

# $\chi_c$ and $\chi_b$ States in Hot Quark-Gluon Plasma

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## ABSTRACT

We have studied the dissociation phenomenon of  $1p$  states ( $\chi_c$  and  $\chi_b$ ) of the charmonium and bottomonium spectra in a hot QCD medium. This study employed a medium modified heavy quark potential encoding the medium effects in the dielectric function to the full Cornell potential. The medium modified potential has a quite different form in the sense that it has a long range Coulomb tail in addition to the usual Yukawa term even above the deconfinement temperature. We further study the flavor dependence of their binding energies and explore the nature of dissociation by employing the perturbative, non-perturbative, and the lattice parametrized form of the Debye masses in the medium-modified potential. Interestingly, perturbative result of the Debye mass predicts the dissociation temperatures closer to the results obtained in lattice correlator studies whereas the lattice parametrized form of the Debye masses gives the results closer to the current theoretical works based on potential studies.

**Keywords:** Quarkonium; Debye Mass; Medium-Modified; Heavy Quark Potential; Binding Energy; Dissociation Temperature

## 1. Introduction

One of the amazing discoveries of experimental measurements at RHIC is the surprising amount of both radial [1-3] and elliptic flow [4-6] exhibited by the outgoing hadrons. Theoretical calculations cannot generate sufficient flow to explain the observations unless partonic cross sections are artificially enhanced by more than an order of magnitude over perturbative QCD predictions [7]. Thus the matter created in these collisions is strongly interacting, unlike the type of weakly interacting quarkgluon plasma expected to occur at very high temperatures on the basis of asymptotic freedom [8-10]. The behavior of the heavy quarkonium states in hot strongly interacting matter was proposed as test of its confinement status, since a sufficiently hot deconfined medium will dissolve any binding between the quark-antiquark pair [11]. Another possibility of dissociation of certain quarkonium states (sub-threshold states at  $T=0$ ) is the decay into open charm (beauty) mesons due to in-medium modification of quarkonia and heavy-light meson masses [12].

Many attempts have been made to understand the dissociation phenomenon of  $Q\bar{Q}$  states in the deconfined medium, using either lattice calculations of quarkonium spectral functions [13-19] or non-relativistic calculations based upon some effective potential [20-25]. These two approaches show poor matching between their predic-

tions because of the uncertainties coming from a variety of sources. None of the approaches give a complete framework to study the properties of quarkonia states at finite temperature. However, some degree of qualitative agreement had been achieved for the S-wave correlators. The finding was somehow ambiguous for the P-wave correlators and the temperature dependence of the potential model was even qualitatively different from the lattice one. Refinement in the computations of the spectral functions have recently been done by incorporating the zero modes both in the S- and P-channels [26,27]. It was shown that, these contributions cure most of the previously observed discrepancies with lattice calculations. This supports the use of potential models at finite temperature as an important tool to complement lattice studies.

The production of  $J/\psi$  and  $\Upsilon$  mesons in hadronic reactions occurs in part through production of higher excited  $c\bar{c}$  (or  $b\bar{b}$ ) states and their decay into quarkonia ground state. Since the lifetime of different sub-threshold quarkonium states is much larger than the typical life-time of the medium which may be produced in nucleus-nucleus collisions; their decay occurs almost completely outside the produced medium. This means that the produced medium can be probed not only by the ground state quarkonium but also by different excited quarkonium states. Since, different quarkonium states have different sizes (binding energies), one expects that

higher excited states will dissolve at smaller temperature as compared to the smaller and more tightly bound ground states. These facts may lead to a sequential suppression pattern in  $J/\psi$  and  $\Upsilon$  yield in nucleus-nucleus collision as the function of the energy density. So, if one wants to interpret the  $J/\psi$  suppression pattern observed in nuclear collisions at CERN SPS and RHIC, as a signature of the formation of the QGP, one requires a right understanding of the dissociation of  $\chi_c$  and  $\chi_b$  in the QGP medium. This is due to the fact that a significant fraction ( $\sim 30\%$ ) of the  $J/\psi$  yield observed in the collisions is produced by  $\chi_c$  decays [28-30]. The  $J/\psi$  yield could show a significant suppression even if the energy density of the system is not enough to melt directly produced  $J/\psi$  but is sufficient to melt the higher resonance states because they are loosely bound compared to the ground state  $J/\psi$ . This motivates the special attention to the excited states  $\chi_c$  and  $\chi_b$ .

