty$, the asymptotic behavior of the metric tensor is specified by Equations (25). Matching the obtained solution to these equations, we can conclude that $C_1 = 1, C_3 = C_5 = 0$ and $\varepsilon_e = r_g$. Thus, for the external space, parameter ε_e is equal to the gravitational radius in Equation (20) and the solution becomes

$$g_{11}^e = 1 + r_g\left(\frac{1}{r} - \frac{2C_2}{r^3}\right) + r_g^2\left(-\frac{2C_4}{r^3} + \frac{15}{8r^2} + \frac{7C_2^2}{2r^6} - \frac{5C_2}{2r^4}\right) \tag{53}$$

$$g_{22}^e = r^2\left[1 + r_g\left(-\frac{1}{2r} + \frac{C_2}{r^3}\right) + r_g^2\left(\frac{C_4}{r^3} - \frac{9}{16r^2} - \frac{C_2^2}{4r^6} - \frac{C_2}{4r^4}\right)\right] \tag{54}$$

$$g_{44}^{e}=1-\frac{r_g}{r}+r_g^2\frac{C_2}{4r^4} \tag{55}$$

The integration constants that enter this solution should be found from the boundary conditions on the sphere surface. To formulate the boundary conditions, we need to consider the internal ($0\le r\le R$) problem. Assume that the sphere consists of the perfect fluid for which $\sigma_r=\sigma_\theta=-p$. Thus, $E_1^1=E_2^2=-\chi p$ and $E_4^4=\chi\mu c^2$. In addition to Equations (36) and (37), decompose the pressure as

$$p=\varepsilon_i p_1+\varepsilon_i^2 p_2 \tag{56}$$

Then, we should take in Equations (37)

$$\left(E_1^1\right)_1=\left(E_2^2\right)_1=-\chi p_1,\quad \left(E_1^1\right)_2=\left(E_2^2\right)_2=-\chi p_2 \tag{57}$$

Substituting the first of these equations in Equation (39), we get $p_1'=0$. Taking into account the boundary condition for the pressure $p_1(r=R)=0$, we can conclude that $p_1=0$. This result explains the problem discussed in Section 2.3 in connection with the linearized solution. The pressure corresponding to this solution is zero which means that the linearized solution does not describe gravitation in solids.

To proceed, specify the parameter ε_i for the internal problem. Assume that $\varepsilon_i=\chi\mu c^2$. Then, the corresponding equation in Equations (37) yields

$$\left(E_4^4\right)_1=1,\quad \left(E_4^4\right)_2=0 \tag{58}$$

Then, Equation (45) becomes

$$\left\{r\left[3f_2^i+r\left(f_2^i\right)'\right]\right\}'=-r^2$$

The solution of this equation is

$$f_2^i=-\frac{r^2}{15}+\frac{B_1}{r}+\frac{B_2}{r^3}$$

This solution is regular at the sphere center $r=0$ if $B_1=B_2=0$. Using the first equation of Equations (43), we get

$$f_2^i=-\frac{r^2}{15},\quad f_1^i=\frac{2r^2}{15} \tag{59}$$

Taking $p_1=0$ in Equations (57) and applying Equation (44), we arrive at

$$3f_2^i+r\left(f_2^i\right)'+r\left(f_4^i\right)'=0$$

Substituting the first equation of Equations (59) and integrating, we have

$$f_4^i=\frac{1}{6}\left(r^2+B_3\right) \tag{60}$$

Now, we can find the pressure in the fluid. Recall that $p_1=0$, so that

$$p=\varepsilon_i^2 p_2=\chi^2\mu^2c^4 p_2 \tag{61}$$

Here, p_2 can be determined from Equation (40) which can be transformed with the aid of Equations (57), (58) and (60) to

$$\chi p_2' + \frac{r}{6} = 0$$

The solution of this equation which satisfies the boundary condition $p_2(r = R) = 0$ is

$$p_2 = \frac{1}{12\chi}\left(R^2 - r^2\right)$$

Substituting this result in Equation (61), we finally get

$$p = \frac{\chi\mu^2 c^4}{12}\left(R^2 - r^2\right) \tag{62}$$

This solution must coincide with Equation (23) corresponding to the Newton theory. Matching Equations (23) and (62), we can conclude that $\chi = \chi_0$, where χ_0 is given by Equation (6). Thus, Equation (6) is proved for the second-order asymptotic solution for the spherically symmetric problem. Using the obtained result and applying Equations (19) and (42) for r_g and *m*, we can obtain the final form for parameter ε_i, *i.e.*,

$$\varepsilon_i = \chi\mu c^2 = \frac{3r_g}{R^3} \tag{63}$$

Thus, for the internal problem, the parameter is also expressed in terms of the gravitational radius.

Determine the metric tensor components for the second-order approximation. Consider Equation (47) whose left-hand part is zero because of Equations (58). Substituting the first equation of Equations (59), we obtain the following equation:

$$\left(\varphi_2^i\right)'' + \frac{5}{r}\left(\varphi_2^i\right)' + \frac{3}{r^2}\varphi_2^i = -\frac{r^2}{15}$$

The solution of this equation is

$$\varphi_2^i = -\frac{r^4}{525} + \frac{B_4}{r} + \frac{B_5}{r^3}$$

Using the regularity condition and the second equation in Equations (43), we get $B_4 = B_5 = 0$ and

$$\varphi_2^i = -\frac{r^4}{525}, \quad \varphi_1^i = \frac{3r^4}{175} \tag{64}$$

The function $\varphi_4^i(r)$ can be found from Equation (46) in which $\left(E_1^1\right)_2 = -\chi p_2$. Using Equations (59), (60), (62) and (64) and integrating, we find

$$\varphi_4^i = \frac{1}{60}\left[r^2\left(\frac{r^2}{4} + \frac{5}{2}R^2 + \frac{5}{3}B_3\right) + B_6\right] \tag{65}$$

Finally substituting Equations (59), (60), (64) and (65) in Equations (36) and using Equation (63) for ε_i, we arrive at the following expressions for the metric tensor components of the internal space:

$$g_{11}^i = 1 + \frac{2r_g r^2}{5R^3} + \frac{27r_g^2 r^4}{175R^6} \tag{66}$$

$$g_{22}^{i} = r^2\left(1 - \frac{r_g r^2}{5R^3} - \frac{3r_g^2 r^4}{175R^6}\right) \tag{67}$$

$$g_{44}^{i} = 1 + \frac{r_g}{2R^3}\left(r^2 + B_3\right) + \frac{3r_g^2}{20R^6}\left[r^2\left(\frac{r^2}{4} + \frac{5}{2}R^2 + \frac{5}{3}B_3\right) + B_6\right] \tag{68}$$

Determine the integration constants using the boundary conditions in Equations (31). Equating the terms with r_g in Equations (53) and (66) and doing the same for the terms including r_g^2, we get

$$C_2 = \frac{3}{10}R^2,\quad B_3 = -3R^2,\quad C_4 = \frac{9R}{14},\quad B_6 = \frac{11}{4}R^4$$

Thus, the boundary conditions (31) for g_{11} and g_{44} are satisfied. The boundary condition for g_{22} is satisfied because of Equation (41). Finally, we arrive at the following expressions for the components of the metric tensor in the external and internal spaces:

$$g_{11}^{e} = 1 + \overline{r}_g\left(\frac{1}{\overline{r}} + \frac{3}{5\overline{r}^3}\right) + \overline{r}_g^2\left(\frac{15}{8\overline{r}^2} - \frac{9}{7\overline{r}^3} - \frac{3}{8\overline{r}^4} + \frac{63}{200\overline{r}^6}\right)$$

$$g_{22}^{e} = r^2\left[1 + \overline{r}_g\left(-\frac{1}{2\overline{r}} + \frac{3}{10\overline{r}^3}\right) + 9\overline{r}_g^2\left(-\frac{1}{16\overline{r}^2} + \frac{1}{14\overline{r}^3} - \frac{1}{120\overline{r}^4} - \frac{1}{400\overline{r}^6}\right)\right]$$

$$g_{44}^{e} = 1 - \frac{\overline{r}_g}{\overline{r}} + \frac{3\overline{r}_g^2}{40\overline{r}^4}$$

$$g_{11}^{i} = 1 + \frac{2}{5}\overline{r}_g\overline{r}^2 + \frac{27}{175}\overline{r}_g^2\overline{r}^6 \tag{69}$$

$$g_{22}^{i} = r^2\left(1 - \frac{1}{5}\overline{r}_g\overline{r}^2 - \frac{3}{175}\overline{r}_g^2\overline{r}^4\right)$$

$$g_{44}^{i} = 1 + \frac{\overline{r}_g}{2}\left(\overline{r}^2 - 3\right) + \frac{3\overline{r}_g^2}{80}\left[\overline{r}^2\left(\overline{r}^2 - 10\right) + 11\right]$$

Here,

$$\overline{r} = \frac{r}{R},\quad \overline{r}_g = \frac{r_g}{R}$$

For real objects, the ratio $\overline{r}_g$ is rather small. For example, for Earth $\overline{r}_g = 1.4\times10^{-6}$, for Sun $\overline{r}_g = 4.25\times10^{-6}$, for the largest of the observed visible stars—red supergiant UI Scutti ($R = 11.9\times10^{11}\,\text{m}$, $m = 64\times10^{30}\,\text{kg}$), $\overline{r}_g = 8\times10^{-9}$ [7].

Thus, the classical expression for the gravitational constant in Equation (6) is proved, but under the following conditions:

- The problem is spherically symmetric and static.
- The sphere consists of a perfect fluid with constant density.
- The asymptotic equations are of the second order, not of the first.
- The coordinate condition in Equation (41) is valid.

If the last of these conditions is violated, the result can be different. For example, take the coordinate condition in the form $g_{22} = r^2$ or $f_2 = \varphi_2 = 0$ which corresponds to the Schwarzchild solution of the spherically symmetric static problem for a fluid sphere [3]. Using decompositions in Equations (36) and (37) under the conditions $f_2 = \varphi_2 = 0$, we arrive at the following equations analogous to Equations (44)-(47):

$$\left(E_1^1\right)_1 = \frac{1}{r^2}\left(f_1 - rf_4'\right) \tag{70}$$

$$\left(E_4^4\right)_1 = \frac{1}{r^2}\left(rf_1\right)' \tag{71}$$

$$\left(E_1^1\right)_2 = \frac{1}{r^2}\left(\varphi_1 - f_1^2\right) - \frac{\varphi_4'}{r} + \frac{f_4'}{r}\left(f_1 + f_4\right) \tag{72}$$

$$\left(E_4^4\right)_2 = \frac{1}{r^2}\left[r\left(\varphi_1 - f_1^2\right)\right]' \tag{73}$$

Consider the external problem for which $E_i^i = 0$. Integration of Equations (71) and (73) yields

$$f_1^e = \frac{C_1}{r}, \quad \varphi_1^e = \left(\frac{C_1}{r}\right)^2 \tag{74}$$

Using Equations (74), we can transform Equations (70) and (72) to

$$\left(f_4^e\right)' = \frac{C_1}{r^2}, \quad \left(\varphi_4^e\right)' = \left(f_1^e + f_4^e\right)\left(f_4^e\right)'$$

Integration gives

$$f_4^e = -\frac{C_1}{r} + C_2, \quad \varphi_4^e = -\frac{C_1 C_2}{r} + C_3 \tag{75}$$

Using Equations (74) and (75), we can present Equations (36) for the external space as

$$g_{11}^e = 1 + \varepsilon_e \frac{C_1}{r} + \varepsilon_e^2\left(\frac{C_1}{r}\right)^2, \quad g_{22}^e = r^2, \quad g_{44}^e = 1 + \varepsilon_e\left(-\frac{C_1}{r} + C_2\right) + \varepsilon_e^2\left(-\frac{C_1 C_2}{r} + C_3\right)$$

Comparing these expressions with Equations (25), we can conclude that $C_1 = 1$, $C_2 = C_3 = 0$ and $\varepsilon_e = r_g$. Thus, the components of the metric tensor for the external space become

$$g_{11}^e = 1 + \frac{r_g}{r} + \left(\frac{r_g}{r}\right)^2, \quad g_{22}^e = r^2, \quad g_{44}^e = 1 - \frac{r_g}{r} \tag{76}$$

This result demonstrates a specific feature of the Schwarzchild solution for the external space—it does not include the integration constants which allow us to satisfy the boundary conditions on the sphere surface. This result follows from the form of the coordinate condition used to obtain the solution. Indeed, the Einstein equation, Equation (28), includes g_{22}'' which is zero if $g_{22} = r^2$. So, Equation (28), being initially of the second order, reduces to the equation of the first order, and its solution contains only one integration constant which is found from the asymptotic condition. The second constant that could be used to satisfy the boundary condition for g_{11} is missing and this condition is not satisfied in the Schwarzchild solution [8].

Consider the internal space for which, as earlier, assume that $\varepsilon_i = \chi\mu c^2$. Then, Equations (71) and (73) allow us to find f_1^i and φ_1^i. The regular solutions of these equations are

$$f_1^i = \frac{r^2}{3}, \quad \varphi_1^i = \frac{r^4}{9}$$

Thus, the radial component of the metric tensor for the internal space becomes

$$g_{11}^i = 1 + \frac{\chi\mu c^2}{3} r^2 + \left(\frac{\chi\mu c^2}{3}\right)^2 r^4 \tag{77}$$

Matching this expression with the first equation in Equations (76), we can conclude that the boundary condition on the sphere surface $g_{11}^e(r=R) = g_{11}^i(r=R)$ can be satisfied if $\chi\mu c^2 R^3 = 3r_g$. Formally, Equation (77) can be used to find χ. Indeed, using Equation (20) for r_g, we arrive at the following expression for the gravitational constant:

$$\chi = \frac{6mG}{R^3 \mu c^4} \tag{78}$$

If m is specified by Equation (42), then $\chi = \chi_0$, where χ_0 is specified by Equation (6). However, this not the case for the Schwarzchild solution for which

$$\begin{aligned} m &= 4\pi\mu \int_0^R \sqrt{g_{11}} g_{22} \mathrm{d}r \\ &= 4\pi\mu \int_0^R \sqrt{\left[1 + \frac{\chi\mu c^2}{3} r^2 + \left(\frac{\chi\mu c^2}{3}\right)^2 r^4\right]} r^2 \mathrm{d}r \\ &\approx 4\pi\mu \int_0^R \left(1 + \frac{\chi\mu c^2}{6} r^2 + \frac{\chi^2\mu^2 c^4}{18} r^4\right) r^2 \mathrm{d}r \\ &= \frac{4}{3}\pi\mu R^3 \left(1 + \frac{\chi\mu c^2 R^2}{10} + \frac{\chi^2\mu^2 c^4 R^4}{42}\right) \end{aligned}$$

Substituting this result in Equation (78), we arrive at

$$\chi = \chi_0 \left(1 + \frac{\chi\mu}{10} c^2 R^2 + \frac{\chi^2\mu^2}{42} c^4 R^4\right)$$

As can be seen, $\chi \neq \chi_0$. Moreover, the obtained equation cannot be used to find χ because χ is the gravitational constant and cannot depend on R and μ. Thus, the asymptotic analysis of the Schwarzchild solution does not allow us to derive the proper expression for the gravitational constant.

4. Light Ray Deviation in the Vicinity of Sun

Having proposed the new metrics in Equations (69), we need to check whether it allows us to predict the experimentally found shift angle which specifies the light ray deviation from the straight trajectory in the vicinity of Sun. For the gravitating sphere with radius R, this angle can be determined from the following equation [4]:

$$\alpha = 2I - \pi \tag{79}$$

in which [9]

$$I = \int_R^\infty \sqrt{\frac{g_{11}}{g_{22}\left(\frac{g_{22} g_{44}^R}{g_{22}^R g_{44}} - 1\right)}} \mathrm{d}r \tag{80}$$

and $g_{ii}^R = g_{ii}(r = R)$.

Consider the linearized solutions for which

$$g_{11} = 1 + f_1, \quad g_{22} = r^2(1 + f_2), \quad g_{44} = 1 + f_4 \tag{81}$$

For the Schwarzchild solution in Equations (76), we have

$$f_1 = \frac{r_g}{r}, \quad f_2 = 0, \quad f_4 = -\frac{r_g}{r} \tag{82}$$

and Equation (81) becomes

$$\begin{aligned} I &= R\int_R^\infty \frac{\mathrm{d}r}{r\sqrt{r^2 - R^2}}\left[1 - \frac{r^2 f_4^R - R^2 f_4}{2(r^2 - R^2)}\right] \\ &= R\int_R^\infty \frac{\mathrm{d}r}{r\sqrt{r^2 - R^2}}\left[1 + \frac{r_g}{2r} + \frac{r_g r}{2R(r + R)}\right] \\ &= \frac{\pi}{2} + \frac{2r_g}{R} \end{aligned} \tag{83}$$

Substitution in Equation (79) yields

$$\alpha = 2\overline{r}_g \tag{84}$$

in which, as earlier, $\overline{r}_g = r_g / R$. Calculation for Sun [4] gives the traditional result $\alpha = 1.75''$ which is in good agreement with the existing experimental data [10].

For the obtained linearized solution in Equations (69),

$$f_1 = r_g\left(\frac{1}{r} - \frac{3r^2}{5R^3}\right), \quad f_2 = r_g\left(-\frac{1}{2r} + \frac{3R^2}{10R^3}\right), \quad f_4 = -\frac{r_g}{r} \tag{85}$$

Substitution in Equation (80) yields

$$I = R\int_R^\infty \frac{\mathrm{d}r}{r\sqrt{r^2 - R^2}}\left[1 - \frac{r^2 f_4^R - R^2 f_4}{2(r^2 - R^2)} + \frac{1}{2}\left(f_1 - f_2 + f_2^R + f_4\right) - \frac{r^2 f_2 - R^2 f_2^R}{2(r^2 - R^2)}\right] \tag{86}$$

The first two terms in this equation are the same that in Equation (83), because function f_4 is the same in Equations (82) and (85). As can be directly checked, the result of integration of the two last terms in Equation (96) is zero. Thus, the result of integration in Equation (86) is the same that in Equation (83), and the shift angle is specified by Equation (84).

Now, calculate the integral in Equation (80) using the second-order approximation of the metric tensor in Equations (69). Omitting rather cumbersome transformations, present the final result which is

$$\alpha = 2\overline{r}_g + \frac{139\pi - 128}{320}\overline{r}_g^2 = 2\overline{r}_g + 0.955\overline{r}_g^2$$

Taking into account that for Sun $\overline{r}_g = 4.25\times10^{-6}$, we can conclude that the second term is negligible in comparison with the first one, and that the shift angle for the obtained metric coefficients in Equations (69) is specified by the traditional Equation (84) which is in good agreement with experiment.

5. Conclusion

As follows from the foregoing analysis, the general relativity theory reduces for low intensity gravitation to the Newton theory only for the empty space. For solids, the linearized equations of the general relativity do not describe gravitation and the second-order asymptotic equations should be applied. For this approximation, the traditional expression for the gravitational constant is found for spherically symmetric static problem under special coordinate condition. The linearized Schwarzchild solution is not reduced to the Newton solution. The obtained solution for the metric tensor of the spherically symmetric empty space yields the traditional result for the angle of light ray deviation in the vicinity of Sun.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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NeoMinkowskian Cosmological Black Hole, Poincaré's Gravific Electron and Density of CBR

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Abstract

In the previous paper (JMP 2014) we showed that there exists a NeoMinkowskian Gravitational Expanding Solution of GR (General Relativity) with CC (Cosmological Constant). We prove now that NeoMinkowskian Vacuum (non-baryonic Fluid), with gravitational (first) density (dark energy) *and gravitational waves* (at light speed), corresponds to the Gravitation Field of a Cosmological Black Hole (CBH). The latter predicts furthermore a basic emission of Radiation (CBR) from Hubble spherical singular Horizon to the inside of CBH (unlike Hawking's emission) at an initial singular time. Our solution is then compatible with a well-tempered Big Bang and Expanding Universe (Escher's Figure, see Penrose, §3) but incompatible with inflation. The latter is based on Hypothesis of a so-called Planck's particle (Lemaitre's primitive atom) characterized by a so-called Planck length. We prove that we can short-circuit this unstable particle with a stable cosmological Poincaré's electron with gravific pressure. It is well known that electron is a stranger in usual Minkowskian vacuum (dixit Einstein). The stranger electron can be perfectly integrated in NeoMinkowskian Radiation fluid and then also (with its mass, charge and wavelength) in (second density of) CBR. Everything happens as if the leptonic mass of the electron were induced by our cosmological field. The unexpected cosmological model proposed here is the only one that *predicts numerical values of* (*second*) *density and temperature of CBR very close to the observed* (*COBE*) *values.*

Keywords

Cosmological Constant, General Relativity, Minkowskian Metric, Cosmological Black Hole, Tachyons, Hyperbolic Horizon, Density of Vacuum, Density of CBR, Poincaré's Gravitational Waves, Poincaré's Electron, De Broglie's Wave Electrodynamics, De Broglie's Subquantum Substratum

Introduction: NeoMinkowskian Gravitational Vacuum as Solution of GR with CC

Let's summarize first how we come to an unexpected Gravitational NeoMinkowskian Expanding Vacuum which "looks like" de Sitter's Expanding Vacuum [1] [2].

(1) Gravitational Density of NeoMinkowskian Vacuum

Let us consider Einstein's equation of General Relativity (*GR*) with Cosmological Constant (*CC*) Λ and Perfect Fluid $T_{\mu\nu} \equiv (p+\rho)\frac{u_\mu u_\nu}{c^2} - pg_{\mu\nu}$ (standard notations: energy density ρ, pressure p and 4-velocity $u_\mu u_\nu$):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \chi T_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} = \frac{8\pi G}{c^4}\left[(p+\rho)\frac{u_\mu u_\nu}{c^2} - pg_{\mu\nu}\right] \qquad (0)$$

Let us introduce Minkowskian Metric (*MM*) $g_{\mu\nu} = \eta_{\mu\nu}$ in (0) with therefore cancellation of Einstein's curvature tensor $G_{\mu\nu} = 0$:

$$G_{\mu\nu} = 0 \quad \Rightarrow \quad \Lambda \eta_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} = \frac{8\pi G}{c^4}\left[(p+\rho)\frac{u_\mu u_\nu}{c^2} - p\eta_{\mu\nu}\right] \qquad (0\text{-}MM)$$

A non-trivial NeoMinkowskian Solution with $\rho = -p$ ($\rho + p = 0$) is then:

$$T_{\mu\nu}^{VACUUM} = \rho\eta_{\mu\nu} = \frac{\Lambda c^4}{8\pi G}\eta_{\mu\nu} \qquad (1)$$

The density of VACUUM, simulated by a Non-Baryonic ($G_{\mu\nu} = 0$) NeoMinkowskian Fluid, depends therefore on the Gravitational Constant (see 14):

$$\rho + p = 0 \quad \Rightarrow \quad \rho = \frac{\Lambda c^4}{8\pi G} = \rho_\Lambda \qquad (1\text{bis})$$

This *Gravitational NEO-Minkowskian Vacuum* (*GMV*) does not correspond to the usual *ElectroMagnetic* Vacuum (*EMV*) (with permittivity, permeability, impedance…) of the standard Minkowskian Vacuum of Special Relativity (*SR*). We reject here any attempt of putsch[1] that consists in canceling (a priori) this gravitational density (1bis): $p = \rho = 0$.

We are looking first for the determination of the global gravitational field (§1) which corresponds to this density and then for Poincaré's Gravific Waves (in the framework of Lorentz Transformation, (*LT*, see §1-3, 8). The operation $p = \rho = 0$ is characteristic of de Sitter's Vacuum whose metric (*dSM*) and not that of Minkowski ($MM \neq dSM$). In basic equation (0), NeoMinkowskian Vacuum is based on cancellation ($\rho + p = 0$) of Einstein's Tensor ($G_{\mu\nu} = 0$) whilst deSitterian Vacuum is based on cancellation ($p = \rho = 0$) of Tensor of Perfect Fluid ($T_{\mu\nu} = 0$):

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0 \qquad (0\text{-}dSM)$$

At first glance, these two approaches to Vacuum (0-*dSM* and 0-*MM*) are very far apart. We first show (in introduction, see also *JMP* previous paper) that they are indeed very close (6).

[1]Given that Einstein's (local) curvature tensor $G_{\mu\nu} = 0$. is canceled, it is generally admitted that Global NeoMinkowskian space is flat and so the basic density (1bis) is canceled. Let us note that CC dimensionally corresponds to a global (negative) curvature, see §3-3-2).

The only way to develop the role of CC in NeoMinkowskian Vacuum is to start from that of CC in Riemanian Vacuum.

(2) Cosmological Constant (CC) in Expanding Riemannian Vacuum

In cosmological literature we never find (1) but we always find (anti-1) with Riemannian Metric (RM) $g_{\mu\nu}$ ($G_{\mu\nu} \neq 0$):

$$T_{\mu\nu}^{DarkEnergy} = \frac{\Lambda c^4}{8\pi G} g_{\mu\nu} \quad \Rightarrow \quad \rho + p = 0 \qquad \text{(anti-1)}$$

The cosmological term $\Lambda g_{\mu\nu}$ (first member) is associated, by the cosmologists, with the density term (second member) $\frac{8\pi G}{c^4}\rho g_{\mu\nu}$. This so-called "component of vacuum" (Dark Energy) is then associate to $\rho + p = 0$ (next to other components: radiation component, matter component…).

In order to avoid the fateful. $G_{\mu\nu} = 0 \;\Rightarrow\; g_{\mu\nu} = \eta_{\mu\nu}$ (1), the strategy of cosmologists consists of multiplying (0) into several equations (phases or periods): one equation for matter, one equation for radiation, one equation for curvature, one equation for dark energy (before the matter and then $G_{\mu\nu} = 0$)… (each equation corresponding to a period of the inflation).

We have in our strategy "one and only one equation (0)" with one and only one solution (1 bis, 9, 12)[2].

In order to find the characteristics of Dark Energy, cosmologists develop the thermodynamics underlying the relationship $\rho + p = 0$:

$$\rho + p = 0 \;\Rightarrow\; H = U + pV = 0 \;\Rightarrow\; \mathrm{d}H = \mathrm{d}U + p\mathrm{d}V = 0 \;\Rightarrow\; \rho + p = 0 \quad \text{(1-\&-anti-1)}$$

Local transformation $\rho + p = 0$ leads by integration to a global Volume $U = \rho V$. By differentiation $\mathrm{d}U + p\mathrm{d}V = 0$ we return to the local starting point $\rho + p = 0$ ($\frac{\mathrm{d}U}{\mathrm{d}V} = \rho$) if and only if $\mathrm{d}V \neq 0$.

The variable volume results from a well known property of Enthalpy ($H = U + pV$ isentropic $\mathrm{d}S = 0$ and non isochoric variation $\mathrm{d}V \neq 0$).

Nothing prevents then introducing a variability over time and therefore a growing energy in an expanding empty universe:

$$\rho + p = 0 \quad \Rightarrow \quad U(t) = \rho V(t) \qquad (2)$$

which can be developed $\mathrm{d}V \neq 0$ on the basis of any metric $g_{\mu\nu}$ included $\eta_{\mu\nu}$. This is our starting point (2) with a very binding determination of the metric: $g_{\mu\nu} = \eta_{\mu\nu}$ (as we will see it, NeoMinkowskian constraint of inaccessible singular ($<c$ or $>c$) velocity is very strong).

(3) Cosmological Constant (CC) in Exponential Expanding NeoMinkowskian Vacuum

The first Minkowskian constraint on (2), in a pseudoEuclidean space-time $4D$ framework, involves a spherical (Euclidean) $3D$ symmetry:

$$V(t) = \frac{4}{3}\pi R^3(t) \quad \Rightarrow \quad \dot{V}(t) = \frac{4}{3}\pi 3R^2(t)\dot{R}(t) \quad \Rightarrow \quad \frac{\dot{U}(t)}{U(t)} = \frac{\dot{V}(t)}{V(t)} = 3\frac{\dot{R}(t)}{R(t)} \quad \text{(2 bis)}$$

The non-static NeoMinkowskian GMV has to be treated with a *radial* scale

[2]We will see (§5) that another density that corresponds (see §4) to the same solution.

factor $R(t)$ and a *radial* velocity $\dot{R}(t)$ and therefore with a radial ratio $\frac{\dot{R}(t)}{R(t)}$. Given that we have indifferently a constant density of energy or a constant density of mass $\rho_M = \frac{\rho}{c^2} = \rho_\Lambda$ as well, NeoMinkowskian spherical symmetry imposes Einstein's equivalence relation between (dark) pseudo-energy $U(t)$ and (non-baryonic) pseudo-mass $M(t)$:

$$M(t)c^2 = U(t) = \frac{4}{3}\pi\rho R^3(t) \tag{3}$$

We have thus a global variable pseudo-mass a $M(t) = \frac{4}{3}\pi\frac{\rho}{c^2}R^3(t)$ and a local constant pseudo-mass density $\frac{\rho}{c^2}$ (in radial Friedman's equation it is usually the opposite situation).

Nothing prevents a priori that this pseudo-mass from being sensitive to Newton law of gravitation.

On one hand a static model with potential energy $E_P = -\frac{GM}{R}$ is excluded because unstable (collapse).

On the other hand a dynamical model, based on an equilibrium between kinetics energy E_c and potential energy E_P, is possible:

$$\frac{1}{2}\dot{R}(t)^2 - \frac{GM(t)}{R(t)} = \frac{1}{2}\dot{R}(t)^2 - \frac{4}{3}\pi G\frac{\rho}{c^2}R(t)^2 = 0 \tag{4}$$

(*YP*12 with common numbering in the French and English version of our previous research [1] [2])[3]. We simplified with a Non-Zero enigmatic Non-Baryonic (*NB*, $G_{\mu\nu} = 0$) (micro) mass m_{NB}:

$$\frac{1}{2}m_{NB}\dot{R}(t)^2 - \frac{GM(t)m_{NB}}{R(t)} = 0 \tag{4 bis}$$

We can then determine the desired ratio (2 bis):

$$\frac{\dot{R}(t)}{R(t)} = \sqrt{\frac{1}{3}\frac{8\pi G}{c^2}\rho} = c\sqrt{\frac{\Lambda}{3}} = H_\Lambda \tag{5}$$

From $\rho + p = 0$ we return to *CC* with[4] $\Lambda = \frac{8\pi G\rho}{c^4}$.

That means that our dynamical equation (4, YP12) is an alternative to historical Einstein's determination of constant χ in (0) with weak Newtonian Gra-

[3]A dynamic fatal objection seems to be however formulated here at this stage: Our model (YP12, 4) is neither Minkowskian nor NeoMinkowskian because gravitational law with especially kinetics energy would non relativistic. That would be true for a material (baryonic) point (§3). We urge the reader to be cautious (and patient, §3) because no one has tried until now to apply scalar (temporal) Newton law of gravitation to a space point (we are in the framework of *GR*) that is to say a *non-baryonic (non-material) pseudo-mass*. Nobody really knows (at this stage) if this law (4) is incompatible or not with *MM*.

[4]With $\Lambda = 0$ we return to static Minkowskian usual solution $\frac{\dot{R}(t)}{R(t)} = 0$.

vitation Field (metric $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$). Indeed from (0) $\Lambda\eta_{\mu\nu} = \chi T_{\mu\nu} = \chi\rho\eta_{\mu\nu}$ we have $\rho = \frac{\Lambda}{\chi}$ and from (5) $\chi = \frac{\Lambda}{\rho} = \frac{8\pi G}{c^4}$. The determination of constant χ with NeoMinkowskian gravitation field (4) is then much more direct than that with the weak gravitation field.

On the basis of YP12 we rediscover gravitational (critical) density (1-bis) in function of (measurable) Hubble's constant (see numerical value in annex 1 with Gauss cgs-units):

$$\rho_M = \frac{\rho}{c^2} = \rho_\Lambda = \frac{3H_\Lambda^2}{8\pi G} \tag{5 bis}$$

Our non-usual solution consists therefore in an Exponential Expanding (*EE*) of Global Vacuum with a Hubble (measurable) Constant which defines the kinematics underlying the future gravitational dynamics, §1):

$$R(t) = R(0)\mathrm{e}^{H_\Lambda t}, \quad \dot{R}(t) = \dot{R}(0)\mathrm{e}^{H_\Lambda t}. \tag{6}$$

It was at this point that we had arrived in our previous work [1] [2], (6) in keeping with recent observations of an accelerating (see 10) expanding universe [3].

Nevertheless the originality of our NeoMinkowskian *EE* of Vacuum, very close to deSitterian *EE* of Vacuum (with zero density, §1-4), did not seem obvious yet.

So the binding singularity of light velocity, underlying any Minkowskian (or NeoMinkowskian) solution, does not appear in (6). Except if we impose a structural speed of NeoMinkowskian space-time $\dot{R}(0) = c$ in Initial Conditions (*IC*).

This is the reason why we suggest to examine first, on a kinematic point of view, the contrast "deSitterian *IC* (§1-1) versus NeoMinkowskian *IC*" (§1-2).

1. [TACHYON] Only Can Escape (to the Outside) from Cosmological Horizon of NeoMinkowskian Black Hole

The structure of the paper will follow 3-partition of NeoMinkowskian space-time: Tachyon ($v > c$) §1, Photon ($v = c$) §2, Bradyon ($v < c$) §3.

1.1. Undetermined Initial Conditions (IC) in deSitterian EE (Inflation)

Let's take a look at deSitterian solution that consists of canceling the second member: $T_{\mu\nu} = 0$ or $p = \rho = 0$ in (0):

$$0 = G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{\dot{a}^2(t)}{a^2(t)} - \frac{\Lambda}{3} \tag{0-dSM}$$

On the left we write Einstein's equation (0) and on the right radial Friedman's equation[5] with Robertson-Walker's metric. The latter is based on two factors (scale factor $a(t)$ and curvature factor *K*).

[5]Let us note that NeoMinkowskian solution (1 or 5) is also a solution of Friedman's equation with $a(t) = 1$ ($K = 0$). The scale factor disappears. Everything seems static. Unless the scale factor is hidden in a scale hyperbola (§2-3). We note that our model does not correspond to pseudo-hyperbolic value of parameter $K = -1$ in *RW* (see §3-3).

The simplest deSitterian solution is then Parabolic ($K=0$) Vacuum (*dSV*) without matter $\rho=0$, without structural light velocity c and even without gravitational constant *G*: the factor $\frac{8\pi G}{c^4}$ being eliminated from both Einstein's and Friedman's equation (Hubble's constant is the same (true constant) connected with *CC* in both cases $c\sqrt{\frac{\Lambda}{3}}=H_\Lambda$):

$$\frac{\dot{a}(t)}{a(t)}=c\sqrt{\frac{\Lambda}{3}}=H_\Lambda \quad\Rightarrow\quad a(t)=a(0)\mathrm{e}^{H_\Lambda t},\ \dot{a}(t)=\dot{a}(0)\mathrm{e}^{H_\Lambda t} \qquad \text{(anti-6)}$$

We propose the numbering "anti"-6 for three reasons:

1) deSitterian Hubble constant is not connected with density ($\rho=0$) and so to a determined gravitational dynamic (5 bis).

2) deSitterian *IC* are undetermined, $a(0),\dot{a}(0)$, for the radial distance $a(0)$ and radial velocity $\dot{a}(0)$ as well. We can choose to introduce the radius of the so-called Planck's particle (Planck's length, see annex 1), Poincaré's radius of electron, Bohr's radius of atom, Erathostène's radius of Earth, diameter of solar system, diameter of Milky Way as well.

3) In de Sitter's model (*dSV*) there is no singular *IC* (Horizon). It means that the spherical surface (for example the radius of Hubble) $a(0)$ is continually moving (instantly) at the time $t=0$ with velocity $\dot{a}(0)$ (for example ($\dot{a}(0)=c$).

1.2. Determined IC in NeoMinkowskian EE: From Singular Velocity to Singular Cosmological Horizon

If we set the light speed $\dot{R}(0)=c$ in (6) to an initial time, say zero ($t=0$), then this *singular* constant speed immediately implies, with $H_\Lambda R_H=c$ a *singular* constant Radius $R(0)=R_H=\sqrt{\frac{3}{\Lambda}}$. (*H* for Horizon and for Hubble).

The only physical (and logical) consistent interpretation of this double discontinuity (Horizon) involves a singular EMISSION of light, at the (fixed) speed c, from a singular spherical surface Horizon whose Radius $R(0)=R_H$ is fixed in $t=0$ (unlike deSitterian continuity).

Unlike de Sitter's initial radius, NeoMinkowskian initial radius ($R(0)=R_H$) must be the Horizon of Hubble (binding determination induced by *CC*).

Unlike usual Minkowskian theory, NeoMinkowskian theory (6) has a DOUBLE HORIZON (c,R_H): *Horizon of Velocity and Horizon of Space*.

