Journal of Mathematical Finance, 2017, 7, 412-436
http://www.scirp.org/journal/jmf
ISSN Online: 2162-2442
ISSN Print: 2162-2434

Return Predictability and Strategic Trading under Symmetric Information

Ming Guo[1], **Hui Ou-Yang**[2]

[1]ShanghaiTech University, Shanghai, China
[2]Cheung Kong Graduate School of Business, Beijing, China
Email: guoming@shanghaiTech.edu.cn, houyang@ckgsb.edu.cn

How to cite this paper: Guo, M. and Ou-Yang, H. (2017) Return Predictability and Strategic Trading under Symmetric Information. *Journal of Mathematical Finance*, **7**, 412-436.
https://doi.org/10.4236/jmf.2017.72022

Received: November 4, 2015
Accepted: May 20, 2017
Published: May 23, 2017

Abstract

This paper develops a rational equilibrium model of strategic trading under symmetric information in which there is a liquidity provider and a strategic trader. The strategic trader considers the impact of his trades, the liquidity provider sets the stock price competitively, and there is a possibility that the value of the stock payoff will be revealed perfectly before the terminal date. Under certain conditions, we find that a buy (sale)-order by the strategic trader leads to positive (negative) stock returns in the future and that there exists a positive contemporaneous relationship between the stock return and the trades of the strategic trader. Under other conditions, we demonstrate that the stock exhibits positive (negative) returns following buying (selling) by the liquidity provider. We then introduce a trend chaser into the rational model. If trend chasing is weak, we show that the mechanical trend chaser can actually make a profit. If trend chasing is strong, the strategic trader is able to raise the stock prices by buying initially to attract the trend chaser and sells to the trend chaser later for profits.

Keywords

Strategic Trading under Symmetric Information, Tend Chasing, Return Predictability

1. Introduction

Extensive empirical studies have demonstrated that non-informational trading affects stock prices and returns[1]. Many of them have further shown that non-informational trading can lead to certain predictable patterns of stock returns or can forecast future stock returns. [9] documents positive excess returns in the month following intense buying by individuals and negative excess returns after

[1]See, for example, [1]-[9].

individuals sell. They suggest that the documented patterns are consistent with the notion that risk-averse individuals provide liquidity to meet institutional demand for immediacy. [2] shows that the trading by retail investors moves prices and that over a short period of time, stocks heavily bought by retail traders earn strong positive future returns whereas stocks sold earn negative returns. See also [7] and [8] on the relationships between buying and selling by small traders and future stock returns, as well as [10] and [11] on the contemporaneous correlation between buying and selling by retail investors and stock returns.

Notice that the empirical results of [2] and [9] seem to be contradictory with each other. In addition, using trade level data from the stock market in Pakistan, [12] find evidence for a specific trade-based pump and dump price manipulation scheme. "When prices are low, colluding brokers trade amongst themselves to artificially raise prices and attract positive-feedback traders. Once prices have risen, the former exit leaving the latter to suffer the ensuring price fall." They further demonstrate that the price manipulation cannot be attributed to superior information by the brokers, rather, it is mainly the result of pure price manipulation by the colluding brokers to take advantage of positive feed-back traders.

Inspired by the above-mentioned empirical findings, this paper develops an equilibrium model of strategic trading under symmetric information. In the basic version of the model, there is a strategic trader, who trades strategically and his trades affects the equilibrium price, and a competitive liquidity provider, who provide liquidity to other investors in the market. Both traders are risk averse[2]. We consider a four-period model in which trading takes place three times, with the strategic trader initiating a buy or a sale order and the liquidity provider clearing the market by setting the stock price competitively. Both traders are rational in the sense that they maximize their expected utility functions. To generate sustained trading, we assume that there is a probability that both traders receive a signal that reveals the stock payoff perfectly in each period before the fourth period. When this signal arrives or once the stock payoff is known, there will be no more trading due to symmetric information. As a result, the game ends and both market participants consume their entire wealth. Before the revelation, we assume that the stock payoff follows a normal distribution.

When the probability of observing the signal is zero, we find that the no-trade theorem of [13], which assumes a competitive model, still holds under our strategic model. That is, after the first round of trading, both traders reach Pareto optimal risk sharing, and therefore, no additional trading in future periods occurs. When the probability is positive, however, we show that the strategic trader trades gradually to achieve optimal risk sharing as well as to minimize the market impact costs of his trades. Depending on the risk aversion and endowment of the strategic trader and the liquidity provider, we obtain four sets of results as follows.

First, the liquidity trader holds a large long position initially, and the strategic

[2]The strategic trader can be interpreted as a proprietary trading desk, a mutual fund, or a hedge fund. The liquidity provider can be interpreted as an individual investor or a market maker.

trader can afford to take on more risk associated with the stock payoffs. Hence, the strategic trader buys the stock and the liquidity trader sells the stock. The stock price increases throughout the periods until it converges up to the fundamental value of the stock at the terminal date. Second, when the liquidity trader has a negative endowment and tends to cover his short position, the strategic trader initiates stock sales and the liquidity trader buys from the strategic trader. The stock price decreases in the first two periods until converging down to the fundamental value of the stock at the terminal date. In these two cases, to achieve optimal risk sharing the strategic trader buys or sells gradually to minimize the market impact of his trades. This is the reason that stock prices and returns exhibit predictability and the trades by the strategic trader can forecast stock returns. In particular, a buy (sale) order by the strategic trader leads to higher (lower) stock returns in the future. These results provide potential explanations for the empirical findings of [2] [7] [8], in which the trades by retail traders are systematically correlated and the retail traders would correspond to our strategic trader. The contemporaneous relationships between stock returns and trades by the strategic trader are positive, which are consistent with the empirical results of [11] [14], and others.

Third, the strategic trader has a negative endowment and tends to cover his short position, so he buys the stock from the liquidity provider. The stock price, which is above the fundamental value due to a negative risk premium, increases in the first two periods, then declines to the fundamental value in the third period. Fourth, the strategic trader has a positive endowment and tends to reduce his position. Due to risk sharing and a positive risk premium, the stock prices are all below the fundamental value. Since the strategic trader sells the stock and the liquidity provider buys the stock, the stock price declines in the first two periods and then increases to the fundamental value of the stock in the last period.

These results suggest that following the buying (selling) by the liquidity provider, the stock exhibits positive (negative) returns, providing potential explanations for the empirical findings of [9].

To capture the empirical results of [12], we introduce a trend chaser or a positive feed-back trader into our rational model. This trend chaser follows a prespecified trading rule, which is proportional to the difference between the current stock price and the previous stock price. In other word, the trend chaser buys (sells) when the price increases (decreases). The trades of both the strategic trader and the trend chaser affect stock prices. We show that if the trend chasing intensity is large, the strategic trader purchases the stock in the first two periods, pushing up stock prices. The strategic trader then sells the stock to the trend chaser in the third period so that the stock price subsequently falls. Specifically, because of the trades by the strategic trader, the stock price difference between the second period and the first period is large enough, so that the trend chaser will buy the stock in large quantities in the third period. Due to a large demand by the trend chaser in the third period, the stock price will remain high. Therefore, the strategic trader profits by selling the stock in the third period at the expense of the trend chaser. In this case, the stock price increases first and then declines in the last period. If the trend chasing intensity is low, however, the strategic trader may not have incentives to manipulate the stock prices, and as a result, the trend chaser can actually make a profit. This is due to the fact that the trend chaser shares risk with both the strategic trader and the liquidity provider.

For completeness, we show that our results hold in the presence of a Kyle-type noise trader. We also show that although there is a positive probability that the stock payoff will be revealed perfectly in each period, the no-trade theorem of [13] still holds in a competitive model in which the trades by the initiating trader do not affect the stock price. In other words, strategic trading is essential to overcome the no-trade theorem.

This paper develops perhaps the first rational model of strategic trading under symmetric information in which both the strategic trader and the liquidity provider are utility maximizers. The traditional inventory models, represented by [15] [16] [17][3], solve a risk-averse market maker's maximization problem but the stock demand by strategic traders is exogenously assumed. [20] [21] develop competitive trading models under symmetric information. To generate sustained trading beyond the first period, Grossman and Miller assume that one trader receives a positive stock supply in the first period and another trader receives a negative stock supply in the second period[4], whereas Campbell, Grossman, and Wang assume that the aggregate risk aversion of market participants changes over time.

Our model in the presence of a trend chaser is related to the literature on price manipulation, which is almost exclusively information based. See, for example, [1] [23] [24] [25] [26]. [26] considers a behavior price manipulation model under symmetric information. In this model, there are three types of traders whose trading strategies are all exogenously assumed.

The rest of this paper is organized as follows. Section 2 spells out the model assumptions. Section 3 characterizes the equilibrium. Section 4 solves the maximization problems of the strategic trader and the liquidity provider. Section 5 presents the main results. Section 6 concludes the paper. The appendices extend the model to incorporate a noise trader as well as studies the competitive limit of our strategic model.

2. Model Specification

We consider a four-period model in which there are three types of traders: a risk-averse strategic trader[5], a risk-averse liquidity provider who clears markets by setting equilibrium stock prices competitively[6], and a trend chaser. When we

[3]See also [18] [19].

[4][22] extends the Grossman-Miller model to incorporate capital and margin constraints.

[5]A representative strategic trader captures the notion of many strategic traders who collude in trading. This assumption is consistent with the empirical findings of [2] [7] [12], in which brokers collude or retail investors herd in trading.

[6]Equivalently, we can assume that there are a continuum of competitive liquidity traders, but we normalize the number to be 1.

set the trend chaser's demand for stock to be zero, the model reduces to the basic version in which both the strategic trader and the liquidity provider are rational agents. There is one risk free bond and one risky stock available for trading, and trading takes place at times 1, 2, and 3. At time 4, the game ends and all participants receive payments according to their stock holdings.