In the studies of the bulk properties of the QCD plasma phase [31-34], deviations from perturbative calculations were found at temperatures much larger than the deconfinement temperature. This calls for quantitative non-perturbative calculations. The phase transition in full QCD appears as a crossover rather than a "true" phase transition with related singularities in thermodynamic observables (in the high-temperature and low density regime) [35-40]. Therefore, it is not reasonable to assume that the string-tension vanishes abruptly at or above  $T_c$  and one should study its effect on the behavior of quarkonia even above the deconfinement temperature. This issue, usually overlooked in the literature, was certainly worth to investigate. This is exactly what we have done in our recent work [41,42] where we have obtained the medium-modified form of the heavy quark potential by correcting the full Cornell potential (linear plus Coulomb), not only its Coulomb part alone as usually done in the literature, with a dielectric function encoding the effects of the deconfined medium. We found that this approach led to a long-range Coulomb potential with an (reduced) effective charge [41] in addition to the usual Debye-screened form employed in most of the literature. With this effective potential, we investigated the effects of perturbative and non-perturbative contributions to the Debye mass on the dissociation of quarkonium states. We subsequently used this study to determine the binding energies and the dissociation temperatures of the ground and the first excited states of charmonium and bottomonium spectra.

Our starting potential at  $T = 0$  was Cornell potential which has no terms to account the spin-dependence forces in QCD [43], so the medium-modified potential [41] also has no spin-dependent terms. As a consequence, Schrodinger equation with the above medium-modified potential gives the same energy eigenvalues for the first

excited states  $\psi'$  and  $\chi_c$  in charmonium spectroscopy ( $\Upsilon'$  and  $\chi_b$  states in bottomonium spectroscopy) making them degenerate. This is certainly not desirable since their masses are not the same (in fact, mass of  $\psi'$  is slightly higher than  $\chi_c$  whereas the latter state is more tightly bound than the former). Therefore the determination of the binding energies of  $1p$  states, viz.,  $\chi_c$ ,  $\chi_b$  and their dissociation temperatures like the ground and first excited states is not directly possible as had been done in our earlier work by employing the medium modified potential [41]. The principal quantum number ( $n$ ) of  $\psi'$  and  $\chi_c$  are same but their spin quantum number and as well as their total angular momentum are not the same. So, their quantum states should be denoted by all four quantum numbers ( $nlsj$ ) and the difference in their binding energies (or in their total masses) should be originated from a spin-dependent correction terms.

We have done this job in two fold way. First, we have determined the binding energy for  $\psi'$  by employing the medium-modified potential [41] into the Schrodinger equation. Then we obtain the binding energy for  $\chi_c$  by adding the correction terms to the binding energy of  $\psi'$ . In our analysis, correction terms will be obtained by adopting a variational treatment of the relativistic two-fermion bound-states in quantum electrodynamics (QED) [44-47] taking into account the spin-dependent terms for the corresponding quantum numbers of  $\psi'$  and  $\chi_c$  states. In this endeavor, coupled integral equations for a relativistic two-fermion system are derived variationally within the Hamiltonian formalism of QED using an improved ansatz that is sensitive to all terms in the Hamiltonian [45,46].

The paper is organized as follows. In Section 2, we review the work on the medium modified Cornell potential and dissociation of  $1s$  and  $2s$  states of charmonium and bottomonium spectra. In Section 3, we discuss how to determine the binding energies of  $\chi_c$  and  $\chi_b$ . In Section 4, we study the melting of  $\chi_c$  and  $\chi_b$  in a hot QCD medium and determine their dissociation temperatures. Finally, we conclude in Section 5.

## 2. In-Medium Modifications to Heavy-Quark Potential

The interaction potential between a heavy quark and antiquark gets modified in the presence of a medium and it plays a vital role in understanding the fate of quark-antiquark bound states in the QGP medium. This issue has well been studied and several excellent reviews exist [48,49] which dwell both on the phenomenology as well as on the lattice QCD. In these studies, they assumed the melting of the string motivated by the fact that there is a phase transition from a hadronic matter to a QGP phase. As a consequence they modified the Coulomb part of the

potential only so they used a much simpler form (screened Coulomb) for the medium modified potential in the deconfined phase. But recent lattice results indicates that there is no genuine phase transition at vanishing baryon density, it is rather a cross-over, so there is no reason to assume the melting of string at the deconfinement temperature. We have addressed this issue in our recent work [41] where we developed an effective potential once one corrects the full Cornell potential with a dielectric function embodying medium effects. We recall the basic details which are relevant for the present demonstration.