The complete explicit solution (6), that takes into account imposed *IC* in $t=0$ ($R(0)=c$ and $R(0)=R_H$) is then:

$$\frac{\dot{R}_{Tachy}(t)}{R_{Tachy}(t)}=H_\Lambda=\sqrt{\frac{8\pi G\rho}{3}} \Rightarrow R(t)=R_H\mathrm{e}^{H_\Lambda t},\ \dot{R}(t)=c\mathrm{e}^{H_\Lambda t} \Rightarrow c=H_\Lambda R_H \quad \text{(6 bis)}$$

For a time later $t>0$ we will have therefore supra-luminous speeds $\dot{R}(t)>c$ of TACHYONS (the first member of Feinberg's trio) that move away from the singular Horizon $R(0)=R_H$ We have a law of Hubble but for tachyons $\dot{R}(t)=H_\Lambda R(t)$! (for bradyons see, 19 bis).

Our tachyonic model of an expanding Universe fits well in the framework of *GR* where nothing forbids that the speed of a space point is greater than a singular (light) speed. And therefore nothing forbids that the speed of a space point is ALWAYS greater than a singular (light) speed.

Given that the velocity *MUST* be super-luminous $\dot{R}(t)>c$ for tachyons if and only if we have *no negative time* $t>0$ the original time $t=0$ is then a (true) singularity.

The latter must logically correspond to a finite time $T_H=H_\Lambda^{-1}$ in the same way that we have a finite space Horizon. Our NeoMinkowskian model with a finite time is then compatible with a Big Bang.

Given that the Hubble Horizon R_H is given at initial time $t=0$, our NeoMinkowskian model is NOT compatible with (deSitterian) inflation of a supposed "Planck's particle" that delete ($R_H \rightharpoonup l_P \rightharpoonup 0$) Hubble's Horizon[6].

The paper could stop here: We deduce the first expanding *GR*-Solution with STRICT NeoMinkowskian inequality ($R(t)>R_H-\dot{R}(t)>c$): Only tachyons can escape from the Horizon.

But what are they escaping from? (§1-3)

Unlike deSitterian zero density, NeoMinkowskian non-zero gravitational density (1bis) defines a gravitational field (constant *G*). What is this searched gravitational field *GMV* (§1-3)?

The paper could then not stop here because we are looking for a full description (kinematic and dynamic) of NeoMinkowskian space-time (tachyon, photon, bradyon).

1.3. NeoMinkowskian Cosmological Black Hole (CBH) and Poincaré's Gravific Waves

So far we have developed the (Radial) Kinematic (6, 6 bis) aspect of the NeoMinkowskian solution. Let's now develop the Gravific (Scalar) Dynamic aspect. In order to do that, let us now introduce our Kinematic NeoMinkowskian *IC* in Dynamic *YP*12 (5) at singular time $t=0$:

$$\frac{1}{2}\dot{R}(t)^2-\frac{GM(t)}{R(t)}=\frac{1}{2}\dot{R}(0)^2-\frac{GM(0)}{R(0)}=\frac{1}{2}c^2-\frac{GM_H}{R_H}=0 \tag{7}$$

With a new gravific dynamic *IC* $M_H=\frac{4}{3}\pi\frac{\rho}{c^2}R_H^3$ we deduce the *threshold* escape speed:

$$c^2=\frac{2GM_H}{R_H} \tag{8}$$

An Horizon from which a PHOTON cannot escape (only Tachyon can es-

[6]Since there is neither observed expansion of the galaxies themselves nor the solar (or atomic) system itself, the existence a priori of a Hyperbolic Horizon is then consistent with current observations (§3-3).

cape) is by definition *a Schwarzshild's Horizon* of a BLACK HOLE.

More precisely: A Cosmological Black Hole (*CBH*) whose Universal Schwarzshild's Horizon is Hubble's Horizon.

In parallel with a 3-partition of the speed space (tachyons, photons and bradyons), our *CBH* involves a 3-partition of space itself (out, on and in).

THE searched GRAVITATIONAL FIELD *GMV* which corresponds to gravitational density (1, 1 bis) is given by A UNIVERSAL NEOMINKOWSKIAN BLACK HOLE (*CBH*).

Underlying Minkowskian Metric *MM* must be then written as follows:

$$\mathrm{d}s^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2 = \mathrm{d}r^2 - \frac{2GM_H}{R_H}\mathrm{d}t^2 \tag{9}$$

It was enough to think about it!

According to *GR* transformations of coordinates with *MM* are LINEAR LORENTZ TRANSFORMATION (*LT*) (see §2-3, 18 scale hyperboles).

On such grounds it is no longer possible to claim that *MM* is incompatible with gravitational density (1 bis), with gravitational field in vacuum (8) and then also with possible existence of GRAVITATIONAL WAVES at light velocity.

This is exactly Poincaré's position in 1905 with his "Gravific Waves"[7]: "*Quelles modifications elle* [*la transformation de Lorentz*] *nous obligerait*? *apporter aux lois de la gravitation. C'est ce que j'ai cherché à déterminer. J'ai été conduit à supposer que la propag ation de la gravitation* [*ondes gravifique dixit Poincaré*] *n" est pas instantanée mais qu'elle se fait à la vitesse de la lumière*" (introduction) [4].

So we are led to rehabilitate Poincaré's work on *LT* in 1905 but on the basis... of Einstein's *GR* in 1916 (with *CC* 1917). So there is no polemical intention on our part (see annex 2).

Formula (8) looks like Laplace's (Mitchell's) non-relativistic formula of static Stellar Black Hole (*SBH*). There is neither singular escape velocity ($v^2 = c^2 = \frac{2GM_S}{R_S}$) nor singular Horizon in non-relativistic Laplace's approach.

IN SUMMARY: The only way to make relativistic Laplace's formula (9 with 10) consists in claiming that alone tachyonic points can escape from event Horizon. But we need then a gravitational field (*CBH*) which does not exist in usual *SR*. YP-12 involves that the desired *GMV* is then *CBH* in NeoMinkowskian global solution (with a negative hyperbolic curvature, see §3).

At this stage our NeoMinkowskian Black Hole (*CBH*) seems very close to that

[7]We purposely used the term GRAVIFIC WAVE used by Poincaré in 1905. Laplace considered that a velocity CAN be super-luminous: "La gravitation se déplace au moins 300 fois plus vite que la lumière. Poincaré criticizes Laplace in 1905 by proposing that the speed of a gravitational wave must be the limit (singular) speed of light on the basis of Lorentz Transformation (*LT*)". Poincaré's position on gravific waves is non orthodox because *SR* is reputed without gravitation, without density (1 bis, the putsch note 1) and then without gravific waves, note 8).

He shows also (in §6 of the same paper) that ELECTRON undergoes a GRAVIFIC pressure (§5-2) in the framework of *LT*. Thanks to NeoMinkowskian approach, we will be able to synthesize the G-wave and the G-pressure (§5-2).

of cosmology of FLAT Universe (Tatum[8], [5]).

Let us precise in order to complete the dynamic that we have also an acceleration $\ddot{R}(t)$ deduced with a second derivation:

$$\ddot{R}(t)-H_\Lambda^2 R(t)=0 \quad \Rightarrow q_\Lambda=-\frac{R(t)\ddot{R}(t)}{\dot{R}(t)^2}=-1 \quad \Rightarrow \quad \frac{R(0)\ddot{R}(0)}{\dot{R}(0)^2}=\frac{R_H\alpha_M}{c^2}=1 \quad (10)$$

with a new basic invariant constant α_M determined in $t=0$. *YP*-12 introduces a global singular pseudo-mass M_H of *CBH* (singular dark Energy $U_H=M_Hc^2$ which might seem unrealistic if it did not fit with a singular acceleration α_M. This basic acceleration α_M cannot be applied to the constant singular velocity of emission of photon but can be applied to tachyons (centripetal α_M) and to bradyons (centrifugal α_M) as well (§3.3).

So we have to continue our path from very formal and abstract maximal (centripetal) acceleration for tachyon, to a very physical and concrete minimal (centrifugal) acceleration for bradyon (25)[9]. We will show that this hyperbolic acceleration is a universal invariant (§3-3) and therefore all the above dynamic gravitational equations are perfectly relativistic in NeoMinkowskian meaning.

Finally we check the consistence of our approach with the deduction of the "second" usual cosmological parameter (Hubble H_Λ and acceleration q_Λ):

$$q_\Lambda=\frac{\rho}{p}=-1 \quad \rightleftarrows \quad \alpha_M=H_\Lambda c=\frac{2GM_H}{R_H^2} \quad (11)$$

In truth they are here both sides of the same coin. Let us insist on the necessity of not confusing $q_\Lambda=-1$ with $K=-1$. In the first case it is a reference to a basic global hyperbolic motion ($q_\Lambda=-1$) with constant acceleration α_M whilst in the second case it is a reference to the pseudo-hyperbolic value of local curvature ($K=-1$) parameter in RW metric (note 5).

By conferring a positive role to an essential component (tachyons) of the NeoMinkowskian non-baryonic framework ($G_{\mu\nu}=0$) the unique solution (1 or 9) can henceforth be written:

$$T_{\mu\nu}^{VACUUM}=\frac{\Lambda c^4}{8\pi G}\eta_{\mu\nu}^{-}=\rho_\Lambda\begin{pmatrix}-1&0&0&0\\0&1&0&0\\0&0&1&0\\0&0&0&1\end{pmatrix} \quad (12)$$

Supra-luminous speed of tachyon is perfectly allowed in *GR* for space-points.

[8]We have somehow established a theoretical horizon for formulas of flat universe ($K=0$ in *RW*, see note 5). The problem is that this Horizon must be Hyperbolic (see §3).

[9]Outside the *CBH* (minimal light velocity) we have a minimal R_H coupled with a maximal (centripetal) α_M. Inside the *CBH* we will have exactly the opposite (minimal centrifugal α_M §3-3). The objection that the acceleration cannot be a relativistic invariant is inadmissible because both theories are not in competition at this stage. The question will arise when the bradyon that matches to tachyon will be defined. To reassure the reader, we will have (§3) STRICT inequalities in both cases (tachyons and bradyons) but in inverted sense ($R(t)>R_H-\dot{R}(t)>c$ and $r(t)<R_H-v(t)<c$). It will be solve with hyperbolic acceleration which is a relativistic invariant ((§3-3). IDEM for Newton's laws (dynamic and gravitation as well).

Our *GR*-solution (1-1bis-1-ter) is always valid whatever the value of velocity (> = and <) because the basic operation $\rho + p = 0$ makes disappear the factor $(p+\rho)\frac{u_\mu u_\nu}{c^2}$ with 4-velocities (the temporal component is here −1 in 00). The positive role of tachyons induces internal density of dark energy ($\rho_\Lambda = \frac{3M_H c^2}{4\pi R_H^3}$).

Is this the only possible density? Is there another density defined by $\eta^+_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ (see §3-3) which is not directly the unique solution (1, 12) but which is coupled (§4 see the ratio anti-32 & §5) with the density (1-bis, 1-ter)?

1.4. A Cosmological Hidden Non-Baryonic Micro-Mass?

Both Newton s laws (Gravitation and Dynamic) are coupled ($t = 0$):

$$F_G = \frac{c^4}{2G} = M_H \alpha_M = M_H \frac{2GM_H}{R_H^2} \tag{13}$$

(density 1bis can be write $\rho_\Lambda = \frac{3F_G}{4\pi R_H^2}$). We have not only a Non-Baryonic (*NB*) Macro-Mass M_H but also a *NB* micro-mass m_{NB} (4 bis):

$$\frac{1}{2} m_{NB} c^2 - \frac{GM_H m_{NB}}{R_H} = 0 \quad (YP12) \qquad \Rightarrow m_{NB} c^2 R_H = \frac{2GM_H m_{NB}}{R_H^2} \quad \text{(4 ter)}$$

We see that we have next to a Macro Force (13) a Newton's law of dynamic a micro force (14):

$$f_G = m_{NB}\alpha_M = \frac{2GM_H m_{NB}}{R_H^2} = 2m_{NB} G \frac{M_H}{R_H^2} \tag{14}$$

If F_G (13) is determined, f_G (14) is, at this stage, undetermined (*NB* micro-mass will be precise in §4). This will be a very important point for Poincaré's "dynamic of electron" (§5) with a gravitational force ($m_{NB} = m_e$?).

1.5. From Static Stellar Black Hole (SBH) to Dynamic Cosmological Black Hole (CBH)

Let us show now that *CBH*, with non-static *MM* (13) is the cosmological limit of Schwarzschild's static metric (Stellar Black Hole, *SBH*). The latter is written (Outside the *SBH*):

$$\mathrm{d}s^2 = \left(1 - \frac{R_S}{r}\right)\mathrm{d}r^2 - \frac{1}{1 - \frac{R_S}{r}} c^2 \mathrm{d}t^2, \quad r > R_S \quad \text{(Schwarz-out)} \tag{15}$$

coupled with formula of Laplace ($R_S = \frac{2GM_S}{c^2}$). It is well known that the infinite behavior $r \mapsto \infty$ of (schwarz-out), brings back to the usual static Minkowskian limit $\mathrm{d}s^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2$. A sad destiny for the *SBH* which seems to be evaporated!

In dynamic NeoMinkowskian limit of RG with CC we have $SBH \xrightarrow[r\mapsto\infty]{} CBH$ *with* $R_S \mapsto R_H$ ($M_S \mapsto M_H$).

$$\mathrm{d}s^2 = \lim_{r\mapsto\infty}\left[\left(1-\frac{R_S}{r}\right)\mathrm{d}r^2 - \frac{1}{1-\frac{R_S}{r}}c^2\mathrm{d}t^2\right] = \mathrm{d}r^2 - \frac{2GM_H}{R_H}\mathrm{d}t^2 \quad \text{(Mink-out) (15-out)}$$

This is a happy destiny for Stellar *SBH* which becomes Universal *CBH* (9).

It has been showed (Kruskal) that "singularity of Schwarzschild" R_S is not a true physical singularity. We have to analyze the essential difference with the singularity (of Horizon) of Hubble R_H which is a true physical (hyperbolic) singularity (§3).

The structure of the paper will follow Feinberg's partition of NeoMinkowskian space time: Tachyon (OUT) §1, Photon (ON) §2, Bradyon (IN) §3 [6].

2. [PHOTON] Emitted to the Inside: From CBH to CBR (Cosmological Background Radiation)

NeoMinkowskian Universal coupling (R_H, c) involves NECESSARILY *a basic emission* of waves (with constant velocity *c*) from the spherical surface of radius R_H (at singular initial time $t = 0$).

Let us write LikeLight interval:

$$\mathrm{d}s^2 = \mathrm{d}r^2 - \frac{2GM_H}{R_H}\mathrm{d}t^2 = 0 \quad \Rightarrow \quad \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{\frac{2GM_H}{R_H}} = c \quad \text{(Mink-on)} \qquad \text{(16-on)}$$

(If there is no emission we have to apply the acceleration to the singular speed (of the photon) which becomes therefore no longer singular)

In what sense of radial direction should this radiation be emitted? To the outside or to the Inside of *CBH*?

Given that only tachyons can escape to the outside (§1), the photons (§2) can only be emitted TO THE INSIDE from the border R_H and then in the bradyonic universe.

This is logically unstoppable.

Unless we invoke quantum fluctuations (Hawking 1974) in order to justify an emission of Black Radiation to the outside from event horizon of *SBH* (§2-1).

Our basic emission of light resembles that of Hawking but it is not that of Hawking. Except on one point: *the emitted radiation MUST be a Black Radiation*. Indeed we have an emission of radiation from a global spherical surface of last diffusion, based on an isentropic transformation (2 & 40).

Our *CBH* is consistent with a singularity of the type Big Bang coupled with an emission of the type of *CBR* (Cosmological Black (Background) Radiation (a *CBR* at the Horizon of the *CBH*?). We will show in the next paragraph that is also compatible with an expansion of the type Hubble (R_H is an Hyperbolic Horizon, §3).

We focus attention on a very important point that (Mink-on, 16) defines a NeoMinkowskian STRUCTURAL space-time velocity (von Ignatowski): it can correspond to light wave (photon) or gravific wave (graviton) as well (see Table 1).

2.1. From Hawking's (Local) Black Radiation to (Global) CBR Black Radiation

Let us remember that Hawking's black radiation [7]. is emitted from Horizon of Events of *SBH* to the outside (see also Unruh, [8]). The expression "singularity of Schwarzschild" is henceforth outdated (still used in the old scientific literature). We confirm this point by making $\mathrm{d}s^2 = 0$ in Schwarzschild's metric (15):

$$\left(1-\frac{R_S}{r}\right)^2 \mathrm{d}r^2 - c^2\mathrm{d}t^2 = 0,\quad \frac{\mathrm{d}r}{\mathrm{d}t} = \lim_{r\mapsto R_S} \frac{c}{1-\frac{R_S}{r}} \mapsto \infty \quad \text{(Schwarz-on)} \tag{16}$$

We are indeed getting a non singular speed as large as we wish for *SBH*. But a speed of what? It is not speed of light *c*. Given that we are in the framework of *GR*, this supra-luminous speed can only be that of space point and therefore to the speed of propagation of gravitation (see Laplace, note 8).

In contrast with (Schwarz-on), for *CBH* the singular Horizon of Hubble R_H is coupled (R_H, c) with singular Horizon of light velocity *c* (Zero interval, or LightLike interval, see Table 1, we repeat 16-on, just above):

$$\mathrm{d}r^2 - \frac{2GM_H}{R_H}\mathrm{d}t^2 = 0,\quad \frac{\mathrm{d}r}{\mathrm{d}t} = \sqrt{\frac{2GM_H}{R_H}} = \dot{R}(0) = c,\quad r\mapsto R_H \quad \text{(Mink-on) (16-on)}$$

Our Black Hole emits not only CBR (see Table 1) but also gravific waves at the speed of light (Poincaré, 1905, [4]) to the inside.

Inside the *SBH* we have:

$$\mathrm{d}s^2 = c^2\frac{1}{\frac{R_S}{r}-1}\mathrm{d}t^2 - \left(\frac{R_S}{r}-1\right)\mathrm{d}r^2,\quad r<R_S \quad \text{(Schwarz-in)} \tag{17}$$

Table 1. Stellar black hole versus cosmological black hole.

Schwarzschild's Horizon of *SBH* $c^2 = \frac{2GM_S}{R_S}$	Hubble's Horizon of *CBH* $c^2 = \frac{2GM_H}{R_H}$
(Schwarz-out)	spacelike TACHYON (Mink-out)
$r > R_S$ (15)	$R(t) > R_H$, $\dot{R}(t) > c$, $\mathrm{d}s^2 = \mathrm{d}r^2 - c^2\mathrm{d}t^2$
Hawking's Black Radiation (outside)	lightlike PHOTON (Graviton?) (CBR inside)
$r \mapsto R_S$, $\frac{\mathrm{d}r}{\mathrm{d}t} \mapsto \infty$ (16) (Schwarz-on)	$R(0) = R_H$, $\dot{R}(0) = c$, $0 = \mathrm{d}r^2 - c^2\mathrm{d}t^2$ (Mink-on)
	timelike BRADYON (Galaxy or Electron)
$r < R_S$ (17) (Schwarz-in)	$r(t) < R_H$, $\dot{r}(t) < c$, $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2$ (Mink-in)

see Mink-in (17-in) in Table 1.

The density (1-1bis) is then internal density of *CBH* (see §3-3, §4) which can be written in cubic form of density of dark energy in $\frac{1}{R_H^3}$ and radiation form in $\frac{1}{R_H^4}$:

$$\rho_\Lambda = \frac{3M_H c^2}{4\pi R_H^3} = \frac{3G}{2\pi}\frac{M_H^2}{R_H^4} \tag{1 ter}$$

In the last case it's as if our universe M_H is "in collision" with itself M_H.

2.2. Tri-Partition of NeoMinkowskian Space-Time: TACHYON, PHOTON (GRAVITON) AND BRADYON (Table 1)

We now present a summary (and also a plan) by transposing to our NeoMinkowskian World (spacelike, lightlike, timelike) the 3-partition of zones of usual *SBH* $r > R_S$, $r = R_S$, $r < R_S$ (see Table 1 first column, 15, 16, 17).

In second column of Table 1 we have 3-PARTITION according to velocity and according to space itself.

To each member of Feinberg's Trio (OUT, ON and IN) is attached a specific *MM* (15-out, 16-on, 17-in).

We already have examined the tachyons (§1) and (almost, §2-3) the photons (§2). Finally we will examine non-baryonic bradyons galactic point (§3) and electron §4-5).

If it is almost impossible for an observer to get into a usual *SBH*, it is radically impossible for a (bradyonic) observer to get out of a *CBH*.

Fortunately the interior of *CBH* is precisely the *Universe or the World itself*. Fortunately also the observer is in the right place to measure the *CBR* emission (see abstract).

This the reason why we suggest going beyond the notion the notion of " Hole" and replace it with that of the "W(hole) World" or "The Black Whole Universe" given that it is filled with Cosmological black radiation of the kind *CBR* (emitted from Horizon of Hubble of *CBH* (in $t = 0$).

2.3. Cosmological NeoMinkowskian Scale Hyperbolas and Perfect Cosmological Principle

Given that cosmological kinematics is radial ($r = x$) we can reduce space-time at $2D$ usual diagram (*Otx*) with scale unit Minkowskian Hyperbolas $x^2 - c^2t^2 = 1$ and $c^2t^2 - x^2 = 1$. Which is not usual in NeoMinkowskian space-time is that we have two singular Unit Hyperbolas directly induced by the *CC*.

Let us follow the photon that is emitted (towards the inside (§3-2) in $t = 0$) from R_H. It is received in O at time T_H (no negative time, see §1 6ter).

Both points $(R_H, 0)$ and $(0, T_H)$ determine both singular Unit scale hyperbolas $2D$ (more precisely half hyperbola in the first quadrant, respectively along axis *Ox* and along axis *Ot*):

$$x^2 - c^2t^2 = R_H^2 \quad \text{(bradyonic)} \quad x = ct \quad \text{(photo-gravitonic)}$$
$$c^2t^2 - x^2 = T_H^2 \quad \text{(tachyonic)} \tag{18}$$

We rediscover the 3-partition with singular asymptote ($c = 1$): a point on this asymptote can be photonic or gravitonic as well [4]. We can transform the coordinates of a point-graviton with LT into another point-graviton on asymptote (with zero interval).

The first Hyperbole along *Ox* (Tangent velocity WITHIN the light cone) is a bradyonic Hyperbole (see the Worldline[10] of uniform acceleration §3) whilst the second Hyperbole, along *Ot*, is a tachyonic Hyperbole (Tangent velocity OUT the light cone).

NeoMinkowskian hyperboles are compatible (a priori, see §3) with a Finite Time ($T_H = H_\Lambda^{-1}$) underlying "BIG BANG" but also with a "PERFECT COSMOLOGICAL PRINCIPLE". Indeed we align a finite time T_H on a finite space R_H whilst in Steady State of Gold-Bondi-Hoyle, they align infinite time T_H on a infinite space R_H. In our Hyperbolic Structure (Figure of Escher, see §3), NeoMinkowskian space-time determine in fact a new kind of Steady-Sate.

We must now focus on bradyonic hyperbole (18. It must now be shown that Hyperbole (18) is the global (intergrated) form of local (differential) metric: $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2$ (Mink-in, see Table 1).

(The structure of the paper will follow Feinberg's 3-partition of NeoMinkowskian space time: Tachyon §1, Photon-Graviton §2, Bradyon §3)

3. [BRADYON] Galaxies That Are Approaching a Hyperbolic Horizon (Escher Figures)

Let the tachyons go (We caught the comet by the tail!) and install us in the inside of *CBH* (in the Black Universe) in order to complete NeoMinkowskian pace-time with non-baryonic ($G_{\mu\nu} = 0$) bradyons ("as long as we have not everything, we have nothing"). The Whole Hole is the Whole Universe if we can express ourselves in this way.

3.1. Boundary Conditions for Bradyons, Law of Hubble and Double Special Relativity (THESIS DSR)

In order to introduce the second constant R_H in *SR* we have therefore to define a *DSR* (*Doubly Special Relativity*) with. Unlike standard *DSR* the second constant R_H is not usual Planck's length but Hubble's length R_H directly induced by *GR* with *CC* (if we cancel *CC* (0) the second horizon disappears $R_H = R \to \infty$).

Everything happens as if the bradyons $r(t) < R_H - v(t) < c$ constitute a kind of inverted image (a kind of "Mirror Effect") of tachyons $R(t) > R_H - \dot{R}(t) > c$ with STRICT inequalities in both cases.

The *Boundary Condition* of coexistence of both Horizons (c, R_H) (with standard notations for space points $v = u_1 = v = c\beta$) is as follows:

[10]Minkowski himself (1908) considered that his theory was a cosmological theory of the Whole World (a trajectory becomes a WorldLine).

$$\frac{r(t)}{R_H} = \beta(t) = \frac{v(t)}{c} < 1 \tag{19}$$

Inside the *CBH* the (radial) law of Hubble, coupling "large velocity-large distance ($r = x$)", must therefore govern the expanding of galactic space points of the cosmological fluid (at a fixed time):

$$v = c\beta = H_\Lambda r \qquad \text{(19 bis)}$$

This CONDITION OF RADIALITY[11] (coupling "large velocity-inter-galactic distance") is quite basic. There is no expansion for "short" (non-large) distance and then for Planck's length.

The problem is now that it is difficult to see how a *MM* (see Table 1 $\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}r^2$, Mink-in) could be at the basis of such expanding kinematics (this is our THESIS).

Remember that so far there is only one solution (12) with spacelike signature $(-1,1,1,1)$ and that we are looking for a coupled solution with timelike signature $(+1,-1,-1,-1)$.

Thanks to the "mirror effect" we have a guide with the *acceleration* α_M (10, 11). Indeed if we have tachyonic minimal Horizon R_H (coupled with maximal acceleration α_M), we have now bradyonic minimal acceleration α_M (coupled with maximal Horizon R_H).

3.2. Accelerated Motion of Material Point and Lobachevskian Velocity Space (HYPOTHESIS SR)

Pauli remarks that the Worldline of usual (standard) relativistic Uniformly Accelerated Rectilinear Motion *UARM* in *SR* coincide with usual scale hyperbola (along Ox, 18). Indeed we have the following hyperbolic worldline *UARM* (Minkowski 1908) in inertial system K ($t = 0$, $v = 0$, $x = R$):

$$r^2 - c^2t^2 = x^2 - c^2t^2 = R^2 = \frac{c^4}{\alpha^2}, \quad \frac{R\alpha}{c^2} = 1 \qquad \text{(anti-18)}$$

In usual Minkowskian *UARM* ($R \to \infty$, $\alpha \to 0$) α represents a constant centrifugal (proper in K') acceleration $\alpha = \frac{\mathrm{d}\gamma(t)v(t)}{\mathrm{d}t}$ from which Minkowski deduced in 1908 by a double integration (18 bis). Minkowski's proper time of accelerated particle interests us particularly because it is directly connected with $c^2\mathrm{d}t^2 - \mathrm{d}x(t)^2$ ($v = \frac{\alpha t}{\sqrt{1+\frac{\alpha^2t^2}{c^2}}}$ with $t = \tau = 0$):

$$\mathrm{d}\tau = \mathrm{d}t\sqrt{1-\frac{v^2(t)}{c^2}} = \mathrm{d}t\sqrt{1-\beta^2(t)} \tag{20}$$

[11]This is the basic concept of Radial: Expansion only applies on a very large scale (distance and velocity). In fact this paper is devoted to translations (dark energy). The next will be devoted to rotation (dark matter, see conclusion). The one-third factor $\frac{1}{3}$ ($c\sqrt{\frac{\Lambda}{3}} = H_\Lambda$) are there to remind us that we are in space $3D$ (see also initial ratio, 2 bis, electron and density-pressure for *EM* radiation §5).

Explicitly we have $\tau=\int_0^t\sqrt{1-\frac{v^2(t)}{c^2}}\mathrm{d}t=\frac{c}{\alpha}\ln\left(\frac{\alpha}{c}t+\sqrt{1+\frac{\alpha^2t^2}{c^2}}\right)=\frac{c}{\alpha}\sinh^{-1}\frac{\alpha t}{c}$.

And then a hyperbolic sinus written with hyperbolic (index h for hyperbolic) velocity $w_h=\frac{\alpha\tau}{c}$ [12]:

$$\tau=\frac{c}{\alpha}\sinh^{-1}\frac{\alpha t}{c}\quad\Rightarrow\quad t=\frac{c}{\alpha}\sinh\frac{\alpha\tau}{c}\tag{21}$$

Using for hyperbola (anti-18) the hyperbolic polar coordinates (R,w_h) (Varicak, Borel):

$$x=R\cosh w_h,\quad t=\frac{R}{c}\sinh w_h\tag{21}$$

$$\Rightarrow\quad \mathrm{d}x=R\mathrm{d}w_h\sinh w_h,\quad \mathrm{d}t=\frac{R}{c}\mathrm{d}w_h\cosh w_h\tag{21-bis}$$

we deduce by differentiation (we refer 18 for Hyperboles, 19 for inequalities, 20 for metric):

$$\mathrm{d}s^2=c^2\mathrm{d}t^2-\mathrm{d}x(t)^2=R^2\mathrm{d}w_h^2\tag{20-MM?}$$

which corresponds to proper time (20) and "looks like" to Mink-in signature $(+1,-1,-1,-1)$.

We specify that 20-MM is not a true *MM* because *either* "w_h is variable with fixed R (one hyperbole)" *or* "R is variable with fixed w_h (a continuum set of hyperboles $x\le R$)".

This is the reason why Rindler () introduces a *NON-MM* for his cosmological model based on (*UARM*):

$$\mathrm{d}s^2=R^2\mathrm{d}w_h^2(t)-\mathrm{d}R(t)^2\tag{20-Rindler}$$

There is no singular hyperbole R_H (18)[13] ($\mathrm{d}R\neq 0$).

Already in 1913, M. Born shows that the basic equations of *UARM* $x\ge R$ are also that of Rigid Motion $x\le R$ of the axis *Ox* (*Or*) of Rigid motion of the "rod" *OR* $x\le R$ (a kind of Einstein's successive boost).

With Rigid Motion (a continuum set of hyperboles $x\le R$) we are very close to concept of metric (see demonstration, §3-3).

Let us now return to the motion of accelerated material point and focus on his hyperbolic velocity which is defined by the inverse hyperbolic tangent (see 21)

$$\begin{aligned}&\frac{\mathrm{d}x(t)}{\mathrm{d}t}=\beta(t)=\tanh w_h(t)\quad\Rightarrow\quad w_h(t)=\tanh^{-1}\beta(t)\\&\Rightarrow\quad w_h(t)=\ln\sqrt{\frac{1+\beta(t)}{1-\beta(t)}}=\ln k_h(t)\end{aligned}\tag{22}$$

We have then a centrifugal proper acceleration which is derivative, with respect to proper time, of hyperbolic velocity:

[12]Or Rapidity of Robb: $w_1+w_2=w$.

[13]Rindler introduces, on the same basis (*UARM*) induces also a Black Hole but not a *CBH*.

$$\frac{c\mathrm{d}w_h(t)}{\mathrm{d}\tau} = \alpha \tag{23}$$

We have therefore here an accelerated Global (hyperbolic) motion which seems to correspond to (21-MM)? This kinematic *UARM* without dynamic (gravitation) is very strange (see Pauli, §3-3-3).

Let us now take a look to the geometry underlying *UARM*.

It is well known that in pseudoEuclidean *SR*, the space itself is Euclidean whilst the "space of velocity" is non-Euclidean, in Lobachevskian meaning (relativistic addition of velocities)

More precisely according to Borel [9] global "space of speed" is characterized by a negative curvature $-\frac{1}{c^2}$ given by the inverse of square of light velocity (velocity of light being the radius of curvature.

It is then useful to develop the geometrical aspect of the hypothesis. In Beltrami's disc model for Hyperbolic geometry we introduce Cayley-Klein's hyperbolic DISTANCE d_h induced from the CROSS-RATIO formula is given by [10]:

$$d_h = \tanh^{-1}\frac{d}{d_H} \tag{24}$$

where d is usual distance defined smaller than $d < d_H$ the radius Horizon. If we apply this definition (d_h, d and d_H) to "space of velocity" (w_h, v and c) we rediscover (22):

$$cw_h = c\tanh^{-1}\beta = c\ln\sqrt{\frac{1+\beta}{1-\beta}} \tag{22 bis}$$

We underline that in usual *SR* the velocity space is a true complete Hyperbolic space (with distance introducing by cross ratio) in strong sense of the term non-Euclidean (Penrose [11]).

This is not the case of the pseudo-hyperbolic spaces defined in Friedman's equation with $K = -1$ (where scale factor is given by a sinh (20) and not a tanh (24) which defines a Non-Euclidean Distance (24) (see §3-3-2) (Note in 21 bis that the time and the space are respectively given by a sinh and a cosh).

3.3. DEMONSTRATION: From SR without Gravitation to Gravitational DSR

The demonstration must be done in three stages (Gravitation and Geometry):

1) Hubble constant with its corresponding global Gravitation field (§3-3-1).

2) Law of Hubble itself with its corresponding global Geometry (§3-3-2).

3) Finite Time from negative Lobatchevskian curvature: tanh versus sinh (§ 3-3-3).

3.3.1. NeoEinsteinian Principle of GLOBAL Equivalence between Gravitational Field and Minimal Acceleration

Let us introduce now NeoMinkowskian singular bradyonic hyperbole (18) in *GR* with *CC* or, in other words, the second invariant (maximal) space R_H (or minimal centrifugal acceleration α_M) in *DSR*:

$$r^2 - c^2t^2 = x^2 - c^2t^2 = R_H^2 = \frac{c^4}{\alpha_M^2}, \quad \frac{R_H \alpha_M}{c^2} = 1 \tag{18-R_H}$$

Let's look closely at the limit (19) $r < R_H$. In truth we have first $R < R_H$. (anti-18, 18-R_H). Given that we have (Rindler 20, [12]) $r \le R$ we deduce finally (19) $r < R_H$.

We have therefore now a true metric (Born's rigid motion becomes an expanding motion of space):

$$\mathrm{d}s^2 = c^2\mathrm{d}t^2 - \mathrm{d}x^2 = R_H^2 \mathrm{d}w_h^2 \tag{20 G-MM}$$

with hyperbolic coordinate (R_H, w_h) on the basis ($\mathrm{d}R_H = 0$). In tensorial form

$$\eta_{\mu\nu}^{+} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ (20 G-MM).}$$

This *MM* (17-in) corresponds to a global gravitation field.[14] (*G-DSR*) with Hubble Horizon:

$$\frac{\mathrm{d}w_h(t)}{\mathrm{d}\tau} = H_\Lambda = \frac{\alpha_M}{c} = \frac{2GM_H}{cR_H^2} \tag{23-G-DSR}$$

The answer to first Pauli's question ("*To which global gravitational field in GR does this UARM corresponds*[15]? [13])" is then (23) and therefore (33). We answer with a *global* (*cosmological*) NeoEinsteinian principle of equivalence (23) between Milgrom's minimal centrifugal acceleration ($\alpha > \alpha_M$) and expanding gravitation field within the *CBH* (Universe).

Part one of the thesis (§3-3-1) is thus demonstrated: we have Hubble constant but we do not yet have Hubble law itself (19bis) (§3-3-2). We are at this stage close (except Planck's mass) to cosmology of flat Universe [5].

3.3.2. CC Induces in G-DSR a Global Negative Lobachevskian Curvature of Space (Escher)

Until now our NeoMinkowskian Universe seems to be a Flat Universe, in the meaning of Tatum [5] because LOCAL Einsteinian curvature tensor is zero ($G_{\mu\nu} = 0$). There is no contradiction because we will prove now that there is a GLOBAL curvature for Euclidean 3D space (Penrose).

After the *G* of Gravitation let us now deal the *G* of Geometry. What about the geometry of space $3D$ itself in *G-DSR*?

Let us apply formula of Cayley-Klein's hyperbolic distance (24), respectively with (s_h, r and R_H), which can be considered here as the singular RADIUS OF

[14]We thus ended the forced cancellation of the gravitational NeoMinkowskian density (the putsch, note 1). We follow the same path as Unruh ([8]: Behind Hawking's local radiation there is a kinematic of uniform HYPERBOLIC acceleration. We transform therefore Unruh "local" effects into global (cosmological) effects (we change also the sense—and the meaning—of the emission).

[15]Historically Pauli did not find this field because he was using the *GR* WITHOUT *CC*!

ESCHER DISC (or Sphere):

$$s_h = R_H \tanh^{-1}\frac{r}{R_H} = R_H \ln\sqrt{\frac{1+\frac{r}{R_H}}{1-\frac{r}{R_H}}} = R_H \ln\sqrt{\frac{1+\beta}{1-\beta}} = R_H w_h \tag{25}$$

The thesis, the (global) law of Hubble (19-19 bis)

$$c\beta = c\frac{r}{R_H} = H_\Lambda r \tag{25-G-DSR}$$

is then demonstrated (large distance-large velocity)! The GLOBAL (negative) curvature ($\frac{-1}{R_H^2} = -\frac{\Lambda}{3}$) of the Whole Universe ($3D$) is then given by the CC itself (this is the end for the putsch, note 1).