Without loss of generality, we assume that the interest rate for the bond is zero and that the price of the bond is always 1. At time 4, the stock pays off. Before that time, the strategic trader and the liquidity provider only know that the stock payoff follows a normal distribution with mean D_0 and variance σ_D^2. There is no information asymmetry between the strategic trader and the liquidity provider. To generate trading beyond the first period, we assume that in each period, there is a probability of q that the strategic trader and the liquidity provider will receive a signal that reveals the true value of the stock payoff perfectly[7]. Both the strategic trader and the liquidity provider have the same probability of receiving a perfect signal regarding the stock payoff.

If the perfect signal arrives, trading stops and the game ends. The reason is that when the stock payoff is perfectly known to both the strategic trader and the liquidity provider, the equilibrium stock price is equal to the true value of the stock payoff, and therefore, no additional trading occurs. We then assume that both market participants consume their wealth. In other words, there is an uncertainty about the timing of the traders' consumption. Equivalently, there is a probability of $(1-q)$ that the game will move onto the next period. In each period, if the signal does not arrive, the strategic trader will choose his optimal portfolio.

The strategic trader initiates trading and chooses his optimal trading strategies. The trend chaser picks his trading quantities following a pre-specified trading rule. The liquidity provider chooses her optimal positions and clears markets by setting the prices competitively based on the order flows submitted by the strategic trader and the trend chaser. Because there is no information asymmetry, it does not make any difference to the liquidity provider whether she observes the order flows separately or the total order flows only. Symmetric information allows the liquidity provider to solve for the order flows of the strategic trader, and the order flows of the trend chaser are pre-specified in terms of stock prices that are known to the liquidity provider.

The strategic trader and the liquidity provider have quadratic utility functions of $E(W_s)-1/2\gamma_s Var(W_s)$ and $E(W)-1/2\gamma Var(W)$, respectively[8]. γ_s and γ denote their respective risk-aversion coefficients, and W_s and W denote their respective wealth that the strategic trader and the liquidity provider consume whenever the game ends. The initial endowments of the strategic trader and the liquidity provider are given by X_0 and Y_0, respectively. Because the liquidity provider is risk averse, the order flows of the strategic trader and the trend chaser

[7]Our model differs from the asymmetric information model of [27] in which one trader receives a perfect signal at time 0 and the market maker must infer the private signal from the total order flows submitted by the strategic trader and the noise trader.

[8]See, for example, [28] for an early use of the quadratic utility function.

affect stock prices in equilibrium. As a result, the strategic trader chooses the optimal trading strategies taking into account the impact of his trades on prices. The stock price in period i is denoted by P_i, where $i \in [1, 2, 3, 4]$.

The demand for stock by the trend chaser is assumed to be proportional to the stock price change between two consecutive periods. Obviously, this trader does not trade at time 1. Because the liquidity provider sets stock prices only after observing order flows, both the strategic trader and the trend chaser do not know the price when they submit orders even at time 2. As a result, the trend chaser does not trade until time 3. We assume that the total quantities traded by the trend chaser are given by $Z_3 = g(P_2 - P_1)$, where g is a positive constant.

In summary, in each period $i, i \in [1, 2, 3]$, the strategic trader trades X_i shares of the stock to maximize his expected utility, and the trend chaser trades Z_3 shares of the stock (in period 3). Based on the order flows, the liquidity provider buys or sells Y_i shares to maximize her expected utility and clears markets by setting the equilibrium prices.

3. Equilibrium

In this section, we specify the equilibrium stock price and the market clearing condition in each period. We consider only a linear equilibrium in which the prices are linear functions of the order flows of the stock.

3.1. Stock Prices

At the terminal date 4, the stock is liquidated and market participants are paid according to their stock holdings. The equilibrium stock prices in other periods are given by

$$P_1 = D_0 + \gamma\sigma_D^2\left[-k_1 Y_0 + k_2 X_1 + h_1 X_0\right], \tag{1}$$

$$P_2 = D_0 + \gamma\sigma_D^2\left[-k_3 Y_0 + k_4 X_1 + k_5 X_2 + h_2 X_0\right], \tag{2}$$

$$P_3 = D_0 + \gamma\sigma_D^2\left[-k_6 Y_0 + k_7 X_1 + k_8 X_2 + k_9 X_3 + k_{10} Z_3 + h_3 X_0\right], \tag{3}$$

where the k's and h's are constants to be determined in equilibrium. Because the liquidity provider sets prices based on her observed order flows of the strategic trader and the trend chaser, P_i depends only on the order flows before and in period i, where $i \in [1, 2, 3]$. Note that the liquidity provider's holdings in the stock do not appear in the price functions, because they become redundant once the market clearing conditions are imposed.

3.2. Market Clearing Conditions

Since the market clears in each period, the sum of the positions of the strategic trader, the trend chaser, and the liquidity provider must be equal to zero, that is,

$$0 = Y_1 + X_1, \tag{4}$$

$$0 = Y_2 + X_2, \tag{5}$$

$$0 = Y_3 + X_3 + Z_3. \tag{6}$$

4. Equilibrium Solutions

Using the pricing functions and the market clearing conditions, we next solve rigorously the dynamic maximization problems of the strategic trader and the liquidity provider to determine their optimal trading strategies as well as the coefficients in the pricing functions. We first derive the general expressions for the solutions in terms of various parameters and then employ numerical solutions to obtain the concrete results.

4.1. The Liquidity Provider's Maximization Problems

We solve the liquidity provider's optimization problems using backward induction. We first solve the maximization problem in period 3, which is a one-period problem. Taking the optimal solutions for this period as given, we then solve the maximization problem in period 2. Taking the optimal solutions from periods 2 and 3 as given, we next solve the maximization problem in period 1.

Whenever the stock payoff is revealed in period i, the liquidity provider consumes all her wealth. We denote this wealth by W_i, where $i \in \{2,3,4\}$. We denote her wealth after trading in period i by $\bar{W}_i$, where $i \in \{1,2,3\}$. The initial wealth is denoted by W_0. We next derive W_i.

Lemma 1.

$$W_2 - W_0 = Y_0(D - D_0) + Y_1(D - P_1), \tag{7}$$

$$W_3 - W_0 = Y_0(D - D_0) + Y_1(D - P_1) + Y_2(D - P_2), \tag{8}$$

$$W_4 - W_0 = Y_0(D - D_0) + Y_1(D - P_1) + Y_2(D - P_2) + Y_3(D - P_3). \tag{9}$$

Proof. The positions in bond and stock in period i are denoted by B_i and Y_i, respectively, where $i \in 0,1,2,3$, with B_0 and Y_0 being the initial endowments in bond and stock, respectively. We have that $\bar{W}_i = B_i + \left(\sum_{j=1}^{i} Y_j\right) P_i$ after trading occurs in period i. We know that $B_1 = B_0 - P_1 Y_1$ and $B_2 = B_1 - P_2 Y_2$. When the q probability event happens in period 2, $W_2 = (B_0 - Y_1 P_1) + (Y_0 + Y_1) D$. Hence, $W_2 - W_0 = Y_0(D - P_0) + Y_1(D - P_1)$. In period 3, $W_3 = (B_2 - Y_2 P_2) + (Y_0 + Y_1 + Y_2) D = (B_0 - Y_1 P_1 - Y_2 P_2) + (Y_0 + Y_1 + Y_2) D$. Therefore, we have that $W_3 - W_0 = Y_0(D - P_0) + Y_1(D - P_1) + Y_2(D - P_2)$. Similarly, we can derive the expression for W_4. Q.E.D.

At time 3, there is one period to go. The liquidity provider's maximization problem is given by

$$\max_{Y_3} \left[E\left(W_4 \middle|_{\mathcal{F}_3}\right) - \frac{1}{2}\gamma Var\left(W_4 \middle|_{\mathcal{F}_3}\right) \right]. \tag{10}$$

Recall that the liquidation value of the stock, D, follows a normal distribution with mean of D_0 and variance of σ_D^2. Taking the expectation gives

$$\max_{Y_3} \left[(D_0 - P_3) Y_3 + (D_0 - P_2) Y_2 + (D_0 - P_1) Y_1 + D_0 Y_0 - \frac{1}{2}\gamma\sigma_D^2 (Y_0 + Y_1 + Y_2 + Y_3)^2 \right]. \tag{11}$$

The first-order condition (FOC) with respect to Y_3 yields

$$Y_3 = \frac{(D_0 - P_3)}{\gamma\sigma_D^2} - (Y_0 + Y_1 + Y_2). \tag{12}$$

Note that the first term is the familiar demand function for the stock, which increases with the expected excess return for investing in the stock, $(D_0 - P_3)$, and decreases with both the risk aversion of the liquidity provider and the risk of the stock payoff. Because the liquidity provider is risk averse, the second term shows that her demand for the risky stock decreases with her cumulative holdings in the stock. Using the market clearing conditions and the equilibrium pricing functions specified in Section 3, we obtain

$$k_6 = k_7 = k_8 = k_9 = k_{10} = 1,\ h_3 = 0. \tag{13}$$

In period 2, the liquidity provider's expected utility depends on whether the liquidation value D of the stock will be revealed in period 3. If the q probability event happens, then the liquidity provider sets the price to be the true value of the stock payoff, and the game ends. The liquidity provider will then consume her entire wealth. It can be derived that the liquidity provider's expected utility is given by

$$\begin{aligned}\max_{Y_2}\Big\{ & q\Big[(D_0 - P_2)Y_2 + (D_0 - P_1)Y_1 + D_0Y_0 - \frac{1}{2}\gamma\sigma_D^2(Y_0 + Y_1 + Y_2)^2\Big] \\ & + (1-q)\big[(D_0 - P_3)Y_3 + (D_0 - P_2)Y_2 + (D_0 - P_1)Y_1 + D_0Y_0\big] \\ & - (1-q)\frac{1}{2}\gamma\sigma_D^2(Y_0 + Y_1 + Y_2 + Y_3)^2\Big\}.\end{aligned} \tag{14}$$

The FOC with respect to Y_2 yields

$$Y_2 = \frac{\big[(1-q)(P_3 - P_2) + q(D_0 - P_2)\big]}{q\gamma\sigma_D^2} - (Y_0 + Y_1), \quad q \neq 0. \tag{15}$$

To understand this demand function, we rearrange Equation (15) as: $(Y_0 + Y_1 + Y_2)q\gamma\sigma_D^2 = \big[(1-q)(P_3 - P_2) + q(D_0 - P_2)\big]$. The right hand side of this equation represents the expected profit for investing in the stock, and the left hand side represents the risk premium associated with the q event that the stock payoff D will be revealed. At time 2, D follows a normal distribution of $N(D_0, \sigma_D^2)$.