Usually, in finite-temperature QFT, medium modification enters in the Fourier transform of heavy quark potential as

$$\tilde{V}(k) = \frac{V(k)}{\varepsilon(k)}, \quad (1)$$

where  $\varepsilon(k)$  is the dielectric permittivity given in terms of the static limit of the longitudinal part of gluon self-energy [50]:

$$\varepsilon(k) = \left(1 + \frac{\Pi_L(0, k, T)}{k^2}\right) \equiv \left(1 + \frac{m_D^2}{k^2}\right). \quad (2)$$

The quantity  $V(k)$  in Equation (1) is the Fourier transform (FT) of the Cornell potential. The evaluation of the FT of the Cornell potential is not so straightforward and can be done by assuming  $r$ —as distribution ( $r \rightarrow r \exp(-\gamma r)$ ). After the evaluation of FT we let  $\gamma$  tends to zero. Now the FT of the full Cornell potential can be written as

$$V(k) = -\sqrt{\frac{2}{\pi}} \frac{\alpha}{k^2} - \frac{4\sigma}{\sqrt{2\pi}k^4}. \quad (3)$$

Substituting Equations (2) and (3) into (1) and then evaluating its inverse FT one obtains the  $r$ -dependence of the medium modified potential [42]:

$$V(r, T) = \left(\frac{2\sigma}{m_D^2} - \alpha\right) \frac{\exp(-m_D r)}{r} - \frac{2\sigma}{m_D^2 r} + \frac{2\sigma}{m_D} - \alpha m_D \quad (4)$$

This potential has a long range Coulombic tail in addition to the standard Yukawa term. Interestingly, high temperature behavior of quarkonia is rather governed by the former term in the potential with the (reduced) effective charge in analogous to the fine structure constant in QED. The constant terms are introduced to yield the correct limit of  $V(r, T)$  as  $T \rightarrow 0$  (it should reduce to the Cornell form). Such terms could arise naturally from the basic computations of real time static potential in hot QCD [51] and from the real and imaginary time

correlators in a thermal QCD medium [52].

It is worth to note that the potential in a hot QCD medium is not the same as the lattice parametrized heavy quark free-energy in the deconfined phase which is basically a screened Coulomb form [53-55] because one-dimensional Fourier transform of the Cornell potential in the medium yields the similar form as used in the lattice QCD to study the quarkonium properties which assumes the one-dimensional color flux tube structure [56]. However, at finite temperature that may not be the case since the flux tube structure may expand in more dimensions [53]. Therefore, it is better to consider the three-dimensional form of the medium modified Cornell potential which have been done exactly in the present work. We have compared our in-medium potential with the color-singlet free-energy [57] extracted from the lattice data and found that it agrees with the lattice results except for the non-perturbative result of the Debye masses [41].

We have thus employed this medium-modified effective potential (4) to study the binding energies and the dissociation temperatures for the ground and the first excited states of  $c\bar{c}$  and  $b\bar{b}$  spectroscopy in our earlier work [41]. Let us now proceed to the determination of the binding energies and the dissociation temperatures for  $\chi_c$  and  $\chi_b$  states in Sections 3 and 4, respectively.

### 3. Binding Energy of $\chi_c$ and $\chi_b$

The solution of the Schrödinger equation with the potential (4) numerically gives the energy eigenvalues for the ground states and the first excited states in charmonium ( $J/\psi, \psi'$  etc.) and bottomonium ( $\Upsilon, \Upsilon'$  etc.) spectra. These energy-eigen values are known as ionization potentials/binding energies which become temperature-dependent through the Debye masses and decrease with the increase in temperature.

Apart from the ground and the first excited states, there are other important states ( $1p$ ) in the charmonium and bottomonium spectra *viz*  $\chi_c$  and  $\chi_b$  which contribute significantly in the suppression of the ground state quarkonia ( $J/\psi$  and  $\Upsilon$ ) in RHIC experiments through their decays into  $J/\psi$ 's and  $\Upsilon$ 's. Although both  $\psi'$  and  $\chi_c$  are the first excited states of the charmonium spectra but they are not degenerate. In fact,  $\psi'$  is more massive than  $\chi_c$  but  $\chi_c$  is more tightly bound than  $\psi'$ . So the entire binding energy of  $\chi_c$  will not come from the above calculation, the additional contribution will come from the spin-dependent quantum corrections.