THIS IS AN EXTRAORDINARY RESULT: In *G-DSR* it is not only the "velocity space" (22 bis) but the "space itself" (25) that is Lobachevskian.

We obtain a whole (Global) Lobatchevskian "distance" (a scale factor) with hyperbolic function of tanh (22 and 24) and not a scale factor defined with sinh (21 bis and Friedman's model with $K = -1$ (see Penrose [11]). There is no contradiction between the cancellation of the local curvature factor $K = 0$ in RW metric (note 5).

Beltrami's abstract disc is concretized by famous aesthetic hyperbolic Escher's disc. The fact that the W(hole) Universe (with our observer) is inside a *CBH* is not tragic because first we are enlightened by *CBR* and then we fit in a harmonious (in Penrose's meaning) hyperbolic figure (disc) of Escher (Cayley-Klein' s hyperbolic distance). Hyperbolic Universe is then compatible with radial expanding (without end) of galaxies and with Perfect Cosmological Principle (a horizon of finite space is aligned on a horizon of finite time).

While the tachyons move away as they escape (without end) from gravitational field (25) the galaxies move away infinitely (hyperbolic velocity) as they approach (without end) the (hyperbolic) horizon (§3-3-2) according to the law of Hubble (19-19 bis).

Milgrom's minimal acceleration [14] or Hubble's constant are then effects of hyperbolic (negative) curvature of space $3D$.

By taking into account the *CC* (Einstein 1917) in *GR* (Einstein 1915) and therefore a minimal acceleration α_M in equation of geodesic (Einstein 1915):

$$\frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + \Gamma^\mu_{ij}u^i u^j = \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} + 0 \neq 0 \quad \Leftrightarrow \quad \frac{\mathrm{d}u^\mu}{\mathrm{d}\tau} \neq 0 \tag{26}$$

Exactly as in the flat universe [5], Christoffel Γ^μ_{ij} cancel (with again disappearance of $u^i u^j$ (see 0-MM) but a radial component of acceleration does not cancel:

$$\frac{\mathrm{d}u^1}{\mathrm{d}\tau} \neq 0 = \alpha_M = c^2\sqrt{\frac{\Lambda}{3}} \tag{26 bis}$$

So geodesics represent inertial systems with Straight lines "in global curved space" (or Hyperbolic Straight Lines). All our border relations are validated by hyperbolic global curvature (14, 14 bis, 7, 8, 12).

We have now not only a (Radial) Kinematic *UARM* (*DSR* based on the Duo (c, R_H), but also a double (Scalar) Dynamic Newton's laws (*G-DSR*) (8, 13 and 14):

$$F_G = \frac{c^4}{2G} = M_H \alpha_M, \quad f_G = m_{NB}\alpha_M \qquad \text{(13 bis-14 bis)}$$

Both are hyperbolic invariant but the second (see 14, §1-4) between a micromass m_{NB} and macroacceleration α_M is enigmatic.

3.3.3. Hyperbolic Tangent versus Hyperbolic Sinus: The "Big Bang", An Effect of (Negative) Curvature?

We have to apply the same hyperbolic definition (24) for space (s_h, r and R_H), and time (τ_h, t and T_H) (contrast between sinh (21) and tanh (27) is obvious, see note 5):

$$\tau_h = \tanh^{-1}\frac{t}{T_H} = T_H \ln\sqrt{\frac{1+\frac{t}{T_H}}{1-\frac{t}{T_H}}} = T_H \ln\sqrt{\frac{1+\beta}{1-\beta}} = T_H w_h, \quad s_h = c\tau_h \qquad (27)$$

The comobile (proper) time (in *K'*)[16] τ_h in function of a parametric time t (in *K*). Parameter values of universal time in *K* are included between 0 and T_H ($0 < t < T_H$) in bradyonic Universe ("whole hole").

THIS IS AN EXTRAORDINARY RESULT!: *The finite time* T_H ("Big Bang") *becomes an effect of the* (*negative*) *curvature of hyperbolic straight line of time* (27).

Physical comobile time τ_h (also valid for comobile distance $s_h = c\tau_h$) is obviously INFINITE.

Since then the effective expanding universe (with infinite s_h and infinite τ_h) is a **Steady State of Hoyle** with (see §2-3) Perfect Cosmological Principle (with constant density and *CBR* fluid defined at rest in *K*, see our deduction of the observed Temperature §6).

The (double) derivative (10) with respect to the comobile time τ (represented here with a point):

$$\dot{k}_{Bradyons}(t) = H_\Lambda k_{Bradyons}(t), \quad \ddot{k}(t) - H_\Lambda^2 k(t) = 0 \qquad (28)$$

which is internal *EE* corresponding to basic external *EE YP*12 (4), (6-6 bis).

[16]Pauli was intrigued by the notion of proper acceleration in proper system *K'* which follows the fluid (medium) at rest in *K* (in *SR*): *In relativistic kinematics we will naturally describe by as "uniformly accelerated" a motion for which in a system K' moving with the medium or particle is always of the same magnitude* α. *The system K' is a different one at each instant; for one and the same Galilean system K the acceleration of such a motion is not constant in time* [13]. In *SR* *"The system K' is a different one at each instant"* (successive *K-K'* Lorentz boost). Pauli considers then a single global (hole) system *K'* is a non-Galilean system and moves therefore to *GR* (without *CC*. which was not fashionable in his day, see note 16). According to Pauli, the proper time becomes then a COMOBILE time relative to (bradyonic) medium.

($k = \mathrm{e}^{H_\Lambda \tau}, \dot{k} = H_\Lambda \mathrm{e}^{H_\Lambda \tau}$). There is a "*principle of correspondence*" tachyons-bradyons (taking care of the correspondence of times):

$$R(t) \leftrightarrow R_H k(t), \quad \dot{R}(t) \leftrightarrow R_H \dot{k}(t) = ck(t) \tag{28 bis}$$

The "fatal" objection of the note 3 is therefore refuted: Our basic equation *YP*12(4) is perfectly relativistic (in *GR* and in *DSR* meaning as well[17]).

In *G-DSR* Doppler Galactic Redshift formula $z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_R - \lambda_E}{\lambda_E}$) with fixed time (Reception and Emission), $t_s = t_R - t_E$ is given by hyperbolic factor k_h which coincide with *radial* (note 10) or longitudinal Doppler factor of Bondi [15] k_B:

$$\begin{aligned} k_B &= 1 + z = \frac{r(t_R)}{r(t_E)} = \frac{\lambda_R}{\lambda_E} = \mathrm{e}^{w_h} = \sqrt{\frac{1+\beta}{1-\beta}} \\ &\simeq 1 + \beta + \frac{1}{2}\beta^2 + \frac{1}{2}\beta^3 + O(\beta^4) \\ &\simeq 1 + H_\Lambda \frac{r}{c} + \frac{1}{2} H_\Lambda^2 \frac{r^2}{c^2} + \cdots \end{aligned} \tag{29}$$

We notice a quadruple contrast.

In contrast with radial Doppler formula in *SR* the velocity β is the velocity of a space-point.

In contract with the other Redshift z formulas in *GR*, velocity β is strictly infra-luminous (<1).

In contrast with de Sitter, where there is not singular velocity for space-points) in *G-DSR* there is a double singular velocity for light wave (Photon) and for gravific wave (Graviton), as well.

In contrast with de Sitter, the expanding (29) *G-DSR* is only valid for large distances coupled with large velocities[18].

4. Intermediate Conclusions: Lepton Electron Is Not a Baryon

Our NeoMinkowskian non-baryonic solution of *GR* with *CC* defines a Universe characterized by a (very large) finite Hyperbolic Hubble Horizon R_H at its (singular) origin. Hyperbolic Universe is incompatible with inflation but compatible with the most undeniable observations: Galactic expansion and emission of non-galactic black radiation (*CBR*) a (very long) finite time R_H ago.

[17]Among the two speeds $(c\beta_{bradyon}, ck_{tachyon})$ only the first is physical (<*c*), the second (>*c*) is a purely mathematical construction (a phase velocity). According to de Broglie (§5-4), the supra-luminous velocity of electron (velocity of phase $\frac{c^2}{v}$) is coupled with its infra-luminous (velocity of group $v < c$) of electron. Can we replace the galactic point (Macro-force, 13) with an electron (micro-force 14)?

[18]Both elongation of lengths and elongation of wavelengths are in the same boat exactly as in of Poincaré's Elongated Light Ellipsoids By applying the basic condition large velocity $v \approx c$ -large distance $r \approx R_H$ they acquire a cosmic scope. According to Poincaré elongation (expansion) with a longitudinal factor *k* is a consequence of Lorentz' contraction of factor γ^{-1}. Elongated Light Ellipsoids will be not developed here (see annex 2).

Our paper could stop here.

The situation is somewhat disappointing because at this stage, we are not providing any new (numerical) information on the emission of the basic *CBR*.

The physicist must always be in search of unity: On one side we have Neo-Minkowskian Vacuum with Gravitational Waves $(c, G, \Lambda, R_H, \alpha_M)$ whilst in the other side we have usual *EM* Minkowskian Vacuum (withOUT Gravitational Waves) with usual *EM* constants (*c*, permittivity ε, permeability μ, impedance Ω, WITHOUT CHARGE *e*). Both sets seem irremediably disjoint.

The usual series of "Gc" $\frac{c^2}{2G}$, $\frac{c^4}{2G}$, $\frac{c^5}{2G}$ respectively linear density, cosmological force ($F_G = \frac{c^4}{2G}$, 13), power of vacuum…(see L. Kostro) is not a real synthesis (only *c* is in common, see 30).

Is there a missing element that is neither on one side (Macro) nor the other side (micro)?

ANSWER: the charge *e*. And therefore the concrete electron with charge *e*, mass m_e and "classical" radius r_e which is absent in *SR* and (until now) in *G-DSR* as well.

According to Einstein's famous quotation: "*electron* (*e*, m_e, r_e) *is a stranger in classical electrodynamics*"(Minkowskian Vacuum is without charge).

Until now NeoMinkowskian model is without non-baryonic light emitter.

Without the missing link, Poincaré's gravific electron (pressure of ether, §5), we cannot, at this stage make, any numerical evaluation or prediction.

4.1. A (Stable) "Cosmological" Electron to Short Circuit the (Unstable) Primitive Atom?

Let's test the candidate electron [16] in our equations $m_{NB} = m_e$ (14, 14 bis):

$$f_G = m_e \alpha_M = 2 m_e G \frac{M_H}{R_H^2} \qquad (14\ \text{ter})$$

This electro-dynamics (hyperbolic) force involves a connection between Macro (M_H) and micromass m_e. It defines a "Cosmological (Global) Electron" which could answer to second question of Pauli (see first question, §3-3-1). "*To which global EM field does ELECTRON in UARM corresponds*?".

Pauli's answer (p 93): "*Hyperbolic motion thus constitutes a special case for which there is no formation of a wave zone nor any corresponding radiation*". Pauli find a ZERO magnetic field (and then no radiation[19]) and hastens to specify that locally (parabolic) the electron emitted a radiation (see note 16 and also numerical annex).

In NeoMinkowskian limit of *GR with CC*, we can suspect a link between this

[19]Einstein's Boost is based in *SR* on infinitely slow acceleration ($\alpha_M \to 0$), Einstein 1905 Pauli's deduction is logical because *UARM* consists in a series of infinitely slow boost globally without emission). In *G-DSR* we have a Big (cosmological) Boost with slow acceleration (α_M) globally with emission (*CBR*). We remark that galactic Milgrom's minimal acceleration is here moved to cosmological radically relativistic (*DSR* and *GR*) framework.

cosmological electron and cosmological radiation? The paper could then not stop here.

Note for this intermediate conclusions that Lorentz's electron has remained perfectly unperturbed (*stable and elementary*) for more than a century. Moreover Poincaré's discovers an enormous (negative) pressure (§5-3) that ensures stability of the free electron in 1905.

Our future integrated (stable) electron is the ideal candidate to Short-Circuit (unstable) Planck's particle (Lemaître's primitive atom)?

Reset then by using following writing of *MM*:

$$\mathrm{d}s^2_{tachyon} = \frac{2GM_H}{R_H}\mathrm{d}t^2 - \mathrm{d}r^2 \quad (10) \qquad \Rightarrow \quad \mathrm{d}s^2_{electron} = \frac{e^2}{m_e r_e}\mathrm{d}t^2 - \mathrm{d}r^2 \qquad (30)$$

We could then proceed to an original electro-gravific synthesis under the presidency of c (in other word a new non-usual approach of Einstein's unitary field):

$$c^2 = \frac{2GM_H}{R_H} = \frac{e^2}{m_e r_e} \qquad (30\ \text{bis})$$

4.2. The First and the Second Density

This enigmatic force f_G of synthesis will have to be placed, next to $F_G = \frac{c^4}{2G} = F_\Lambda$ (13) in the framework of usual electrostatic force f_e coupled with (electro-)gravitational force f_{Ge}:

$$F_G = \frac{c^4}{2G} \quad (1) \qquad f_e = \frac{e^2}{r_e^2} \quad (2) \qquad f_{Ge} = \frac{Gm_e^2}{r_e^2} \quad (3) \qquad (31)$$

with the ratio of forces $\frac{f_e}{f_{Ge}} = \frac{e^2}{Gm_e^2} = \kappa_{Ge} \approx 4.1604\times10^{42}$ (see annex, numerical values Gauss-cgs) which becomes a basic universal constant. We have the following trio with:

$$F_G \xleftarrow{\kappa_{Ge}} f_e \xrightarrow{\kappa_{Ge}} f_{Ge} \qquad (32)$$

An electric force well framed by two gravific forces $\frac{2F_G}{f_e} = \frac{f_e}{f_{Ge}} = \kappa_{Ge}$. Most physicists judge that the third force is ridiculous small compared to the other two (we note that f_G is very close to f_{Ge} (see annex 1)).

This balance of forces change completely if we consider the corresponding balance of densities (pressures) ρ_Λ (1 bis) and $w_{Ge} \sim \frac{Gm_e^2}{r_e^4}$ (the ratio $\kappa_{Ge} = \frac{w_e}{w_{Ge}} = \frac{f_e}{f_{Ge}}$ does not change)

$$w_e \xleftarrow{\kappa_{Ge}} \rho_\Lambda \xleftarrow{\Omega} w_{Ge} \qquad (\text{anti-32})$$

The ratio $\frac{w_{Ge}}{\rho_\Lambda}$ (see 12, the "unique" solution) being directly defined with

$3\left(\frac{f_{Ge}}{f_G}\right)^2$ of the order of $\Omega \simeq 5\times 10^{-5}$ (49 and 74, see numerical annex 1).

How can we justify the entry into scene of electron with this *second* density (pressure) in our NeoMinkowskian Universe?

At the beginning of §3 we claimed that "we caught the comet (devil) by the tail (tachyons)". We have now reached the head with bradyons:

$$T_{\mu\nu}^{VACUUM} = \rho_\Lambda \eta_{\mu\nu}^{-} = \rho_\Lambda \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ (tachyons)}$$

$$\eta_{\mu\nu}^{+} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ (bradyons)} \tag{33}$$

We draw attention to the fact that there is "tachyonic" FIRST density (dark energy) but until now no "bradyonic" (electro-photonic) SECOND density (see 57 bis).

There is no multiplier coefficient for timelike metric (Mink-in) unlike (12) for spacelike metric (1-1 bis, the Unique solution whatever the velocity). However for bradyons (and photons) inside the (W)Hole Universe we have another metric and why not another perfect fluid *EM* tensor, another density, another pressure with $T_{\mu\nu}^{EM} = p_{em}\eta_{\mu\nu}$?

In other words: would the metric (33-bradyons) be hidden in Perfect Neo-Minkowskian fluid?

5. The Second Density: From Electronic Density to Density of Gravific Waves

Cosmologists distinguish three different types of Fluid which corresponds to three periods of the universe: 1) the dust or inconsistent matter ($p=0$), 2) the dark energy $p+\rho=0$ (anti-1) and 3) the so-called "Radiation" $p_{em}=\frac{1}{3}w_{em}$ (35) (generally reported, in cosmological literature to a "radiative period of Universe")[20].

In cosmological literature the fluid (37) is always called "Fluid of Radiation" (always written in Riemannian metric $g_{\mu\nu}$).

$$p_{em}=\frac{1}{3}w_{em} \quad\Rightarrow\quad T_{\mu\nu}^{EM}=\frac{4}{3}w_{em}\frac{u_\mu u_\nu}{c^2}-\frac{1}{3}w_{em}g_{\mu\nu} \tag{34}$$

Nothing to do (at first sight) with electron.

[20]There are other fluids such as the ultra-relativistic electronic gas which seem not taken in consideration by cosmologists. So far no trace of the electron (charge e and mass m_e) in cosmological usual representations.

5.1. Hidden Electron and Hidden Graviton in NeoMinkowskian Perfect Fluid "Radiation"

We absolutely need to introduce the light (emission of *CBR* in $t=0$) in Neo-Minkowskian Fluid:

$$T_{\mu\nu}^{EM}=\frac{4}{3}w_{em}\frac{u_\mu u_\nu}{c^2}-\frac{1}{3}w_{em}\eta_{\mu\nu} \tag{35}$$

It is generally claimed that, if we replace $g_{\mu\nu}=\eta_{\mu\nu}$ in (34) from a Riemanian Fluid to a NeoMinkowskian Fluid the gravitation (and then gravific waves) would be eliminated (see the putsch note 1). It is obviously wrong because NeoMinkowskian limit imposes only a LIGHTLIKE 4-vector. The latter can correspond to a photon or could correspond to a (hypothetical) graviton (like light) as well.

In other words, for a classical light wave or a (hypothetical) "classical gravific wave" as well:

$$\lambda_{Ph}=\frac{\hbar}{P_{ph}},\nu_{ph}=\frac{E}{\hbar}\quad\left(\lambda_{Ph}\nu_{ph}=c\right)\qquad\lambda_{Ge}=\frac{a}{P_{Ge}},\nu_{Ge}=\frac{E_{Ge}}{a}\quad\left(\lambda_{Ge}\nu_{Ge}=c\right) \tag{35-36}$$

(a being dimensionally an ACTION). We are therefore perfectly entitled to write next to "Radiation" Fluid (35) a "Gravitation" Fluid (36):

$$T_{\mu\nu}^{GW}=\frac{4}{3}w_{Ge}\frac{u_\mu u_\nu}{c^2}-\frac{1}{3}w_{Ge}\eta_{\mu\nu}\quad\Rightarrow\quad p_{Ge}=\frac{1}{3}w_{Ge} \tag{36}$$

where the index *GW* refers to Poincaré's Gravitational Waves and index *Ge* refers to Gravific-electron because there is a hidden electron in "Radiation" fluid of cosmologists (§5-1).

Photon and Graviton are compatible with $G_{\mu\nu}=0$ (without rest mass) whilst Electron $p_e<\frac{1}{3}w_e$ (with leptonic rest mass) seems incompatible with $G_{\mu\nu}=0$.

The relationship (36) $T_{\mu\nu}^{GW}$ in *MM* (33) is much more restrictive than it seemed at first glance.

We will prove that Poincaré's *GW* not only COULD exist but that they MUST exist (determined action a and λ_{Ge}).

The difficulty being here more logical than mathematical, we resume the situation with the following Table 2 (tachyons are gone):

Table 2. Photon, electron and graviton.

New trio ("GraPhoTron")	ON or IN singularity
(spacelike)	($T_{\mu\nu}=\rho\eta_{\mu\nu}$, $G_{\mu\nu}=0$)
LIGHTLIKE (PHOTON or GRAVITON)	$p_{em}=\frac{1}{3}w_{em}$, $p_{Ge}=\frac{1}{3}w_{Ge}$
	$\left\|E_{Ph}^2-P_{Ph}^2c^2\right\|=0$ and Hypothetical $\left\|E_{Ge}^2-P_{Ge}^2c^2\right\|=0$
timelike ELECTRON	$p_e<\frac{1}{3}w_e$, $\left\|\gamma^2c^2-\gamma^2\beta^2c^2\right\|=c^2$

5.1.1. Hidden Electron in (Cosmological) Fluid "Radiation"

For the first time we take now explicitly in consideration TIMELIKE 4-vectorial writing for velocities in the so called Radiation fluid (38).

Let us try to transpose the notations of the previous Table 2 $\dot{R}(t),\dot{r}(t)$ using those of radial timelike 4-vector velocity that can be reduced (u_0,u_1) to temporal u_0 and spatial component u_1 (we stay at $3D$ because we have $w_{em}=3p_{em}$).

Given that it is impossible to put directly the velocity of light c in such a timelike 4-vector, the radical singularity of our basic border "$v=c$" is then *structurally* inscribed in temporal component $u_0=c$ for an "electronic point" (at rest) $u_1=0$.

There is then a hidden electron in Cosmological tensor of "Radiation" (a cosmological electron?). Let us see this crucial point in details:

$$T_{\mu\nu}^{EM}=\begin{pmatrix} w_{em} & 0 & 0 & 0 \\ 0 & p_{em} & 0 & 0 \\ 0 & 0 & p_{em} & 0 \\ 0 & 0 & 0 & p_{em} \end{pmatrix}=\begin{pmatrix} \frac{4}{3}w_{em} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}-p_{em}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \quad (37)$$

1) The first tensor (left member) is usual EM tensor of radiation with null trace $p_{em}=\frac{1}{3}w_{em}$. This is the reason why the perfect fluid (38) is called "Radiation" in cosmological literature.

2) The second tensor looks like that of an "electron" at rest $u_1=0$ which would hide behind its density (see Poincaré, §5-2).

3) The third tensor (of pressure) with timelike MM is exactly *the one we were looking* (33) in Gravitational *G-DSR* (§3-3, 23-G and 33) with a (possible) way to the determination of the missing coefficient (or the second density).

This triple statement is not very original because it corresponds exactly to that of Poincaré in 1905 ("La dynamique de l'électron", §5-2). So far we have, at this stage an abstract electronic point but not yet a concrete electron (e,m_e,r_e).

Let us remark that we can put in (35 or 37) the pressure to the left:

$$T_{\mu\nu}^{EM}+p_{em}\eta_{\mu\nu}=\frac{4}{3}w_{em}\frac{u_\mu u_\nu}{c^2} \quad (37\ \text{bis})$$

In details:

$$\begin{pmatrix} w_{em} & 0 & 0 & 0 \\ 0 & \frac{1}{3}w_{em} & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_{em} & 0 \\ 0 & 0 & 0 & \frac{1}{3}w_{em} \end{pmatrix}+p_{em}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}=\begin{pmatrix} \frac{4}{3}w_{em} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad (37\ \text{bis})$$

EM_RADIATION +?GRAVIFIC? (positive) PRESSURE- $\Rightarrow$?FREE ELECTRON?

Given that the putsch was canceled, we have also from (36) a hidden electron in Gravitation Field:

$$T^{GW}_{\mu\nu} + p_{Ge}\eta_{\mu\nu} = \frac{4}{3} w_{Ge} \frac{u_\mu u_\nu}{c^2} \tag{37 TER}$$

This analysis shows that in "Radiation" tensor of Cosmologists there is not only a hidden electron but also a connection of this electron with gravific waves.

5.1.2. Hidden Electron in (Cosmological) Black Radiation

The concrete radiation in our cosmological problematic is black radiation in *CBR*. Let us remark, in this respect, that the situation of concrete electron (e, m_e, r_e) is exactly the same. All formulas of Planck's black body are with Planck's constant h and without (e, m_e, r_e).

The concrete electron (e, m_e, r_e) is hidden (behind or below h) while the black radiation is emitted by electronic oscillators[21].

Everything happens as if concrete electron is excluded both in the fluid and in the black body (for example in the formula of Stephan-Boltzmann, see §6 and annex 1).

Let us finally note that according cosmological usual "*Isentropic Expansion of Spherical CBR*" (see basic Equation (1)-anti(1)):

$$\begin{aligned} &\mathrm{d}U + \frac{1}{3} w_{em} \mathrm{d}V = T\mathrm{d}S = 0, \\ &\mathrm{d}\left(w_{em} \frac{4}{3} \pi R^3 \right) + \frac{1}{3} w_{em} \mathrm{d}\left(\frac{4}{3} \pi R^3 \right) = 0 \end{aligned} \tag{38}$$

the variable density w_{em} depends on variable Radius (in $\frac{1}{R^4}$) connected with variable Temperature T (in degrees Kelvin K) with formula of Stephan-Boltzmann (see annex 1):

$$w_{em} = \sigma_{Stephan} T^4 \quad 39)$$

(see annex 1, 80)

$$w_{em} \sim \frac{1}{R^4} \quad \Rightarrow \quad T^4 \sim \frac{1}{R^4} \quad \Rightarrow \quad RT = cte \tag{40}$$

Remember that we have a fixed hyperbolic horizon R_H (directly induced by *CC*, $\rho_\Lambda = \frac{3G}{2\pi} \frac{M_H^2}{R_H^4}$) which could correspond to a new basic constant Temperature T_K of our Black Universe (§6) do not confuse with Time of Hubble T_H). Everything happens as if Radiation Fluid and Black Radiation were the two sides of the same coin (electronic).

5.2. Synthesis between Poincaré's Gravific Electron and Poincaré's Gravific Waves

We have to find a new synthesis for our Trio: ELECTRON-PHOTON

[21]Given that non-baryonic ($G_{\mu\nu} = 0$) Redshift is based on light emission by galactic... baryons, we are missing non-baryonic emitters. Leptonic electron (a renowned emitter!) could be a good candidate §3-6 and §4).

(L-Wave)-GRAVITON (G-Wave). In order to do that we need to start a concrete electron (e, m_e, r_e).

5.2.1. Poincaré's Historical Induction of the Mass of Electron with Gravific Pressure

In 1905 in his basic paper on "La Dynamique de l'Electron, Poincaré is looking for a determination of mass m_e electron from its *EM* emitted field (July, §6 Lorentz' Contraction, see note 20). He discovers that a purely *EM* induction of the mass is not possible because we have to take into account a strange *Non-EM* pressure (probably) of gravitational origin [4].

From Energy-Impusion tensor $T_{\mu\nu}^{EM}$ Poincaré notes (with *LT*) that energy and impulsion of a purely *EM* Electron are not transformed ($E_0 = mc^2$) as the components of a timelike 4-vector: it appears parasitic factors 1/3, 4/3:

$$E_e = \gamma\left(1+\frac{1}{3}\beta^2\right)E_0, \quad P_e = \frac{4}{3}\gamma\beta E_0 \xrightarrow{\text{Gravific Pressure}} E_e = \gamma E_0, \quad P = \gamma\beta E_0 \tag{41}$$

Poincaré then adds to the *EM* tensor a *Non-EM* tensor in such a way that these parasitic thirds are eliminated (41). *Mathematically it means that the diagonal terms of new tensor* ($T_{\mu\nu}^{EM} + T_{\mu\nu}^{Non\text{-}EM}$) *are compensated, except the first one* (00) $w_{em} = \frac{1}{8\pi}E_l^2 = \frac{1}{8\pi}\left(\frac{e}{r^2}\right)^2$ (Electric field E_l) in the system of electron at rest ($w_{em} = \frac{E_l^2 + H^2}{8\pi}$) with $H = 0$, see 45 and Landau 31-5, p106, [17]).

Usually 4-tensor Energy-Impulsion can be reduced to a 4-vector Energy-Impulsion only in the absence of a charge (Minkowskian Vacuum is without charge, see introduction). Thanks to Poincaré's Gravific Pressure, 4-tensor Energy-Impulsion can be reduced to a 4-vector Energy-Impulsion also in presence of a (spherical) charge e (41). The basis idea is to define the electron (w_e) from its field (w_{em}):

$$w_e = w_{em} \tag{42}$$

This internal density w_e (40) in the electron (according to Poincaré) is explicitly written by Langevin (in 1913) on the basis of the model of surface charge distribution in the spherical radius r_e of Poincaré's electron or "hole in ether"[22] (in details we have after integration $\frac{4}{3}\pi r_e^3 w_e = \frac{1}{3}\frac{e^2}{2r_e}$) (1913):

$$w_e = \frac{1}{8\pi}\frac{e^2}{r_e^4} \tag{43}$$

Poincaré does not write in 1905 any formula for its internal density (or pressure) but specifies (in the sentence where he claims gravitational origin) that the density is proportional to the "fourth power of experimental mass m_e of elec-

[22]Such a name "classical radius of electron" is inappropriate in Poincaré's theory because he designates his electron as a "Hole in the ether". In modern language this is similar to that of quantum theory of field (QED of Dirac-Feynman): Poincaré's electron would be "singularity in the field". From which cosmological field the mass of electron is inducted? We are waiting for the answer from *QED*. We have a *WED* answer (§5-6).

tron". With basic relation (32 bis) we find indeed the proportionality with m_e^4 announced by Poincaré:

$$r = \frac{e^2}{m_e c^2} = r_e \quad \Rightarrow \quad w_e = \frac{1}{8\pi}\frac{m_e^4 c^8}{e^6} = \frac{1}{8\pi}\frac{m_e}{r_e^3} \quad \Rightarrow \quad m_e = 8\pi w_e r_e^3 \tag{44}$$

The mass m_e of free electron is therefore entirely induced from cubic (in volume) density of mass w_e by taking into account the surface distribution of the charge. This classical attempt to integrate the concrete electron (r_e, e, m_e) in the framework of *SR* was historically not followed because such a gravific ether is obviously unthinkable from the dominant Einsteinian point of view (1905, June, removal of ether). The few physicists who became interested (Langevin, von Laue, Born, Fermi) in Poincaré's supplementary Potential (overthrowed by qnantum theory), interpreted as a purely AD HOC anti-electrostatic pressure without any (Wave) ElectroDynamics perspective underlying Poincaré's continuous approach.

According to Poincaré the main physical argument for the gravitational origin of his pressure is that it must be attractive (anti-electrostatic). With the same argument being that, in a rather enigmatic way, that his gravific pressure must be *negative* (nothing to do here with $p + \rho = 0$).

In scientific literature about Poincaré's pressure (Langevin, von Laue, Fermi ...) the *EM* factor 1/3 between w_{em} and p_{em} is obviously NOT extended (except Born see below) between gravific p_e and gravific w_e.

5.2.2. Poincaré's Negative Pressure and NeoMinkowskian Perfect Fluid

Poincaré's historical deduction (§6 Lorentz contraction, [4]) has apparently nothing to do with equation of Perfect fluid (formulated apparently by von Laue or Born ten years after 1905). And therefore with (35 bis-36 bis). Let us try to extract, with the perfect fluid, the deepest root of Poincaré reasoning: the role of NEGATIVE pressure in the addition of two basic tensors (*EM* + *Non-EM*) (we do a reconstitution as in a judicial investigation).

Poincaré wonders: "Which tensor should I add to the first in order to remove the diagonal terms in the third"? (*except the first one* (00), 42):

$$\begin{pmatrix} w_{em} & 0 & 0 & 0 \\ 0 & \frac{1}{3}w_{em} & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_{em} & 0 \\ 0 & 0 & 0 & \frac{1}{3}w_{em} \end{pmatrix} + \begin{pmatrix} ?\,w_{em} & 0 & 0 & 0 \\ 0 & p_{em}\,? & 0 & 0 \\ 0 & 0 & p_{em}\,? & 0 \\ 0 & 0 & 0 & p_{em}\,? \end{pmatrix} = \begin{pmatrix} ?\,w_{em} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \tag{45}$$

M (The first is classical *EM* with zero trace). Poincaré's mathematical answer would be logically:

$$\begin{pmatrix} \frac{1}{3}w_{em} & 0 & 0 & 0 \\ 0 & p_{em} = -\frac{1}{3}w_{em} & 0 & 0 \\ 0 & 0 & p_{em} = -\frac{1}{3}w_{em} & 0 \\ 0 & 0 & 0 & p_{em} = -\frac{1}{3}w_{em} \end{pmatrix} \tag{46}$$

and therefore mathematically the gravific pressure indeed must be *negative* $p = -\frac{1}{3}w_e$ because it is then NOT an *EM* pressure $p = +\frac{1}{3}w_e$ (non zero trace $\frac{4}{3}w_e$). Physically his gravific pressure is anti-electrostatic repulsion.

Let us develop the addition (*EM* + *Non-EM*) (with 45):

$$\begin{pmatrix} w_{em} & 0 & 0 & 0 \\ 0 & \frac{1}{3}w_{em} & 0 & 0 \\ 0 & 0 & \frac{1}{3}w_{em} & 0 \\ 0 & 0 & 0 & \frac{1}{3}w_{em} \end{pmatrix} + \begin{pmatrix} \frac{1}{3}w_{em} & 0 & 0 & 0 \\ 0 & -\frac{1}{3}w_{em} & 0 & 0 \\ 0 & 0 & -\frac{1}{3}w_{em} & 0 \\ 0 & 0 & 0 & -\frac{1}{3}w_{em} \end{pmatrix} \tag{47}$$

$$= \begin{pmatrix} \frac{4}{3}w_{em} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

This is exactly the perfect fluid (39) with an electron at rest (§5-1).

EM_RADIATION + GRAVIFIC NEGATIVE PRESSURE- $\Rightarrow$ FREE ELECTRON?

(see 37 bis) Therefore Poincaré's historical (long) deduction is the same as our deduction $g_{\mu\nu} = \eta_{\mu\nu}$ from Riemanian Fluid to NeoMinkowskian Fluid (35-36-37).

There is however a CRUCIAL CONTRAST because (35-36-37) is formulated with *MM*:

$$p_e \eta_{\mu\nu} = \frac{1}{3}\frac{1}{8\pi}\frac{e^2}{r_e^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{48}$$

and therefore the pressure is no longer negative!

Poincaré's negative gravific pressure becomes in NeoMinkowskian Fluid a positive pressure with *MM* (*see* 37 *bis and* 37 *ter*).

Nothing prevents then to affirm (dixit Born) that everything returns in a purely *EM* order with $p_e = +\frac{1}{3}w_e$ aligned on radiation $p_{em} = \frac{1}{3}w_{em}$!

Equation (35-36-37) exerts a huge constraint that did not exist in Poincaré's historical calculation. The gravitational origin of Poincaré's pressure seems to have evaporated but the basic relationship of purely *EM* fluid (Born) is contradictory because we should have $p_e < \frac{1}{3}w_e$ for any electron (see §5-2-4).

The situation seems hopeless so we must ask questions (we propose the three following questions).

5.2.3. Did Poincaré (Langevin) Choose the Right Density?

Did Poincaré (Langevin) choose the right density? In other words: Is the coefficient (density w_e) considered the right one in (33)?

The missing link (see 33) density of the anti-electrostatic force is very huge 10^8 g/cm^3 (stability of elementar electron) and not very credible in the role of density of radiation, Reported to black radiation of *CBR* this first attempt involves $w_e = w_{CBR}$ (see 40, the density involves a temperature) we obtain about 10^{15} K!

At this stage we have not discovered yet the missing coefficient.

Moreover Langevin-Poincaré's density (35-10) is connected with gravitational theory but the constant G is hidden (in 48).

Constant G is however NOT hidden in the third formula of gravific density.

Remember (§4) that there are three densities that we can report (logically) to Feinberg's trio (tachyon, electron, photon (+graviton):

$$\rho_\Lambda = \rho = \frac{\Lambda c^4}{8\pi G} \quad (1)$$

$$w_e = \frac{1}{8\pi}\frac{e^2}{r_e^4} \quad (2)$$

$$w_{Ge} = \frac{G}{8\pi}\frac{m_e^2}{r_e^4} = \frac{w_e}{\kappa} = w_{Photon} \quad (3) \qquad (49)$$

1) The first tachyonic density is the density of dark Energy (CC).

2) The second density is internal mass density of electron (Poincaré's formula, 44).

3) The third enigmatic (Ge) Gravifico-electronic (very tiny) density (40) with very weak gravitational long range force (in contrast with the first, huge gravitational long range force).

The ratio $\kappa_{Ge} = \frac{w_e}{w_{Ge}} = \frac{f_e}{f_{Ge}}$ does not change.

This third density could be reported (logically) to Photon or Graviton, Radiation or Gravitation. The question is: a density of what? (this is not a density of mass in electron, see 44).

It can only be a density of light waves or density of gravific waves.

5.2.4. Can an Electron Moving at the Speed of Light Turn into … Photon?

Let us go back to Born's deadlock. Such an alignment of electron $p_e = \frac{1}{3}w_e$ on photon $p_{em} = \frac{1}{3}w_{em}$ is impossible. Indeed if we follow the equation of fluid of radiation (35-37-38) (this is not the case of historical Poincaré's demonstration) it is a real problem because we have now, from the symmetry underlying Neo-Minkowskian fluid, $p_e = \frac{1}{3}w_e$ while we should have $p_e < \frac{1}{3}w_e$!