In period 1, there is a probability of q that the liquidation value of the stock will be revealed in period 2 and a probability of $(1-q)$ that the game moves on to period 3. We obtain the expected utility of the liquidity provider as

$$\begin{aligned}\max_{Y_1}\Big\{ & q\Big[(D_0 - P_1)Y_1 + D_0Y_0 - \frac{1}{2}\gamma\sigma_D^2(Y_0 + Y_1)^2\Big] \\ & + q(1-q)\big[(D_0 - P_2)Y_2 + (D_0 - P_1)Y_1 + D_0Y_0\big] \\ & - q(1-q)\frac{1}{2}\gamma\sigma_D^2(Y_0 + Y_1 + Y_2)^2 \\ & + (1-q)(1-q)\big[(D_0 - P_3)Y_3 + (D_0 - P_2)Y_2 + (D_0 - P_1)Y_1 + D_0Y_0\big] \\ & - \frac{1}{2}(1-q)(1-q)\gamma\sigma_D^2(Y_0 + Y_1 + Y_2 + Y_3)^2\Big\}.\end{aligned} \tag{16}$$

The FOC with respect to Y_1 yields

$$Y_1 = \frac{\left[(1-q)(P_2 - P_1) + q(D_0 - P_1)\right]}{q\gamma\sigma_D^2} - Y_0. \tag{17}$$

4.2. The Strategic Trader's Maximization Problems

Like the liquidity provider, the strategic trader consumes whenever the game ends or the stock payoff is revealed. When the stock payoff is revealed, his wealth in period i is denoted by W_i^X, where $i \in \{2,3,4\}$. The initial wealth is denoted by W_0. Using similar derivations to those of the liquidity provider's wealth processes, we obtain

$$W_2^X - W_0^X = X_0(D - P_0) + X_1(D - P_1), \tag{18}$$

$$W_3^X - W_0^X = X_0(D - P_0) + X_1(D - P_1) + X_2(D - P_2), \tag{19}$$

$$\begin{aligned} W_4^X - W_0^X &= X_0(D - P_0) + X_1(D - P_1) \\ &\quad + X_2(D - P_2) + X_3(D - P_3). \end{aligned} \tag{20}$$

The strategic trader's maximization problem in period 3 is given by

$$\begin{aligned} \max_{X_3} \Big[& X_1(D_0 - P_1) + X_2(D_0 - P_2) + X_3(D_0 - P_3) \\ & -\frac{1}{2}\mu(X_0 + X_1 + X_2 + X_3)^2 \Big], \quad \mu = \gamma_s\sigma_D^2. \end{aligned} \tag{21}$$

Substituting the conjectured price functions into this equation, the FOC yields

$$D_0 - P_3 - \lambda X_3 - \mu(X_1 + X_0 + X_2 + X_3) = 0, \tag{22}$$

where $\lambda = \gamma\sigma_D^2$. Rearranging the FOC gives

$$X_3 = -\frac{1}{2\lambda + \mu}\left[\lambda(-Y_0 + X_1 + X_2 + Z_3) + \mu(X_0 + X_1 + X_2)\right]. \tag{23}$$

It can be verified that the second-order condition (SOC) is negative. Hence, Equation (23) yields the optimal solution.

The maximization problem in period 2 is given by

$$\begin{aligned} \max_{X_2} \Big\{ (1-q)\Big[& X_0 D_0 + X_1(D_0 - P_1) + X_2(D_0 - P_2) \\ & + X_3(D_0 - P_3) - \frac{1}{2}\mu(X_0 + X_1 + X_2 + X_3)^2 \Big] \\ & + q\Big[X_0 D_0 + X_1(D_0 - P_1) + X_2(D_0 - P_2) - \frac{1}{2}\mu(X_0 + X_1 + X_2)^2 \Big] \Big\}. \end{aligned} \tag{24}$$

The FOC yields

$$\begin{aligned} & (D_0 - P_2 - k_5\lambda X_2) - \mu q(X_0 + X_1 + X_2) - (1-q)\mu(X_0 + X_1 + X_2 + X_3) \\ & -(1-q)(1 + g\lambda k_5)\lambda X_3 = 0, \end{aligned} \tag{25}$$

and the SOC gives

$$\begin{aligned}\mathrm{SOC}_2 &= -2k_5\lambda - \mu q - (1-q)\mu\left[1 - \frac{1}{2\lambda+\mu}\left(\lambda+\mu+g\lambda^2k_5\right)\right],\\ &+(1-q)\frac{\lambda}{2\lambda+\mu}(1+g\lambda k_5)\left(\lambda+\mu+g\lambda^2k_5\right).\end{aligned} \tag{26}$$

To solve the FOC (25) for the strategic trader's optimal trading strategy X_2 in the second period, we assume that X_2 is a linear function of X_0, X_1 and Y_0, which takes the form of

$$X_2 = g_0Y_0 + g_1X_1 + g_2X_0, \tag{27}$$

where g_0, g_1, and g_2 are constants to be determined in equilibrium, and Y_0 is the initial endowment of the liquidity provider.

The optimization problem in period 1 is given by

$$\begin{aligned}\max_{X_1}\Bigg\{&(1-q)^2\Bigg[X_0D_0 + X_1(D_0-P_1) + X_2(D_0-P_2) + X_3(D_0-P_3)\\ &-\frac{1}{2}\mu(X_1+X_0+X_2+X_3)^2\Bigg] + (1-q)q\Bigg[X_0D_0 + X_1(D_0-P_1)\\ &+X_2(D_0-P_2) - \frac{1}{2}\mu(X_1+X_0+X_2)^2\Bigg]\\ &+q\left[X_0D_0 + X_1(D_0-P_1) - \frac{1}{2}\mu(X_1+X_0)^2\right]\Bigg\}.\end{aligned} \tag{28}$$

The FOC with respect to X_1 gives

$$\begin{aligned}0 = &-(P_1 + k_2\lambda X_1 - D_0) - q\mu(X_0+X_1)\\ &-(1-q)\left[k_4X_2\lambda + q\mu(X_0+X_1+X_2)\right]\\ &+(1-q)^2\mu(1+k_s)(X_0+X_1+X_2+X_3)\\ &-(1-q)^2\left\{\lambda X_3\left[1+k_s+g\lambda(k_4-k_2)\right] - k_sP_3\right\},\end{aligned} \tag{29}$$

$$k_s = -\frac{1}{2\lambda+\mu}\left[(\lambda+\mu) + g\lambda^2(k_4-k_2)\right].$$

The SOC gives

$$\begin{aligned}\mathrm{SOC} = &-2k_2\lambda - q\mu\\ &-(1-q)\left[g_1k_4\lambda + q\mu(1+g_1) + \mu(1-q)(1+g_1+k_t)(1+k_s)\right]\\ &-(1-q)^2k_s\lambda\left[1+g_1+g\lambda k_5 + k_t + g\lambda(k_4-k_2)\right]\\ &-(1-q)^2k_t\lambda\left[1+k_s+g\lambda(k_4-k_2)\right],\end{aligned} \tag{30}$$

where $k_t = k_s - \dfrac{(\lambda+\mu)g_1 + g\lambda^2k_5g_1}{2\lambda+\mu}$.

Recall that P_1, P_2, and P_3 are linear functions of $X_i, i\in[1,2,3]$, with $k_j, j\in[1,2,3,\cdots,10]$, being the coefficients. We have shown that $k_j = 1$, $j\in[6,7,8,9,10]$, $h_i, i\in[1,2,3]$ and $k_i, i\in[1,2,3,4,5]$, are functions of g_0, g_1, and g_2 only. X_2 is a function of g_0, g_1, g_2, Y_0, X_0, and X_1. The FOC (22) of the maximization problem in period 3 shows that X_3 is a function of X_0, X_1, X_2, and P_3. Substituting the pricing functions of P_3 into Equation (22), we see that X_3 can be expressed in terms of g_0, g_1, g_2, X_0, and X_1 as in Equation (23). The FOC (25) of the maximization problem in period 2 shows that X_2 is a function of X_0, X_1, X_3, and P_2. Substituting the pricing function of P_2 and the expression for X_3 into this equation and rearranging it yield that X_2 can be expressed as a linear function of Y_0, X_1, and X_0. Plugging this expression for X_2 into $X_2 \equiv g_0 Y_0 + g_1 X_1 + g_2 X_0$ yields a linear function of X_0, X_1, and Y_0. Because this equation holds for any Y_0, X_1, and X_0, comparing the coefficients in front of Y_0, X_1, and X_0 yields three equations for g_0, g_1, and g_2. Solving these equations gives the solutions for g_0, g_1, and g_2. Plugging the expressions for X_2, X_3, P_2, and P_3 into the FOC (29) of the maximization problem in period 1 yields the expression for X_1 as a linear function of X_0 and Y_0. The closed-form solutions to these equations do exist but they are extremely complicated. We next solve for g_0, g_1, and g_2 numerically with certain parameters and simultaneously ensure that the SOCs are all satisfied under those parameter values.

5. Main Results

The inputs for numerical calculations are the stock endowments of the liquidity provider Y_0 and the strategic trader X_0, the expected value of the stock payoff, D_0, the probability q that the stock payoff will be revealed perfectly in periods 1, 2, and 3, g, $\lambda = \gamma \sigma_D^2$, and $\mu = \gamma_s \sigma_D^2$. We next present the results both with and without a trend chaser in the market.