Some authors have studied the relativistic two-particle Coulomb problem, based on approximations to the Bethe-Salpeter equations. Others have started with effective Lagrangians based on perturbative expansions of the relativistic Lagrangians. However, we choose the varia-

tional methods [44-47] where coupled integral equations for a relativistic two-fermion system are derived variationally within the Hamiltonian formalism of quantum electrodynamics, using an improved ansatz that is sensitive to all terms in the Hamiltonian. The equations are solved approximately to determine the eigenvalues and eigenfunctions, at arbitrary coupling, for various states of the two-particle system. In the variational treatment of the relativistic two-fermion bound-state system in QED, the total energy in a quantum state ( $nlsj$ ) consists of Bohr like terms, relativistic correction in the kinetic energy and most importantly the spin-dependent terms which take into account the non-degeneracy between the sub-states. The total energy up to the fourth order in fine-structure constant  $\alpha$  is written [45,46]:

$$E_{nlsj} = 2m_Q - \frac{1}{2}\mu \frac{\alpha^2}{n^2} + \Delta K_{nl} + \Delta V_{nlsj}, \tag{5}$$

where

$$\Delta K_{nl} = -\frac{\mu^4 \alpha^4}{m_Q^3} \left( \frac{2}{(2l+1)n^3} - \frac{3}{4n^4} \right) \tag{6}$$

is the  $\alpha^4$  correction to the kinetic energy and the correction to the spin-dependent potential energy is

$$\Delta V_{nlsj} = -b_{nlsj} \frac{\alpha^4 \mu^3}{n^3}, \tag{7}$$

where the coefficients  $b_{nlsj}$  for the different quantum states ( $nlsj$ ) are tabulated in Ref. [45,46]. The fine structure constant ( $\alpha$ ) in QED will be replaced by the effective charge ( $2\sigma/m_D$ ) in our model. Using the appropriate values of the quantum numbers and the coefficients corresponding to  $\psi'$  and  $\chi_c$  states in charmonium spectra ( $\Upsilon'$  and  $\chi_b$  in bottomonium spectra), we obtain the correction term which is to be added to the binding energy of  $\psi'$  is

$$\Delta E_{c,b}^{corr} = \frac{m_{c,b} \sigma^4}{6m_D^8} \tag{8}$$

So the binding energy of  $\chi_c(\chi_b)$  is

$$E_{bin}(\chi_c, \chi_b) = E_{bin}(\psi', \Upsilon') + \Delta E_{\chi_c, \chi_b}^{corr} \tag{9}$$

where  $m_D$  is the Debye mass for which we choose a gauge invariant, non-perturbative form by Kajantie *et al.* [58] which is obtained by computing the non-perturbative contributions of  $O(g^2T)$  and  $O(g^3T)$  from a 3-D effective field theory for QCD as

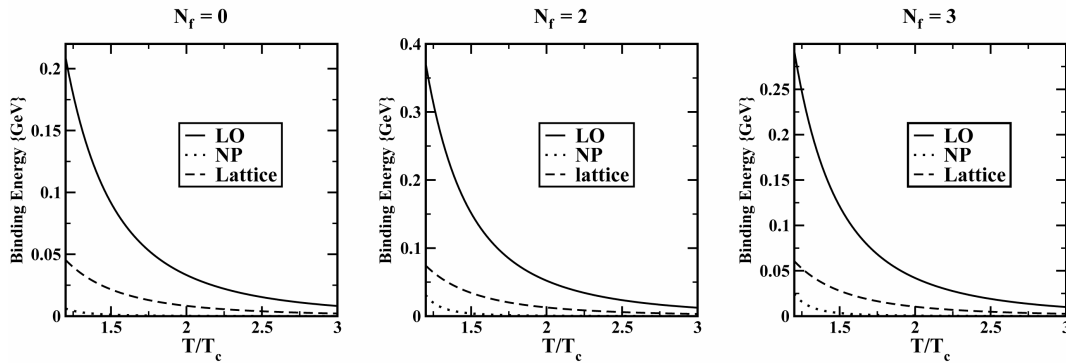
$$m_D^{NP} = m_D^{LO} + N_c g^2 T 4\pi \ln m_D^{LO} g^2 T + c_{N_c} g^2 T + d_{N_c, N_f} g^3 T + O(g^4 T), \tag{10}$$