In Summary, with underlying symmetry in "radiation" fluid (35) that: involves "$p = \frac{1}{3}w$" for photon and for electron as well. Consequently we have

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"$E = Pc$" for electron (with non zero proper mass) and photon (with zero proper mass) as well! In other words, can an electron moving at the speed of light turn into ... photon? No! Ask the (almost) same question but otherwise.

5.2.5. Does the "Perfect Ultra-Relativistic Electron" ("PURE") Exist?

For photonic gas we have *rigorous* relationship $p_{ph} = \frac{1}{3} w_{ph}$ whilst for an Ultra-Relativistic (hot) electronic gas we have an *approximate* relationship $p_e \approx \frac{1}{3} w_e$. "Ultra-relativistic" usually means that the proper energy (mass at rest) $E_0 = m_0 c^2$ of the electron becomes negligible (almost zero) compared to its kinetic energy $\gamma \gg 1$).

Perfect Ultra-Relativistic ("$v = c$") Electron (*PURE*) with zero mass seems impossible because leptonic electron has a (proper) mass. The situation seems desperate because (35) leads inexorably to a contradiction between $p_e \approx \frac{1}{3} w_e$ and the limit $p_e = \frac{1}{3} w_e$.

That is to say: Does the "PURE" exist?

If limit electron ($v = c$) or Cosmological Electron does not exist, then the primitive atom exists.

All the above is based on (35 and 35 bis)[23]. So let's forget for a few moments (35) and let's take a look on (36 and 36 bis).

Remember that Poincaré never establishes any direct link between his GRAVIFIC waves (introduction) and his GRAVIFIC pressure on electron (§6) [4]).

5.3. From ELECTRON TO GRAVITON: Synthesis towards (Second) Density of Gravific Waves

Let's reverse the reasoning from (36) that defines the Graviton (or Gravific Wave) by assuming that there is (maybe) a link with "PUR" electron (*Ge*).

If *PURE* exists then it defines a Gravific wave (lightlike Graviton, SEE previous **Table 2** and [16]):

$$\left\| E_{Ge}^2 - P_{Ge}^2 c^2 \right\| = 0, \quad \lambda_{Ge} = \frac{a}{P_{Ge}}, \quad \nu_G = \frac{E_{Ge}}{a} \quad \left(\lambda_{Ge} \nu_{Ge} = c\right) \tag{35-36}$$

where $\left(P_{Ge}, E_{Ge}\right)$ are respectively impulsion and energy of "light" electron (the graviton) where the "Action" a remains to be determined (52).

Let's locate the problem at the level of basic relation (in NeoMinkowskian approach) for electron[24]:

[23]Such a passage (from 35) to the limit, $\beta \to 1$ ($v \to c$), $m_0 \to 0$, $\gamma \to \infty$) from a timelike 4-vector $\left\| \gamma^2 m_0^2 c^4 - \gamma^2 m_0^2 \beta^2 c^4 \right\| = E_0^2 = \left(m_0 c^2\right)^2$ to a lightlike 4-vector is obviously impossible (indeterminate (35). This PUR or light Electron ($v = c$) is an Horror (exactly as Einstein's LichtKomplex (1905) is a terror (see annex 2, historical epilogue).

[24]It doesn't exist such a relationship for baryons. Evidence: the quarks.

$$r_e = \frac{e^2/c}{m_e c} = \frac{e^2/c}{P_{Ge}} \quad \Rightarrow \quad \frac{r_e}{c} = \frac{e^2/c}{m_e c^2} = \frac{e^2/c}{E_{Ge}} \tag{50}$$

The first $P_{Ge} = m_e c$ is an Horror (a "PURE" Horror) whilst the second $E_{Ge} = m_e c^2$ is a Wonder (curious discrimination!).

Given that Poincaré's concept of "Hole in ether" recalls de Broglie's Diffraction of an electronic wave in "hole of a screen", let us associate to Poincaré's length r_e a de Broglie's *wave* (λ_{Ge}, ν_{Ge}) [16]:

$$\lambda_{Ge} = \frac{e^2/c}{P_{Ge}} = \frac{e^2/c}{m_e c} \quad \rightleftharpoons \quad \nu_{Ge} = \frac{E_{Ge}}{e^2/c} = \frac{m_e c^2}{e^2/c} \tag{51}$$

that defines logically a wavelength and then a frequency and finally a "lightlike gravitational wave" $\lambda_{Ge}\nu_{Ge} = c$ or a lightlike graviton $E_{Ge} = P_{Ge} c$ (a radius is a length that becomes a wavelength and therefore a wavelength)

$$P_{Ge} = \frac{e^2/c}{\lambda_{Ge}} \quad \rightleftharpoons \quad E_{Ge} = \left(e^2/c\right)\nu_{Ge} \tag{52}$$

where Impulsion P_{Ge} Energy E_{Ge} with Action *a* is now determined ($a = e^2/c$ replaces $\hbar$). CQFD.

Does the (Cosmological) "PURE" exist? The answer is: YES but it's not the photon: it is the GRAVITON.

The wavelength associated with the Graviton λ_{Ge} is not the wavelength λ_e (Compton) of a (quantum) electron:

$$P_e = \frac{\hbar}{\lambda_e} \quad \rightleftharpoons \quad E_e = \hbar\nu_e \tag{53}$$

A GRAVITON is A PUR (WAVE $v = c$) ELECTRON WHOSE THE WAVELENGTH IS λ_{Ge}.

Frequency ν_{Ge} will be connected with angular velocity of Thomas $\omega_T = 2\pi\nu_{Ge}$ see conclusions[25].

The limit electron does not give a photon but a graviton[26].

Concretely it means that gravific waves (1905, intro) are inseparable from a very special gravific density connected to Poincaré's electron (1905, §6).

The lightlike graviton is without proper mass. The mass of electron is in fact carried by the *G*-wave in (51). We suggest calling this mass m_e (without proper mass $G_{\mu\nu} = 0$) a "comobile mass" of graviton (without charge[27]) exactly like galactic fluid (see also comobile time, §3-5).

In NeoMinkowskian *G-DSR* the "Wonder" $E_{Ge} = m_e c^2$ and the "Horror" $P_{Ge} = m_e c$ will be on the same boat (end of the discrimination, 50).

Let us specify also the ratio with wavelength of Compton and radius of Bohr (with fine structure constant):

[25]See proportionality Energy-Frequency of LichtKomplex of the young Einstein in annex 2.

[26]Given that RADIAL Kinematics corresponds in this paper, to SCALAR Dynamics, Graviton might have something to do with SCALAR Boson (Englert, ULB).

[27]Charge disappears in a pure coefficient of proportionality e^2/c (see §5-4).

$$\frac{\lambda_e}{\lambda_{Ge}} = \frac{\hbar}{e^2/c}, \quad \frac{\lambda_{Bohr}}{\lambda_{Ge}} = \left(\frac{\lambda_e}{\lambda_{Ge}}\right)^2 = \left(\frac{\hbar c}{e^2}\right)^2 \qquad (53 \text{ bis})$$

(see 59, ratio of densities).

Objection: This is a Graviton without Gravity (without constant *G*)?

Answer: *G* is in the amplitude (energetic intensity) of the *G*-wave. Gravity (*G*) is directly inscribed in density of energy of *G*-interaction between two point electron (a "double electron" $m_e m_e$, see 16-on) separated by basic wavelength λ_{Ge} (a kind of "string"):

$$f_{Ge} = G\frac{m_e^2}{\lambda_{Ge}^2} \ (30\text{-}3) \qquad w_{Ge} = \frac{G}{8\pi}\frac{m_e^2}{\lambda_{Ge}^4} \ (49\text{-}3) \qquad (54)$$

So (49-3) is a density of Poincaré's Gravific Waves. Most physicists think that the gravitational (density of) force between electrons (separated with λ_{Ge}) is perfectly negligible. This verdict was true before 1965 and it's still true after 1965.

Poincaré had the right reasoning (§5-2) but not the right density (see 49-3, in 54 and §6).

5.4. GRAVIFIC SUBQUANTUM SUBSTRATUM: WED (Poincaré-de Broglie) versus QED (Dirac-Feynman)

de Broglie, specialist of the diffraction of the wave electron, distinguishes (in 1957) three basic levels in physics[28].

1) The first level is (macroscopic) according to de Broglie is classical physics (dynamic and thermodynamics).

2) The second level is (microscopic) Quantum physics (baryonic or atomic matter $G_{\mu\nu} \neq 0$).

3) The third level (hypomicroscopic) is the deepest level (photonic-electronic, non baryonic $G_{\mu\nu} = 0$): "*the deepest level is Hypomicrophysics SubQuantum Substratum* co*nstituted by this Vacuum a huge reservoir of underlying energy of which we still know almost nothing*" (*in French*: *Le niveau le plus profond, hypomicrophysique ou subquantique pourrait-on dire, constitué par ce "vide" réservoir immense d'énergie sous-jacente dont nous ignorons encore presque tout*)

The third level "*Hypomicrophysics SubQuantum Substratum*" is particularly suitable for our problematic.

According to de Broglie, relativistic Wave Mechanics or Wave ElectroDy-

[28]Cours professé par de Broglie à la Sorbonne durant l'année scolaire 1957-1958 [16]:

1) *le niveau macrophysique des phénomènes macroscopiques directement observables? notre échelle qui est le domaine propre de la Physique dite classique* (*the macrophysic level of macroscopic phenomena directly observable on our scale which is the proper domain of so-called classical physics*).

2) *le niveau microphysique ou quantique qui est celui des molécules, des atomes, des noyaux ou plus généralement des particules élémentaires, qui est le domaine propre de la Physique quantique.* (*the microphysical or quantum level which is that of molecules, atoms, nuclei or more generally elementary particles, which is the proper domain of quantum physics*):

The first level is (macroscopic) according to de Broglie is classical physics.

namic (WED) (§5) should preside over the destiny of Quantum Mechanics or Quantum ElectroDynamic (QED).

We know a little more today with NeoMinkovskian CONTINUUM which adds a decisive gravitational component ($G\text{-}WED$) to de Broglie's subquantum substratum [16]. The unified field ($G\text{-}DSR$ is a $G\text{-}WED$) is photonico-electro-gravific (§ 5-4).

We suggest here to continue with relativistic mind of de Broglie. The only difference is that we use both SR and GR (and therefore $G\text{-}WED$) in order to determine (we put in evidence sub-h*ypo*) *the SUB* (*microphysics quantum stratum*).

The most fundamental principle of QED (microphysics) is that the LEAST action corresponds to h (or $\hbar$). In $G\text{-}WED$ (Hypomicrophysics) we have the following LEAST action (52):

$$\left(\frac{e^2}{c}\right)_{WED} \ll \hbar_{QED} \tag{55}$$

the subquantum "continuum" of action $\left(\frac{e^2}{c}\right)_{WED}$, in harmony with continuous spectrum of CBR, is smaller (SUB) than the " quantum" of action.

The fine structure constant becomes then a decisive factor between $G\text{-}WED$ and QED in its two forms (Sommerfeld or Planck Einstein):

$$\frac{\hbar c}{e^2} \approx 137,\cdots \qquad \frac{hc}{e^2} \approx 860,\cdots \tag{55 bis}$$

In order to treat of (the density of) non-baryonic SUBquantum VACUUM $G\text{-}WED$ is then better adapted:

$$G\text{-}WED_{Poincare\text{-}deBroglie} \gg QED_{Dirac\text{-}Feynman} \tag{55 ter}$$

Given that we have a direct basic connection (graviton) between electron and photon *without the positron* (annihilation e^+e^-), Poincaré-de Broglie's $G\text{-}WED$ involves then a (cosmological) *Break of symmetry* (in cosmological observation there is no antimatter, QED).

However, as long as we have no precision (56) about SUB formula of density of gravitational waves (49-3, 54) exactly as we have density of light waves, this inequality (55) seems purely formal.

6. Theoretical Deduction of the (Second) Density and the Temperature of CBR

After the first attempt (42, $w_e = w_{em}$), let us now introduce (second attempt) $w_{Ge} = w_{CBR}$ the density of Gravific Waves (53) and then also that of CBR (54 and annex 1):

$$w_{Ge} = \frac{G}{8\pi}\frac{m_e^2}{\lambda_{Ge}^4} = w_{CBR} \simeq 3.8\times 10^{-34}\ \text{g/cm}^3 \tag{56}$$

With Stefan-Boltzman's formula (39, 40 and 80) and numerical annex 1) we suggest a theoretical[29] deduction of the absolute temperature of *CBR* ($R_H T_K = cte$):

$$T_K \approx 2.6\ \text{K} \tag{57}$$

very close to COBE observation (see annex 1).

The right missing coefficient in (33) (with constant *G*) (instead of 48, Poincaré 1905):

$$p_{Ge}\eta_{\mu\nu} = \frac{1}{3} w_{Ge}\eta_{\mu\nu} = \frac{1}{3}\frac{G}{8\pi}\frac{m_e^2}{\lambda_{Ge}^4}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \tag{58}$$

We can now complete (33):

$$\rho_\Lambda \eta^{-}_{\mu\nu} = \rho_\Lambda \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \text{ (dark Energy)}$$

$$w_{Ge} h^{+}_{\mu\nu} = w_{Ge} \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \text{ (CBR)} \tag{58 bis}$$

We have not only the density of *CBR* but also the basic ratio (see 12) with critical density (see anti-32 and 75) directly deducted from NeoMinkowskian limit of *GR* with *CC*:

$$\frac{w_{CBR}}{\rho_\Lambda} = \Omega_{CBR} = 5.22 \times 10^{-5} = \frac{1}{3}\left(\frac{f_{Ge}}{f_G}\right)^2 \tag{59}$$

(very close to observation 5.38×10^{-5}). The coincidence is obviously not perfect (with the factor 4/3 it's even better, 48). For the evaluation of the cosmic substratum density our *WED* error is of the order 10 to the power −1. It must be compared (see Unruh) with that of *QED* that is of the order of 10 to the power 120:

"*The cosmological constant problem arises because the magnitude of vacuum energy density predicted by quantum mechanics is about* 120 *orders of magnitude larger than the value implied by observations of accelerating cosmic expansion.*" [8]

This *CC* problem reported by Unruh disappears with *WED*. We prove that his monstrous error comes from the misuse (extrapolation) of baryonic quantum theory to a radically non-baryonic subquantum vacuum (confusion between level 2 and 3 according to de Broglie). Note also that this last ratio (59) is also very close to (53) $\left(\frac{\lambda_{Ge}}{\lambda_e}\right)^2 \approx \left(\frac{1}{137}\right)^2$.

[29]No inflationary model predicts such a result (to our knowledge). Flat Cosmology (Tatum) predicts this temperature using not the Planck's particle but Planck's mass. It is very curious (see 83).

7. Conclusions: From Cosmological Electron to Galactical Electron?

Rather than the Quantization of *GR* (main stream) we chose here the *GR*-ization of Quantum (electron).

The NeoMinkowskian synthesis between Poincaré's GRAVIFIC waves and Poincaré's GRAVIFIC pressure on electron is now complete.

Planck's unstable cosmological particle is then shorted (short-circuited) by stable cosmological electron which is in fact the graviton.

Hyperbolic NeoMinkowskian Universe not only predicts a *CBR* but also the Temperature observed of this *CBR*.

We follow Penrose [18] on two essential points:

1) "*There is something particularly elegant about hyperbolic geometry*". *G-DSR* (introduction and §1-2-3)

2) "*I should say that I do not really believe these* (*inflationary*) *theories*". *G-DSR* (§4 intermediate conclusions and §5 conclusions)

The most beautiful result of Lobatchevskian interpretation of "Big Bang" or "Big Boost" may be: *The finite time* T_H *is an effect of the curvature of hyperbolic straight line of time* (§3-3-3, 27).

Cosmology joins geology: catastrophism or inflationism (with a Big exceptional Bang of primitive atom) against unifomitarism or gradualism (with Big Well tempered Boost).

We are aware of the incompleteness of this present exploration.

In our approach everything seems RADIAL: there is no RADIAN (no aberration). We have a cosmological electron (Translation) but not a galactical electron (Rotation).

Nothing is circular (*UCM*), everything seems rectilinear (*UARM*). Everything is longitudinal, nothing is transversal: the magnetism seems gone (der verdammte magnetismus, sagte der junge Albert zu Mileva). It could come back in force transversely.

An alternative to the escape velocity (Dark Energy) of this paper however exists: the orbiting velocity (Dark Matter). The only difference is given by a factor 1/2 (in *YP*12).

The composition of two *LT* not in the same direction involves Thomas' Rotation (angular velocity connected with frequency of wave electron, 52).

According to Pauli's, Thomas' rotation is a decisive correction of factor 1/2 in Dirac' spin 1/2 of electron.

A Gravitational REVOLUTION in Dirac's electronic spin? De revolutionibus orbium spinorium ...?

(Remember also that all this was done in the absence of baryons ($G_{\mu\nu} = 0$) and also especially in the absence of neutrinos).

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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1. Numerical Annex 1: From Cosmological Electron to Gravific Density of CBR

1.1. Some Cosmological Constants and Poincaré's Cosmological Electron

Critic density of tachyonic dark energy from Hubble's constant (1 bis):

$$\rho_\Lambda = \frac{3}{8\pi}\frac{\left(2.168\times10^{-18}\right)^2}{6.67428\times10^{-8}} = 8.6412\times10^{-30}\ \mathrm{g/cm^3} \tag{60}$$

Kinematic and Dynamic values: $R_H = 1.38\times10^{28}\ \mathrm{cm}$,

$\alpha_M = 7.3543\times10^{-8}\ \mathrm{cm/s^2}$, $M_H = 9.311\times10^{55}\ \mathrm{g}$

with ($\rho_\Lambda = \frac{\Lambda c^4}{8\pi G} = \frac{3H_\Lambda^2 c^2}{8\pi G} = \frac{3\alpha_M^2}{8\pi G} = \frac{3F_G}{4\pi R_H^2} = \frac{3G}{2\pi}\frac{M_H^2}{R_H^4} = \frac{3M_H c^2}{4\pi R_H^3}$).

A coincidence:

$$\Lambda\cdot M_H = 10^{-56}\ \mathrm{cm^{-2}}\times10^{56}\ \mathrm{g}\approx 1\ \mathrm{g/cm^2}$$

Enigmatic force between an electronic mass and a cosmological acceleration (Poincaré's force):

$$f_G = m_e\alpha_M = 2m_e G\frac{M_H}{R_H^2} = 9.11\times10^{-28}\times7.35\times10^{-8} = 6.3\times10^{-35}\ \mathrm{cgs} \tag{61}$$

f_G is very close to f_{Ge} (63, 31).

1.2. Game of Units (of Thrones?)

It should not be confused Planck's black body theory $(c,k_B,\hbar)$ with Hypothesis of the so-called Planck's cosmological particle $(c,G,\hbar)$. The latter is an unstable primitive atom (inflation) based on a game of units with 3 constants $(c,G,\hbar)$ (tiny Planck's length $\sqrt{\frac{hG}{c^3}} = l_{Pl} = 10^{-33}\ \mathrm{cm}$, huge Planck's density $\rho_{Pl} = 10^{+94}\ \mathrm{g/cm^3}$ and medium Planck's mass $l_{Pl}\approx10^{-5}\ \mathrm{g}$), $G = 6.67428\times10^{-8}$.

We have also a game of Stoney's units (*i.e.* Stoney mass $\sqrt{\frac{e^2}{2G}}\approx10^{-6}\ \mathrm{g}$) with also 3 constants $\left(c,G,e^2/c\right)$ connected to Planck's units by constant of fine structure $\frac{hc}{e^2}\approx137$, ….

These "games of units" are short-circuited by a basic electronic (or gravific) length $\lambda_{Ge} = \frac{e^2}{m_e c^2} = r_e = 2.8289\times10^{-13}\ \mathrm{cm}$ and therefore also by the mass of electron $m_e = 9.109\times10^{-28}\ \mathrm{g}$ and the charge $e = 4.803\times10^{-10}$.

The stable cosmological electron (in fact the graviton) is hidden in electro-gravific correspondence (under the presidency of velocity c of light, see 30 & 32, $c = 2.99792\times10^{10}\ \mathrm{cm/s}$)

$$c^2 = \frac{2GM_H}{R_H} = \frac{e^2}{m_e r_e} \tag{62}$$

We have to show that f_G (14, 61) is perfectly integrated in new (cosmological) theory of unified field.

1.3. Ratio of Forces

Next to the macro-force ($F_G = \frac{c^4}{2G}$) both classical forces respectively electrostatic and gravific f_e & f_{Ge},

$$f_e = \frac{e^2}{r_e^2} = 2.8826 \times 10^6 \text{ cgs}, \quad f_{Ge} = \frac{Gm_e^2}{r_e^2} = 6.9216 \times 10^{-37} \text{ cgs} \tag{63}$$

there exists an electrogravific connection (basic constant κ_{Ge}) between "infinitely large" (M_H and R_H) and "infinitely small" (m_e and r_e).

$$\frac{f_e}{f_{Ge}} = \frac{e^2}{Gm_e^2} = \kappa_{Ge} \approx 4.1604 \times 10^{42} \tag{64}$$

We have three forces $\left(F_G, f_e, f_{Ge}\right)$ where f_e is well supervised (κ_{Ge}) by the other two $f_e^2 = 2F_G f_{Ge}$:

$$\frac{2F_G}{f_e} = \frac{f_e}{f_{Ge}} = \frac{e^2}{Gm_e^2} = \kappa_{Ge}, \quad \frac{2F_G}{f_{Ge}} = \kappa_{Ge}^2 \tag{65}$$

See §4 intermediate conclusions:

$$F_G \xleftarrow{\kappa_{Ge}} f_e \xrightarrow{\kappa_{Ge}} f_{Ge} \tag{32}$$

At this stage f_G, very close to f_{Ge}, is missing (70).

1.4. Ratio of Lengths and Integration of the Missing Force

Ratio of linear density respectively for Universe and electron is also given by fundamental physical constant of NeoMinkowskian continuum:

$$\frac{2M_H}{m_e} = \kappa_{Ge} \frac{R_H}{r_e}, \quad \frac{\frac{M_H}{R_H}}{\frac{m_e}{r_e}} = \frac{1}{2}\kappa_{Ge} \tag{66}$$

Numerically let us remark that we have approximately $\frac{R_H}{r_e} \approx \kappa_{Ge}$ and then $\frac{2M_H}{m_e} \approx \kappa_{Ge}^2$. More precisely let us introduce ϵ in such a way that:

$$\frac{\epsilon R_H}{r_e} = \kappa_{Ge}, \quad \epsilon \approx 90 \tag{67}$$

with the current measures $\epsilon \approx 90$:

$$\epsilon = \kappa_{Ge} \frac{r_e}{R_H} = 2 \frac{M_H r_e^2}{m_e R_H^2} \tag{68}$$

Hypothetical "acceleration" $\alpha_m = G\frac{m_e}{r_e^2}$ is very close to cosmological acceleration α_M:

$$f_G = m_e \alpha_M = m_e 2G \frac{M_H}{R_H^2}, \quad f_{Ge} = m_e \frac{Gm_e}{r_e^2} = m_e \alpha_m \tag{69}$$

Exactly the same ratio ϵ for the lengths (68):

$$\frac{\alpha_M}{\alpha_m} = \frac{f_G}{f_{Ge}} = 2\frac{M_H r_e^2}{m_e R_H^2} = \epsilon \approx 90 \tag{70}$$

The set of formulas with (61) is therefore perfectly coherent (see the final ratio of densities 49, 72, anti-32):

1.5. Ratio of Densities

Leaving aside the first historical attempt (42-43) $w_{Electron} = \frac{1}{8\pi}\frac{e^2}{r_e^4} = w_e$ (2) with the ratio ($\kappa_{Ge} = \frac{w_{El}}{w_{Ph}}$), it remains two densities (see anti-32):

$$\rho_{Tachyon} = \frac{\Lambda c^4}{8\pi G} = \frac{3\alpha_M^2}{8\pi G} \quad (1) \qquad w_{Photon} = \frac{G}{8\pi}\frac{m_e^2}{r_e^4} = w_{Ge} = \frac{1}{8\pi G}\alpha_m^2 \quad (2) \tag{71}$$

Our theoretical deduction of the basic (couplage, 12) ratio with G-interaction between respectively, "two Universes" and "two Electrons" $\frac{3G}{2\pi}\frac{M_H^2}{R_H^4}$ and $\frac{G}{8\pi}\frac{m_e^2}{r_e^4}$:

$$\frac{\rho_\Lambda}{w_{Ge}} = \frac{\frac{3\alpha_M^2}{8\pi G}}{\frac{1}{8\pi G}\alpha_m^2} = 3\left(\frac{\alpha_M}{\alpha_m}\right)^2 = 3\left(\frac{f_G}{f_{Ge}}\right)^2 = 3\epsilon^2 \approx 19200 \tag{72}$$

Two basic DENSITIES are (density of mass):

$$\rho_\Lambda = \frac{\rho}{c^2} \approx 8.6412\times10^{-30}\ \mathrm{g/cm^3}, \quad \frac{w_{Ge}}{c^2} \simeq 3.8\times10^{-34}\ \mathrm{g/cm^3} \tag{73}$$

The second in details:

$$\frac{w_{Ge}}{c^2} = \frac{1}{8\pi}\times\frac{6.67428\times10^{-8}\times\left(9.11\times10^{-28}\right)^2}{\left(2.8289\times10^{-13}\right)^4\times\left(2.99\times10^{10}\right)^2} \approx 3.85\times10^{-34}\ \mathrm{g/cm^3} \tag{74}$$

Or the usual inverse ratio:

$$\frac{w_{CBR}}{\rho_\Lambda} = \Omega_{CBR} = 5.22\times10^{-5} = \frac{1}{3}\left(\frac{f_{Ge}}{f_G}\right)^2 \tag{75}$$

With Stefan-Boltzman's formula with $w_{Ge} = w_{CBR}$ we find an absolute temperature of (57).

Observed CBR ($COBE$) values:

$$w_{COBE} = 4.6\times10^{-34}\ \mathrm{g/cm^3} \tag{76}$$

$$\frac{\rho_\Lambda}{w_{CBR}} = \frac{8.6412\times10^{-30}}{4.5908\times10^{-34}} = 18823 \tag{77}$$

or inverted:

$$\Omega_{CBR} = \frac{w_{CBR}}{\rho_\Lambda} = 5.38 \times 10^{-5} \quad (78)$$

Finally let us note that (74-76) are very close to $\left(\frac{1}{137}\right)^2$ the ratio (53) between $\lambda_{Poincare} = \lambda_{Ge}$ and λ_{Bohr}.

So with $\epsilon = \Omega_{CBR} = 1$ it would remain only one "density-pressure":

$$\rho_\Lambda = 3 p_{CBR} \quad (79)$$

Which of the two? Ask the question (between mass of electron and Hubble constant) is answer it. CONJECTURE 1: If our universe R_H was about 10^2 larger than the current evaluation from Hubble constant we would have (79). Another CONJECTURE is possible (see annex 2).

1.6. Law of Stefan: Radiation and Gravitation

$$w_{CBR} = \sigma_{Stephan} T^4, \quad \sigma_{Stephan} = 7.56564 \times 10^{-15}\,\text{cgs} \quad (80)$$

If we admit (56) we have:

$$T^4 = \frac{w_{Ge}}{\sigma_B} = \frac{G}{8\pi} \frac{m_e^2}{r_e^4} \frac{15\hbar c^3}{8\pi^3 k_B^4} = G \frac{m_e^6 c^8}{e^8} \frac{15\hbar^3 c^3}{64\pi^4 k_B^4} = G \frac{m_e^2 m_e^4 c^8}{e^2} \frac{15\left(\hbar^3 c^3\right)}{64\pi^4 \left(e^6\right) k_B^4} \quad (81)$$

This formula (*CBH* or *CBR*) might have something to do with Hawking's formula for *SBH* [9] (especially in his Tatum cosmological interpretation (mean mass) [11]).

$$\frac{64\pi^4}{15} k_B^4 T^4 = G \frac{m_e^2 \left(m_e c^2\right)^4}{e^2} 137^3 \Rightarrow \frac{\pi}{15} \kappa_{Ge} k_B^4 T^4 = \left(m_e c^2\right)^4 \frac{137^3}{4\pi^3} \quad (82)$$

$$\left(\frac{m_e c^2}{k_B T}\right)^4 = \frac{\pi}{15} \kappa_{Ge} \left(\frac{4\pi}{137}\right)^3 \quad (83)$$

The electron is so integrated in *CBR* and thus also in universal substratum. Observers located inside the universe can be happy they have light with electricity!

2. Annex 2 Historical Epilogue: The "Fine Structure" of Special Relativity (Poincaré-Einstein, Conjecture 2)

It seems that the question of the limit ($v = c$) "Perfectly UltraRelativistic Electron" (PURE) has never been considered in physics.

It's wrong because this question is already considered in 1905 in the two papers of Poincaré and Einstein [4] [18].

We have shown that from a historical point of view there was not only one theory of *SR* but two theories of *SR* (quasi-simultaneous, 1905) theories of *SR*: that of Einstein (June) and that of Poincaré (July).

Both theories are very close but not confused. So there is a "Fine Structure" of *SR* in epistemological meaning (with quotes [19]). Both theories are based on LT

and invariance (limit) of light velocity but they are singularities that seemed irreducible: In Poincaré's *SR* we have for examples: Light elongated ellipsoids [20][30], gravific pressure on electron, gravific waves .... In Einstein's *SR* we have for example: Doppler formula and LichtKomplex. Poincaré's point of view is then very close to NeoMinkoskian Limit of ... Einstein's *GR*.

2.1. Einstein's "LichtKomplex" and Poincaré's Graviton

LichtKomplex are introduced by Einstein in June basic paper §8 [5] three months after his famous LichtQuant (1905). The latter became (with impulsion) the photon for which Einstein got in 1922 the Nobel Prize. Einstein's LichtKomplex has (almost) nothing to do with that.

Einstein's LichtKomplex were considered as horrors (or terrors) by Lorentz, and Planck (and most of physicists). LichtKomplex were rejected by the community of physicists because they presuppose that a certain amount of matter travels at the speed of light. Einstein eliminated them in all subsequent presentations of his *SR* (already in 1907 ...). Their radical elimination will persist even after 1922. They were ejected from both History of Physics and Physics itself.

Why?

Both they use LT of a spherical volume but Poincaré considers (1905 §1) a sphere driven with an electronic point (invariance of charge and action e^2/c) whilst Einstein's considers (1905 §8) a sphere driven with a photonic point (LichtKomplex). BY MAKING $v = c$ (SIC) in Poincaré's spherical Electron., Einstein deduces the existence of spherical particles "whose energy transforms proportionally the frequency". The coefficient of proportionality is then not $\hbar$ but e^2/c (see 52).

Thanks to *GR* (Einstein 1915) with *CC* (Einstein 1917) with Poincaré's (Neo-Minkowskian) Limit, we now know that when (the young) Einstein makes $v := c$ (reSIC) in Poincaré's electron, he determines not a photon but a graviton (see 52). The history of physics is highly nonlinear.

Let us finally not that Einstein uses also Langevin's formula (43) for density (1/8π), see also Landau 31-5, p106 [17].

2.2. Poincaré's Velocity with Respect to the Gravific Ether (Conjecture 2)

The fine structure (without quotes) constant is thus hidden in the synthesis between the two *SR* (a "Double" *SR*). called by Einstein "factor 900":

$$\frac{h}{e^2/c} \approx 860 \tag{84}$$

[30]Electron, atom, solar system, galaxy are not in expansion. Poincaré could seem "inflationist" at this respect, "Mon lit est en expansion ...". Et il précise "mais je ne peux pas m'en apercevoir"! Le principe einsteinien (quantique) de l'identité des unités de mesure (des atomes) est à cet égard plus clair. Le fait est que Poincaré avait une théorie de l'expansion (ellipsoédes lumineux allongés, note 18) en 1907 basée sur ... la contraction de Lorentz (20 ans avant l'univers de Hubble.

With Poincaré's [21] (relative) speed with respect to the gravific ether:[31] (*CBR*), we have the right to formulate a second conjecture (conjecture of "Big Boost"):

$$c/860 \approx 360 \text{ km/s} \tag{85}$$

This is very close of the observed *COBE* value. A dipolar effect on 3 K. of the order of 3 mK ($10^{-3} \approx \frac{1}{900}$). Compatible with the inflationist hot *BB* 10^{+3} (3000K)?

If we were at rest with respect to the ether, the "quantum" h and the "continuum" e^2/c would be thus reconciled.

[31]Einstein had suppressed in his *SR* (1905) the ether (with the possibility of measuring a speed with respect to it). Poincaré did not remove the ether because it was a (gravitational) source of the mass of the electron. Note that Einstein reset an ether in 1922 (see L. Kostro) but he could not make the connection with *CBR* discovered (1965) after his death (1955).

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How Do Electric and Magnetic Fields Move?

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Abstract

In nature, there are two fundamentally different types of motion of the electric and magnetic fields: dynamic and kinematic. A typical manifestation of the first type of motion takes place in a plane harmonic EM-wave. For already more than a century the question about the ratio of the phases of the electric and magnetic fields, oscillating in such a wave, remains open. From time to time in this regard, fierce disputes arise. The point is that far from any phase difference turns out to be compatible with the full system of Maxwellian equations. Maxwell's classical theory as applied to such a wave leads to the conclusion that the electric and magnetic vectors in it oscillate harmoniously with zero phase shift. In the framework of this theory, a rigorous mathematical proof is given.

Keywords

Electromagnetic Wave, Transverse Oscillations in Phase, Longitudinal Immobility of Field Vectors, Electro-Kinematics, Magneto-Kinematics

1. Introduction

After Heinrich Hertz, a famous German physicist, discovered experimentally the existence of electromagnetic waves propagating in vacuum, all physicists believed that the movement of energy in space occurs due to the periodic process of mutual induction of electric and magnetic fields. The following picture of the phenomenon looked quite logical. When the magnetic field at a given point in space decreases, the electric field increases, reaching a maximum on the moment of zeroing the magnetic, and vice versa. The zero value of the magnetic vector corresponds to the maximum rate of its change, which in turn implies the maximum value of the induced electric vector, and also vice versa. The beautiful idea of a mutual transfer of energy from a magnetic form to an electric one and back proved to be extremely tenacious. Outbreaks of this "faith" sometimes still occur [1], although rigorous proof of its falsity has already been found.

Too scholastic, schooling-wise, even trivial as this might seem, the question about phase shift between electric and magnetic vectors in EM-wave has a peculiar, just Shakespearian tension for the Maxwell's theory of electromagnetism: "To be" —Figure 1(a), or "not to be" —Figure 1(b).

2. Solution of the Problem

Show that in a plane harmonic EM-wave propagating in a vacuum, there is no periodic conversion of energy between magnetism and electricity. It is known that in such a wave, the electric and magnetic vectors are orthogonal to the direction of propagation and mutually orthogonal, therefore, we choose the coordinate system in the following way. The x axis is oriented along the propagation direction, the y axis is parallel to the electric vector, the z axis is parallel to the magnetic vector so that there is a right-oriented triple of unit basis vectors (Figure 1). In this system, the electric vector $\mathbf{E}$ has coordinates $(0,E,0)$, and the magnetic one $\mathbf{H}$ has coordinates $(0,0,H)$. The "electric" and "magnetic" coordinates depend on time and space according to a harmonic law. We can take $E = E_0 \cos\left(\omega t - \frac{\omega}{c} x\right)$, then $H = H_0 \cos\left(\omega t - \frac{\omega}{c} x + \delta\right)$. Here, E_0 and H_0 are half-amplitude values, and δ is an unknown phase shift.

Since there are no currents in a vacuum, both vectors obey two Maxwell equations, which in a Gaussian system of units have the form:

$$curl\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{H}}{\partial t}, \quad curl\mathbf{H} = \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}.$$

In the orthonormal coordinate system (x,y,z), the curl of the vector field $\mathbf{F} = (F_x, F_y, F_z)$ is a vector whose coordinates are expressed in terms of the spatial partial derivatives of the coordinates of the field vector as follows:

$$curl\mathbf{F} = \left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}, \frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}, \frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right).$$

In our case, from all the spatial partial derivatives of the electric vector, non-zero one would be only

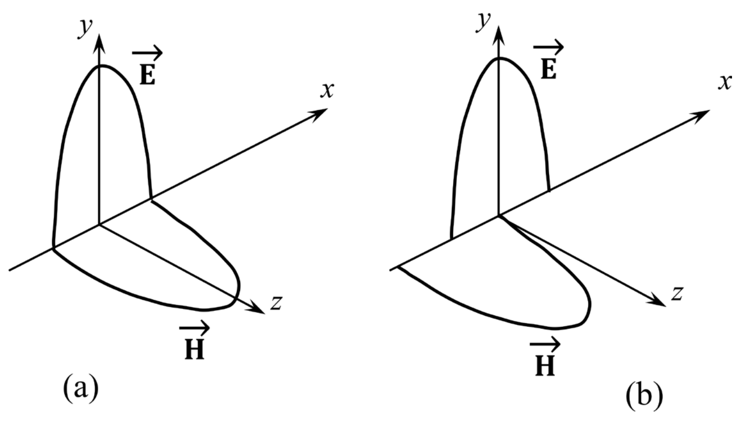

Figure 1. Phase shift in EM-wave.

$$\frac{\partial E_y}{\partial x} = \frac{\omega}{c} E_0 \sin\left(\omega t - \frac{\omega}{c} x\right),$$

therefore vector

$$curl\mathbf{E} = \left(0, 0, \frac{\omega}{c} E_0 \sin\left(\omega t - \frac{\omega}{c} x\right)\right). \tag{1}$$

Let us find the partial time derivative of the magnetic vector. Since only the third its coordinate is different from zero, then

$$\frac{\partial \mathbf{H}}{\partial t} = \left(0, 0, -\omega H_0 \sin\left(\omega t - \frac{\omega}{c} x + \delta\right)\right). \tag{2}$$

From a comparison of (1) and (2), it is seen that a necessary and sufficient condition for the satisfying of the first of the two above Maxwell equations is a multiple of the constant 2π value of the phase shift $\delta = 0, \pm 2\pi, \pm 4\pi, \cdots$.