5.1. Results without a Trend Chaser or $g = 0$

In this setup, we have a rational model in which the strategic trader initiates trades to achieve optimal risk sharing with the liquidity provider. Because of the market impact cost, the strategic trader trades gradually to minimize the market impact of his trades. We next present four sets of results, depending on the initial endowments and the risk aversions of the strategic trader and the liquidity provider.

Case 1: Figure 1 presents the results for the case in which the strategic trader initiates a buy order and keeps buying in all three periods, or X_1, X_2, and X_3 are all positive. Equivalently, the liquidity provider sells the stock in all periods to clear markets. The strategic trader and the liquidity trader share risk optimally. The equilibrium stock price keeps going up until it converges up to the fundamental value D_0, which is set to be 0 in all calculations without loss of generality. We thus have $P_1 < P_2 < P_3 < 0$. The buy orders by the strategic trader lead to positive stock returns in the future, and the stock returns exhibit predictable patterns. The strategic trader, who has no endowment in the stock, can afford to take on additional allocation of stock. Hence, the strategic trader initiates a buy order, and the liquidity provider sells to the strategic trader and clears the market by setting equilibrium prices. With an uncertainty about the timing of consumption and the market impact of the strategic trader's trades, the strategic trader trades gradually to achieve optimal risk sharing with the liquidity provider as well as to minimize the market impact costs of his trades. As a result, the

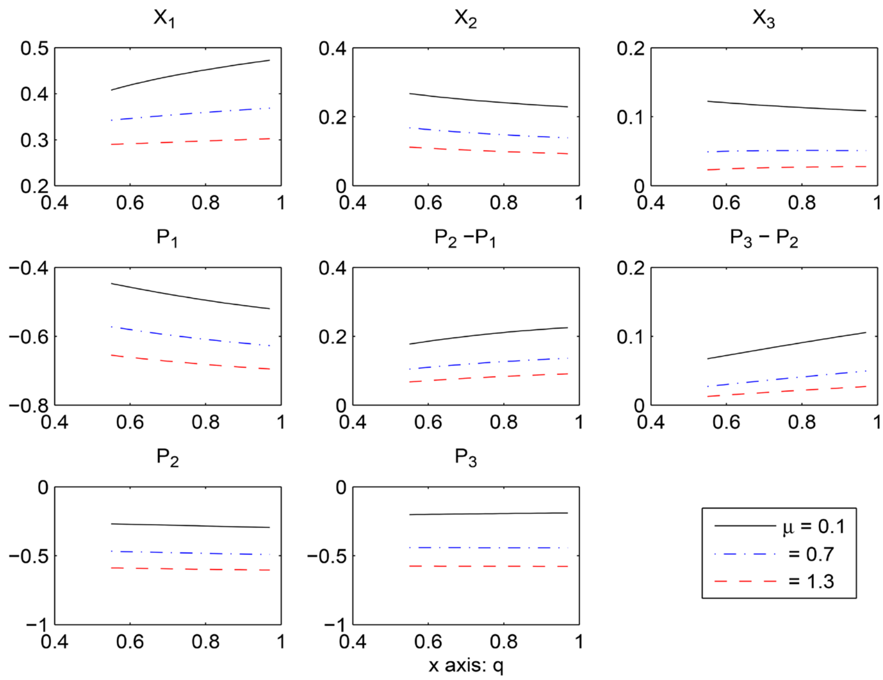

Figure 1. Price dynamics ($D_0 = 0$, $Y_0 = 1$, $X_0 = 0$, $\lambda = 1$, and $g = 0$).

stock return exhibits predictability, and the strategic trader's trade can be used to forecast future stock returns.

Case 2: Figure 2 presents the results for the case in which the strategic trader initiates a sale order and keeps selling in all three periods, that is, X_1, X_2, and X_3 are all negative. Since the liquidity provider has a negative endowment, she tends to cover his short position. Hence, the liquidity provider buys the stock in all periods to clear markets. To achieve optimal risk sharing between the two traders, the strategic trader sells to the liquidity provider. Due to the negative risk premium associated with the liquidity provider's negative endowment, the stock prices in the first three periods are greater than the fundamental value 0. The equilibrium price keeps going down until it converges to the fundamental value, or $P_1 > P_2 > P_3 > 0$. The strategic trader initiates a sale order and trades gradually to share risk with the liquidity provider as well as minimize the market impact of his trades. The gradual trading by the strategic trader leads to the stock return predictability. As a result, the trade by the strategic trader can be used to forecast future stock returns. To compensate the liquidity provider, the sale-orders by the strategic trader lead to negative stock returns in the future, and there exists a positive contemporaneous relationship between stock returns and strategic trader's orders.

Our results in the above two cases provide potential explanations for the empirical findings of [2] [7] [8], in which their herding retail investors would correspond to our strategic trader. They find that the buy (sale) orders of retail investors lead to positive (negative) stock returns in the future. In addition, the positive contemporaneous relationship between the trades of the strategic trader and stock returns is consistent with the empirical results of [10] [11], and others.

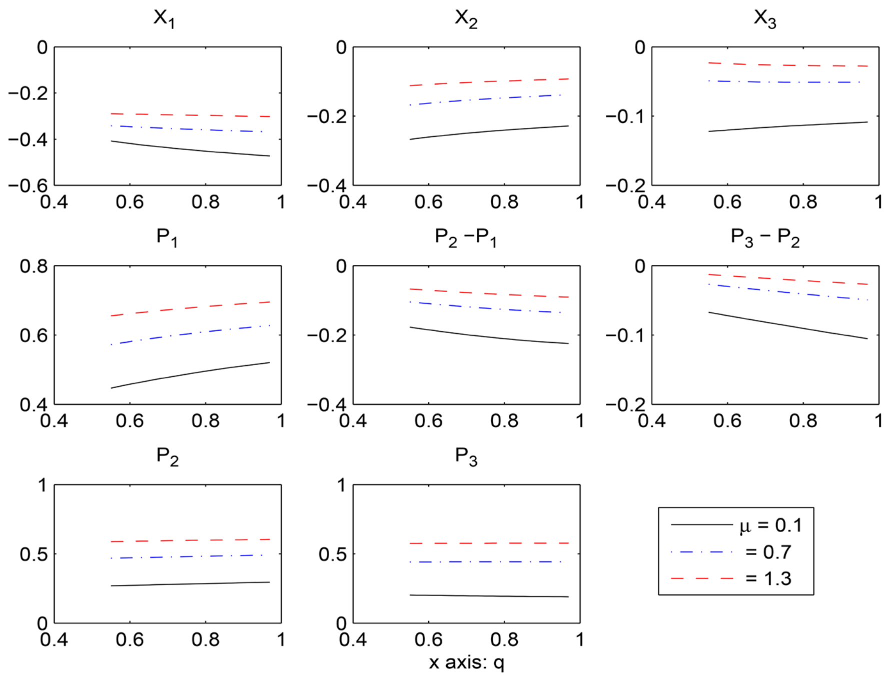

Figure 2. Price dynamics ($D_0 = 0$, $Y_0 = -1$, $X_0 = 0$, $\lambda = 1$, and $g = 0$).

Case 3: In **Figure 3**, the strategic trader has a short position in initial stock endowment. To achieve optimal risk sharing, the strategic trader tends to cover his short position and initiates a buy order. He may keep buying in all three periods. The liquidity provider sells the stock to clear markets. To minimize the market impact of his trades, the strategic trader trades gradually, and his buy orders decline over time. Due to a negative risk premium, the stock prices in the first three periods are above the fundamental value D_0. In particular, they increase in the first three periods and then come down to D_0 at the terminal date. There is a downward price reversal in the last period.

Case 4: In **Figure 4**, the endowment of the strategic trader is positive and that of the liquidity provider is zero, so the strategic trader initiates a sale order. To minimize market impact, the strategic trader sells the stock gradually to the liquidity provider in all three periods. Due to a positive risk premium, the stock prices are lower than D_0. The price decreases in the first three periods, then goes up to D_0 at the terminal date. There is an upward price reversal in the last period.

In the above two cases, the sale (buy) orders by the liquidity provider lead to a negative (positive) price reversal in the last period. [9] documents positive excess returns after individuals buy and negative excess returns after individuals sell. They interpret the individuals in their sample as liquidity providers. Our results regarding the liquidity provider offer potential explanations for the empirical findings of [9].

In sum, under a symmetric information framework, we find that a combination of optimal risk sharing, strategic trading, and stochastic timing of consumption generates not only sustained trading beyond the first period but also the

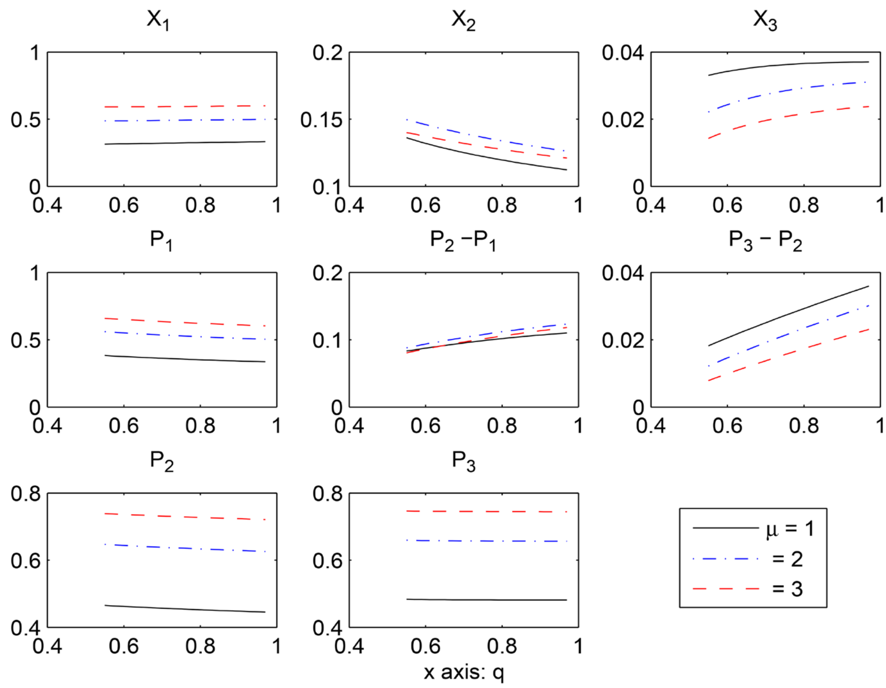

Figure 3. Price dynamics ($D_0 = 0$, $Y_0 = 0$, $X_0 = -1$, $\lambda = 1$, and $g = 0$).