where the leading-order (LO) perturbative result,

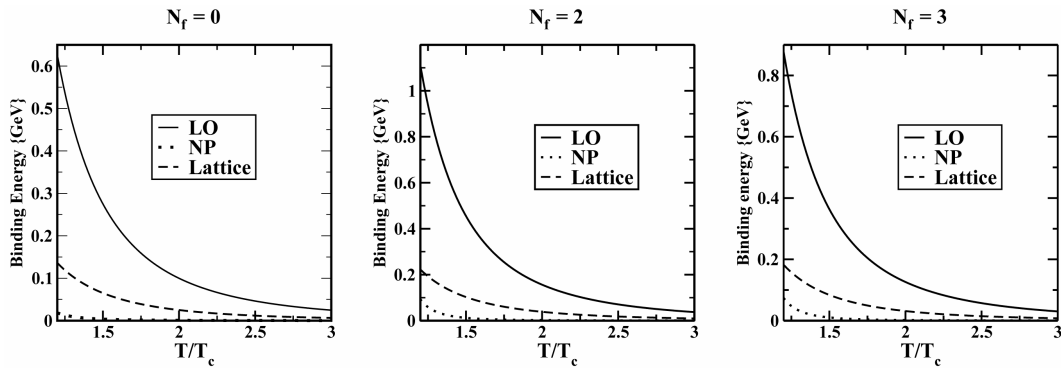
$m_D^{LO} = g(T)T \sqrt{\frac{N_c}{3} + \frac{N_f}{6}}$ , has been known for a long time [59-61]. The logarithmic part of the  $O(g^2)$  correction can be extracted perturbatively [62], but  $c_{N_c}$  and the higher order corrections are non-perturbative. We wish to explore the effects of the different terms in the Debye mass on the binding energy of  $\chi_c$  and  $\chi_b$ . We have used the two-loop expression for the QCD coupling constant at finite temperature from Ref. [63] and the renormalization scale from Ref. [64].

The effects of each terms in the Debye mass (10) cannot always be explored separately due to the following reason: in the weak coupling regime, the soft scale ( $=gT$ ) at the leading-order related to the screening of electrostatic fields is well separated from the ultra-soft scale ( $=g^2T$ ) related to the screening of magnetostatic fields. In such regime, it appears meaningful to see the contribution of each terms in the Debye mass separately. But when the coupling becomes large enough (which is indeed the case), the two scales are no longer well separated. So while looking for the next-to-leading corrections to the leading-order result from the ultra-soft scale, it is not a wise idea to stop at the logarithmic term, since it becomes crucial the number multiplying the factor  $1/g$  to establish the correction to the LO result. In fact the Debye mass in the NLO term is always smaller than the LO term because of the negative (logarithmic) contribution ( $\log(1/g)$ ) to the leading-order term, while the full correction (all  $g^2T$  terms) to the Debye mass results positive. So, we consider only three forms of the Debye masses, *viz.* leading-order result ( $m_D^{LO}$ ), non-perturbative form ( $m_D^{NP}$ ), and lattice parametrized form ( $m_D^L = 1.4m_D^{LO}$ ) extracted from lattice free energy [25] to study the dissociation phenomena.

Thus we have finally computed the binding energies for  $\chi_c$  and  $\chi_b$  and plotted them in **Figures 1** and **2**, respectively where different curves denote the choice of the Debye masses used to calculate the binding energy from Equation (9). We consider three cases for our analysis: pure gluonic, 2-flavor and 3-flavor QCD. There is a common observation in all figures that the binding energies show strong decrease with the increase in temperature. In particular, binding energies obtained from  $m_D^{LO}$  and  $m_D^L$  give realistic variation with the temperature. The temperature dependence of the binding energies show a quantitative agreement with the results based on the spectral function technique calculated in a potential model for the non-relativistic Green's function [64]. On the other hand, when we employ non-perturbative form of the Debye mass ( $m_D^{NP}$ ) the binding energies become unrealistically small compared to its zero temperature value and also compared to the binding energies employing  $m_D^{LO}$  and  $m_D^L$ . This anomaly can



**Figure 1.** Dependence of  $\chi_c$  binding energy (in GeV) on temperature  $T/T_c$ .



**Figure 2.** Dependence of  $\chi_b$  binding energy (in GeV) on  $T/T_c$ .

be understood by the fact that the value of  $m_D^{NP}$  is significantly larger than both  $m_D^{LO}$  and  $m_D^L$  so that the binding energies become substantially smaller. This observation indicates that the present form of the non-perturbative corrections to the Debye mass may not be the complete one, the situation may change once the  $O(g^4T)$  non-perturbative contributions to Debye mass are incorporated and then evaluate the binding energy. Thus, the study of temperature dependence of binding energy is poised to provide a wealth of information about the nature of dissociation of quarkonium states in a thermal medium which will be reflected in their dissociation temperatures discussed in the next section.