Let us verify this condition for the second of the equations. We have

$$curl\mathbf{H} = \left(0, -\frac{\partial H_z}{\partial x}, 0\right) = \left(0, -\frac{\omega}{c} H_0 \sin\left(\omega t - \frac{\omega}{c} x + \delta\right), 0\right), \tag{3}$$

and

$$\frac{\partial \mathbf{E}}{\partial t} = \left(0, -\omega E_0 \sin\left(\omega t - \frac{\omega}{c} x\right), 0\right). \tag{4}$$

Comparing between (3) and (4) shows the fairness of the above mentioned condition also for the second of Maxwell's equations. Thus, we can draw the following conclusion. In a plane harmonic EM wave propagating in vacuum, the electric and magnetic vectors oscillate *in phase*. At any point in space, they simultaneously pass through zero and simultaneously reach their semi-amplitude values ($E_0 = H_0$ in the Gaussian system of units). A complete energy exchange between the electric and magnetic components requires a specific phase shift that is equal to the definite value π/2, but Nature does not want to provide even a partial energy transfer between the electric and magnetic forms.

3. Conclusions

As we can see, the relationship between the phases of the electric and magnetic vectors in a plane EM wave has far from just a simple "educational" value. The correct solution to this issue is rather of more philosophical, even worldview significance. The adequacy of our perception of the surrounding reality depends on the accepted answer.

Certainly, intuition is a wonderful thing, but, alas, it isn`t omnipotent. In addition to the example of intuitionistic failure discussed above, there is another similar case in the doctrine of electromagnetism. In essence, it has folk rather than scientific character, and consists of a magical belief that the fields in the EM wave move at the speed of light. In this misconception, the main role is played by confusion with energy transfer in space. In fact, electrodynamics implements a complete analogy with waves on the surface of a pond. Because of the stone falling into the pond, water moves only in the vertical direction, but these coordinated shifting are transporting horizontally (at a speed of meters per second) the energy that destroys the bank. Similarly, in an EM wave the electric field and magnetic field do not move anywhere in the source's own system, and only their coordinated transverse oscillations provide energy transport in the longitudinal direction [2]. Such behavior of field vectors returns to life the idea of worldwide ether.

As for the true spatial displacements of the electric and magnetic fields, they objectively exist, but are not related to electrodynamics. The study of such movements is carried out in the framework of electro-magneto-kinematics—a specific region in the doctrine of electricity and magnetism [3] [4] [5] [6] [7]. Two Russian engineers, N. Zaev and V. Dokuchaev, opened this new page in the electromagnetic doctrine half a century ago due to an experiment with a rotating electromagnet.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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The List of Used Variables

$\mathbf{E}$ —electric vector, $\mathbf{H}$ —magnetic vector, c—light speed in vacuum, ω —circular frequency.

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An Alternative Model of Proton and Neutron

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Abstract

Based on a model of fermions which implies a model of photons, a model of the neutron is constructed by merging two photons of equal energy propagating in opposite directions. The fermion model is outlined, and the merging of two photons is described in detail. The radius of the neutron obtained in this way is $R_n = 0.84008\ldots$ fm. This value is four times the reduced Compton wavelength of the neutron. Assuming the same model for the proton, one obtains a value of $R_p = 0.84123\ldots$ fm, which agrees with the most recent experimental value for the charge radius of the proton within the given limits of error. The neutral charge of the neutron is reproduced, and the positive charge of the proton follows within the model, if the proton is formed via the anti-neutron by losing one electron. $S = \pm\hbar/2$, and zero dipole moment, is also reproduced for proton and neutron. Further, a value of the magnetic moment of the neutron of $\mu = -2.00\mu_N$ (μ_N: nuclear magnetic moment), and of the proton of $\mu = 2.666\ldots\ \mu_N$ is predicted. The deviation by ca. 5% from the recommended respective values of ($-1.9130\mu_n$), and ($2.793\mu_n$) is ascribed to the (g-2)-anomaly. Finally, the relation of the model with the established description of the nucleons in terms of three quarks bound by gluons is shortly discussed.

Keywords

Quantum Physics, Modeling of Nucleons, Classical Probability

1. Introduction

The fermion model developed in three recent publications [1] [2] [3], is used in the present paper to construct a model of the nucleons. In paragraph 2, it is outlined. Qualitatively, it describes particles in terms of paths of a quantum around a fixed point in space. The position of the particle is the position of this point, and its mass is the localized energy of the motion. It takes an observation time of at least one period of the periodic motion of the quantum until the particle "exists" as an observable entity. This time is very short, and depends as $1/m$ on the mass of the considered fermion. In case of the electron it is on the order of the period of the well-known "Zitterbewegung" [4] [5] [6]. For longer observation times the particle described in this way has internal structure, and therefore can own properties. Properties of the particle at rest can be obtained from the stationary spatial density distribution of the quantum, which reflects the topology of its path. Mass, size, spin, and magnetic moment of the electron have been correctly described in this way. For a description of properties of the moving particle, a more detailed use of the motion of the quantum in space and time is necessary. The emergence of the property "de Broglie wavelength" and the implied interference phenomena of the moving particle have also been demonstrated to be a natural consequence within the model [3].

Similar models as the one developed in [1] [2] [3] have been proposed and discussed earlier in the literature [see f.i., [7] [8] [9] [10]]. What we intend to prove in this paper, is the fact that, our fermion model, sketched above qualitatively, allows to construct nucleons and to calculate their properties. In paragraph 2, we describe the fermion model in sufficient detail to demonstrate its application to the construction of the nucleons. In paragraph 3, the "merging" into nucleons is outlined, and results are reported. In paragraph 4, we summarize the results and address shortly the question of a relation between the proposed model and the established theory.

2. The Fermion Model

The basic quantity determining the spatial distribution is the reduced Compton wavelength

$$L = \hbar/mc\,, \tag{1}$$

with ($\hbar$) being the quantum of action, (m) the mass of the fermion, and (c) the velocity of light. The distribution of the quantum in a topology that characterizes a fermion, arises when a ring of radius ($L/2$) containing the quantum is rotated around an axis that lies in the plane of the ring, and goes through one point on the ring (see **Figure 1**).

The population probability of the quantum on the ring is uniform. The possible positions of the quantum form the surface of a torus (see **Figure 2**). The positions are described by the position vector

$$\boldsymbol{r} = L\left\{\cos^2\left(\frac{\varphi_t}{2}\right)\cos(\varphi_c), \cos^2\left(\frac{\varphi_t}{2}\right)\sin(\varphi_c), \sin(\varphi_t)/2\right\} \tag{2}$$

The angles (φ_c) and (φ_t) are, respectively, the azimuth-, and the torus angle. The probability density on the surface of the torus is independent of (φ_c), and its dependency on the torus angle (φ_t) is determined by the condition that the probability on the ring is uniform (**Figure 1**). The probability for the position vector to point into a certain area ($\mathrm{d}\varphi_c \mathrm{d}\varphi_t$) thus becomes simply $\rho = 1/(4\pi^2)$, independent of the angles. Averages of functions of the position vector, $F(\boldsymbol{r})$, can

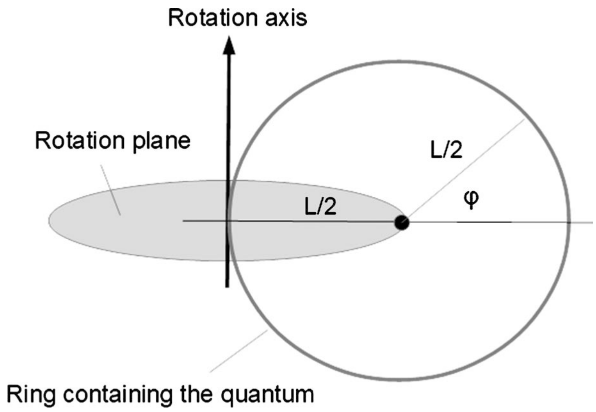

Figure 1. The figure shows the plane containing the quantum. The quantum is distributed with constant probability density on the ring. The plane containing the ring rotates around the rotation axis in the way shown in the figure. The rotation frequency is $\omega_c = c/(L/2)$, and the center of the ring has speed (c). The angle (φ) in the figure is the torus angle (φ_t) used in the text, and the angle defining the angular position of the ring is the angle ($\varphi_c = \omega_c t$) used in the text.

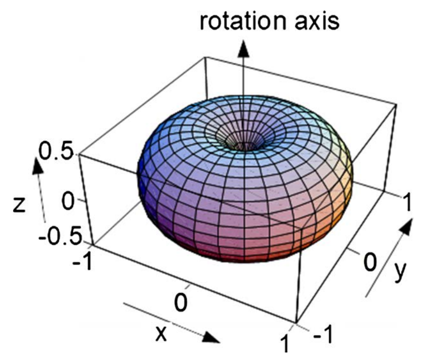

Figure 2. The possible positions of the quantum during its cyclic motion around the rotation axis. The coordinates are given in units of L. The shape of the entity formed after one period of the "Zitterbewegung" is a torus on whose surface the quantum circles. The radius of the entity, which we identify as the fermion, has a radius $R = L = \hbar/mc$.

therefore be obtained as the integral

$$F(r)_{av} = \left(\frac{1}{4\pi^2}\right)\iint F(\boldsymbol{r})\,\mathrm{d}\varphi_c\mathrm{d}\varphi_t \tag{3}$$

The momentum ($p = mc = \hbar/L$) perpendicular to the ring (see **Figure 1**) is ascribed to the quantum, so that its momentum vector becomes

$$\boldsymbol{p} = \left(\frac{\hbar}{L}\right)\left\{-\sin(\varphi_c), \cos(\varphi_c), 0\right\} \tag{4}$$

This leads to an instantaneous angular momentum caused by the quantum at position ($\boldsymbol{r}$), which is given by $\boldsymbol{S} = \boldsymbol{r} \times \boldsymbol{p}$, so that the property "spin" can be obtained simply as the average

$$\boldsymbol{S}_{av} = \left(\frac{1}{4\pi^2}\right)\iint \boldsymbol{r} \times \boldsymbol{p}\,\mathrm{d}\varphi_c\mathrm{d}\varphi_t \quad (\varphi_c, \varphi_t \text{ from } 0 \text{ to } 2\pi). \tag{5}$$

The result, using (2), is $\boldsymbol{S}_{av} = \{0, 0, \hbar/2\}$, which identifies the spin as the average instantaneous angular momentum caused by the quantum during one period of its motion on the torus surface.

The instantaneous spin vectors during the motion form a distribution shown in **Figure 3**.

This distribution was shown to explain the results of spin measurements in directions forming an angle with the average spin of the particle [1], and led to the construction of our fermion model.

If the elementary charge (e) is ascribed to the quantum, the average magnetic moment caused by the rotation of the ring can be calculated. The frequency of rotation is $\nu = c/(L\pi)$, the well-known "Zitterbewegung" frequency. With this frequency, the current around the axis is $I = -ec/(L\pi)$. The classical expression for the magnetic moment is $\mu = IA$, with (A) being the average area enclosed by the current. The average of (A), is easily calculated using position vector (2) of the quantum on the torus. The result is $A_{av} = \pi L^2/2$. The magnetic moment calculated in this way becomes

$$\mu_{av} = -e(c/L)(L^2/2) = -ecL/2 = -e\hbar/(2m) = -\mu_{\text{classic}} \tag{6}$$

This agrees with the value obtained for the Dirac electron. We point out for later use that the anomaly of the magnetic moment of the electron is given by ($\alpha/\pi = 0.00232\cdots$), so that the magnetic moment becomes
$\mu = -\mu_{\text{classic}}(1+(\alpha/\pi))$, with ($\alpha$) being the fine structure constant.

The fermion model outlined above is completely general. The predicted properties, spin $S = \hbar/2$, and magnetic moment, $\mu_{av} = \mu_{\text{classic}}$, are independent of the mass, and the radius of the described particle is $R = \hbar/(mc)$. For fermions of arbitrary mass, therefore, we have the general relation

$$Rm = \hbar/c \tag{7}$$

We point out that, the radius predicted by (7) depends on the relative velocity

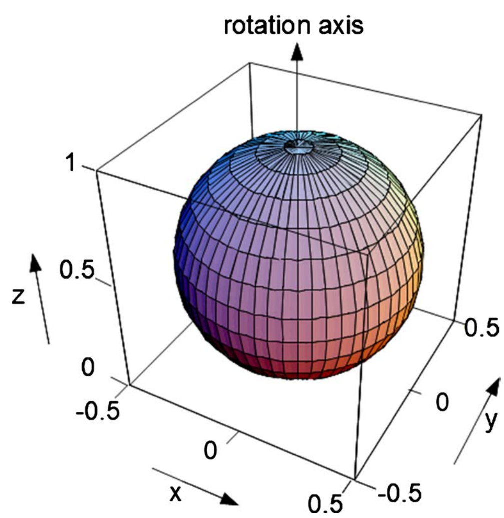

Figure 3. The instantaneous spin vector in units of ($\hbar$) during one period of the motion of the quantum with momentum (mc) around the center of the particle. The average is the observable spin vector $S_{av} = \{0, 0, \hbar/2\}$.

(v), because the mass in the model is the relativistic mass $m = m_0 \Big/ \sqrt{1-(v/c)^2}$, so that ($R$) decreases as $R = (\hbar/m_0 c)\sqrt{1-(v/c)^2}$, and reproduces the point-like character of the electron observed in scattering experiments at velocities approaching (c).

3. The Nucleons

If we apply the general relation (7) to the mass (M) of the proton, we find that the radius $R = \hbar/(Mc)$ is much smaller than the recommended value. This means that, within the described fermion model, the proton is not a single fermion. Although, within the established "standard model of elementary particles"(SM), the proton is described as being composed of three quarks bound by gluons, we consider—as the most obvious possibility within our fermion model—the "merging" of two quanta of equal energy (mc^2) into a particle of mass ($M = 2m$). Since the "merging" leads to a topology of the paths of two quanta that does not allow to distinguish different "particles" at all, the relation to the standard model is not clear, especially, because quarks do not exist as free particles. This will be shortly discussed in paragraph 4 in the light of the results to be described below.

Below we describe how the two quanta are "fused" into a new particle of mass (M) containing two quanta in periodic motions around a fixed point in space. The topology of the paths describing the synchronized motion of the two quanta, then, determines the properties of the new particle.

We consider two quanta which have equal probability on a ring in a plane perpendicular to their velocity vector. If the velocity is the velocity of light (c), the distributions on the rings are static. The radius of the ring is equal to the reduced Compton wavelength ($L = \hbar/(mc)$), and therefore determines the momentum (mc), and the energy ($mc^2 = \hbar\omega$). Each ring, with momentum ($mc = \hbar/L$), and speed (c) along the normal to the plane defined by the ring, represents the properties of a photon. We consider a situation, where the two quanta can "fuse". This is illustrated in **Figure 4**.

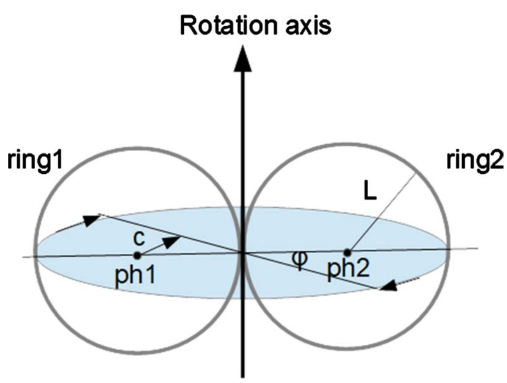

Figure 4. The situation when two photons represented by the two rings, have opposite directions, meet, and "stick together" in the way shown in the figure. The two rings then rotate as a whole around the center, which is at rest in space. The two photons, indicated as (ph), keep their velocity (c), and rotate on circles of radius (L) with frequency $\nu = c/(2\pi L)$. The radius of the "entity" formed is $R = 2L$.

The situation is described in the legend of the figure. The position vectors of the two quanta on the two rings are $\boldsymbol{r}_1 = \{x, y, z\}$, and $\boldsymbol{r}_2 = \{-x, -y, \pm z\}$, and are given by relation (2), except that (L) has to be replaced by ($2L$):

$$\boldsymbol{r}_1 = 2L\left\{\cos^2\left(\frac{\varphi_t}{2}\right)\cos(\varphi_c), \cos^2\left(\frac{\varphi_t}{2}\right)\sin(\varphi_c), \sin(\varphi_t)/2\right\} \tag{8}$$

$$\boldsymbol{r}_2 = 2L\left\{-\cos^2\left(\frac{\varphi_t}{2}\right)\cos(\varphi_c), -\cos^2\left(\frac{\varphi_t}{2}\right)\sin(\varphi_c), \pm\sin(\varphi_t)/2\right\} \tag{8a}$$

The common torus formed in the periodic motion of the two quanta represents an "entity" of energy Mc^2, and of radius $2L = 4\hbar/Mc$ at rest in space. We identify this torus with the neutron. A parametric plot of the positions of the two quanta using relations (8) is shown in **Figure 5**. The nucleon radii predicted in this way are:

$$R_{\text{neutron}} = 2L = 4\hbar/Mc = 0.84007\cdots\text{ fm} \tag{9}$$

$$R_{\text{proton}} = 0.84123\cdots\text{ fm} \tag{9a}$$

This proton value is—within the limits of error—equal to the most recent value for the charge radius of the proton, which has been obtained from an evaluation of Lamb-shift measurements on myonic hydrogen using quantum electrodynamic methods [11]. To our knowledge our result (9a) is the only purely theoretical prediction available today. Therefore, if the complete agreement between the value given in (9a), and the experimental result, is accepted to be not "accidental", it has to be concluded that the "Ansatz" implied in our fermion model, which is alternative to quantum mechanics and standard model, merits further attention.

The spin caused by the common motion of the two quanta can be obtained by defining the relative coordinate as $\boldsymbol{R} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, and a reduced mass $m_1 m_2/(m_1 + m_2) = m/2$. For these reduced quantities, the momentum becomes, $\boldsymbol{P} = \hbar/(4L)\{-\sin(\varphi_c), \cos(\varphi_c), 0\}$, and the instantaneous spin is $\boldsymbol{S} = \boldsymbol{P} \times \boldsymbol{R}$.

The average spin calculated for ($\boldsymbol{R}$), and ($-\boldsymbol{R}$), using expression (8) becomes

$$S_{\text{proton}} = \{0, 0, \hbar/2\} \text{ and } \{0, 0, -\hbar/2\} \tag{10}$$

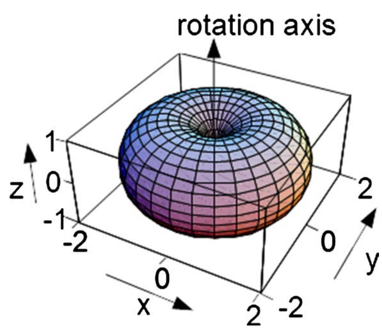

Figure 5. Representation of the nucleon by the possible positions of the two quanta during their periodic motion. The coordinates are given in units of (L). The instantaneous positions of the two quanta are on opposite sides of the axis, and the radius of the nucleon is four times its reduced Compton wavelength.

Assigning a charge (Q) to the position ($\boldsymbol{R} = \boldsymbol{r}_1 - \boldsymbol{r}_2$), we obtain from the general relation $\mu = IA$, with the current $I = Qc/(4\pi L)$, and the average surface (A) calculated with (8, 8a) two different magnetic moments, depending on the (+)-, or (−)-sign chosen in (8a) for the z-coordinate. We obtain, respectively,

$$\begin{aligned} \mu(a) &= 6(Q/e)\mu_n \\ \mu(b) &= 8(Q/e)\mu_n \end{aligned} \tag{11}$$

In (11), (μ_n) is the classical nuclear magnetic moment.

We have not investigated the role of the quantity "charge" in the context of our model. We point out, however, that with $Q/e = \pm 1/3$ one obtains possible magnetic moments of

$$\mu(a) = \pm 2\mu_n \tag{12}$$

The (±)-signs we ascribe to the possible formation of particle and anti-particle. The magnetic moment for the *particle* neutron is negative. If this required (−) sign is chosen, $\mu(a) = -2\mu_n$ deviates by only ca. 5% from the established value of the neutron magnetic moment of $\mu_{\text{neutron}} = -1.91\cdots\mu_n$. We therefore conclude that the model predicts

$$\mu_{\text{neutron}} = -2\mu_n \tag{13}$$

The total charge of the neutron is zero, so that for the charges (q_1) and (q_2) of the quanta relation $q_1 + q_2 = 0$ is required. With the condition $\boldsymbol{R} = \boldsymbol{r}_1 - \boldsymbol{r}_2$, which implies $Q = q_1 - q_2 = -(1/3)e$, we thus obtain $q_1/e = -1/6$ and $q_2/e = 1/6$, for the neutron. In addition we point out that the (+)-sign in relation (8) characterizes the neutron (antineutron).

The proton magnetic moment has the (+)-sign, and by loss of one electron from the neutron, the total charge increases to $+1e$. For symmetry reasons we assume that the extra charge (e) is added equally to the charges $q_1 = -(1/6)e$ and $q_2 = (1/6)e$, leading to $q_1 = (1/3)e$, and $q_2 = (2/3)e$, for the proton. The required positive (Q) can only be obtained for $Q = q_2 - q_1 = (1/3)e$, which corresponds to the topology given by $\boldsymbol{R} = \boldsymbol{r}_2 - \boldsymbol{r}_1$. This shows that the formation of a proton proceeds by loss of an electron from an anti-neutron. Relation (11), and $Q = (1/3)e$ lead to $\mu(b) = (8/3)\mu_n$. Since this value deviates by only ca. 5% from the established value of the proton magnetic moment of 2.79 … μ_n, we therefore conclude that the model predicts a value of the proton magnetic moment of

$$\mu_{\text{proton}} = \mu(b) = (8/3)\mu_n = 2.67\mu_n \tag{14}$$

We point out that the proton in this way is characterized by the (−)-sign in relation (8), so that the proton topology differs from the neutron topology. Within the model, therefore, the transition (anti-neutron → proton +e^-) is accompanied by a topology change. This topology change probably characterizes the loss of an antineutrino implied in the transition.

We expect the values (13, 14) to be correct at the level where the magnetic moment for the fermion becomes equal to μ_{classic}. For the neutron we derive a correction factor caused by the ($g - 2$) anomaly by comparing the predicted value

(13), with the recommended value of $1.9130\mu_n$, as $k = 1 - 1.9130/2 = 0.0435$. For the proton formed from the anti-neutron by loss of an electron we include the established correction factor of (0.00232), and obtain

$$\mu_{\text{proton}} = (8/3)(1 + 0.0435 + 0.00232)\mu_n = 2.7888\mu_n, \tag{15}$$

which agrees very closely with the recommended value. To our knowledge, a calculation from first principles of the magnetic moments of the nucleons is not yet available.

4. Summary

We have shown in this paper that our extended fermion model is able to predict the proton radius in "complete" agreement with the most recent experimental value of the charge radius. The radius predicted is—simply—four times the reduced Compton wavelength of the proton. This result

$$R_{\text{proton}} = 4\hbar/Mc = 0.84123\cdots \text{ fm} \tag{16}$$

is of special interest because it confirms the result of the most accurate value obtained in 2010 from an evaluation of Lamb-shift measurements on myonic hydrogen using quantum electrodynamic methods [11]. This value is

$$R_{\text{proton}} = (0.84184 \pm 0.0007) \text{ fm}. \tag{17}$$

Since this value disagrees with the actual recommended value, of

$$R_{\text{proton}} = (0.8875 \pm 0.0051) \text{ fm}, \tag{18}$$

by more than the given limits of error, there existed since (2010) for some years a so-called "proton-radius puzzle". In (2019), new electron-proton scattering experiments [12], which yielded a value of

$$R_{\text{proton}} = (0.831 \pm 0.007) \text{ fm}, \tag{19}$$

have explained the "puzzle" as having been due to unknown experimental artefacts of earlier experiments [12]. To our knowledge, our predicted value (16) is the only theoretical value available today.

The spin ($\pm\hbar/2$) of the nucleons is predicted, and values of the magnetic moments of proton and neutron are obtained. These values deviate by ca. 5% from the established recommended values. This deviation we ascribe to the (g-2)-anomaly also presents for the electron. By determining the correction factor for the neutron from our predicted value (13), one obtains from (14) for the proton a predicted value of

$$\mu_{\text{proton}} = 2.7888\mu_n, \tag{20}$$

which is very close to the recommended value. To our knowledge, a calculation from first principles of the magnetic moments of the nucleons is not yet available.

It is remarkable that the quark "particles", invoked in the rather involved description of the nucleons within the standard model (SM), appear in the form of charges (1/3)e and $(2/3)e$ in our model as a consequence of the topology of the autonomous motion of two quanta. Different "particles", cannot be distinguished in our model, where the topology of the paths of two quanta defines *one* "entity", which, for sufficiently long observation times, can be viewed as *one* particle, in the present case as one of the nucleons. The validity of the standard model, and of the quantum chromodynamics treatment of the nuclei is, of course, not questioned. On the other hand, the successes of the fermion model, demonstrated for the electron in references [1] [2] [3], and for the nucleons in this paper, suggest that there exists a close relation between the two descriptions, which deserves further investigations.

The outline given of the model shows that, it follows from an "Ansatz" which is "alternative" to quantum mechanics and (SM): instead of "particles", *quanta* are the basis, which form photons, fermions, and nucleons, in autonomous periodic motion. With this description, the nucleons are not composed of particles, but are rather "elementary", in a similar way as fermions are. These elementary particles have internal structure and can own properties. For their radius (R) we have the general relation

$$RM = 4\left(\hbar/c\right), \tag{21}$$

with (M) being the relativistic mass, which implies a velocity dependence of the radius.

The model is completely general because it does not involve any free parameters.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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Relativistic Correction of the Rydberg Formula

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Abstract

The relationship $E = -K$ holds between the energy E and kinetic energy K of the electron constituting a hydrogen atom. If the kinetic energy of the electron is determined based on that relationship, then the energy levels of the hydrogen atom are also determined. In classical quantum theory, there is a formula called the Rydberg formula for calculating the wavelength of a photon emitted by an electron. In this paper, in contrast, the formula for the wavelength of a photon is derived from the relativistic energy levels of a hydrogen atom derived by the author. The results show that, although the Rydberg constant is classically a physical constant, it cannot be regarded as a fundamental physical constant if the theory of relativity is taken into account.

Keywords

Rydberg Formula, Rydberg Constant, Classical Quantum Theory, Energy-Momentum Relationship in a Hydrogen Atom, Relativistic Kinetic Energy

1. Introduction

In the classical quantum theory of Bohr, the energy levels of the hydrogen atom are given by the following formula [1] [2].

$$E_{\mathrm{BO},n} = -\frac{1}{2}\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{m_e e^4}{\hbar^2}\cdot\frac{1}{n^2} \tag{1a}$$

$$= -\frac{\alpha^2 m_e c^2}{2n^2}, \quad n = 1, 2, \cdots. \tag{1b}$$

Here, E_{BO} refers to the total mechanical energy predicted by Bohr. Also, α is the following fine-structure constant.

$$\alpha = \frac{e^2}{4\pi\varepsilon_0 \hbar c}. \tag{2}$$

Bohr thought the following quantum condition was necessary to find the energy levels of the hydrogen atom.

$$m_e v_n \cdot 2\pi r_n = 2\pi n\hbar. \tag{3}$$

The energy of the hydrogen atom is also given by the following formula.

$$E = -K = \frac{1}{2}V(r) = -\frac{1}{2}\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r}. \tag{4}$$

If E in Equation (1b) is substituted into Equation (4), then the following formula can be derived as the orbital radius of the electron.

$$r_n = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2} n^2, \quad n = 1, 2, \cdots. \tag{5}$$

The photonic energy emitted during a transition between energy levels $\left(E_{\mathrm{BO},n} - E_{\mathrm{BO},m}\right)$ and wavelength λ for principal quantum numbers m and n can be expressed as follows.

$$E_{\mathrm{BO},n} - E_{\mathrm{BO},m} = h\nu = \frac{hc}{\lambda} = hcR_\infty\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \ m = 1, 2, \cdots, \ n = m+1, m+2, \cdots. \tag{6}$$

Here, R_∞ is the Rydberg constant, which is defined by the following equation.

$$R_\infty = \frac{\alpha^2 m_e c}{2h} = \frac{2\pi^2 m_e e^4}{ch^3}. \tag{7}$$

The Rydberg formula can be derived from Equation (6) as indicated below.

$$\frac{1}{\lambda} = \frac{E_{\mathrm{BO},n} - E_{\mathrm{BO},m}}{hc} = R_\infty\left(\frac{1}{m^2} - \frac{1}{n^2}\right), \quad m = 1, 2, \cdots, \quad n = m+1, m+2, \cdots. \tag{8}$$

2. Relationship Enfolded in Bohr's Quantum Condition

This section to Section 4 are excerpts from another paper, but this material is repeated because it is needed here. The Planck constant h can be written as follows [3]:

$$\hbar = \frac{h}{2\pi} = \frac{m_e c \lambda_C}{2\pi}. \tag{9}$$

Here, λ_C is the Compton wavelength of the electron.

When Equation (9) is used, the fine-structure constant α can be expressed as follows.

$$\alpha = \frac{e^2}{2\varepsilon_0 m_e c^2 \lambda_C}. \tag{10}$$

Also, the classical electron radius r_e is defined as follows.

$$r_e = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}. \tag{11}$$

If r_e/α is calculated here,

$$\frac{r_e}{\alpha} = \frac{\lambda_C}{2\pi}. \tag{12}$$

If Equation (5) is written using r_e and α, the result is as follows.

$$r_n = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2} n^2 = \frac{e^2}{4\pi\varepsilon_0 m_e c^2}\left(\frac{4\pi\varepsilon_0 \hbar c}{e^2}\right)^2 n^2 = \frac{r_e}{\alpha^2} n^2. \tag{13}$$

Next, if $\hbar$ in Equation (9) and r_n in Equation (13) are substituted into Equation (3),

$$m_e v_n \cdot 2\pi \frac{r_e}{\alpha^2} n^2 = 2\pi n \frac{m_e c \lambda_C}{2\pi}. \tag{14}$$

If Equation (12) is also used, then Equation (14) can be written as follows.

$$m_e v_n \cdot 2\pi \frac{r_e}{\alpha^2} n^2 = 2\pi n \frac{m_e c r_e}{\alpha}. \tag{15}$$

From this, the following relationship can be derived [4].

$$\frac{v_n}{c} = \frac{\alpha}{n}. \tag{16}$$

3. The Relation between Kinetic Energy and Momentum Derived from the STR Relationship

The energy-momentum relationship in the special theory of relativity (STR) holds in an isolated system in free space. Here, if m_0 is rest mass and m relativistic mass, the relationship can be written as follows.

$$\left(m_0 c^2\right)^2 + p^2 c^2 = \left(mc^2\right)^2. \tag{17}$$

What is the relationship between relativistic kinetic energy and momentum if this relationship holds?

Incidentally, Sommerfeld once defined kinetic energy as the difference between the relativistic energy mc^2 and rest mass energy $m_0 c^2$ of an object [5]. That is,

$$K = mc^2 - m_0 c^2 = m_0 c^2 \left[\frac{1}{\left(1-\beta^2\right)^{1/2}} - 1\right], \quad \beta = \frac{v}{c}. \tag{18}$$

Sommerfeld believed that Equation (18), which can be derived from Equation (17), can also be applied to the electron in a hydrogen atom.

First, it is clear that the following formula holds [4].

$$\left[m_0 c^2 + \left(mc^2 - m_0 c^2\right)\right]^2 = \left(mc^2\right)^2. \tag{19}$$

Expanding the left side of this equation yields the following.

$$m_0^2 c^4 + \left(m^2 c^4 - m_0^2 c^4\right) = \left(m_0 c^2\right)^2 + \left(m + m_0\right)\left(mc^2 - m_0 c^2\right)c^2. \tag{20}$$

Using this, Equation (19) becomes as follows.

$$\left(m_0 c^2\right)^2 + \left(m + m_0\right)\left(mc^2 - m_0 c^2\right)c^2 = \left(mc^2\right)^2. \tag{21}$$

Since this equation and Equation (17) are equal, the following relationship must hold when Equation (18) is taken into account.

$$p^2 = (m_0 + m)(mc^2 - m_0c^2) = (m_0 + m)K. \tag{22}$$

The following formula is obtained from this.

$$K_{\mathrm{re}} = \frac{p_{\mathrm{re}}^2}{m_0 + m}. \tag{23}$$

Here, K_{re} is relativistic kinetic energy and p_{re} relativistic momentum. The "re" in K_{re} and p_{re} stands for "relativistic".

Equation (23) is the formula for relativistic kinetic energy. Classical (non-relativistic) kinetic energy, in contrast, is defined as follows.

$$K_{\mathrm{cl}} = \frac{1}{2}m_0v^2 = \frac{p_{\mathrm{cl}}^2}{2m_0}. \tag{24}$$

In classical theory, mass does not depend on velocity. That is, Equation (23) and Equation (24) are the same if $m = m_0$.

4. Energy-Momentum Relationship of the Electron Derived with Another Method

The author has previously derived the following relationships applicable to the electron constituting a hydrogen atom [6].

$$(m_{\mathrm{e}}c^2)^2 - p_n^2c^2 = (m_nc^2)^2. \tag{25}$$

Here,

$$m_nc^2 = m_{\mathrm{e}}c^2 - K_{\mathrm{re},n}. \tag{26}$$

m_n is the mass of an electron in a state where the principal quantum number is n.

These energy relationships can be illustrated as follows (**Figure 1**).

In this paper, Equation (25) will be derived more simply by using a method different from that used previously. The logic of Equations (19) to (23) is borrowed to accomplish that purpose.

Now, it is clear that the following equation holds.

$$\left[m_nc^2 + (m_{\mathrm{e}}c^2 - m_nc^2)\right]^2 = (m_{\mathrm{e}}c^2)^2. \tag{27}$$

Expanding and rearranging this equation, the following equation is obtained.

$$(m_nc^2)^2 + (m_{\mathrm{e}} + m_n)(m_{\mathrm{e}}c^2 - m_nc^2)c^2 = (m_{\mathrm{e}}c^2)^2. \tag{28}$$

Next, the relativistic kinetic energy of the electron can be defined as follows by referring to Equation (23).

$$K_{\mathrm{re},n} = m_{\mathrm{e}}c^2 - m_nc^2 = \frac{p_{\mathrm{re},n}^2}{m_{\mathrm{e}} + m_n}. \tag{29}$$

From this,

$$(m_{\mathrm{e}} + m_n)(m_{\mathrm{e}}c^2 - m_nc^2) = p_{\mathrm{re},n}^2. \tag{30}$$

Finally, Equation (28) matches Equation (25).