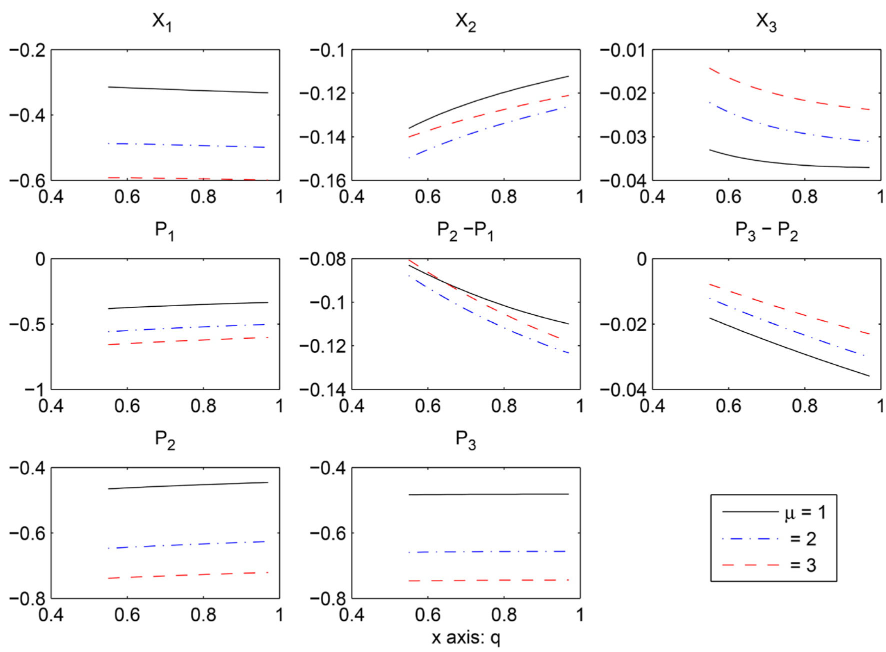

Figure 4. Price dynamics ($D_0 = 0$, $Y_0 = 0$, $X_0 = 1$, $\lambda = 1$, and $g = 0$).

predictability of stock returns. We are able to reconcile two seemingly contradictory empirical findings in a parsimonious rational model. In particular, the buying (selling) by one group of traders leads to positive (negative) stock returns in the future and the buying (selling) by another group of traders leads to negative (positive) stock returns.

When the probability of observing the signal is zero, we find that the no-trade theorem of [13], which assumes a competitive model, still holds under our strategic model. That is, after the first round of trading, both traders reach Pareto optimal risk sharing, and consequently, there will be no additional trading in future periods.

For completeness, we have shown that our results hold in the presence of a Kyle-type noise trader. We have also shown that the no-trade theorem of [13] holds in a competitive model in which the trades by the initiating trader do not affect the stock price, although there is a positive probability that the stock payoff will be revealed perfectly in each period. In other words, strategic trading is essential to overcome the no-trade theorem. The detailed solutions are presented in the appendices.

5.2. Results with a Trend Chaser

Khwaja and Mian (2005) find that pure price manipulation in the absence of private information can generate a pump and dump price pattern. Specifically, a group of colluding brokers drive up the stock price initially and then sell the stock to trend chasers in the market. The stock price subsequently falls as the brokers exit the market. This empirical test provides a great opportunity for the application of our basic model. In this subsection, we introduce a trend chaser into the basic model. Specifically, the trend chaser trades according to a prespecified trading rule given by $Z_3 = g\left(P_2 - P_1\right)$, where Z_3 denotes the trend chaser's demand for stock and g is a constant. Because the trend chaser does not observe the stock price when he submits his order at time 2, he does not trade until the third period.

Figure 5 and Figure 6 present the results for different values of g and q. Figure 5 considers the case of weak trend chasing (a small g value), and Figure 6 considers the case of strong trend chasing. For comparison, we include the case of no trend chasing or $g = 0$.

Figure 5 shows that the stock price increases in the first three periods and then converges up to the fundamental value of D_0. In this case, the strategic trader has no share of stock and the liquidity provider has one share of stock in the initial endowment. Consequently, the stock price is below D_0 due to a positive risk premium. As in Case 1 without the trend chaser, to minimize the market impact costs of his trades, the strategic trader buys the stock from the liquidity provider gradually in the first three periods. Note that P_2 is greater than P_1, so the trend chaser buys the stock at time 3. Because P_3 is less than D_0, the trend chaser makes a profit. In this case, the behavioral trend chaser can survive in this economy. Intuitively, the trend chaser and the strategic trader help the liquidity provider to reduce stock risk, hence, their expected profit can be positive due to risk bearing. This result provides a potential justification for the existence of some trend chasers.

When trend chasing is strong, however, the trading by the strategic trader is quite different. Figure 6 shows that X_1 and X_2 are positive but X_3 is negative, and that the stock price increases in the first three periods and then decreases to D_0. In other words, the strategic trader purchases the stock in the first two periods,

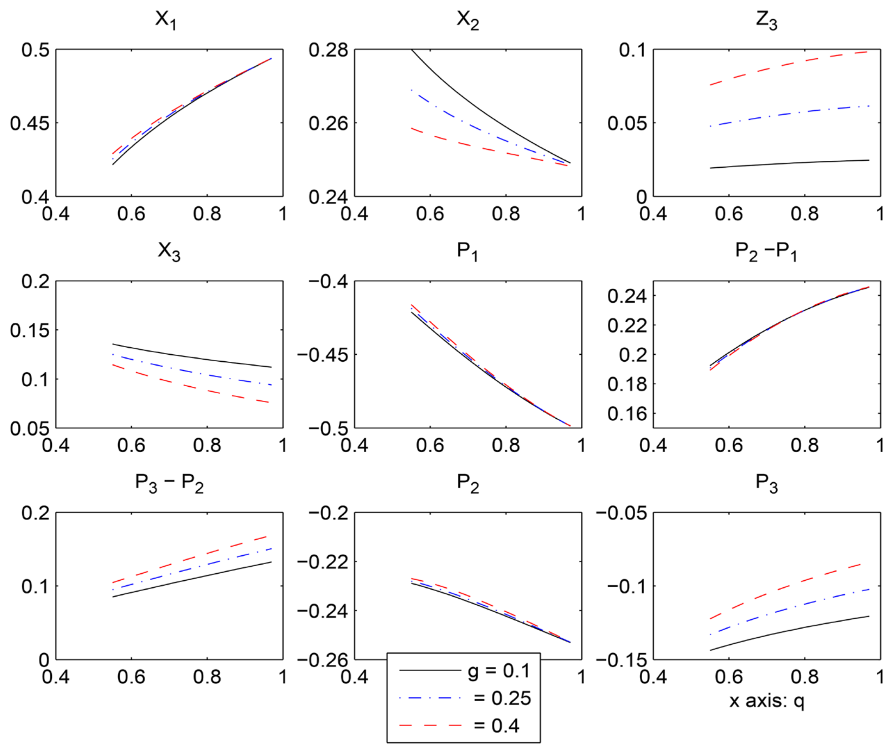

Figure 5. Price dynamics with weak trend chasing ($D_0 = 0$, $Y_0 = 1$, $X_0 = 0$, $\lambda = 1$, and $\mu = 0.01$).

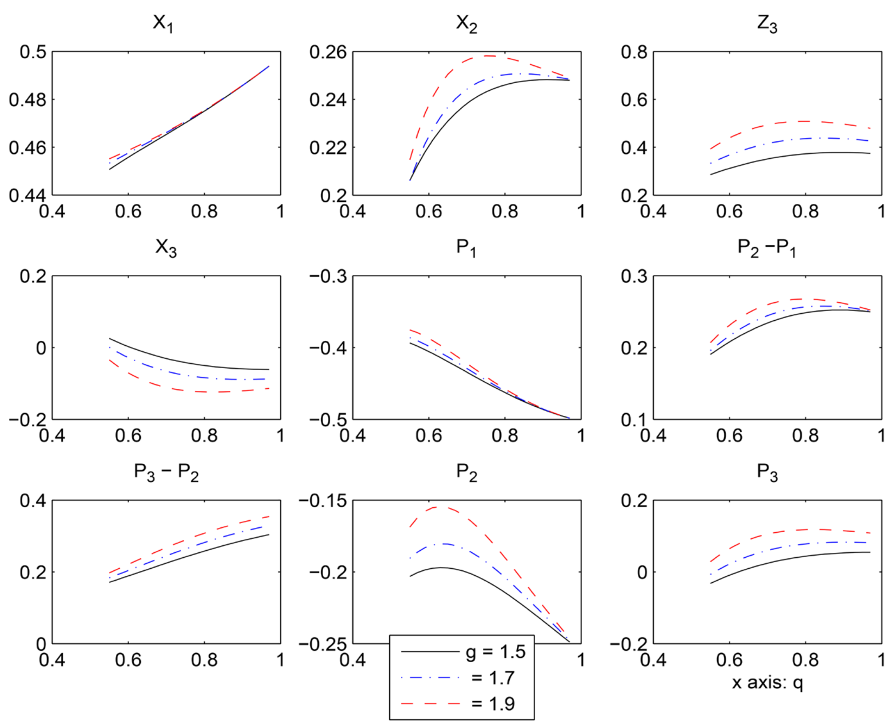

Figure 6. Price dynamics with strong trend chasing ($D_0 = 0$, $Y_0 = 1$, $X_0 = 0$, $\lambda = 1$, and $\mu = 0.01$).

pushing up stock prices, and then sells the stock at a high price to the trend chaser at time 3. Although the strategic trader sells the stock at time 3, the stock price still remains high due to the strong buying by the trend chaser ($Z_3 \gg |X_3|$) at the same time.