In addition, we take advantage of all the available lattice data, obtained not only in quenched QCD ( $N_f = 0$ ), but also including two and, more recently, three light flavors. We are then in a position to study also the flavor dependence of the dissociation process.

#### 4. Dissociation Temperatures

It has been customary to consider a state dissociated when its binding energy becomes zero. In principle, a state is dissociated when no peak structure is seen, but the widths shown in the spectral functions from the current potential model calculations are not physical. Broadening of the states as the temperature increases is

not included in any of these models. In [64] authors have argued that no need to reach zero binding energy ( $E_{bin} = 0$ ) to dissociate, but when  $E_{bin} \leq T$  a state is weakly bound and thermal fluctuations can destroy it. However, others have set a more conservative condition for dissociation [65]:  $2E_{bin}(T) < \Gamma(T)$ , where  $\Gamma(T)$  is the thermal width of state. However, we now calculate the upper bound of the dissociation temperature by the condition for dissociation:

$$E_{bin}(\chi_{c,b}) = T_D, \tag{11}$$

where the string tension ( $\sigma$ ) is  $0.184 \text{ GeV}^2$  and critical temperatures ( $T_c$ ) are taken as  $270 \text{ MeV}$ ,  $203 \text{ MeV}$  and  $197 \text{ MeV}$  for pure gluonic, 2-flavor and 3-flavor QCD medium, respectively [66]. However, the choice of the mean thermal energy ( $T$ ) is not rigid because even at low temperatures  $T < T_D$  (say) the Bose/Fermi distributions of partons will have a high energy tail with partons of mechanical energy  $> |E_{bin}(\chi_c, \chi_b)|$ .

The dissociation temperatures for  $\chi_c$  and  $\chi_b$  are listed in **Table 1** with the Debye mass in the leading-order. It is found from **Table 1** that  $\chi_c$ 's are dissociated at  $1.12T_c$ ,  $1.30T_c$ , and  $1.26T_c$  for the pure, 2-flavor, and 3-flavor QCD, respectively whereas  $\chi_b$ 's are dissociated comparatively much higher temperature which seems justifiable because of their higher binding energy. This

**Table 1. Upper(lower) bound on the dissociation temperature ( $T_D$ ) for  $\chi_c$  and  $\chi_b$  (in unit of  $T_c$ ) using the leading-order term in Debye mass,  $m_D^{LO}$ .**

State	Pure QCD	$N_f=2$	$N_f=3$
$\chi_c$	1.12 (0.90)	1.30 (1.10)	1.26 (1.02)
$\chi_b$	1.40 (1.10)	1.64 (1.33)	1.60 (1.30)

finding agrees with the results obtained from the lattice correlator studies [15]. This is an interesting observation in the literature on the flavor (system) dependence of the dissociation temperature. This dependence is essential while calculating the screening energy density (energy density of the system at the dissociation temperature) in various descriptions of QGP ( $N_f = 0, 2, 3$ ) for the study of  $J/\psi$  survival within the screening scenario in an expanding QGP. On the other hand, employing lattice parametrized form,  $m_D^L$  for the Debye mass we obtain the values (**Table 2**) much smaller than the leading-order results where  $\chi_c$  is dissociated below  $T_c$  and  $\chi_b$  is dissociated just above  $T_c$ . However, this finding agrees with the results from the potential model studies [64] but does not agree with the lattice correlator studies. Finally, when we use non-perturbative form of the Debye mass, the dissociation temperatures come out to be unrealistically small which defy physical interpretation. Summarizing the results, we conclude that as we move from perturbative to non-perturbative domain, the binding energies become smaller and smaller. As a consequence the dissociation temperatures obtained also become smaller. This is due to the hierarchy in the Debye masses:  $m_D^{LO} < m_D^L \ll m_D^{NP}$ . In fact,  $m_D^L$  is 1.4 times greater than  $m_D^{LO}$  while  $m_D^{NP}$  is much greater than both  $m_D^{LO}$  and  $m_D^L$ .