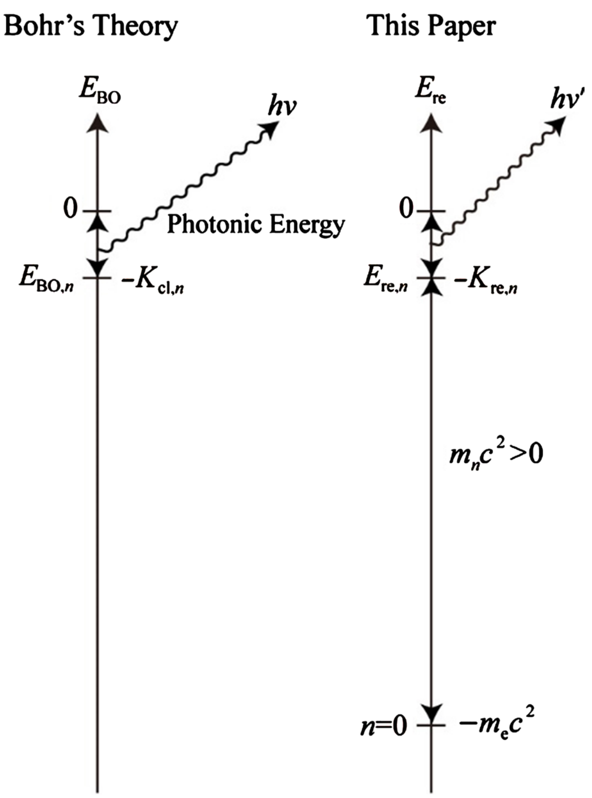

Figure 1. Energy levels of a hydrogen atom derived from Bohr's classical quantum theory and this paper: According to the virial theorem, $E_{\mathrm{BO},n}=-K_{\mathrm{cl},n}$ and $E_{\mathrm{re},n}=-K_{\mathrm{re},n}$. An electron at rest in free space emits a photon when it is taken into a hydrogen atom. Also, the electron acquires the same amount of kinetic energy as the energy of the emitted photon. If $E_{\mathrm{BO}}=0, E_{\mathrm{re}}=0$ are described using an absolute energy scale, then the electron is at rest in free space, and this corresponds to the state of having a rest mass energy of $m_e c^2$.

Incidentally, $p_{\mathrm{re},n}=m_n v_n$ [4], and thus it is clear that the following equation holds.

$$p_{\mathrm{re},n}c=m_n v_n c. \tag{31}$$

Here, if we substitute $p_{\mathrm{re},n}c$ in Equation (31) into Equation (25) and rearrange, then the following value is obtained.

$$m_n=m_e\left(1+\frac{v_n^2}{c^2}\right)^{-1/2}. \tag{32}$$

If the relation in Equation (16) is used here, Equation (32) becomes as follows.

$$m_n=m_e\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}=m_e\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}. \tag{33}$$

Hence, the energy levels of a hydrogen atom $E_{\mathrm{re},n}$ are:

$$E_{\mathrm{re},n}=-K_{\mathrm{re},n}=m_n c^2-m_e c^2=m_e c^2\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}-1\right],\quad n=0,1,2,\cdots. \tag{34}$$

5. Relativistic Energy of a Hydrogen Atom Derived from Equation (16)

When both sides of Equation (16) are squared, and then multiplied by $m_{\mathrm{e}}/2$,

$$\frac{1}{2}\frac{m_{\mathrm{e}}v_n^2}{c^2}=\frac{1}{2}\frac{m_{\mathrm{e}}\alpha^2}{n^2}. \tag{35}$$

Hence,

$$E_{\mathrm{BO},n}=-\frac{1}{2}m_{\mathrm{e}}v_n^2=-\frac{\alpha^2 m_{\mathrm{e}}c^2}{2n^2}. \tag{36}$$

If Equation (16) is taken as a departure point, the energy levels of the hydrogen atom derived by Bohr can be derived immediately. Equation (16) has tremendous power. However, from a relativistic perspective, $(1/2)m_{\mathrm{e}}v_n^2$ is an approximation of the kinetic energy of the electron. Therefore, the energy in Equation (1) is also an approximation of the true value.

Next, let's try to derive the energy levels in the hydrogen atom from Equation (16). If both sides of Equation (16) are first squared, and then both sides are multiplied by $m_n^2/(m_{\mathrm{e}}+m_n)$.

$$\frac{m_n^2 v_n^2}{m_{\mathrm{e}}+m_n}=\frac{\alpha^2c^2}{n^2}\frac{m_n^2}{m_{\mathrm{e}}+m_n}. \tag{37}$$

Here, the left side of Equation (37) is the relativistic kinetic energy of the electron, and thus the energy levels are:

$$E_{\mathrm{re},n}=-K_{\mathrm{re},n}=-\frac{m_n^2 v_n^2}{m_{\mathrm{e}}+m_n}=-\frac{\alpha^2c^2}{n^2}\frac{m_n^2}{m_{\mathrm{e}}+m_n}. \tag{38}$$

Next, if Equation (33) is taken into account, the right side of Equation (38) is as follows.

$$E_{\mathrm{re},n}=-\frac{\alpha^2c^2}{n^2}\times\left(\frac{n^2m_{\mathrm{e}}^2}{n^2+\alpha^2}\right)\times\frac{1}{m_{\mathrm{e}}\left[1+\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right]}. \tag{39}$$

Next, the numerator and denominator of Equation (39) are multiplied by:

$$1-\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}.$$

When this is done, Equation (39) is as follows.

$$E_{\mathrm{re},n}=-\frac{\alpha^2}{n^2}\left(\frac{n^2}{n^2+\alpha^2}\right)m_{\mathrm{e}}c^2\times\left(\frac{n^2+\alpha^2}{\alpha^2}\right)\left[1-\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}\right] \tag{40a}$$

$$=m_{\mathrm{e}}c^2\left[\left(\frac{n^2}{n^2+\alpha^2}\right)^{1/2}-1\right] \tag{40b}$$

$$=m_{\mathrm{e}}c^2\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}-1\right]. \tag{40c}$$

Equation (34) can also be derived taking Equation (16) as a starting point. The discussion thus far in this section has provided an explanation by quoting another paper.

Now, if a Taylor expansion is performed on the right side of Equation (34),

$$E_{\mathrm{re},n} = m_e c^2 \left[\left(1 - \frac{\alpha^2}{2n^2} + \frac{3\alpha^4}{8n^4} - \frac{5\alpha^6}{16n^6} + \cdots \right) - 1 \right] \tag{41a}$$

$$= -m_e c^2 \left(\frac{\alpha^2}{2n^2} - \frac{3\alpha^4}{8n^4} + \frac{5\alpha^6}{16n^6} - \cdots \right) \tag{41b}$$

$$= -\frac{\alpha^2 m_e c^2}{2n^2} \left(1 - \frac{3\alpha^2}{4n^2} + \frac{5\alpha^4}{8n^4} - \cdots \right). \tag{41c}$$

Comparing Equation (41c) and Equation (1b), it is evident that Equation (1) is an approximation of Equation (34). That is,

$$E_{\mathrm{re},n} \approx E_{\mathrm{BO},n}. \tag{42}$$

Next, **Table 1** summarizes the energies of a hydrogen atom obtained from Equation (1) and Equation (34) [7].

The following values of CODATA were used when calculating energies.

$$\alpha = 7.2973525693 \times 10^{-3}.$$

$$c = 2.99792458 \times 10^{8}\ \mathrm{m \cdot s^{-1}}.$$

$$m_e = 9.1093837015 \times 10^{-31}\ \mathrm{kg}.$$

The results derived from section 3 to 5 are summarized here in **Table 2**.

In deriving the energy levels of a hydrogen atom, Sommerfeld began from Einstein's energy-momentum relationship. However, that is a mistake. The Einstein relation that holds in an isolated system in free space is not applicable in the space inside a hydrogen atom where there is potential energy. The author derived, for the first time, Equation (25) that is applicable to an electron in a hydrogen atom.

6. Discussion

In the sections up to the previous section, the groundwork was laid for finding a formula for the wavelength of a photon emitted from a hydrogen atom.

The differences in energy between different energy levels in the hydrogen atom can be found with the following formula.

Table 1. Comparison of the energies of a hydrogen atom predicted by Bohr's classical quantum theory and this paper.

	Bohr's Energy Levels	This Paper
$n = 1$	−13.60569 eV	−13.60515 eV
2	−3.40142 eV	−3.40139 eV
3	−1.511744 eV	−1.511737 eV

Table 2. Formulas and energies derived from the standpoint of STR and Sommerfeld, and formulas and energies derived by the author: Equations marked at the right with an asterisk are quoted from Reference [4]. When discussed by depicting a classical picture, like the Bohrmodel, the electron moving within the atom becomes lighter as its velocity increases.

	Formula of STR and Sommerfeld (SO)	This Paper
Kinetic Energy	$K_{\text{re}} = mc^2 - m_0c^2$. (18)	$K_{\text{re},n} = m_e c^2 - m_n c^2$. (26)
	$K_{\text{re}} = \dfrac{p_{\text{re}}^2}{m_0 + m}$. (23)	$K_{\text{re},n} = \dfrac{p_{\text{re},n}^2}{m_e + m_n}$. (29)
Energy-Momentum Relationship	$(m_0c^2)^2 + p^2c^2 = (mc^2)$. (17) Holds in isolated systems in free space	$(m_e c^2)^2 - p_n^2c^2 = (m_n c^2)^2$. (25) Applicable to an electron in a hydrogen atom
Energy Levels of a Hydrogen Atom	$E_{\text{SO}} = -K_{\text{re}} = m_0c^2 - mc^2$. (18)	$E_{\text{re},n} = -K_{\text{re},n} = m_n c^2 - m_e c^2$. (34)
	$E_{\text{SO},n} = m_e c^2\left[1-\left(\dfrac{n^2}{n^2-\alpha^2}\right)^{1/2}\right], n = 1, 2, \cdots$. *	$E_{\text{re},n} = m_e c^2\left[\left(\dfrac{n^2}{n^2+\alpha^2}\right)^{1/2} - 1\right], n = 0, 1, 2, \cdots$. (40b)
	$E_{\text{SO},n} \approx -\dfrac{\alpha^2 m_e c^2}{2n^2}\left(1+\dfrac{3\alpha^2}{4n^2}+\dfrac{5}{8}\dfrac{\alpha^4}{n^4}\right)$. *	$E_{\text{re},n} \approx -\dfrac{\alpha^2 m_e c^2}{2n^2}\left(1-\dfrac{3\alpha^2}{4n^2}+\dfrac{5\alpha^4}{8n^4}\right)$. (41c)
E_1	$E_{\text{SO},1} = -13.60624$ eV. *	$E_{\text{re},1} = -13.60515$ eV. *

$$E_{\text{re},n} - E_{\text{re},m} = \left(m_n c^2 - m_e c^2\right) - \left(m_m c^2 - m_e c^2\right) = m_n c^2 - m_m c^2 = h\nu = h\frac{c}{\lambda}$$
$$= m_e c^2\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} - \left(1+\frac{\alpha^2}{m^2}\right)^{-1/2}\right], \quad n = m+1, m+2, \cdots. \tag{45}$$

The following equation is also known.

$$\lambda_C = \frac{h}{m_e c}. \tag{46}$$

Taking into account Equation (46),

$$m_e c^2 = \frac{hc}{\lambda_C}. \tag{47}$$

Based on this, Equation (45) can be written as follows.

$$\frac{1}{\lambda} = \frac{E_{\text{re},n} - E_{\text{re},m}}{hc}$$
$$= \frac{1}{\lambda_C}\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2} - \left(1+\frac{\alpha^2}{m^2}\right)^{-1/2}\right], \ m = 0, 1, 2, \cdots, \ n = m+1, m+2, \cdots. \tag{48}$$

Equation (48) is the formula for wavelength, taking into account Equation (25). If the Taylor expansion of Equation (48) is taken, the following formula is obtained.

$$\frac{1}{\lambda} = \frac{1}{\lambda_C}\left[\left(1-\frac{\alpha^2}{2n^2}+\frac{3\alpha^4}{8n^4}-\frac{5\alpha^6}{16n^6}+\cdots\right)-\left(1-\frac{\alpha^2}{2m^2}+\frac{3\alpha^4}{8m^4}-\frac{5\alpha^6}{16m^6}+\cdots\right)\right] \tag{49a}$$

$$= \frac{\alpha^2}{2\lambda_C}\left[\left(\frac{1}{m^2}-\frac{3\alpha^2}{4m^4}+\frac{5\alpha^4}{8m^6}-\cdots\right)-\left(\frac{1}{n^2}-\frac{3\alpha^2}{4n^4}+\frac{5\alpha^4}{8n^6}-\cdots\right)\right] \tag{49b}$$

Table 3. Wavelengths of photons emitted due to transitions between different energy levels: the 3 values in the table, in order from the top, are the values found from Equation (8), the values found from Equation (48), and the actual measured values.

	$m = 1$	2
$n = 2$	121.502 nm 121.508 nm 121.6 nm	
3	102.517 nm 102.522 nm 102.6 nm	656.112 nm 656.121 nm 656.29 nm

$$=\frac{\alpha^2 m_e c}{2h}\left[\left(\frac{1}{m^2}-\frac{3\alpha^2}{4m^4}+\frac{5\alpha^4}{8m^6}-\cdots\right)-\left(\frac{1}{n^2}-\frac{3\alpha^2}{4n^4}+\frac{5\alpha^4}{8n^6}-\cdots\right)\right] \tag{49c}$$

$$=R_\infty\left[\left(\frac{1}{m^2}-\frac{3\alpha^2}{4m^4}+\frac{5\alpha^4}{8m^6}-\cdots\right)-\left(\frac{1}{n^2}-\frac{3\alpha^2}{4n^4}+\frac{5\alpha^4}{8n^6}-\cdots\right)\right] \tag{49d}$$

$$\approx R_\infty\left(\frac{1}{m^2}-\frac{1}{n^2}\right)+R_\infty\left(\frac{3\alpha^2}{4n^4}-\frac{3\alpha^2}{4m^4}-\frac{5\alpha^4}{8n^6}+\frac{5\alpha^4}{8m^6}\right). \tag{49e}$$

It is evident here that the approximation value of Equation (49e) matches the Rydberg formula Equation (8). That is,

$$\frac{1}{\lambda_C}\left[\left(1+\frac{\alpha^2}{n^2}\right)^{-1/2}-\left(1+\frac{\alpha^2}{m^2}\right)^{-1/2}\right]\approx R_\infty\left(\frac{1}{m^2}-\frac{1}{n^2}\right),\quad n=m+1,m+2,\cdots. \tag{50}$$

Here, if the wavelengths of photons emitted due to transitions between different energy levels are calculated using Equations (8) and (48), the results are as indicated in Table 3.

Precision up to 4 significant digits is required for experiments. Therefore, there will be no problems even if the approximation of Equation (8) is used instead of Equation (48) to calculate wavelengths.

However, Equation (8) for calculating the wavelength of the spectra of a hydrogen atom is strange because it does not include the Compton wavelength of the electron.

7. Conclusions

In classical quantum theory, the wavelength of a photon emitted due to a transition by an electron to a different energy level is calculated using Equation (8). However, this paper has shown that Equation (48) is a formula more exact than calculating the wavelength of the photon.

Thus, it has been shown that the existing Equation (8) is an approximation for Equation (48).

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Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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How to Explicitly Calculate Feynman and Wheeler Propagators in the ADS/CFT Correspondence

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Abstract

We discuss, giving all necessary details, the boundary-bulk propagators. We do it for a scalar field, with and without mass, for both the Feynman and the Wheeler cases. Contrary to standard procedure, we do not need here to appeal to any unfounded conjecture (as done by other authors). Emphasize that we do not try to modify standard ADS/CFT procedures, but use them to evaluate the corresponding Feynman and Wheeler propagators. Our present calculations are original in the sense of being the first ones undertaken explicitly using distributions theory (DT). They are carried out in two instances: 1) when the boundary is a Euclidean space and 2) when it is of Minkowskian nature. In this last case we compute also three propagators: Feynman's, Anti-Feynman's, and Wheeler's (half advanced plus half retarded). For an operator corresponding to a scalar field we explicitly obtain, for the first time ever, the two points' correlations functions in the three instances above mentioned. To repeat, it is not our intention here to improve on ADS/CFT theory but only to employ it for evaluating the corresponding Wheeler's propagators.

Keywords

ADS/CFT Correspondence, Boundary-Bulk Propagators, Feynman's Propagators, Wheeler's Propagators

1. Introduction

Propagators and correlators are one of the essential tools to work, for example, in Quantum Field Theory (QFT) and String Theory (ST), in particular, in formulating the correspondence ADS/CFT (Anti-de Sitter/Conformal Field Theory). This correspondence was established by Maldacena [1] in 1998 and is universally regarded as a very useful model for many purposes.

The bibliography on this subject, for scalar fields, is quite extensive. We give here just a small representative in [2]-[12]. For a more complete bibliography the reader is directed to the report [13].

One of the ADS/CFT correspondence's prescriptions (see [2]) will allow us to evaluate the correlators on the boundary of ADS space. The first boundary-bulk propagator was calculated by Witten a few months after the appearance of [1], entitled *Anti de Sitter space and holography*. In this case the boundary is a Euclidean space [2] [3].

In this work, instead, we evaluate the boundary-bulk propagators for the case in which the boundary is a Minkowskian space. In such regards, remark that some attempts have been made before in [14] [15] [16].

1.1. The Wheeler Propagator

The *Feynman's propagator* for a free real scalar field is a time-ordered correlation function of two scalar fields $\Phi(x)$ and $\Phi(y)$ in the vacuum state

$$G_F(x-y) = \langle 0|\hat{T}\Phi(x)\Phi(y)|0\rangle. \tag{1.1}$$

This propagator is a *Green function* of the *Klein-Gordon* equation, and is discussed in almost any text-book on quantum mechanics. Not so well-known at all is the *Wheeler propagator*. In fact, to provide a fairly complete description of it constitutes one of the present goals.

More than half a century ago, J. A. *Wheeler* and R. P. *Feynman* published a work [17] in which they represented electromagnetic interactions by means of a *half advanced* and *half retarded* Green functions. The charged medium was supposed to be a *perfect absorber*, so that no radiation could possibly escape the system.

We are going to call this kind of *Green function* a "*Wheeler function*" (or propagator). It has been used before by *P. A. M. Dirac* [18], when trying to avoid some run-away solutions, in which one finds rapid increases that cannot be controlled. Later on, in 1949, J. A. Wheeler and R. P. Feynman showed that, in spite of the fact that the Green function contains an advanced part, the results do no contradict causality [19].

A causal, unitary, and Lorentz invariant quantification of tachyons was performed in reference [20]. The corresponding propagator is precisely a Wheeler's one.

The same happens with complex mass particles that appear in higher order supersymmetric models [21]. For these particles, the propagator is also a Wheeler's propagator.

We review some precedent work below.

1.2. The Starinets and Son Paper

The main previous attempt to try to calculate boundary-bulk propagators in the

Minkowskian boundary for the Anti-de Sitter space [in the ADS/CFT correspondence] was made by Son and Starinets (SS) in 2002 [22]. However, SS needed to formulate a conjecture that we show here to become unnecessary if one uses the full distributions-theory of type $\mathcal{S}'$ (of Schwartz). SS literally state (the necessary symbols will be explained later in the text) "We circumvent the difficulties mentioned above by putting forward the following conjecture

$$G^R(k) = -2F(k,z)\big|_{z_B} \qquad (3.15)\text{"}$$

For this conjecture no rigorous mathematical basis is presented. Instead, we will nor need here any conjecture at all. SS' work was entitled "Minkowski-space correlators in AdS/CFT correspondence: recipe and applications".

1.3. The Freedman *et al.* Paper

We must also mention the work of Freedman *et al.* [23], in which the authors deal with the case of a Euclidean boundary. Freedman, however, did not treat the case of a Minkowskian boundary, at least in the way that Son and Starinets did. To repeat, we make full use here of distribution theory. This does not entail, of course, a simple $i\epsilon$ prescription, but a much more elaborate treatment, that has not been performed before in this field. Let us also remark, as this is an important point for us, that in this paper we do not evaluate renormalized correlation functions.

1.4. Our Treatment

As stated above, in the present effort we evaluate, without any a la Starinets and Son conjecture, the boundary-bulk propagators corresponding to the following three cases i) Feynman, ii) Anti-Feynman, and iii) Wheeler (half advanced plus half retarded). We do this both for massless and massive scenarios (a scalar field involved). Later we calculate the two points correlators (TPC) for operators corresponding to this scalar field in the three instances previously mentioned. We clarify that in this paper we do not evaluate the renormalized TPC.

We demonstrate as well that the Feynman propagator must be a function of $\rho + i0$ (see below for the notation) in momentum space, and therefore a function of $x^2 - i0$ in configuration space. We show that something similar happens with the Anti-Feynman propagator. *For the first time ever*, we calculate the Wheeler's propagator (half advanced plus half retarded) as well.

As usual, we use here regularity conditions 1) at the origin (Dirichlet's) and 2) of rapid decay at infinity (boundary condition). This applies, for instance, to Equations (2.8), (2.9), and (2.10).

It may be asserted that propagators are always to be interpreted in a distributional sense, but most authors do not employ, in dealing with them, the FULL distribution theory developed by Laurent Schwartz [24] and Israelovich M. Guelfand *et al.* [25].

Note also that, until the 90'*s, the only field propagators that had been calcu-*

lated were Anti-de Sitter (spatial) ones.

1.5. Organization of This Work

The paper is organized as follows: Section 2 deals with the Euclidean case. In it, the three different propagators referred to above cannot be distinguished (neither in the massive nor in the massless instances).

In Section 3, we tackle similar scenarios as those of Section 2, but now in Minkowski's space, where the three propagators can be distinguished.

In Section 4, we compute in Euclidean space the TPC for a scalar operator corresponding to a scalar field via Witten's prescription.

In Section 5, we generalize the calculations of Section 4 to Minkowski's space. We obtain in this fashion the two-point correlations functions corresponding to the three different propagators of our list above.

Finally, some conclusions are drawn in Section 6.

2. Euclidean Case

2.1. Massless Scalar Field Propagator

The Klein-Gordon equation in $ADS_{\nu+1}$ for the scalar field $\phi(z,\boldsymbol{x})$ reads, in Poincare coordinates,

$$z^2\partial_z^2\phi(z,\boldsymbol{x})+(1-\nu)z\partial_z\phi(z,\boldsymbol{x})+z^2\nabla^2\phi(z,\boldsymbol{x})-\Delta(\Delta-\nu)\phi(z,\boldsymbol{x})=0, \quad (2.1)$$

where $\Delta(\Delta-\nu)\geq 0$ plays the role of m^2. We exclude tachyons form of this treatment. Here Δ is the conformal dimension, ν the boundary's dimension, and $\boldsymbol{x}$ their coordinates. The Fourier transform in the variables $\boldsymbol{x}$ of the field $\phi(z,\boldsymbol{x})$ is

$$\hat{\phi}(z,\boldsymbol{k})=\int\phi(z,\boldsymbol{x})e^{i\boldsymbol{k}\cdot\boldsymbol{x}}d^\nu x. \quad (2.2)$$

Using (2.2), (2.1) takes the form

$$z^2\partial_z^2\hat{\phi}(z,\boldsymbol{k})+(1-\nu)z\partial_z\hat{\phi}(z,\boldsymbol{k})-\left[z^2k^2+\Delta(\Delta-\nu)\right]\hat{\phi}(z,\boldsymbol{k})=0. \quad (2.3)$$

We analyze now the massless case given by $\Delta=0,\nu$. For it we have the motion equation

$$z^2\partial_z^2\hat{\phi}(z,\boldsymbol{k})+(1-\nu)z\partial_z\hat{\phi}(z,\boldsymbol{k})-z^2k^2\hat{\phi}(z,\boldsymbol{k})=0, \quad (2.4)$$

or equivalently (for $z\neq 0$),

$$\partial_z^2\hat{\phi}(z,\boldsymbol{k})+\frac{1-\nu}{z}\partial_z\hat{\phi}(z,\boldsymbol{k})-k^2\hat{\phi}(z,\boldsymbol{k})=0. \quad (2.5)$$

In the variable *z*, this equation is of the Bessel type (see [26])

$$F''(z)+\frac{1-2\alpha}{z}F'(z)-\left[k^2+\frac{\mu^2-\alpha^2}{z^2}\right]F(z)=0 \quad (2.6)$$

The pertinent solution (that does not diverge when the argument tends to infinity) is

$$F(z)=z^\alpha\mathcal{K}_\mu(kz). \quad (2.7)$$

Thus, the solution of (2.5) becomes

$$\hat{\phi}(z,k) = z^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(kz). \tag{2.8}$$

One easily verifies that, for infinitesimal z [26],

$$\mathcal{K}_{\frac{\nu}{2}}(kz) = \frac{2^{\frac{\nu}{2}-1}\Gamma\left(\frac{\nu}{2}\right)}{(kz)^{\frac{\nu}{2}}} + O\left((kz)^{-\frac{\nu}{2}+2}\right). \tag{2.9}$$

Equation (2.9) is just a Bessel-McDonald distribution (defined by Guelfand [25]) in Euclidean space. As a consequence,

$$\lim_{z\to 0} z^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(kz) = \frac{2^{\frac{\nu}{2}-1}\Gamma\left(\frac{\nu}{2}\right)}{k^{\frac{\nu}{2}}}. \tag{2.10}$$

In other words, the solution is regular at the origin and vanishes at infinity (in the variable z). Accordingly, we have, for the field in the bulk, the solution

$$\phi(z,\boldsymbol{x}) = \frac{z^{\frac{\nu}{2}}}{(2\pi)^{\nu}} \int a(\boldsymbol{k}) \mathcal{K}_{\frac{\nu}{2}}(kz) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^{\nu}k. \tag{2.11}$$

This solution must reduce itself to the field $\phi_0(x)$ on the boundary, so that

$$\phi(0,\boldsymbol{x}) = \phi_0(\boldsymbol{x}) = \frac{2^{\frac{\nu}{2}-1}\Gamma\left(\frac{\nu}{2}\right)}{(2\pi)^{\nu}} \int a(\boldsymbol{k}) k^{-\frac{\nu}{2}} \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^{\nu}k = \frac{1}{(2\pi)^{\nu}} \int \hat{\phi}_0(\boldsymbol{k}) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^{\nu}k. \tag{2.12}$$

From this last equation we can obtain $a(k)$ as a function of $\hat{\phi}_0$ and then write

$$\phi(z,\boldsymbol{x}) = \frac{z^{\frac{\nu}{2}} 2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu} \Gamma\left(\frac{\nu}{2}\right)} \int k^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(kz) \hat{\phi}_0(\boldsymbol{k}) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^{\nu}k, \tag{2.13}$$

or, equivalently,

$$\phi(z,\boldsymbol{x}) = \frac{z^{\frac{\nu}{2}} 2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu} \Gamma\left(\frac{\nu}{2}\right)} \iint k^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(kz) \phi_0(\boldsymbol{x}') \mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \mathrm{d}^{\nu}k \mathrm{d}^{\nu}x'. \tag{2.14}$$

From (2.14) we then obtain an expression of the boundary-bulk propagator

$$K(z,\boldsymbol{x}-\boldsymbol{x}') = \frac{z^{\frac{\nu}{2}} 2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu} \Gamma\left(\frac{\nu}{2}\right)} \int k^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(kz) \mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \mathrm{d}^{\nu}k. \tag{2.15}$$

To carry out the integration in the variable k we appeal to the expressions for the Fourier transform and its inverse obtained by Bochner [27]. For the Fourier transform we have

$$\hat{f}(k)=\int f(\boldsymbol{x})\mathrm{e}^{i\boldsymbol{k}\cdot\boldsymbol{x}}\mathrm{d}^{\nu}x=\frac{(2\pi)^{\frac{\nu}{2}}}{k^{\frac{\nu}{2}-1}}\int_{0}^{\infty}r^{\frac{\nu}{2}}\mathcal{J}_{\frac{\nu}{2}-1}(kr)f(r)\mathrm{d}r, \tag{2.16}$$

and for its inverse

$$f(r)=\frac{1}{(2\pi)^{\nu}}\int\hat{f}(\boldsymbol{k})\mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\mathrm{d}^{\nu}k=\frac{1}{(2\pi)^{\frac{\nu}{2}}r^{\frac{\nu}{2}-1}}\int_{0}^{\infty}k^{\frac{\nu}{2}}\mathcal{J}_{\frac{\nu}{2}-1}(kr)\hat{f}(k)\mathrm{d}k. \tag{2.17}$$

Using these relations we have now

$$K(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\frac{(2\pi)^{\frac{\nu}{2}}}{|\boldsymbol{x}-\boldsymbol{x}'|^{\frac{\nu}{2}-1}}\int_{0}^{\infty}k^{\nu}\mathcal{K}_{\frac{\nu}{2}}(kz)\mathcal{J}_{\frac{\nu}{2}-1}\left(k|\boldsymbol{x}-\boldsymbol{x}'|\right)\mathrm{d}k. \tag{2.18}$$

So as to evaluate the last integral we appeal to a result from [26]

$$\int_{0}^{\infty}x^{\mu+\nu+1}\mathcal{K}_{\nu}(bx)\mathcal{J}_{\mu}(ax)\mathrm{d}x=2^{\mu+\nu}a^{\mu}b^{\nu}\frac{\Gamma(\mu+\nu+1)}{\left(a^{2}+b^{2}\right)^{\mu+\nu+1}}, \tag{2.19}$$

Our deduction follows a different, simpler and complete path than that of [2]. Our approach also has a didactic utility.

$$K(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{\Gamma(\nu)}{\pi^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}\left[\frac{z}{z^{2}+(\boldsymbol{x}-\boldsymbol{x}')^{2}}\right]^{\nu}, \tag{2.20}$$

which leads to

$$\phi(z,\boldsymbol{x})=\int K(z,\boldsymbol{x}-\boldsymbol{x}')\phi_{0}(\boldsymbol{x}')\mathrm{d}^{\nu}x', \tag{2.21}$$

an expression that, in turn, leads to

$$\lim_{z\to 0}K(z,\boldsymbol{x}-\boldsymbol{x}')=\delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{2.22}$$

2.2. Massive Field Propagator

We now consider the massive case $\Delta\neq 0,\nu$. The equation of motion for this case reads

$$z^{2}\partial_{z}^{2}\hat{\phi}(z,\boldsymbol{k})+(1-\nu)z\partial_{z}\hat{\phi}(z,\boldsymbol{k})-\left[z^{2}k^{2}+\Delta(\Delta-\nu)\right]\hat{\phi}(z,\boldsymbol{k})=0, \tag{2.23}$$

or equivalently,

$$\partial_{z}^{2}\hat{\phi}(z,\boldsymbol{k})+\frac{1-\nu}{z}\partial_{z}\hat{\phi}(z,\boldsymbol{k})-\left[k^{2}+\frac{\Delta(\Delta-\nu)}{z^{2}}\right]\hat{\phi}(z,\boldsymbol{k})=0. \tag{2.24}$$

The solution for this last equation is

$$\hat{\phi}(z,k)=z^{\frac{\nu}{2}}\mathcal{K}_{\mu}(kz), \tag{2.25}$$

with

$$\mu = \pm\sqrt{\frac{\nu^2}{4} + \Delta(\Delta - \nu)}. \tag{2.26}$$

Since $\mathcal{K}_\mu(z) = \mathcal{K}_{-\mu}(z)$, we select for μ in (2.26) the plus sign. We have then

$$\phi(z, \boldsymbol{x}) = \frac{z^{\frac{\nu}{2}}}{(2\pi)^\nu} \int a(\boldsymbol{k}) \mathcal{K}_\mu(kz) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^\nu k. \tag{2.27}$$

For $\Delta \neq 0$, this solution is not regular at the origin. To overcome this problem we select

$$\phi(\epsilon, \boldsymbol{x}) = \phi_\epsilon(\boldsymbol{x}) = \frac{\epsilon^{\frac{\nu}{2}}}{(2\pi)^\nu} \int a(\boldsymbol{k}) \mathcal{K}_\mu(k\epsilon) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^\nu k = \frac{1}{(2\pi)^\nu} \int \hat{\phi}_\epsilon(\boldsymbol{k}) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^\nu k, \tag{2.28}$$

where ϵ is infinitesimal. From (2.28) we have then

$$a(\boldsymbol{k}) = \frac{\hat{\phi}_\epsilon(\boldsymbol{k})}{\epsilon^{\frac{\nu}{2}} \mathcal{K}_\mu(k\epsilon)}. \tag{2.29}$$

Replacing the result of (2.29) into (2.27) we obtain

$$\phi(z, \boldsymbol{x}) = \frac{1}{(2\pi)^\nu} \left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}} \int \frac{\mathcal{K}_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \hat{\phi}_\epsilon(\boldsymbol{k}) \mathrm{e}^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \mathrm{d}^\nu k, \tag{2.30}$$

or similarly,

$$\phi(z, \boldsymbol{x}) = \frac{1}{(2\pi)^\nu} \left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}} \iint \frac{\mathcal{K}_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \phi_\epsilon(\boldsymbol{x}') \mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \mathrm{d}^\nu k \mathrm{d}^\nu x'. \tag{2.31}$$

From this last equation we see that the propagator is

$$K_m(z, \boldsymbol{x} - \boldsymbol{x}') = \frac{1}{(2\pi)^\nu} \left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}} \int \frac{\mathcal{K}_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')} \mathrm{d}^\nu k. \tag{2.32}$$

As a consequence we can write

$$\phi(z, \boldsymbol{x}) = \int K_m(z, \boldsymbol{x} - \boldsymbol{x}') \phi_\epsilon(\boldsymbol{x}') \mathrm{d}^\nu x'. \tag{2.33}$$

From (2.33) we immediately gather that

$$K_m(\epsilon, \boldsymbol{x} - \boldsymbol{x}') = \delta(\boldsymbol{x} - \boldsymbol{x}'). \tag{2.34}$$

2.3. Wrong but Popular Approach for Approximate Massive Field Propagators

It is instructive to discuss here a popular but non-valid approach for the function $\mathcal{K}(k\epsilon)$. The issue here is that, although ϵ is infinitesimal, it cannot adopt a 0-value. As k is an unbounded variable, when $k \to \infty$, we have $k\epsilon \to \infty$. Notice first that

$$\mathcal{K}_\mu(k\epsilon) = \frac{2^{\mu-1}\Gamma(\mu)}{(k\epsilon)^\mu} + O\left((k\epsilon)^{2-\mu}\right). \tag{2.35}$$

Some people make now the approximation

$$\mathcal{K}_\mu(k\epsilon)=\frac{2^{\mu-1}\Gamma(\mu)}{(k\epsilon)^\mu}. \tag{2.36}$$

From (2.32) one obtains an approximation for the propagator K that can be called M. Ome has then

$$M_m(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{1}{(2\pi)^\nu}\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\frac{\epsilon^\mu}{2^{\mu-1}\Gamma(\mu)}\int k^\mu\mathcal{K}_\mu(kz)\mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\mathrm{d}^\nu k. \tag{2.37}$$

Using again the Bochner formula one arrives at

$$\int k^\mu\mathcal{K}_\mu(kz)\mathrm{e}^{-i\boldsymbol{k}\cdot(\boldsymbol{x}-\boldsymbol{x}')}\mathrm{d}^\nu k=\frac{(2\pi)^{\frac{\nu}{2}}}{|\boldsymbol{x}-\boldsymbol{x}'|^{\frac{\nu}{2}-1}}\int\limits_0^\infty k^{\mu+\frac{\nu}{2}}\mathcal{K}_\mu(kz)\mathcal{J}_{\frac{\nu}{2}-1}(k|\boldsymbol{x}-\boldsymbol{x}'|)\mathrm{d}k. \tag{2.38}$$

By recourse to (2.19) it follows that

$$M_m(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{\epsilon^{\mu-\frac{\nu}{2}}}{\pi^{\frac{\nu}{2}}}\frac{\Gamma\left(\mu+\frac{\nu}{2}\right)}{\Gamma(\mu)}\left[\frac{z}{z^2+(\boldsymbol{x}-\boldsymbol{x}')^2}\right]^{\mu+\frac{\nu}{2}}. \tag{2.39}$$

Defining

$$\gamma=\frac{\nu}{2}+\mu=\frac{\nu}{2}+\sqrt{\frac{\nu^2}{4}+\Delta(\Delta-\nu)}, \tag{2.40}$$

one can write

$$M_m(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{\epsilon^{\gamma-\nu}}{\pi^{\frac{\nu}{2}}}\frac{\Gamma(\gamma)}{\Gamma\left(\gamma-\frac{\nu}{2}\right)}\left[\frac{z}{z^2+(\boldsymbol{x}-\boldsymbol{x}')^2}\right]^\gamma. \tag{2.41}$$

It is then realized that, by construction,

$$M_m(\epsilon,\boldsymbol{x}-\boldsymbol{x}')\neq\delta(\boldsymbol{x}-\boldsymbol{x}'), \tag{2.42}$$

and define

$$N_m(z,\boldsymbol{x}-\boldsymbol{x}')=M_m(z,\boldsymbol{x}-\boldsymbol{x}')\epsilon^{\nu-\gamma}, \tag{2.43}$$

which allows one to write for N_m the expression

$$N_m(z,\boldsymbol{x}-\boldsymbol{x}')=\frac{1}{\pi^{\frac{\nu}{2}}}\frac{\Gamma(\gamma)}{\Gamma\left(\gamma-\frac{\nu}{2}\right)}\left[\frac{z}{z^2+(\boldsymbol{x}-\boldsymbol{x}')^2}\right]^\gamma. \tag{2.44}$$

Therefore, one has constructively proved that

$$\lim_{z\to0}N_m(z,\boldsymbol{x}-\boldsymbol{x}')\neq\delta(\boldsymbol{x}-\boldsymbol{x}'). \tag{2.45}$$

Note that (2.44) is indeed the well known expression for the boundary-bulk propagator for a scalar field in configuration space. However, this expression can only be used as an approximation to the propagator K when $\mu\cong\frac{\nu}{2}$.