We define g as a measure of the likelihood of manipulation. When g increases, the magnitudes of X_1, X_2, X_3, Z_3, P_1, P_2, and P_3 all increase. The strategic trader trades so that the difference between the stock price in the second period and that in the first period is sufficiently large. As a result, the trend chaser will demand a large amount of stock in the third period. Numerically, both $(P_3 - P_2) > 0$ and $(P_2 - P_1) > 0$ increase with g. With a high $(P_3 - P_2)$, the strategic trader can profit more by selling in the third period.

In particular, when g is large enough, P_3 can even exceed D_0 (=0), which can be seen from **Figure 6**. Notice that in the absence of a trend chaser, the stock price in this case is always lower than D_0, because the liquidity provider and the strategic trader are risk averse and they have long positions in the stock. With trend chasing in the market, the strategic trader buys the stock in the first two periods, and the liquidity provider uses her inventory to clear markets. In the third period, it is possible that the trend chaser demands the stock so much that the liquidity provider has to borrow shares to clear the market. Consequently, the liquidity provider prices the stock higher than D_0, driving up the stock price significantly. This phenomenon corresponds to a bubble state as defined in [29].

Our model with a trend chaser under symmetric information produces similar results to those obtained by [29] in an asymmetric information model. Note that our model relies on the rational behaviors of the strategic trader and the liquidity provider to drive up the stock price. In De Long et al., the stock demand of the liquidity provider is exogenously assumed, so their model is a partial equilibrium one. In addition, the assumption of asymmetric information is not supported by [12], who find that strategic trading rather than asymmetric information leads to the pump and dump trading pattern. In [26], traders share the same information as in our model but the trading strategies of all the traders are exogenously assumed. If trading strategies were to be determined optimally, then there would not be any trading after the first round or their results would not hold.

In sum, our model represents perhaps the first rigorous model that generates the pump-and-dump price pattern. These results are due to a combination of strategic trading, trend chasing, and a stochastic consumption date. Absence of any of the three factors will not generate the pump and dump patterns.

When a trend chaser is introduced into our rational model, the expanded model can then be viewed as a trade-based manipulation model in which there is a strategic trader, a competitive liquidity provider, and a mechanical trend chaser. If the intensity of trend chasing is weak, then manipulation by the strategic trader will not be strong. It is possible in this case that the trend chaser can actually make a profit. This result perhaps provides a rationale that trend chasers can survive in the market. If the intensity of trend chasing is strong, however, the strategic trader will trade, leading to a significant price change between the first two periods. As a result, the trend chaser will demand a large quantity of stock in the third period, which maintains the stock price at a high level, while the strategic trader exits the market. In other words, the strategic trader raises stock prices initially to attract trend chasers. Once prices have risen, the strategic trader sells to trend chasers, and prices subsequently fall, generating a pump and dump price scheme.

6. Conclusion

In conclusion, this paper develops a theoretical framework for risk sharing and strategic trading under symmetric information. This framework not only overcomes the no-trade theorem, but also generates stock return predictability.

Acknowledgments

We thank Vincent Ou-Yang for comments.

References

[1] Andrade, S.C., Chang, C. and Seasholes, M.S. (2008) Trading Imbalances, Predictable Reversals, and Cross-Stock Price Pressure. *Journal of Financial Economics*, **88**, 406-423.

[2] Barber, B., Odean, T. and Zhu, N. (2009) Do Retail Trades Move Markets. *The Review of Financial Studies*, **22**, 151-186. https://doi.org/10.1093/rfs/hhn035

[3] Boehmer, E. and Wu, J. (2008) Order Flow and Prices. AFA 2007 Chicago Meetings Paper. https://doi.org/10.2139/ssrn.891745

[4] Chan, K., Hameed, A. and Lau, S.T. (2003) What If Trading Location Is Different from business location? Evidence from the Jardine Group. *Journal of Finance*, **58**, 1221-1246. https://doi.org/10.1111/1540-6261.00564

[5] Coval, J. and Stafford, E. (2007) Asset Fire Sales (and Purchases) in Equity Markets. *Journal of Financial Economics*, **86**, 479-512.

[6] Hendershott, T. and Seasholes, M.S. (2007) Market Maker Inventories and Stock Prices. *American Economic Review*, **97**, 210-214. https://doi.org/10.1257/aer.97.2.210

[7] Hvidkjaer, S. (2008) Small Trades and the Cross-Section of Stock Returns. *Review of Financial Studies*, **21**, 1123-1151. https://doi.org/10.1093/rfs/hhn049

[8] Jackson, A. (2003) The Aggregate Behavior of Individual Investors. Working Paper, London Business School. https://doi.org/10.2139/ssrn.536942

[9] Kaniel, R., Saar, G. and Titman, S. (2008) Individual Investor Trading and Stock Returns. *Journal of Finance*, **63**, 272-310. https://doi.org/10.1111/j.1540-6261.2008.01316.x

[10] Goetzmann, W.N. and Massa, M. (2003) Index Funds and Stock Market Growth. *Journal of Business*, **76**, 1-28. https://doi.org/10.1086/344111

[11] Kumar, A. and Lee, C.M.C. (2006) Retail Investor Sentiment and Return Comovements. *Journal of Finance*, **61**, 2451-2486. https://doi.org/10.1111/j.1540-6261.2006.01063.x

[12] Khwaja, A.I. and Mian, A. (2005) Unchecked Intermediaries: Price Manipulation in an Emerging Stock Market. *Journal of Financial Economics*, **78**, 203-241.

[13] Milgrom, P. and Stokey, N. (1982) Information, Trade, and Common Knowledge. *Journal of Economic Theory*, **26**, 17-27.

[14] Grossman, S.J. and Stiglitz, J.E. (1980) On the Impossibility of Informationally Effi-

cient Markets. *American Economic Review*, **70**, 393-408.

[15] Amihud, Y. and Mendelson, H. (1980) Dealership Market: Market Making with Inventory. *Journal of Financial Economics*, **8**, 31-53.

[16] Garman, M. (1976) Market Microstructure. *Journal of Financial Economics*, **3**, 257-275.

[17] Ho, T. and Stoll, H. (1981) Optimal Dealer Pricing under Transactions and Return Uncertainty. *Journal of Financial Economics*, **9**, 99-117.

[18] Biais, B. (1993) Price Formation and Equilibrium Liquidity in Fragmented and Centralized Markets. *Journal of Finance*, **48**, 157-184. https://doi.org/10.1111/j.1540-6261.1993.tb04705.x

[19] O'Hara, M. and Oldfield, G. (1986) The Microeconomics of Market Making. *Journal of Financial and Quantitative Analysis*, **21**, 361-376. https://doi.org/10.2307/2330686

[20] Campbell, J.Y., Grossman, S.J. and Wang, J. (1993) Trading Volume and Serial Correlation in Stock Returns. *Quarterly Journal of Economics*, **108**, 905-939. https://doi.org/10.2307/2118454

[21] Grossman, S.J. and Miller, M.H. (1988) Liquidity and Market Structure. *Journal of Finance*, **43**, 617-633. https://doi.org/10.1111/j.1540-6261.1988.tb04594.x

[22] Brunnermeier, M.K. and Pedersen, L.H. (2009) Market Liquidity and Funding Liquidity. *Review of Financial Studies*, **22**, 2201-2238. https://doi.org/10.1093/rfs/hhn098

[23] Aggarwal, R. and Wu, G. (2006) Stock Market Manipulation. *Journal of Business*, **79**, 1915-1953. https://doi.org/10.1086/503652

[24] Allen, F. and Gale, D. (1992) Stock-Price Manipulation. *Review of Financial Studies*, **5**, 503-529. https://doi.org/10.1093/rfs/5.3.503

[25] Hong, H., Lim, T. and Stein, J.C. (2000) Bad News Travels Slowly: Size, Analyst Coverage, and the Profitability of Momentum Strategies. *Journal of Finance*, **55**, 265-295. https://doi.org/10.1111/0022-1082.00206

[26] Mei, J., Wu, G. and Zhou, C. (2003) Behavior Based Manipulation: Theory and Prosecution Evidence. Working Paper, NYU. https://doi.org/10.2139/ssrn.457880

[27] Kyle, A.S. (1985) Continuous Auctions and Insider Trading. *Econometrica*, **53**, 1315-1335. https://doi.org/10.2307/1913210

[28] Leland, H.E. and Pyle, D. (1977) Information Asymmetry, Financial Structure, and Financial Intermediation. *Journal of Finance*, **32**, 371-387. https://doi.org/10.2307/2326770

[29] De Long, J.B., Shleifer, A., Summers, L. and Waldmann, R. (1990) Positive Feedback Investment Strategies and Destabilizing Rational Speculation. *Journal of Finance*, **45**, 375-395. https://doi.org/10.1111/j.1540-6261.1990.tb03695.x

Appendix

A. Equilibrium with Noise Traders

In our basic model, optimal risk sharing and strategic trading generate sustained trading under symmetric information. In asymmetric information models such as those of [21] [27], an exogenously specified noisy supply is required for trading to take place. For completeness, we incorporate this feature into our basic model. We assume that in period i there is a stochastic demand given by U_i, where $i \in [1,2,3]$ and that U_i follows an *i.i.d* normal process with a mean of zero and a variance of σ_U^2. We consider a linear equilibrium in which the equilibrium stock prices are linear functions of the order flows for the stock.