However, if we treat the partons in high temperature to be relativistic, we could replace the mean thermal energy by  $3T$  (instead of  $T$ ) to obtain the lower bound for the dissociation temperatures. It is found that all entries in **Tables 1** and **2** have been decreased by 20% - 25% approximately giving the lower bound of the dissociation temperatures (inside the first bracket). To compare our results quantitatively with the recent results [64] based on the spectral function technique calculated in a potential model with a similar description of the system (for 3-flavor QCD with  $T_c = 192$  MeV), we tabulated the upper limit on the dissociation temperatures with the same form of Debye mass used in Ref. [64] in **Table 3** giving a good agreement with their results.

Finally, it is learnt that the inclusion of non-perturbative corrections to the Debye mass ( $m_D^{NP}$ ) lead to unusually smaller value of the dissociation temperatures for both  $\chi_c$  and  $\chi_b$ . This does not immediately imply that the non-perturbative effects should be ignored. It is

**Table 2. Same as Table 1 but with the lattice parametrized form of the Debye mass  $m_D^L$ .**

State	Pure QCD	$N_f=2$	$N_f=3$
$\chi_c$	0.80 (0.60)	0.95 (0.77)	0.93 (0.75)
$\chi_b$	1.00 (0.80)	1.20 (0.97)	1.15 (0.93)

**Table 3. Upper bound of the dissociation temperatures with  $T_c = 192$  MeV and the lattice parametrized form of the Debye mass [64].**

State	$\chi_c$	$\chi_b$
$T_D$	$\leq 0.9T_c$	$1.2T_c$

rather interesting to investigate the disagreement between the non perturbative result obtained with a dimensional-reduction strategy and the Debye mass arising from the Polyakov-loop correlators. Only future investigations may throw more light on this issue.

## 5. Conclusions

In conclusion, we have studied the dissociation of  $1p$  states ( $\chi_c$  and  $\chi_b$ ) in the charmonium and bottomonium spectra in the hot QCD medium. We have employed the medium modified form of the heavy quark-potential in which the medium modification causes the dynamical screening of both the (color) charge and the range of the potential which, in turn, lead to the temperature dependent binding energy of  $\psi'$  and  $Y'$ . We have then studied the temperature dependence of the binding energy of  $\chi_c$  and  $\chi_b$  states in the pure gauge and realistic QCD medium by incorporating the fourth-order corrections (in the screened charge  $\alpha_{eff} = 2\sigma/m_D^2$ ) coming from the spin-dependent terms to the binding energies of  $\psi'$  and  $Y'$  states, respectively. For this purpose, we have adopted a formulation [45,46], in which a variational treatment of the relativistic two-fermion bound-state system in QED [45,46] has been developed to compute the spin-dependent corrections.

Next we have determined the dissociation temperatures of  $\chi_c$  and  $\chi_b$  employing the Debye mass in the leading-order and the lattice parametrized form. Our estimates with the Debye mass extracted from the lattice free energy are consistent with the finding of recent works based on potential models [64] whereas the Debye mass in the leading-order gives the identical results based on the lattice correlator studies. We have further shown that the inclusion of non-perturbative contributions to the Debye mass lower the dissociation temperatures substantially which looks unfeasible to compare to the spectral analysis of lattice temporal correlator of mesonic current [15]. This leaves an open problem of the agreement be-

tween these two kind of approaches. This could be partially due to the arbitrariness in the criteria/definition of the dissociation temperature. To examine this point we have estimated both the upper and lower bound on the dissociation temperatures by fixing the mean thermal energy  $T$  and  $3T$ , respectively. Thus, this study provides us a handle to decipher the extent upto which non-perturbative effects should be incorporated into the Debye mass.

In brief, we obtained the binding energies and the dissociation temperatures of  $\chi_c$  and  $\chi_b$ . This enables us to investigate their flavor dependence and temperature dependence. We have estimated the upper bound on the dissociation temperatures of  $\chi_c$  and  $\chi_b$ . We found that these estimates obtained by employing the lattice parametrized Debye mass show good agreement with the prediction in [64]. On the other hand, estimates with the perturbative result of the Debye mass are consistent with the predictions of lattice correlator studies [15,19,24,28].

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