The above recounted approximation, not very well founded, is precisely the one most people use in current literature to obtain the propagator (2.32). From it, people deduce the approximation (2.44).

Indeed, one of the main goals of our paper is to overcome the problems posed by this approximation. We will try below to do better than current usage, and shall indeed achieve our goal.

3. Minkowskian Case

3.1. Massless Field Propagator

Let us now deal with the case in which the boundary of the $ADS_{\nu+1}$ is the ν-dimensional Minkowskian space. In the massless case the field-equation is

$$z^2\partial_z^2\hat{\phi}(z,k)+(1-\nu)z\partial_z\hat{\phi}(z,k)+z^2k^2\hat{\phi}(z,k)=0, \tag{3.1}$$

where $k^2=k_0^2-\boldsymbol{k}^2=\rho$. Thus, we can write

$$z^2\partial_z^2\hat{\phi}(z,\rho)+(1-\nu)z\partial_z\hat{\phi}(z,\rho)+z^2\rho\hat{\phi}(z,\rho)=0, \tag{3.2}$$

or, rewriting this last equation,

$$z^2\partial_z^2\hat{\phi}(z,\rho)+(1-\nu)z\partial_z\hat{\phi}(z,\rho)-z^2\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^2\hat{\phi}(z,\rho)=0. \tag{3.3}$$

The distribution $(\rho\pm i0)^{\lambda}$ is defined as (see reference [24])

$$(\rho\pm i0)^{\lambda}=\rho_+^{\lambda}+\mathrm{e}^{\pm i\pi\lambda}\rho_-^{\lambda}, \tag{3.4}$$

and can be cast in terms of $H(x)$, the Heaviside step function [24]. We recast now (3.3) in the form of a Bessel equation

$$\partial_z^2\hat{\phi}(z,\rho)+\frac{1-\nu}{z}\partial_z\hat{\phi}(z,\rho)-\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^2\hat{\phi}(z,\rho)=0. \tag{3.5}$$

The solution of this equation that is 1) regular at the origin and 2) vanishes for $\rho\to\infty$, becoming

$$\hat{\phi}(z,k)=z^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]. \tag{3.6}$$

One must take into account that $\lim_{k\to\infty}\mathrm{e}^{ikx}=0$ (see below in this section and [25]).

$$\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]=\frac{2^{\frac{\nu}{2}-1}\Gamma\left(\frac{\nu}{2}\right)}{\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]^{\frac{\nu}{2}}}+O\left(\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]^{-\frac{\nu}{2}+2}\right). \tag{3.7}$$

Equation (3.7) is just a Bessel-McDonald distribution (defined by Guelfand [24]) in Minkowskian space. We have then

$$\phi_{\mp}(z,x)=\frac{z^{\frac{\nu}{2}}}{(2\pi)^{\nu}}\int a(\boldsymbol{k})\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k=\int\hat{\phi}(z,k)\mathrm{e}^{ik\cdot x}\mathrm{d}^{\nu}k. \tag{3.8}$$

From this last equation we deduce that

$$\phi_{\mp}(z,x)=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\int\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]\hat{\phi}_0(k)\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k, \quad (3.9)$$

or, equivalently,

$$\phi_{\mp}(z,x)=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\iint\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right] \times\phi_0(x')\mathrm{e}^{-ik\cdot(x-x')}\mathrm{d}^{\nu}k\mathrm{d}^{\nu}x'. \quad (3.10)$$

The ensuing propagator becomes then

$$K_{\mp}(z,x-x')=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\int\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]\mathrm{e}^{-ik\cdot(x-x')}\mathrm{d}^{\nu}k. \quad (3.11)$$

Thus, the corresponding Feynman's propagator is

$$K_F(z,x-x')=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\int\left[-i(\rho+i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}K_{\frac{\nu}{2}}\left[-i(\rho+i0)^{\frac{1}{2}}z\right]\mathrm{e}^{-ik\cdot(x-x')}\mathrm{d}^{\nu}k. \quad (3.12)$$

Note that the Feynman propagator is a function of $\rho+i0$, as it should. For the anti-Feynman propagator we have instead

$$K_{AF}(z,x-x')=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{(2\pi)^{\nu}\Gamma\left(\frac{\nu}{2}\right)}\int\left[i(\rho-i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}K_{\frac{\nu}{2}}\left[i(\rho-i0)^{\frac{1}{2}}z\right]\mathrm{e}^{-ik\cdot(x-x')}\mathrm{d}^{\nu}k. \quad (3.13)$$

The expression for the Wheeler's propagator (half advanced plus half retarded) is:

$$W(z,x-x')=\frac{1}{2}\left[K_F(z,x-x')+K_{AF}(z,x-x')\right]. \quad (3.14)$$

Using the relations

$$K_F(z,\rho)=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\left[-i(\rho+i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[-i(\rho+i0)^{\frac{1}{2}}z\right], \quad (3.15)$$

and

$$K_{AF}(z,\rho)=\frac{z^{\frac{\nu}{2}}2^{1-\frac{\nu}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\left[i(\rho-i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}}\mathcal{K}_{\frac{\nu}{2}}\left[i(\rho-i0)^{\frac{1}{2}}z\right], \quad (3.16)$$

we can define, as usual, the retarded propagator

$$K_R(z,\rho)=H(k^0)K_F(z,\rho)+H(-k^0)K_{AF}(z,\rho), \quad (3.17)$$

and the advanced propagator

$$K_A(z,\rho) = H(k^0) K_{AF}(z,\rho) + H(-k^0) K_F(z,\rho). \tag{3.18}$$

We are going to show now that $\lim\limits_{k\to\infty} e^{ikx} = 0$ (see [28]). Let $\hat{\phi}$ be a test function belonging to a sub-space $\mathcal{S}$ of Schwartz's one [24] [25]. Its Fourier transform is

$$\phi(k) = \int_{-\infty}^{\infty} \hat{\phi}(x) e^{ikx} dx, \tag{3.19}$$

where ϕ belongs to $\mathcal{S}$. Then one can verify that

$$0 = \lim_{k\to\infty} \phi(k) = \lim_{k\to\infty} \int_{-\infty}^{\infty} \hat{\phi}(x) e^{ikx} dx = \int_{-\infty}^{\infty} \hat{\phi}(x) \lim_{k\to\infty} e^{ikx} dx. \tag{3.20}$$

As a consequence, we obtain

$$\lim_{k\to\infty} e^{ikx} = 0. \tag{3.21}$$

(3.21) is an extremely well-known fact established by Distribution Theory, and can be found in the text-book by Jones [28]. The Feynman propagator is, according to (3.12),

$$K_F(z,x) = \frac{z^{\frac{\nu}{2}} 2^{1-\frac{\nu}{2}}}{(2\pi)^\nu \Gamma\left(\frac{\nu}{2}\right)} \int \left[-i(\rho+i0)^{\frac{1}{2}}\right]^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}\left[-i(\rho+i0)^{\frac{1}{2}} z\right] e^{-ik\cdot x} d^\nu k. \tag{3.22}$$

Since $\mathcal{K}_{\frac{\nu}{2}}$ is exponentially decreasing or oscillating, we can evaluate the integral that defines K_F by means of a Wick rotation over k_0. Therefore we have the change of variables $k_0 = ik_{0E}$, $x_0 = ix_{0E}$, $k_E^2 = k_{0E}^2 + \boldsymbol{k}^2$, and $x_E^2 = x_{0E}^2 + \boldsymbol{x}^2$. Casting the integral that defines the propagator in terms of these new variables, we obtain

$$K_F(z,\boldsymbol{x}_E) = \frac{iz^{\frac{\nu}{2}} 2^{1-\frac{\nu}{2}}}{(2\pi)^\nu \Gamma\left(\frac{\nu}{2}\right)} \int k_E^{\frac{\nu}{2}} \mathcal{K}_{\frac{\nu}{2}}(k_E z) e^{-i\boldsymbol{k}_E\cdot\boldsymbol{x}_E} d^\nu k_E. \tag{3.23}$$

Using Bochner's formula together with (3.19) we have

$$K_F(z,x_E) = \frac{i\Gamma(\nu)}{\pi^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)} \left[\frac{z}{z^2 + x_E^2}\right]^\nu. \tag{3.24}$$

Now, making the change to Minkowskian variables and taking into account that the Fourier transform of a distribution that depends on $\rho - i0$ is a distribution that depends on $x^2 + i0$, we obtain

$$K_F(z,x) = \frac{i\Gamma(\nu)}{\pi^{\frac{\nu}{2}} \Gamma\left(\frac{\nu}{2}\right)} \left[\frac{z}{z^2 - x^2 - i0}\right]^\nu, \tag{3.25}$$

which is the expression of the Feynman propagator in terms of the variables of the configuration space. For the anti-Feynman propagator we analogously find

$$K_{AF}(z,x)=\frac{i\Gamma(\nu)}{\pi^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}\left[\frac{z}{z^2-x^2+i0}\right]^{\nu}. \tag{3.26}$$

3.2. Massive Field Propagator

For the massive case, the field-motion equation is

$$\partial_z^2\hat{\phi}(z,\rho)+\frac{1-\nu}{z}\partial_z\hat{\phi}(z,\rho)-\left\{\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\right]^2+\frac{\Delta(\Delta-\nu)}{z^2}\right\}\hat{\phi}(z,\rho)=0, \tag{3.27}$$

with, again,

$$\mu=\sqrt{\frac{\nu^2}{4}+\Delta(\Delta-\nu)}. \tag{3.28}$$

The pertinent solution is now

$$\hat{\phi}_{\mp}(z,\rho)=z^{\frac{\nu}{2}}\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]. \tag{3.29}$$

The field-expression in configuration space is then

$$\phi_{\mp}(z,x)=\frac{z^{\frac{\nu}{2}}}{(2\pi)^{\nu}}\int a(\boldsymbol{k})\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k. \tag{3.30}$$

Once again we choose

$$\begin{aligned}\phi(\epsilon,x)=\phi_{\epsilon}(x)&=\frac{\epsilon^{\frac{\nu}{2}}}{(2\pi)^{\nu}}\int a(\boldsymbol{k})\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\epsilon\right]\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k\\&=\frac{1}{(2\pi)^{\nu}}\int\hat{\phi}_{\epsilon}(k)\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k,\end{aligned} \tag{3.31}$$

and from (3.23) we obtain

$$a(k)=\frac{\hat{\phi}_{\epsilon}(k)}{\epsilon^{\frac{\nu}{2}}K_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\epsilon\right]}. \tag{3.32}$$

We have then the following relation for the solution

$$\phi_{\mp}(z,x)=\frac{1}{(2\pi)^{\nu}}\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\iint\frac{\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]}{\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\epsilon\right]}\phi_{\epsilon}(x')\mathrm{e}^{-ik\cdot(x-x')}\mathrm{d}^{\nu}k\mathrm{d}^{\nu}x', \tag{3.33}$$

so that the propagator is now

$$K_{m\mp}(z,x-x')=\frac{1}{(2\pi)^{\nu}}\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\int\frac{\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}z\right]}{\mathcal{K}_{\mu}\left[\mp i(\rho\pm i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-i\boldsymbol{k}\cdot(x-x')}\mathrm{d}^{\nu}k. \tag{3.34}$$

The corresponding Feynman's propagator becomes

$$K_{mF}\left(z,x-x'\right)=\frac{1}{(2\pi)^{\nu}}\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\int\frac{\mathcal{K}_{\mu}\left[-i\left(\rho+i0\right)^{\frac{1}{2}}z\right]}{\mathcal{K}_{\mu}\left[-i\left(\rho+i0\right)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-i\boldsymbol{k}\cdot(x-x')}\mathrm{d}^{\nu}k. \quad (3.35)$$

For the anti-Feynman propagator we obtain the expression

$$K_{mAF}\left(z,x-x'\right)=\frac{1}{(2\pi)^{\nu}}\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\int\frac{\mathcal{K}_{\mu}\left[i\left(\rho-i0\right)^{\frac{1}{2}}z\right]}{\mathcal{K}_{\mu}\left[i\left(\rho-i0\right)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-i\boldsymbol{k}\cdot(x-x')}\mathrm{d}^{\nu}k. \quad (3.36)$$

Finally, the definition of Wheeler propagators, half retarded and half advanced, is similar to that of the preceding subsection, this is:

$$W_m\left(z,x-x'\right)=\frac{1}{2}\left[K_{mF}\left(z,x-x'\right)+K_{mAF}\left(z,x-x'\right)\right]. \quad (3.37)$$

3.3. An Approximation

We now evaluate in approximate fashion the propagator $\mathcal{K}_{\mu}\left[-i\left(\rho+i0\right)^{\frac{1}{2}}\epsilon\right]$

$$\mathcal{K}_{\mu}\left[-i\left(\rho+i0\right)^{\frac{1}{2}}\epsilon\right]=\frac{2^{\mu-1}\Gamma(\mu)}{(-i)^{\mu}\left(\rho+i0\right)^{\frac{\mu}{2}}\epsilon^{\mu}}, \quad (3.38)$$

entailing

$$M_{mF}\left(z,x\right)=\frac{z^{\frac{\nu}{2}}}{(2\pi)^{\nu}}\frac{\epsilon^{\mu-\frac{\nu}{2}}}{2^{\mu-1}\Gamma(\mu)}\int\mathcal{K}_{\mu}\left[-i\left(\rho+i0\right)^{\frac{1}{2}}z\right]\left[-i\left(\rho+i0\right)^{\frac{1}{2}}\right]^{\mu}\mathrm{e}^{-ik\cdot x}\mathrm{d}^{\nu}k. \quad (3.39)$$

Effecting again the above Wick's rotation we obtain

$$M_{mF}\left(z,\boldsymbol{x}_E\right)=\frac{iz^{\frac{\nu}{2}}}{(2\pi)^{\nu}}\frac{\epsilon^{\mu-\frac{\nu}{2}}}{2^{\mu-1}\Gamma(\mu)}\int k_E^{\mu}\mathcal{K}_{\mu}\left(k_E z\right)\mathrm{e}^{-i\boldsymbol{k}_E\cdot\boldsymbol{x}_E}\mathrm{d}^{\nu}k_E. \quad (3.40)$$

This integral is evaluated as in the previous cases. One has

$$M_{mF}\left(z,\boldsymbol{x}_E\right)=\frac{i\epsilon^{\mu-\frac{\nu}{2}}}{\pi^{\frac{\nu}{2}}}\frac{\Gamma\left(\mu-\frac{\nu}{2}\right)}{\Gamma(\mu)}\left(\frac{z}{z^2+x_E^2}\right)^{\mu+\frac{\nu}{2}}. \quad (3.41)$$

Changing variables as above we arrive at

$$M_{mF}\left(z,x\right)=\frac{i\epsilon^{\gamma-\nu}}{\pi^{\frac{\nu}{2}}}\frac{\Gamma(\gamma)}{\Gamma\left(\gamma-\frac{\nu}{2}\right)}\left(\frac{z}{z^2-x^2-i0}\right)^{\gamma}, \quad (3.42)$$

where

$$\gamma=\frac{\nu}{2}+\sqrt{\frac{\nu^2}{4}+\Delta\left(\Delta-\nu\right)}. \quad (3.43)$$

Now we return to the inequality

$$M_{mF}(\epsilon,x)\neq\delta(x) \tag{3.44}$$

The following relation is valid for N_F

$$N_{mF}(\epsilon,x)=\epsilon^{\nu-\gamma}M_{mF}(\epsilon,x). \tag{3.45}$$

Proceeding in analogous fashion with the Anti-Feynman propagator we obtain the approximation

$$M_{mAF}(z,x)=\frac{i\epsilon^{\gamma-\nu}}{\pi^{\frac{\nu}{2}}}\frac{\Gamma(\gamma)}{\Gamma\left(\gamma-\frac{\nu}{2}\right)}\left(\frac{z}{z^2-x^2+i0}\right)^{\gamma} \tag{3.46}$$

4. Glaring Mistakes of Son and Starinets' Calculation [22] Corrected

By appeal to the unproved conjecture mentioned in Subsection 1.2, Son and Starinets evaluated the retarded propagator for a scalar field in a work regarded as a standard-bear of the ADS/CFT field. They found

$$K_R(z,\rho)=\frac{N^2k^4}{64\pi^2}\left[\ln\left|k^2\right|-i\pi H\left(-k^2\right)sgn\omega\right] \tag{4.1}$$

We will show below that this result is both wrong and incomplete.

The retarded propagator reads

$$K_R(z,\rho)=H\left(k^0\right)K_F(z,\rho)+H\left(-k^0\right)K_{AF}(z,\rho). \tag{4.2}$$

For $\nu=4$ one has

$$\begin{aligned}&K_2\left[-i(\rho+i0)^{\frac{1}{2}}z\right]\\&=-\frac{1}{4}+i(\rho+i0)^{\frac{1}{2}}z^{-1}+\left[\frac{i\pi}{2}-C-\ln z-\frac{1}{2}\ln(\rho+i0)\right]f(z,\rho)+g(z,\rho),\end{aligned} \tag{4.3}$$

where

$$f(z,\rho)=\sum_{s=0}^{\infty}\frac{(-\rho)^{1+s}}{s!(2+s)!}\left(\frac{z}{2}\right)^{2+2s}, \tag{4.4}$$

and

$$g(z,\rho)=\sum_{l=0}^{\infty}\frac{(-\rho)^{1+l}}{l!(2+l)!}\left(\frac{z}{2}\right)^{2+2l}\left[\sum_{s=0}^{l}\frac{1}{s}+\sum_{s=0}^{2+l}\frac{1}{s}\right]. \tag{4.5}$$

Using [26] we have

$$\begin{aligned}&K_2\left[i(\rho-i0)^{\frac{1}{2}}z\right]\\&=-\frac{1}{4}-i(\rho-i0)^{\frac{1}{2}}z^{-1}-\left[\frac{i\pi}{2}+C+\ln z+\frac{1}{2}\ln(\rho-i0)\right]f(z,\rho)+g(z,\rho).\end{aligned} \tag{4.6}$$

This, Feynman's propagator becomes

$$\begin{aligned}K_F(z,\rho) &= \frac{z^2\rho}{8} - \frac{i}{4}(\rho+i0)^{\frac{1}{2}} z \\ &\quad - \frac{z^2\rho}{2}\left[\frac{i\pi}{2} - C - \ln z - \frac{1}{2}\ln(\rho+i0)\right] f(z,\rho) + g(z,\rho),\end{aligned} \tag{4.7}$$

while the anti-Feynman one turns out to be

$$\begin{aligned}K_{AF}(z,\rho) &= \frac{z^2\rho}{8} + \frac{i}{4}(\rho-i0)^{\frac{1}{2}} z \\ &\quad + \frac{z^2\rho}{2}\left[\frac{i\pi}{2} + C + \ln z + \frac{1}{2}\ln(\rho-i0)\right] f(z,\rho) + g(z,\rho).\end{aligned} \tag{4.8}$$

With the two last results OUR version of Starinets and Son retarded propagator becomes

$$\begin{aligned}K_R(z,\rho) &= \frac{z^2\rho}{8} - \frac{i\pi z^2\rho}{4} sgn(k^0) f(z,\rho) + \frac{z^2\rho}{2}(C+\ln z) f(z,\rho) \\ &\quad - iz\left[H(k^0)(\rho+i0)^{\frac{1}{2}} - H(k^0)(\rho-i0)^{\frac{1}{2}}\right] \\ &\quad + \frac{z^2\rho}{4}\left[H(k^0)\ln(\rho+i0) + H(-k^0)\ln(\rho-i0)\right] f(z,\rho) + g(z,\rho),\end{aligned} \tag{4.9}$$

and for the advanced one

$$\begin{aligned}K_A(z,\rho) &= \frac{z^2\rho}{8} + \frac{i\pi z^2\rho}{4} sgn(k^0) f(z,\rho) + \frac{z^2\rho}{2}(C+\ln z) f(z,\rho) \\ &\quad + iz\left[H(k^0)(\rho-i0)^{\frac{1}{2}} - H(k^0)(\rho+i0)^{\frac{1}{2}}\right] \\ &\quad + \frac{z^2\rho}{4}\left[H(k^0)\ln(\rho-i0) + H(-k^0)\ln(\rho+i0)\right] f(z,\rho) + g(z,\rho).\end{aligned} \tag{4.10}$$

With a little algebra the two propagators reappear as

$$\begin{aligned}K_R(z,\rho) &= \frac{z^2\rho}{8} + \frac{z^2\rho}{2}(C+\ln z) f(z,\rho) - iz\left[H(k^0)(\rho+i0)^{\frac{1}{2}} - H(k^0)(\rho-i0)^{\frac{1}{2}}\right] \\ &\quad + \frac{z^2\rho}{4}\left[H(k^0)\ln(-\rho-i0) + H(-k^0)\ln(-\rho+i0)\right] f(z,\rho) + g(z,\rho),\end{aligned} \tag{4.11}$$

$$\begin{aligned}K_A(z,\rho) &= \frac{z^2\rho}{8} + \frac{z^2\rho}{2}(C+\ln z) f(z,\rho) + iz\left[H(k^0)(\rho-i0)^{\frac{1}{2}} - H(k^0)(\rho+i0)^{\frac{1}{2}}\right] \\ &\quad + \frac{z^2\rho}{4}\left[H(k^0)\ln(-\rho+i0) + H(-k^0)\ln(-\rho-i0)\right] f(z,\rho) + g(z,\rho).\end{aligned} \tag{4.12}$$

Consider now the penultimate term of the retarded propagator. It is

$$\frac{z^2\rho}{4}\left[H(k^0)\ln(-\rho-i0) + H(-k^0)\ln(-\rho+i0)\right] f(z,\rho). \tag{4.13}$$

Considering just the first term ($s=0$) in $f(z,\rho)$ we can write (up to a sign)

$$\frac{z^2\rho}{4}\left[H(k^0)\ln(-\rho-i0) + H(-k^0)\ln(-\rho+i0)\right], \frac{\rho z^2}{8} \tag{4.14}$$

that can be recast as

$$\frac{z^4\rho^2}{32}\ln\left[|\rho|-i\pi sgn\left(k^0\right)H\left(\rho\right)\right]. \tag{4.15}$$

k^2 reads, using Son and Starinets' metrics

$$\rho = k^{02} - \boldsymbol{k}^2 = -k^2:$$

$$\frac{z^4k^4}{32}\ln\left[\left|k^2\right|-i\pi sgn\left(k^0\right)H\left(-k^2\right)\right], \tag{4.16}$$

which coincides with (4.1) after calling $N^2 = 2\pi^2 z^4$.

Thus, expression (4.1) is just a single term of the full expression for the retarded propagator of (4.11). This last propagator verifies $\lim_{z\to\infty} K_R\left(z,\rho\right)=0$ while (4.1) does not. We conclude then that (4.1) CAN NOT be used as a propagator.

Starinets and Son expression (SS) (4.1) cannot be regarded as a propagator for the massless scalar field. The same happens for the Feynman propagator of Eq. (3.21) in page 9 of [22]. These erroneous results demonstrate that their conjecture is inadequate.

4.1. Son and Starinets Surprising Elimination of a Divergence

To justify the results of their paper, in page 22 of [22], Son and Starinets encounter an infinite in their equation (A.22). They eliminate it by setting $(-1)! = 1 = \Gamma(0)$, which is absurd since $\Gamma(z)$ has a pole in $z=0$, and, as a consequence, it has a divergence in this value of *z*. This procedure is mathematically unacceptable. However, it was applauded by many ADS/CFT practitioners. Read and learn!

5. Two Points Correlation Functions in Euclidean Space

5.1. Massless Case

To evaluate the two-point correlation function of a scalar operator, we use the result obtained in [29]. This is

$$\left\langle \mathcal{O}\left(\boldsymbol{x}_1\right)\mathcal{O}\left(\boldsymbol{x}_2\right)\right\rangle = -\int\sqrt{g}\,\partial_\mu K\left(y_0,\boldsymbol{y}-\boldsymbol{x}_1\right)\partial^\mu K\left(y_0,\boldsymbol{y}-\boldsymbol{x}_2\right)\mathrm{d}^{\nu+1}y, \tag{5.1}$$

where $0 \le y_0 = z < \infty$, $y_\mu = x_\mu$, $\mu \neq 0$, and then

$$\left\langle \mathcal{O}\left(\boldsymbol{x}_1\right)\mathcal{O}\left(\boldsymbol{x}_2\right)\right\rangle = -\int\limits_{Boundary}\lim_{z\to 0}\left[z^{1-\nu}K\left(z,\boldsymbol{x}-\boldsymbol{x}_1\right)\partial_z K\left(z,\boldsymbol{x}-\boldsymbol{x}_2\right)\right]\mathrm{d}^\nu x. \tag{5.2}$$

As $\lim\limits_{z\to 0} K\left(0,\boldsymbol{x}_1-\boldsymbol{x}_2\right)=\delta\left(\boldsymbol{x}_1-\boldsymbol{x}_2\right)$, we obtain

$$\left\langle \mathcal{O}\left(\boldsymbol{x}_1\right)\mathcal{O}\left(\boldsymbol{x}_2\right)\right\rangle = -\lim_{z\to 0}\left[z^{1-\nu}\partial_z K\left(z,\boldsymbol{x}_1-\boldsymbol{x}_2\right)\right]. \tag{5.3}$$

Using now the expression for *K* given in Equation (2.20) we have

$$\left\langle \mathcal{O}\left(\boldsymbol{x}_1\right)\mathcal{O}\left(\boldsymbol{x}_2\right)\right\rangle = -\frac{\Gamma\left(\nu+1\right)}{\pi^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\left(\boldsymbol{x}_1-\boldsymbol{x}_2\right)^{2\nu}}. \tag{5.4}$$

Accordingly, we have here arrived to the usual, well-known result.

5.2. Massive Case

For the massive case we obtain, similarly,

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_m = -\int\limits_{Boundary} \left[z^{1-\nu} K_m(z, \mathbf{x}-\mathbf{x}_1)\partial_z K(z, \mathbf{x}-\mathbf{x}_2)\right] \mathrm{d}^\nu x. \tag{5.5}$$

As $K_m(\epsilon, \mathbf{x}-\mathbf{x}_1) = \delta(\mathbf{x}-\mathbf{x}_1)$ we can write

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_m = -\int \delta(\mathbf{x}-\mathbf{x}_1)\left[z^{1-\nu}\partial_z K(z, \mathbf{x}-\mathbf{x}_2)\right]_{z=\epsilon} \mathrm{d}^\nu x. \tag{5.6}$$

Thus we arrive at

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_m = -\left[z^{1-\nu}\partial_z K_m(z, \mathbf{x}_1-\mathbf{x}_2)\right]_{z=\epsilon}. \tag{5.7}$$

Now, we use the expression for K_m given in (3.32) and write

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_m = -\frac{\epsilon^{1-\nu}}{(2\pi)^\nu}\partial_z\left[\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\int \frac{\mathcal{K}_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}^\nu k\right]_{z=\epsilon}, \tag{5.8}$$

or, equivalently,

$$\begin{aligned}\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_m = -\frac{\epsilon^{1-\frac{3\nu}{2}}}{(2\pi)^\nu}\Bigg[&\frac{\nu}{2}z^{\frac{\nu}{2}-1}\int \frac{\mathcal{K}_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}^\nu k \\ &+ z^{\frac{\nu}{2}}\int k\frac{\mathcal{K}'_\mu(kz)}{\mathcal{K}_\mu(k\epsilon)} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}^\nu k\Bigg]_{z=\epsilon}.\end{aligned} \tag{5.9}$$

Using now the following result, given in [26],

$$\mathcal{K}'_\mu(z) = -\frac{\mu}{z}\mathcal{K}_\mu + \mathcal{K}_{\mu-1}, \tag{5.10}$$

we obtain

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle = \epsilon^{-\nu}\left(\mu-\frac{\nu}{2}\right)\delta(\mathbf{x}_1-\mathbf{x}_2) - \frac{\epsilon^{1-\nu}}{(2\pi)^\nu}\int k\frac{\mathcal{K}_{\mu-1}(k\epsilon)}{\mathcal{K}_\mu(k\epsilon)} \mathrm{e}^{-i\mathbf{k}\cdot(\mathbf{x}_1-\mathbf{x}_2)}\mathrm{d}^\nu k. \tag{5.11}$$

Note that we have not renormalized the correlation functions. We will do that using the results of [23] in a forthcoming paper.

6. Two Points Correlation Functions in Minkowskian Space

6.1. Massless Case

Similarly to the Euclidean case we obtain for the Minkowskian one the result

$$\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\rangle_F = i\lim_{z\to 0}\left[z^{1-\nu}\partial_z K_F(z, x_1-x_2)\right]. \tag{6.1}$$

Thus, we obtain for the Feynman's propagator

$$\langle \mathcal{O}(\mathbf{x}_1)\mathcal{O}(\mathbf{x}_2)\rangle_F = -\frac{\Gamma(\nu+1)}{\pi^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)}\frac{1}{\left[(\mathbf{x}_1-\mathbf{x}_2)^2-(x_{10}-x_{20})^2+i0\right]^\nu}. \tag{6.2}$$

For the Anti-Feynman instance one has

$$\left\langle \mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\right\rangle_{AF} = -\frac{\Gamma(\nu+1)}{\pi^{\frac{\nu}{2}}\Gamma\left(\frac{\nu}{2}\right)} \frac{1}{\left[(\boldsymbol{x}_1-\boldsymbol{x}_2)^2-(x_{10}-x_{20})^2-i0\right]^{\nu}}, \tag{6.3}$$

and for Wheeler's situation,

$$\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{W} = \frac{1}{2}\left[\left\langle \mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\right\rangle_{F} + \left\langle \mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\right\rangle_{AF}\right]. \tag{6.4}$$

6.2. Massive Case

Again, following the developments of the Euclidean case, we have, for the Minkowskian instance, the two points Feynman's correlator:

$$\left\langle \mathcal{O}(\boldsymbol{x}_1)\mathcal{O}(\boldsymbol{x}_2)\right\rangle_{mF} = i\left[z^{1-\nu}\partial_z K_{mF}(z, x_1-x_2)\right]_{z=\epsilon}. \tag{6.5}$$

Thus, we have

$$\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mF} = i\frac{\epsilon^{1-\nu}}{(2\pi)^{\nu}}\partial_z\left[\left(\frac{z}{\epsilon}\right)^{\frac{\nu}{2}}\int \frac{K_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}z\right]}{K_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-ik\cdot(x_1-x_2)}\mathrm{d}^{\nu}k\right]_{z=\epsilon}, \tag{6.6}$$

or equivalently,

$$\begin{aligned}\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mF} = i\frac{\epsilon^{1-\frac{3\nu}{2}}}{(2\pi)^{\nu}}&\left[\frac{\nu}{2}z^{\frac{\nu}{2}-1}\int \frac{K_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}z\right]}{K_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-ik\cdot(x_1-x_2)}\mathrm{d}^{\nu}k\right.\\ &\left.-iz^{\frac{\nu}{2}}\int(\rho+i0)^{\frac{1}{2}}\frac{K'_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}z\right]}{K_{\mu}\left[-i(\rho-i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-ik\cdot(x_1-x_2)}\mathrm{d}^{\nu}k\right]_{z=\epsilon}.\end{aligned} \tag{6.7}$$

Using again (5.10) we finally obtain

$$\begin{aligned}&\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{F}\\ &=\epsilon^{-\nu}\left(\mu-\frac{\nu}{2}\right)\delta(x_1-x_2)+\frac{\epsilon^{1-\nu}}{(2\pi)^{\nu}}\int(\rho+i0)^{\frac{1}{2}}\frac{K_{\mu-1}\left[-i(\rho+i0)^{\frac{1}{2}}\epsilon\right]}{K_{\mu}\left[-i(\rho+i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-ik\cdot(x_1-x_2)}\mathrm{d}^{\nu}k.\end{aligned} \tag{6.8}$$

For the Anti-Feynman propagator we obtain in analogous fashion

$$\begin{aligned}&\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mAF}\\ &=\epsilon^{-\nu}\left(\mu-\frac{\nu}{2}\right)\delta(x_1-x_2)-\frac{\epsilon^{1-\nu}}{(2\pi)^{\nu}}\int(\rho-i0)^{\frac{1}{2}}\frac{K_{\mu-1}\left[i(\rho-i0)^{\frac{1}{2}}\epsilon\right]}{K_{\mu}\left[i(\rho-i0)^{\frac{1}{2}}\epsilon\right]}\mathrm{e}^{-ik\cdot(x_1-x_2)}\mathrm{d}^{\nu}k.\end{aligned} \tag{6.9}$$

and for Wheeler

$$\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mW} = \frac{1}{2}\left[\left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mF} + \left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\right\rangle_{mAF}\right]. \tag{6.10}$$

Note again that we have not re-normalized the correlation functions. We will do that using the results of [23] in a forthcoming paper.

7. Conclusions

In this work we have firstly calculated, without using any conjecture, the boundary-bulk Feynman, Anti-Feynman, and Wheeler propagators (half advanced plus half retarded) for both a massless and a massive scalar field, by recourse to the theory of distributions.

We conclusively showed that a previous 2002 work by Son and Starinets [22] (discussing only the Feynman propagator) is wrong.

As further novelties, in the paper we showed that, for massive scalar fields, the expression for the boundary-bulk propagator in Euclidean momentum space does not correspond to the expression used in configuration space, but it is rather a mere approximation.

Subsequently, using the previous results, we have evaluated the correlation functions of scalar operators corresponding to massless and massive scalar fields.

Unlike the results obtained in [22], with the ones obtained here you can calculate the n-points correlation functions from gravity. This is feasible for a scalar operator when n is an arbitrary natural number. This is perhaps our main present contribution.

Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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Comparative Analysis of Existing and Alternative Version of the Special Theory of Relativity

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Abstract

The article explains that: 1) relativistic formulas obtained in the existing version of the special theory of relativity (STR) are incorrect; 2) relativistic formulas obtained in the existing version of the STR are explained incorrect due to the use of the nonexistent in nature principle of light speed non-exceedance; 3) conclusions on physical unreality of imaginary numbers and existence of only our visible Monoverse drawn from relativistic formulas of the existing version of the STR are incorrect due to the use of the incorrect principle of light speed non-exceedance. In other words, the existing version of the STR created in the 20th century is not quite true. Moreover, the correct STR could not be created in the 20th century, since 1) the principle of physical reality of imaginary numbers refuting experimentally the postulated (*i.e.* being an unproven assumption) principle of light speed non-exceedance was published only in the 21st century; 2) experimental data whose mathematical analysis discerned the quaternion structure of the hidden Multiverse consisting of twenty to twenty-two invisible parallel universes in six-dimensional space were obtained by WMAP and Planck spacecraft only in the 21st century; 3) explanation of the way how astronomical observations of constellations of the starry sky in portals can experimentally prove the existence of invisible universes was published only in 2019. Therefore, the article presents an alternative version of the STR, free from the shortcomings of its existing version. Other relativistic formulas that have been obtained in the alternative version of the STR are explainable both at sub-light and hyper-light speeds, and for real and imaginary values of all quantities corresponding to these formulas. Therefore, the principle of light speed non-exceedance is excluded from this version of the STR. For the same reason, the alternative version of the STR states that there is a Multiverse of mutually invisible parallel universes, rather than a Monoverse, since all the mutually invisible parallel universes are relative to each other beyond the event horizon. It also explains how the existence of these invisible parallel universes can be proved by astronomical observations in portals. Moreover, the WMAP and Planck spacecraft data are used in the alternative version of the STR to clarify the structure of the hidden Multiverse. Their mathematical processing has testified that the hidden Multiverse has a quaternion structure and contains twenty-twenty two invisible universes in six-dimensional space.

Keywords

Imaginary Numbers, Special Theory of Relativity, Dark Matter, Dark Energy, Dark Space, Multiverse, Hyperverse

1. Introduction

The existing version of the special theory of relativity (STR) [1] [2] [3] created in the 20th century is a great scientific achievement of physics. But its creators Joseph Larmor [4], Nobel Prize laureate Hendrik Antoon Lorentz [5], Jules Henri Poincaré [6], Nobel Prize laureate Albert Einstein [7] and other outstanding scientists were in advance of their time and could not complete the theory, since they began to develop a theory for completion which in physics there was no necessary knowledge at that time. Therefore STR creators had to replace this missing knowledge with postulates, *i.e.* unproven assumptions. In other words, they had to guess this knowledge. But they did not guess. As a result, the version of the STR that was created in the 20th century and still exists is unfinished. Therefore, the STR was not actually created in the 20th century[1]. In fact, at that time only a task of its creation was set and an attempt to find at least its partially correct solution was made.

Nevertheless, such an unfinished version of the STR is a great scientific achievement of Albert Einstein and other authors of SRT, since it induced physical community to persistent efforts in solving the problem of creating the correct STR.