A.1. Equilibrium Prices

At the terminal date 4, the stock is liquidated and the market participants are paid according to their stock holdings. The equilibrium prices at other times are given by

$$P_1 = D_0 + \gamma\sigma_D^2\left[-k_{\{11\}}Y_0 + k_{\{12\}}X_1 + k_{\{13\}}U_1 + h_1X_0\right], \tag{31}$$

$$P_2 = D_0 + \gamma\sigma_D^2\left[-k_{\{21\}}Y_0 + k_{\{22\}}X_1 + k_{\{23\}}U_1 + k_{\{24\}}X_2 + k_{\{25\}}U_2 + h_2X_0\right], \tag{32}$$

$$\begin{aligned} P_3 = D_0 + \gamma\sigma_D^2\Big[&-k_{\{31\}}Y_0 + k_{\{32\}}X_1 + k_{\{33\}}U_1 + k_{\{34\}}X_2 + k_{\{35\}}U_2 + k_{\{36\}}X_3 \\ &+k_{\{37\}}U_3 + k_{\{38\}}Z_3 + h_3X_0\Big], \end{aligned} \tag{33}$$

where the k's and h's are constants to be determined in equilibrium. As in the basic model, P_i depends only on the order flows before and in period i, where $i \in [1,2,3]$. The liquidity provider sets the prices competitively. Note that under symmetric information, the liquidity provider can distinguish the orders between the strategic trader and the noise traders. Therefore, the impacts of these two orders may be different. We assume different coefficients in the above price functions.

A.2. Market Clearing Conditions

Because the market clears in each period, the sum of the positions of the strategic trader, the noise trader, and the liquidity provider must be equal to zero, that is,

$$0 = Y_1 + X_1 + U_1, \tag{34}$$

$$0 = Y_2 + X_2 + U_2, \tag{35}$$

$$0 = Y_3 + X_3 + U_3 + Z_3. \tag{36}$$

A.3. Solution Procedure

Using the pricing functions and the market clearing conditions, we solve the maximization problems of the strategic trader and the liquidity provider to determine their optimal trading strategies as well as the coefficients in the pricing functions. We solve these dynamic maximization problems by backward induction. We first derive the general expressions for the solutions in terms of various parameters and then obtain concrete results numerically.

The Liquidity Provider's Maximization Problems

We start with solve the maximization problem in period 3, which is a one-period problem. Taking the optimal solutions for this period as given, we then solve the liquidity provider's maximization problem in period 2. Taking the optimal solutions from periods 2 and 3 as given, we next solve the maximization problem in period 1. When the stock payoff is revealed perfectly in period *i*, the stock price will be equal to the stock payoff afterwards. As a result, the game ends and the liquidity provider consumes all her wealth. The wealth processes take the same forms as those in the basic model.

At time 3, the liquidity provider's problem is given by

$$\max_{Y_3}\left[E\left(W_4|_{\mathcal{F}_3}\right)-\frac{\gamma}{2}Var\left(W_4|_{\mathcal{F}_3}\right)\right]. \tag{37}$$

Recall that the liquidation value of the stock, D, follows a normal distribution with a mean of D_0 and a variance of σ_D^2. Taking the expectation gives

$$\begin{aligned}\max_{Y_3}\Big[&\left(D_0-P_3\right)Y_3+\left(D_0-P_2\right)Y_2+\left(D_0-P_1\right)Y_1+D_0Y_0\\&-\frac{1}{2}\gamma\sigma_D^2\left(Y_0+Y_1+Y_2+Y_3\right)^2\Big].\end{aligned} \tag{38}$$

The liquidity provider can figure out the order flow by the strategic trader and the order flow by the noise trader exactly, so the volatility of the noisy supply, σ_U, does not appear directly in her expected utility.

In period 2, the expected utility of the liquidity provider depends on whether the liquidation value D of the stock will be realized in period 3. The liquidity provider's problem is given by

$$\begin{aligned}\max_{Y_2}\Big\{&(1-q)E\left[\left\{E\left(W_4|_{\mathcal{F}_3}\right)-\frac{\gamma}{2}Var\left(W_4|_{\mathcal{F}_3}\right)\right\}\Big|_{\mathcal{F}_2}\right]\\&+q\left[E\left(W_3|_{\mathcal{F}_2}\right)-\frac{\gamma}{2}Var\left(W_3|_{\mathcal{F}_2}\right)\right]\Big\}.\end{aligned} \tag{39}$$

In period 1, there is a probability of q that the liquidation value of the stock will be realized in period 2 and a probability of $(1-q)$ that the game moves onto period 3. The liquidity provider's maximization problem is given by

$$\begin{aligned}\max_{Y_1}\Big\{&(1-q)^2E\left[\left\{E\left(W_4|_{\mathcal{F}_3}\right)-\frac{\gamma}{2}Var\left(W_4|_{\mathcal{F}_3}\right)\right\}\Big|_{\mathcal{F}_1}\right]\\&+q(1-q)\left[E\left(W_3|_{\mathcal{F}_1}\right)-\frac{\gamma}{2}Var\left(W_3|_{\mathcal{F}_1}\right)\right]\\&+q(1-q)\left[E\left(W_2|_{\mathcal{F}_1}\right)-\frac{\gamma}{2}Var\left(W_2|_{\mathcal{F}_1}\right)\right]\Big\}.\end{aligned} \tag{40}$$

Solving the above optimization problems yields the optimal stock demand by the liquidity provider in each period. We summarize the results in the following proposition.

Proposition 1. The optimal trades by the liquidity provider in each period are given by the following equations:

$$Y_3 = \frac{(D_0 - P_3)}{\gamma\sigma_D^2} - (Y_0 + Y_1 + Y_2), \tag{41}$$

$$Y_2 = \frac{\left[(1-q)\left(E[P_3 \mid \mathcal{F}_2] - P_2\right) + q(D_0 - P_2)\right]}{q\gamma\sigma_D^2} - (Y_0 + Y_1), \quad q \neq 0, \tag{42}$$

$$0 = E[P_3 \mid \mathcal{F}_2] - P_2, \quad q = 0, \tag{43}$$

$$Y_1 = \frac{\left[(1-q)\left(E[P_2 \mid \mathcal{F}_1] - P_1\right) + q(D_0 - P_1)\right]}{q\gamma\sigma_D^2} - Y_0, \quad q \neq 0, \tag{44}$$

$$0 = E[P_2 \mid \mathcal{F}_1] - P_1, \quad q = 0. \tag{45}$$

Notice that the Y's take similar forms to those in the basic model without noise traders. Using the market clearing conditions and the equilibrium pricing functions, we obtain

$$k_{\{31\}} = k_{\{32\}} = k_{\{33\}} = k_{\{34\}} = k_{\{35\}} = k_{\{36\}} = k_{\{37\}} = k_{\{38\}} = 1, \quad h_3 = 0. \tag{46}$$

Note that when $q = 0$, we have $E[P_3 \mid \mathcal{F}_M(2)] = E[P_3 \mid \mathcal{F}_M(1)] = E[P_2 \mid \mathcal{F}_M(1)] = P_1$, that is, the prices follow a random walk process.

A.4. The Strategic Trader's Maximization Problems

As in the basic model, the strategic trader's maximization problem in period 3 is given by

$$\max_{X_3} \left\{ E\left[X_0(D_0 - P_0) + X_1(D_0 - P_1) + X_2(D_0 - P_2) + X_3(D_0 - P_3) - \frac{1}{2}\mu(X_1 + X_0 + X_2 + X_3)^2 - \frac{1}{2}\psi X_3^2 \mid \mathcal{F}_3 \right]\right\}, \tag{47}$$

where $\psi = \frac{1}{2}\gamma_s\sigma_U^2$. σ_U appears explicitly in the strategic trader's problem, because when he submits the order, he does not know the exact value of the noise trader's supply, which follows a normal distribution. Substituting the conjectured price functions into this equation, the FOC yields

$$E\left[(D_0 - P_3) \mid \mathcal{F}_3\right] - \lambda X_3 - \mu(X_1 + X_0 + X_2 + X_3) - \psi X_3 = 0, \tag{48}$$

where $\lambda = \gamma\sigma_D^2$. Rearranging the FOC gives

$$X_3 = -\frac{1}{2\lambda + \mu + \psi} \cdot \left[-Y_0 + (X_1 + U_1) + (X_2 + U_2) + Z_3 + \mu(X_0 + X_1 + X_2)\right]. \tag{49}$$

It can be verified that the SOC is negative.

The maximization problem in period 2 is given by

$$\max_{X_2}\left\{(1-q)E\left[W_4^X-\frac{1}{2}\mu\left(X_1+X_0+X_2+X_3\right)^2-\frac{1}{2}\psi X_3^2\mid\mathcal{F}_2\right]\right.$$
$$\left.qE\left[W_3^X-\frac{1}{2}\mu\left(X_1+X_0+X_2\right)^2-\frac{1}{2}\psi X_2^2\mid\mathcal{F}_2\right]\right\}. \tag{50}$$

To solve the FOC for the optimal X_2, we assume that X_2 is a linear function of X_1+U_1, which takes the form of

$$X_2=g_0Y_0+g_1\left(X_1+U_1\right)+g_2X_0+g_3X_1, \tag{51}$$

where g_0, g_1, and g_2 are constants to be determined in equilibrium, and X_0 and Y_0 are the initial endowments of the strategic trader and the liquidity provider, respectively. The optimization problem in period 1 is given by

$$\max_{X_1}\left\{(1-q)^2E\left[W_4^X-\frac{1}{2}\mu\left(X_1+X_0+X_2+X_3\right)^2-\frac{1}{2}\psi X_3^2\mid\mathcal{F}_1\right]\right.$$
$$+(1-q)qE\left[W_3^X-\frac{1}{2}\mu\left(X_1+X_0+X_2\right)^2-\frac{1}{2}\psi X_2^2\mid\mathcal{F}_1\right] \tag{52}$$
$$\left.+qE\left[W_2^X-\frac{1}{2}\mu\left(X_1+X_0\right)^2-\frac{1}{2}\psi X_1^2\mid\mathcal{F}_1\right]\right\}.$$

The solutions to the above optimization problems are summarized in the following proposition.