2. The Logic of Reasoning That Led to Creation of the Existing Version of the STR

Before proceeding to consideration of shortcomings of the existing version of the STR, it would be useful to understand the logic of its creation and the circumstances in which it was created. Then it would become clear that the STR version created in the 20th century was a great scientific achievement, and its errors couldn't be avoided at that time.

First of all, creating the STR it should be determined what space we lived in. However, we have not the slightest idea about other space, than the space of our

[1]We can say that Albert Einstein laid the foundation of SRT and began to build the building of this theory, but due to the lack of necessary experimental data this building was not completed.

home and the nature surrounding it, because we have got no experimental clue. Therefore, it was not possible to guess any other metric of the space, apart from the usual three-dimensional space of real numbers, in the 20th century.

And therefore, relativistic formulas in the existing version of the STR were derived precisely for such a space. They gave some idea of relativistic effects

$$m = \frac{m_0}{\sqrt{1-(v/c)^2}} \tag{1}$$

$$\Delta t = \Delta t_0 \sqrt{1-(v/c)^2} \tag{2}$$

$$l = l_0 \sqrt{1-(v/c)^2} \tag{3}$$

where m_0 is the rest mass of a physical body;

m is the relativistic mass of a moving physical body;

Δt_0 is the rest time of a physical body;

Δt is the relativistic time of a moving physical body;

l_0 is the rest longitudinal length of a physical body;

l is the relativistic longitudinal length of a moving physical body;

v is the velocity of a moving physical body;

c is the speed of light.

Naturally, these formulas and their graphs (**Figures 1(a)-(c)**) should then be explained in the STR. No wonder, explaining the formulas (1)-(3) in the range $0 \le v < c$ of the v argument change, the authors of the existing version of the STR found the only three-dimensional space of real numbers known. At the same time they were confronted with an incomprehensible and insuperable circumstance. Relativistic mass m, relativistic time Δt and relativistic longitudinal length l of a moving physical body calculated by formulas (1)-(3) turned out to

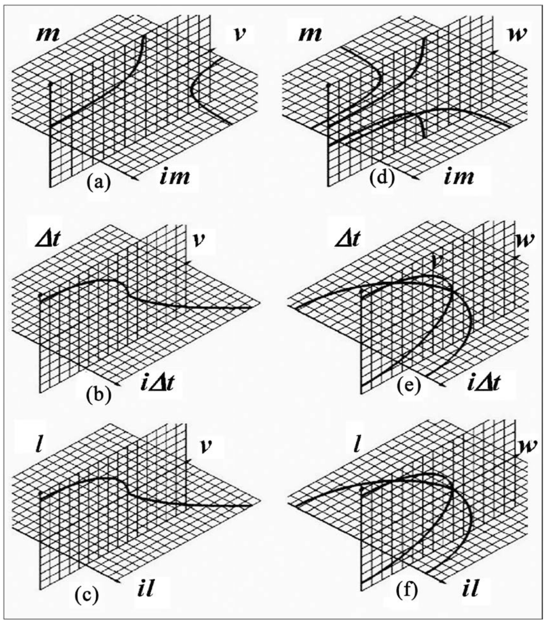

Figure 1. Graphs of functions (1)-(3) and (4)-(6).

assume imaginary values at $c \le v$. No one could explain this. And there was not the slightest hope that this could be somehow explained in the near future, since none of great mathematicians and physicists of that time managed to explain physical meaning of imaginary numbers discovered in mathematics four hundred years before creation of the existing version of the STR. Moreover, in the graphs of **Figures 1(a)-(c)**, it can be seen that at $0 \le v < c$ and $c \le v < \infty$ the functions $m(v)$, $\Delta t(v)$ and $l(v)$ change in a significantly different way. Therefore, the graphs $m(v)$, $\Delta t(v)$ and $l(v)$ shown in **Figures 1(a)-(c)** turned out to be incompletely explained.

All this discouraged the creators of the STR. Therefore, further development of the STR ceased for a century at this stage of its creation. In order to protect the available results from scientific[2] and pseudoscientific[3] criticism, there was made the only right decision at that time to deny physical reality of imaginary numbers, since it was not proved. For this purpose, the STR introduced the postulate of light speed non-exceedance. It was almost refuted by Nobel Prize received by Pavel Alekseevich Cherenkov, Igor Evgenievich Tamm and Ilya Mikhailovich Frank for discovering and explaining Cherenkov radiation [8] arising when electrically charged particles move through a transparent medium at a speed exceeding the speed of light in the medium. However, later the situation was saved by clarifying that the principle of light speed non-exceedance implied only the speed of light in vacuum.

Nevertheless, a certain natural distrust of the postulated principle of light speed non-exceedance has been preserved. Therefore, attempts to refute it were made. The last was the OPERA experiment. On September 23, 2011 the OPERA collaboration published [9] a sensational report on registration of superluminal neutrinos. However, on March 15, 2012 the ICARUS collaboration published [10] a no less sensational report on refutation of the OPERA experiment. This even created illusion of incontrovertibility of the existing version of the STR.

However, creation of such an illusion was conceivably the true goal of the unsuccessful OPERA experiment, since alternative successful experiments [11]-[29], including those conducted in 2008-2010 [12] [13] [14] [15] [16], *i.e.* prior to publication of OPERA experiment results, were not taken into account in the

[2]The STR was criticized by Oliver Heaviside, Nikola Tesla, Nobel Prize laureateFriedrich Wilhelm Ostwald, Nobel Prize laureateJoseph John Thomson, Nobel Prize laureate Svante August Arrhenius, Nobel Prize laureatePhilipp Eduard Anton von Lenard, Nobel Prize laureate Alvar Gullstrand, Nobel Prize laureateWilhelm Carl Werner Otto Fritz Franz Wien, Nobel Prize laureateWalther Hermann Nernst, Nobel Prize laureateErnest Rutherford, 1st Baron Rutherford of Nelson, Nobel Prize laureateJohannes Stark, Nobel Prize laureateFrederick Soddy, Nobel Prize laureatePercy Williams Bridgman, Nobel Prize laureateEdwin Mattison McMillan, Nobel Prize laureateHideki Yukawa, Nobel Prize laureate Hannes Ol of Gösta Alfven and many other outstanding scientists.

[3]For example, decisions on banning criticism of the theory of relativity were made three times in the Soviet Union: in 1934, by the resolution of the Central Committee of the All-Union Communist Party (Bolsheviks) on the discussion of relativism; in 1942, by the resolution of the Presidium of the Academy of Sciences of the Soviet Union on the theory of relativity; and in 1964, by the closed decree of the Presidium of the Academy of Sciences of the Soviet Union that forbade all scientific councils, journals and departments to accept, consider, discuss and publish works criticizing the theory of Albert Einstein.

Internet, although they refuted the principle of light speed non-exceedance and made the OPERA experiment needless. Therefore, assumption that the unsuccessful OPERA experiment was just a promotional event allows us at least somehow explanation of an irrational situation, in which attention was so diligently attracted to the needless and false experiment and successful alternative experiments were ignored.

So, concluding what has been said above, it can be argued that the STR was not created in the 20th century [30] [31], because

- relativistic formulas obtained in its existing version turned out to be incorrect;
- its relativistic formulas were explained incorrectly due to the use of the incorrect principle of light speed non-exceedance;
- conclusions on existence of only our visible Monoverse drawn from its relativistic formulas were incorrect due to the use of the principle of light speed non-exceedance.

Moreover, in the 20th century correct relativistic formulas (10)-(12) could not be obtained and correct version of the STR could not be created, since;

- the principle of physical reality of imaginary numbers refuting experimentally the postulated (*i.e.* being an unproven assumption) principle of light speed non-exceedance was published only in the 21st century;
- experimental data whose mathematical analysis discerned the quaternion structure of the hidden Multiverse were obtained by WMAP and Planck spacecraft only in the 21st century;
- explanation of the way how astronomical observations of constellations of the starry sky in portals can experimentally prove the existence of invisible universes was published only in 2019.

An alternative version of the STR free of the shortcomings of its existing version is presented below in the article.

3. The Logic of Reasoning That Led to Creation of the Alternative Version of the STR

Creation of the alternative version of the STR differed from its generally recognized version. Its basic premise was experimental proof of the principle of physical reality of concrete imaginary numbers, so indisputably refuted by the light speed non-exceedance postulate available in the existing version of the STR. Further, the analysis of WMAP and Planck experimental data using the principle of physical reality of imaginary numbers allowed determining[4] the metric of space we live in and refining the structure of our Multiverse. Our Multiverse turned out to consist of twenty to twenty two mutually invisible parallel universes and have a quaternion structure in six-dimensional space. Therefore, it is called the hidden Multiverse. Notably, existence of invisible parallel universes can be confirmed by astronomical observations made from portals available on

[4]And do not guess it with the postulates, as in the existing version of the STR.

Earth.

Details are given below.

3.1. Proofs of Physical Reality of Concrete Imaginary Numbers

In contrast to existing version of the STR, its alternative version primarily proves the principle of physical reality[5] of concrete imaginary numbers[6]. And even three experimental proofs have been proposed:

- The first one has been obtained in analysis of oscillatory transient processes. Hence, it follows that there would be no tsunami, church bells would not ring and even children's swing wouldn't sway after being pushed by parents [16] [17] [20] [21] [27] [28] [29], if the statement of physical unreality of imaginary numbers contained in the existing version of the STR were true.
- The second one has been obtained in analysis of oscillatory resonant processes. Hence, it follows that there would be no television and telecommunication, radiolocation and radio navigation, as well as many other exact sciences [12] [13] [14] [15] [19] [20] [27] [28] [29], if the statement of physical unreality of imaginary numbers contained in the existing version of the STR were true.
- The third one has been obtained in analysis of forced oscillatory processes in alternating current electric circuits. Hence, it follows that Ohm's law wouldn't exist for alternating current electric circuits [22] [23] [24] [25] [26], [28] [29], if the statement of physical unreality of imaginary numbers contained in the existing version of the STR were true.

All these experimental proofs, unlike the extremely complex and expensive unique OPERA experiment, can be verified in any radio engineering laboratory. Now they are daily confirmed by practical activities of millions of electric and radio engineers. Consequently, they are guaranteedly faithful and absolutely conclusive. Nevertheless, the physical reality of imaginary numbers still has to be proved, even contrary to Ohm's law [23] [24], which indicates the imperfection of modern physical education, since in SRT the principle of not exceeding the speed of light is still assumed to be true[7].

Therefore, it can be argued that physically real imaginary numbers correspond to an invisible world unknown to us, which remains to be known to the science of the future. The utmost importance of imaginary numbers in the science was noted by Sir Roger Penrose: "*The very system of complex numbers has a profound and timeless reality which goes beyond the mental constructions of any particular mathematician… They were put there neither by Cardano, nor by*

[5]Which in the current version of SRT is denied by the postulated principle of non-exceeding the speed of light.

[6]Naturally, it makes sense to talk about physical reality of imaginary, complex and hypercomplex numbers, as well as real numbers, only when it comes to concrete numbers provided with references to units used to measure parameters of corresponding physical objects and processes.

[7]The situation when postulates are sometimes used for lack of experiments is acceptable in science. But when the postulates disprove experiments, such a situation goes beyond common sense and is unacceptable in science.

Bombelly, nor Wallis, nor Coates, nor Euler, nor Wessel, nor Gauss, despite the undoubted farsightedness of these, and other, great mathematicians; such magic was inherent in the very structure that they gradually uncovered".

From the alternative version of the SRT, the principle of not exceeding the speed of light is therefore excluded. And the principle of the physical reality of imaginary numbers, on the contrary, now needs to be recognized as a general scientific one, and in accordance with this principle all theories and hypotheses should be corrected.

Let us show how this can be done, for example, in the STR.

3.2. Relativistic Formulas of the Alternative Version of the STR

Relativistic formulas (1)-(3) of the existing version of the STR are corrected in the alternative version of the STR as follows. Since the principle of physical reality of imaginary numbers disproves the postulate of not exceeding the speed of light, formulas (1)-(3) might be explainable at argument values v that are both lesser and greater than c. However, since they still defy explanation at $c \le v$, formulas (1)-(3) have to be recognized as incorrect. And for the corrected relativistic formulas to be explainable, graphs of functions $m(v)$, $\Delta t(v)$ and $l(v)$ should be comparable at argument values v that are both lesser and greater than c, *i.e.* should be as shown in **Figures 1(d)-(f)**. They correspond to the following formulas

$$m = \frac{m_0 i^q}{\sqrt{1-\left(v/c-q\right)^2}} = \frac{m_0 i^q}{\sqrt{1-\left(w/c\right)^2}} \tag{4}$$

$$\Delta t = \Delta t_0 i^q \sqrt{1-\left(v/c-q\right)^2} = \Delta t_0 i^q \sqrt{1-\left(w/c\right)^2} \tag{5}$$

$$l = l_0 i^q \sqrt{1-\left(v/c-q\right)^2} = l_0 i^q \sqrt{1-\left(w/c\right)^2} \tag{6}$$

where $q = \lfloor v/c \rfloor$ is the "floor" function of argument v/c;

$w = v - qc$ is the local velocity for each universe, which can take values only in the range $0 \le w < c$;

v is the velocity measured from our universe;

c is the speed of light.

Albert Einstein did not exclude such correction of the STR in future. He wrote: "*There is no single idea, which I would be sure that it will stand the test of time*".

3.3. Structure of the Hidden Multiverse

It follows from formulas (4)-(6) that there is a Multiverse [32]-[39], rather than a Monoverse, as stated in the existing version of the STR. And different quantities q in formulas (4)-(6) correspond to different physically real universes. The quantity $q = 0$ in formulas (4)-(6) corresponds (as $i^0 = 1$) to our universe, and the quantity $q = 1$ corresponds (as $i^1 = i$) to the adjacent universe, in which $c \le v < 2c$ and which is therefore invisible from our universe, since it is located beyond the event horizon. Consequently, this is the universe containing tachyons that do not violate the principle of causality [40]-[45].

Let us, therefore, call it a tachyon universe. For the same reasons our universe shall be referred to as a tardyon universe. Subsequently:

- the quantity $q=2$ in formulas (4)-(6) corresponds to the invisible (as $2c \le v < 3c$ for it) tardyon antiverse[8] (as $i^2=-1$);
- the quantity $q=3$ in formulas (4)-(6) corresponds to the invisible (as $3c \le v < 4c$ for it) tachyon antiverse (as $i^3=-i$);
- the quantity $q=4$ in formulas (4)-(6) corresponds to the invisible (as $4c \le v < 5c$ for it) another tardyon universe (as $i^4=1$);
- the quantity $q=5$ in formulas (4)-(6) corresponds to the invisible (as $5c \le v < 6c$ for it) another tachyon universe (as $i^5=i$) etc.

All universes in this Multiverse are mutually invisible and therefore it shall be called the hidden Multiverse. Distribution of physical contents in this hidden Multiverse is described by the function $f_q(x,y,z)+iq$, where x,y,z are the coordinates of physical contents in a corresponding parallel universe, and q is the coordinate of this universe in the fourth spatial dimension.

Moreover, invisible parallel[9] universes do not actually stand still in such a four-dimensional space[10], but continuously drift and very often slightly penetrate into each other in many spots, generating transition zones[11]. Such zones are usually called portals[12] or star gates [46] [47] [48] [49] [50]. **Figure 2** shows an example of structure of such a hidden Multiverse, which, as can be seen, is helical. Numerous bidirectional portals in the structure are indicated by single two-sided arrows.

3.4. Explanation of the Phenomenon of Dark Matter and Dark Energy

The WMAP [51] and Planck [52] spacecraft were launched into space to solve problems that would seem to have nothing to do with the contents of this article. They did not aim to promote the creation of an alternative version of the STR instead of its existing version, which is still considered unshakably true. They were created to study relic radiation produced by the Big Bang.

They also allowed to determine that the universe is composed of:

- 4.6% baryonic matter according to WMAP data (or 4.9% according to Planck data);
- 22.4% dark matter according to WMAP data (or 26.8% according to Planck data);
- 73.0% dark energy according to WMAP data (or 68.3% according to Planck data).

[8]Which contains antimatter, like other antiverses. And it does not annihilate with matter, because tardyon and tachyon universes and antiverses alternate in the hidden Multiverse.

[9]Since they never intersect despite their infinity.

[10]Non-Minkowski space.

[11]In which the quantity q varies by one from one integer value to another, corresponding to adjacent parallel universes.

[12]Which have nothing to do with 'wormholes' in the general theory of relativity.

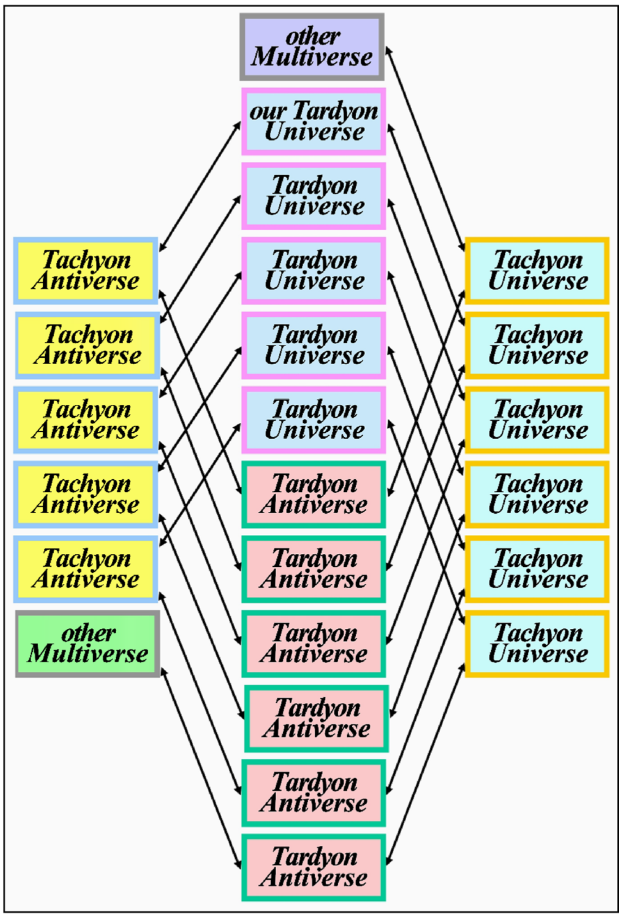

Figure 2. Probable structure of the hidden Multiverse corresponding to the principle of physical reality of complex numbers.

That is, the universe (more precisely, the hidden Multiverse) turned out to be more than 95% composed of dark matter and dark energy. Dark matter was discovered by Jan Hendrik Oort [53] and Fritz Zwikky [54] in 1932-33. Dark energy was discovered by Saul Perlmutter [55], Brian Schmidt [56] and Adam Riess [57] in 1998-1999. They were awarded the Nobel Prize for this discovery. Stressing the importance of the discoveries, the Nobel Prize laureate Adam Riess wrote: "*Humanity is on the verge of a new physics of the Universe. Whether we want it or not, we will have to accept it*". The new physics of the Universe is concerned below.

Despite extremely diligent efforts to study dark matter and dark energy [58]-[64], they still seem completely incomprehensible in the existing version of the STR. Therefore, they were called dark. The famous astrophysicist and professor MichioKaku argued: "*Of course, a whole bunch of Nobel Prizes is waiting for the scientists who can reveal the secrets of the 'dark energy' and 'dark matter'.*"

However, it is easy to see that the phenomenon of dark matter and dark energy is incomprehensible only because its explanation has so far been sought exclusively within the framework of the Monoverse hypothesis corresponding to the existing version of the STR.

In this regard, it would not be out of place to take into account the opinion of Albert Einstein: "*Insanity: doing the same thing over and over again and expecting different results*."

And when using the hypothesis of the hidden Multiverse, the phenomenon of dark matter and dark energy turned out to be quite explainable [20] [25] [26] [29] [35] [36] [37] [45] [65]-[70]:

- dark matter and dark energy are actually a kind of image (gravitational rather than optical or even electromagnetic), something like a shadow, evoked by existence of invisible parallel universes;
- therefore, any physical content, such as molecules, atoms or subatomic particles, will never be found in dark matter and dark energy;
- the dark matter phenomenon is evoked by invisible parallel universes adjacent to our visible universe, whereas the dark energy phenomenon is evoked by other invisible parallel universes of the hidden Multiverse.

Consequently, believing that mass-energy of invisible parallel universes has been substantially averaged over billions of years due to existence of portals, their mass-energy can be accurately assumed to be equal. Therefore, we deduce the following:

- The total number of invisible parallel universes in the hidden Multiverse is $100\%/4.6\% = 21.7$ universes according to WMAP data and $100\%/4.9\% = 20.4$ universes according to Planck data, *i.e.* 20 … 22 universes;
- The number of invisible parallel universes evoking the phenomenon of dark matter is $22.4\%/4.6\% = 4.9$ universes according to WMAP data and $26.8\%/4.9\% = 5.5$ universes according to Planck data, *i.e.* 5 … 6 universes;
- The number of invisible parallel universes evoking the phenomenon of dark energy is $73.0\%/4.6\% = 15.9$ universes according to WMAP data and $68.3\%/4.9\% = 13.9$ universes according to Planck data, *i.e.* 14 … 16 universes.

3.5. Relativistic Formulas of the Alternative Version of the STR (Continued)

Thus, although the WMAP and Planck spacecraft were sent to space for another purpose, the data they received allowed clarifying the structure of the hidden Multiverse and thereby provided experimental support for creation of the alternative version of the STR.

But it is easy to see that results of mathematical processing of WMAP and Planck spacecraft data are inconsistent with formulas (4)-(6) and the structure of the hidden Multiverse shown in **Figure 2**. In **Figure 2** only one tachyon universe and one tachyon antiverse are actually adjacent to each tardyon universe and antiverse, rather than five or six tachyon universes and antiverses as according to the calculations. The thing is that there is no space for placing five or six invisible parallel universes near each tardyon universe or antiverse in the structural diagram given in Figure 2. Consequently, there are three extra dimensions q, r, s in our hidden Multiverse, rather than one q. In this regard, formulas (4)-(6) corresponding to the principle of physical reality of complex numbers should be corrected in accordance with the principle of physical reality of quaternions [71] containing three imaginary units i_1, i_2, i_3 interconnected by the relations

$$i_1^2 = i_2^2 = i_3^2 = 1 \tag{7}$$

$$i_1 i_2 i_3 = i_2 i_3 i_1 = i_3 i_1 i_2 = -1 \tag{8}$$

$$i_1 i_3 i_2 = i_2 i_1 i_3 = i_3 i_2 i_1 = 1 \tag{9}$$

The corrected formulas are written as follows

$$m = \frac{m_0 (i_1)^q (i_2)^r (i_3)^s}{\sqrt{1-\left[v/c-(q+r+s)\right]^2}} = \frac{m_0 (i_1)^q (i_2)^r (i_3)^s}{\sqrt{1-(w/c)^2}} \tag{10}$$

$$\begin{aligned}\Delta t &= \Delta t_0 (i_1)^q (i_2)^r (i_3)^s \sqrt{1-\left[v/c-(q+r+s)\right]^2} \\ &= \Delta t_0 (i_1)^q (i_2)^r (i_3)^s \sqrt{1-(w/c)^2}\end{aligned} \tag{11}$$

$$\begin{aligned}l &= l_0 (i_1)^q (i_2)^r (i_3)^s \sqrt{1-\left[v/c-(q+r+s)\right]^2} \\ &= l_0 (i_1)^q (i_2)^r (i_3)^s \sqrt{1-(w/c)^2}\end{aligned} \tag{12}$$

where q is the total number of parallel universes, penetration into which is made through portals, corresponding to the imaginary unit i_1, with increasing distance from our tardyon universe;

r is the total number of parallel universes, penetration into which is made through portals, corresponding to the imaginary unit i_2, with increasing distance from our tardyon universe;

s is the total number of parallel universes, penetration into which is made through portals, corresponding to the imaginary unit i_3, with increasing distance from our tardyon universe;

v is the velocity measured from our tardyon universe;

c is the speed of light;

$w = v-(q+r+s)c$ is the local velocity for corresponding universe, which can take values only in the range.

And it is quite obvious that the above given WMAP and Planck research data that allowed deriving relativistic formulas (10)-(12) for the alternative version of the STR could be guessed by no postulates. Therefore, the Nobel Prize laureate Stephen Weinberg clearly remarked on the theories created using the postulates: "*Scientific theories cannot be deduced by purely mathematical reasoning*". In other words, no true physical theory can be created without experimental clues.

3.6. Structure of the Hidden Multiverse (Continued)

As can be seen, the results obtained are inconsistent with perceptions generally accepted in relativistic physics and astrophysics. However, Sir Isaac Newton argued: "*No great discovery was ever made without a bold guess*". The same opinion was held by the Nobel laureate Niels Henrik David Bohr who said his catch phrase: "*There is no doubt we have faced a mad theory. But the question is this. Is it really crazy enough to be right*?"

The hidden Multiverse corresponding to formulas (10)-(12) can have the structure shown in **Figure 3**. The structure looks like an open helical ring the ends of which are connected to two other Multiverses. Besides, the hidden Multiverse can be connected to other Multiverses in another way. As can be seen, the quaternion structure [72] differs from the structure shown in **Figure 2** by containing three tachyon universes i_1, i_2, i_3 and three tachyon antiverses i_1, i_2, i_3, which provides three necessary extra dimensions. Thus, six-dimensional space of the hidden Multiverse (**Figure 4**) has actually three extra dimensions q, r, s containing parallel universes, and three dimensions x, y, z containing physical contents of each of these universes. That is, space of such a hidden Multiverse is described by the formula $f_{q,r.s}(x, y, z) + i_1 q + i_2 r + i_3 s$, where the function $f_{q,r,s}(x, y, z)$ describes distribution in coordinates x, y, z of physical content in the corresponding parallel universe having the coordinates q, r, s.

A member of the US National Academy of Sciences Lisa Randall wrote in this regard: "*We can be living in a three-dimensional space sinkhole in a higher-dimensional universe*". Apparently, her assumption was justified.

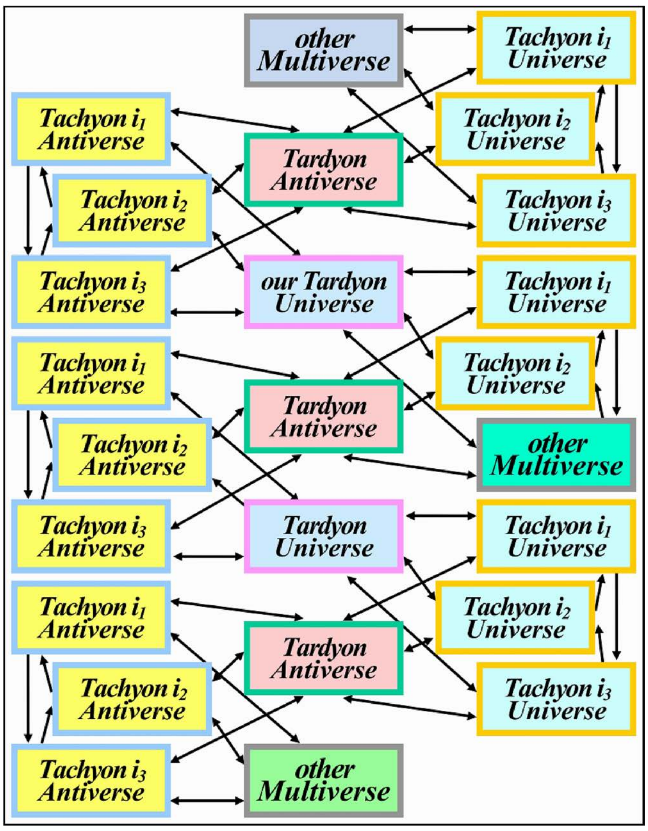

Figure 3. Example of structure of the hidden Multiverse corresponding to the principle of physical reality of quaternions.

3.7. Explanation of the Phenomenon of Dark Space

Previous **Figure 3** shows that our hidden Multiverse is united with other Multiverses through the corresponding portals and forms the Hyperverse together with them. Therefore, it can be argued that other invisible Multiverses of the Hyperverse except our hidden Multiverse form dark space [29] [73]. Herewith, invisible parallel universes of the Multiverses of dark space may presumably be connected to our visible tardyon universe through the corresponding portals, as in **Figure 5(b)**, or may not, as shown in **Figure 5(a)**.

However, availability or lack of such connections cannot be ascertained by astrophysical studies of the WMAP and Planck spacecraft, since otherwise the registered universes would have been classified as universes of dark matter or dark energy.

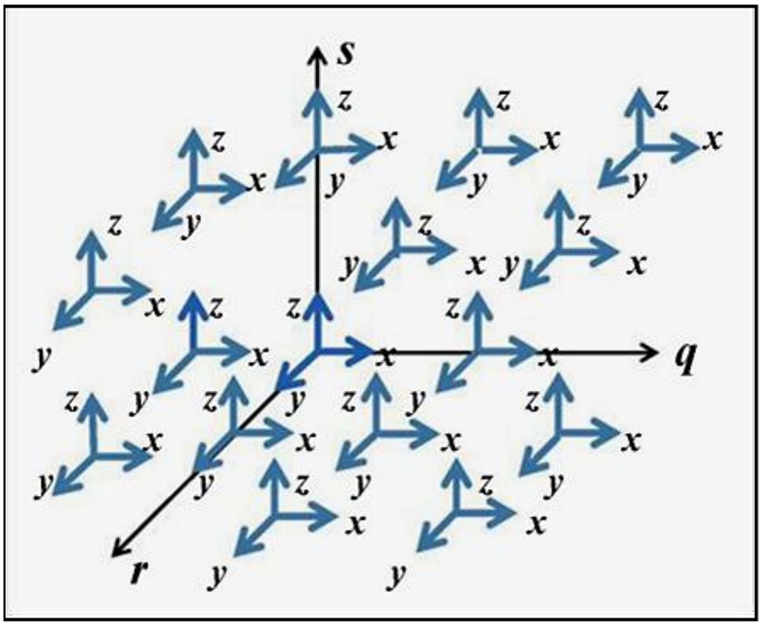

Figure 4. Six-dimensional space of the hidden Multiverse.

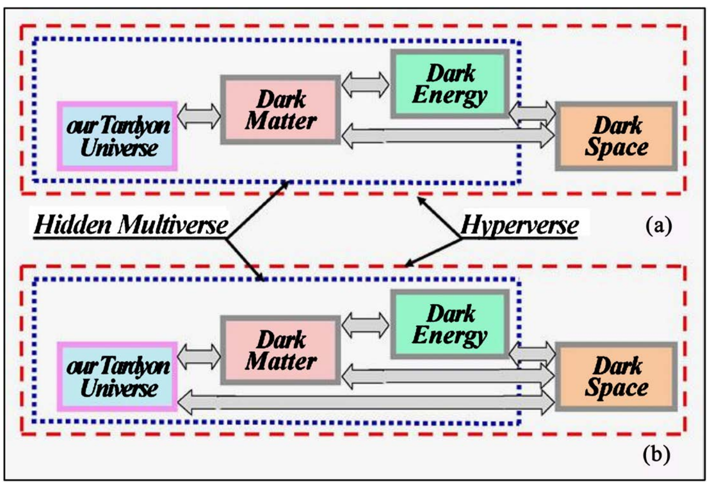

Figure 5. Structure of the Hyperverse.

So, how to make sure of existence of dark space invisible universes? And can this be verified anyway? Apparently, yes. Even in two different ways. First way is astrophysical research of portals. This can be unsafe for people. But for this purpose robotic mobile systems similar to WMAP and Planck spacecraft can be created. Second way is astronomical observations of starry sky constellations made from portals. They are described below.

3.8. Experimental Proof of the Alternative Version of the STR

Thus, we can state that two hypotheses of dark matter and dark energy have been proposed by now. And both are unusual enough to claim to be true in accordance with the criteria of Isaac Newton and Niels Bohr.

The first hypothesis—corresponding to the existing version of SRT—is better known. It suggests that explanation of dark matter and dark energy should be sought in the microcosm. Therefore, it is sought by research at the Large Hadron Collider.

The second hypothesis—corresponding to the alternative version of SRT—is almost unknown set forth in the article. It suggests that explanation of dark matter and dark energy should be sought in the macrocosm. And it is based on the existence of invisible parallel universes of our hidden Multiverse.

There is a good chance that a third hypothesis may be proposed and that it may be even more unusual and appear to be the truest.

But for the time being all of them are just hypotheses. And only an experiment can show which of them will ultimately become a theory [74]. As concerns science, only experiments decide which hypotheses are true or wrong[13]. Therefore, supporters of the first hypothesis search for such a decisive argument at the Large Hadron Collider so persistently. The second hypothesis will also be recognized as true only if it gets experimental confirmation.

It can get the confirmation in the course of astronomical observations of starry sky constellations made from portals [75]. Let's give a comparison to make this idea more clear. The room of our house we are in now is our visible world, whereas the next room is invisible world. However, we can make certain of its existence by TV sounds heard there from. We can see it partially, coming closer to the door and sticking head therein. We can even see the next room entirely, entering it through the door. In this case the room we were in before would become invisible to us.

Similarly, we can partially see the invisible universe, entering the portal. And the further we are, the more we see. The next invisible universe can be seen entirely after entering it through the portal. Thus the last visible universe would become invisible to us. The snag is that portals are invisible too. It is not easy to

[13]In the Thirty Years' War Cardinal Richelieu, driven by the same reasons, ordered to inscribe upon cannons the following text: "Ultima ratio regum". And the last argument of scientists is experiments. Only by experiments can the postulates be confirmed or disproved. But the experiments cannot be either confirmed or disproved by postulates.

get into the next universe through them. Much easier is to get lost in portals and not to return to our world.

Therefore, deepening into portals requires special portal orientation equipment (just as a marine compass used by sailors). Creating the equipment, it should be taken into account that all radio signals fade down with your dipping into the portal and gradually disappear once you are in the adjacent universe.

However, people should not put themselves in danger for such research. Instead, robotic systems can be sent to the portals. They are much easier to create than WMAP and Planck spacecraft.

Moreover, one can see the edge of the adjacent universe even with a shallow dipping into portals. This can be verified by observation of the changed constellation pattern similar to that made by Sir Arthur Stanley Eddington in 1919 [76].

4. Conclusions

Thus, the relativistic formulas obtained in the existing version of the SRT were incorrect due to the lack of experimental support, their conclusion was not completed, they were incorrectly explained, and the conclusions drawn from them about the physical unreality of imaginary numbers and the existence of only our visible Mono-Universe in nature were also incorrect. In other words, the existing version of SRT is incorrect.

Nevertheless, the existing version of STR is a great scientific achievement of Albert Einstein, who created the relativistic physics. Without creating an existing version of the SRT, it would be impossible to create its alternative version.

But in the alternative version of STR, relativistic formulas were created using experimental data obtained already in the 21st century[14]:

- experimentally proven principle of physical reality of imaginary numbers refuting the postulated principle of light speed non-exceedance;
- WMAP and Planck data whose mathematical processing allowed to determine the structure of the hidden Multiverse.

The new relativistic formulas using the principle of physical reality of imaginary numbers are fully explainable. It follows from them that in reality there is not a Monoverse, as stated in all physics textbooks, but a multitude of mutually invisible parallel universes, which together form a hidden Multiverse. And from the data obtained by the WMAP and Planck spacecraft, it follows that the hidden Multiverse exists in six-dimensional space and has a quaternionic structure. In addition, it follows from these data that the invisible Multiverses exist outside the hidden Multiverse, with which they form the Hyperverse.

Such a hypothesis of the hidden Multiverse and Hyperverse made it possible to explain the phenomenon of dark matter and dark energy by a peculiar gravitational shadow of other invisible universes of our visible universe. Such an explanation of the phenomenon of dark matter and dark energy also made it possible to understand why in studies at the Large Hadron Collider it was not possi-

[14]And it is possible that as a result of new experimental data obtained in the future, the relativistic formulas presented in this article will be corrected again, possibly even repeatedly.

ble to detect material carriers of this phenomenon. And the invisible universes, located beyond the borders of the hidden Multiverse, generated the phenomenon of dark space.

The hypothesis of the hidden Multiverse and Hyperuniverse set forth in the alternative version of SRT also explains where antimatter is located and why it does not annihilate with matter, as well as where are tachyons, which do not violate the principle of causality. These explanations are simple and straightforward. Antimatter is found in numerous antiverses, since there are many universes. And tachyons that do not violate the principle of causality are in numerous tachyon universes and anti-universes.

The invisible parallel universes of the hidden Multiverse and Hyperuniverse in six-dimensional space naturally drift relative to each other. Therefore, in many places, neighboring invisible universes are slightly immersed in each other, forming portals through which exchange of their material contents between these universes is possible. On our planet, at least some of the existing anomalous zones are such portals. And from these portals, even with a shallow penetration in them, one can see the edge of the starry sky in neighboring universes with constellations other than we see outside the portals. Thus, such astronomical observations can prove the existence of invisible universes. And for the entire history of the existence of astronomy, there was no more interesting and more important task in it.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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