Proposition 2. *Suppose that*

$$\mathrm{SOC}_2=-2k_{\{24\}}\lambda-\mu q-\psi q-(1-q)\mu\left[1-\frac{1}{2\lambda+\mu}\left(\lambda+\mu+g\lambda^2k_{\{24\}}\right)\right]$$
$$+(1-q)\frac{\lambda}{2\lambda+\mu}\left(1+g\lambda k_{\{24\}}\right)\left(\lambda+\mu+g\lambda^2k_{\{24\}}\right)\le 0, \tag{53}$$

$$\mathrm{SOC}_1=-2k_{\{12\}}\lambda-q\left(\mu+\psi\right)$$
$$-(1-q)\left[g_1k_{\{22\}}\lambda+q\mu\left(1+g_1\right)+\mu(1-q)\left(1+g_1+k_t\right)\left(1+\bar{k}_s\right)\right]$$
$$-(1-q)^2\bar{k}_s\lambda\left[1+g_1+g\lambda k_{\{24\}}k_t+g\lambda\left(k_{\{22\}}-k_{\{12\}}\right)\right] \tag{54}$$
$$-(1-q)^2k_t\lambda\left[1+\bar{k}_s+g\lambda\left(k_{\{22\}}-k_{\{12\}}\right)\right]<0,$$

$$\bar{k}_s=-\frac{1}{2\lambda+\mu}\left[\left(\lambda+\mu\right)+g\lambda^2\left(k_{\{22\}}-k_{\{12\}}\right)\right],$$

$$\bar{k}_t=\bar{k}_s-\frac{\left(\lambda+\mu\right)g_1+g_1g\lambda^2k_{\{24\}}}{2\lambda+\mu}.$$

The optimal trades in periods 2 and 1 satisfy

$$\left(D_0-E\left[P_2\mid\mathcal{F}_2\right]-k_{\{24\}}\lambda X_2\right)-(1-q)\lambda\left(1+g\lambda k_{\{24\}}\right)E\left[X_3\mid\mathcal{F}_2\right]$$
$$-\mu q\left(X_0+X_1+X_2\right)-q\psi X_2-\mu(1-q)\left(X_0+X_1+X_2+E\left[X_3\mid\mathcal{F}_2\right]\right) \tag{55}$$
$$=0,$$

$$\begin{aligned}0 &= \left(D_0 - E\left[P_1 \mid \mathcal{F}_1\right] - k_{\{12\}}\lambda X_1\right) - q\mu\left(X_0 + X_1\right) - q\psi X_1 \\ &-(1-q)\left[k_{\{22\}}\lambda E\left[X_2 \mid \mathcal{F}_1\right] + q\left(X_0 + X_1 + E\left[X_2 \mid \mathcal{F}_1\right]\right)\right] \\ &-(1-q)^2\left\{\mu(1+k_s)\left(X_0 + X_1 + E\left[X_2 + E\left[X_3 \mid \mathcal{F}_2\right] \mid \mathcal{F}_1\right]\right)\right\} \\ &+(1-q)^2\left\{-\lambda\left[1+k_s+g\lambda\left(k_{\{22\}} - k_{\{12\}}\right)\right]E\left[X_3 \mid \mathcal{F}_1\right] + k_s\left(D_0 - E\left[P_3 \mid \mathcal{F}_1\right]\right)\right\}.\end{aligned} \quad (56)$$

Similar to the case without noise trading, we obtain the optimal solutions from the FOCs of the liquidity provider, the strategic trader, and the market clearing conditions. We solve the relevant equations numerically for concrete results. We consider the case in which the endowment of the liquidity provider is larger. In **Figure A1**, we set $Y_0 = 1$, $X_0 = 0.01$, $\gamma = \gamma_s = 1$, and $\sigma_D^2 = 1$. This case corresponds to Case 1 without noise traders. We obtain that the unconditional expected returns (the price changes), $E\left[\left(P_2 - P_1\right)\right]$, $E\left[\left(P_3 - P_2\right)\right]$, and $E\left[\left(D - P_3\right)\right]$, are all positive. Specifically, the expected prices are predictable, and the buying by the strategic trader leads to positive expected stock returns in the future. We can generate all other results obtained in Cases 2 - 4 in the absence of noise traders.

In sum, our results are robust with respect to the introduction of noise traders.

B. Equilibrium with Competitive Traders

We here consider the case in which there are a continuum of competitive traders instead of a strategic trader. We normalize the number of the competitive traders to be 1. The purpose is to show that under this competitive equilibrium, the

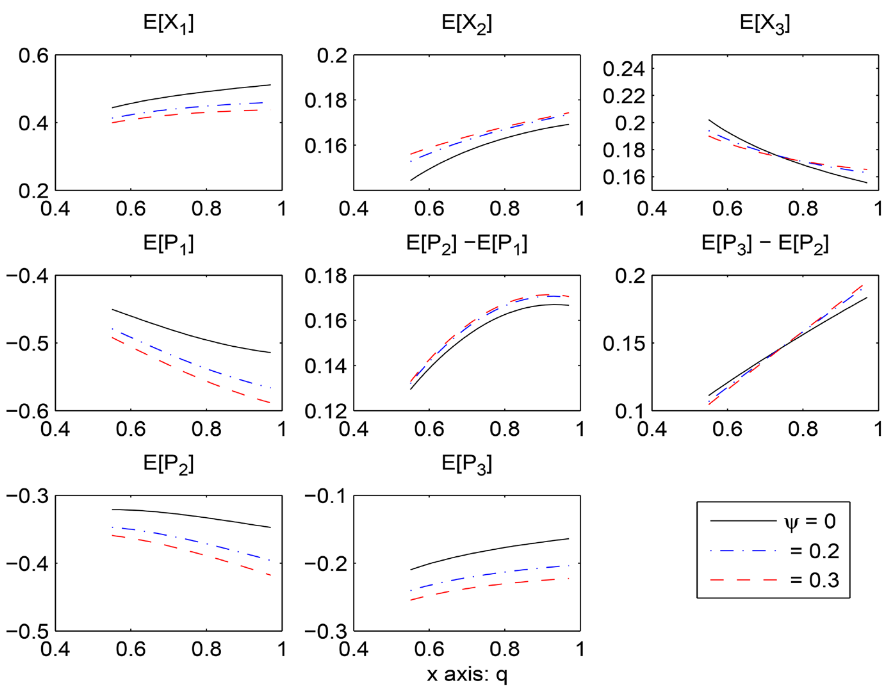

Figure A1. Price dynamics with noise trading ($D_0 = 0$, $Y_0 = 1$, $X_0 = 0$, $\lambda = 1$, $\mu = 0.01$, and $g = 0$).

no-trade theorem of [13] still holds, even with a positive probability q that the stock payoff will be revealed perfectly in each period. This exercise highlights that strategic trading is essential to generate sustained trading under symmetric information.

We take the linear pricing functions as in the strategic equilibrium. The market clearing conditions and the liquidity provider's maximization problems remain the same. In the current competitive equilibrium, the trades by the competitive trader do not affect the pricing directly, that is, when the competitive trader solves his optimal stock demand, he takes the prices as given. Because the solution techniques are essentially the same as in the strategic equilibrium, we omit them here.

The FOCs of the competitive trader yield

$$\begin{aligned}
&X_3 = \frac{\left(D_0 - P_3\right)}{\gamma_s \sigma_D^2} - \left(X_0 + X_1 + X_2\right),\\
&X_2 = \frac{\left[(1-q)\left(P_3 - P_2\right) + q\left(D_0 - P_2\right)\right]}{q\gamma_s \sigma_D^2} - \left(X_0 + X_1\right), \quad q > 0,\\
&X_1 = \frac{\left[(1-q)\left(P_2 - P_1\right) + q\left(D_0 - P_1\right)\right]}{q\gamma_s \sigma_D^2} - X_0, \quad q > 0,\\
&P_3 = P_2 = P_1, \quad q > 0.
\end{aligned} \tag{57}$$

Combining the FOCs and the market clearing conditions yields

$$\begin{aligned}
&\gamma_a \equiv \frac{\gamma\gamma_s}{\gamma + \gamma_s},\\
&\frac{\left(D_0 - P_3\right)}{\sigma_D^2} = \gamma_a\left(X_0 + Y_0\right),\\
&\frac{\left[(1-q)\left(P_3 - P_2\right) + q\left(D_0 - P_2\right)\right]}{q\sigma_D^2} = \gamma_a\left(X_0 + Y_0\right),\\
&\frac{\left[(1-q)\left(P_2 - P_1\right) + q\left(D_0 - P_1\right)\right]}{q\sigma_D^2} = \gamma_a\left(X_0 + Y_0\right),
\end{aligned}$$

where γ_a represents the representative investor's risk aversion coefficient. Simple calculations give

$$P_3 = P_2 = P_1 = \gamma_a \sigma_D^2\left(X_0 + Y_0\right). \tag{58}$$

Consequently, we arrive at

$$\begin{aligned}
&X_1 = \frac{\gamma_a}{\gamma_s}\left(X_0 + Y_0\right) - X_0, \quad Y_1 = \frac{\gamma_a}{\gamma_s}\left(X_0 + Y_0\right) - Y_0,\\
&X_2 = X_3 = Y_2 = Y_3 = 0.
\end{aligned} \tag{59}$$

In sum, we have shown that in this competitive equilibrium, the liquidity provider and the competitive trader achieve optimal risk sharing after only one round of trades. This result is consistent with the no-trade theorem of [13].