

The Valuation of Corruption

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Abstract

Unlike the current measures in the literature, where corruption is constructed as an index, this paper provides a formula for quantifying corruption. By using option pricing techniques, the paper shows that the monetary value of a corrupt activity is equivalent to a regular bond and an embedded European call option. This formula is very important because it could be used to gauge the level resources lost to corrupt activities, and also to determine the level of “tax” that could be levied at corrupt-government officials. Results in the paper show that a government committed to reducing corruption should institute measures that will reduce the level and the volatility of the price of the goods in the parallel markets. The paper also finds that a government could reduce corruption by cutting interest rates, which would spur growth and render corruption as an unprofitable exercise.

Keywords

Corruption, Option Pricing

1. Introduction

Despite the long history of corruption, modern research on economics of corruption began about 40 years ago with the work of [1]. Since then there has been a slew of articles written on the subject. One could also argue that origins of the body of work on corruption can be traced to the seminal paper of [2] on rent-seeking and its impact on economic activities. As [3] suggests, this body of work has three main features: cross country comparisons, data on corruption is derived from perception indices, and corruption is explained as a consequence of a country’s institutionalized policy environment. Although the literature has provided us with valuable insights on the subject, it has drawbacks. First, the use of the corruption index is inappropriate since it is fraught with perception bias, as perceptions of corruption change from country to country and from culture to culture. Second, the literature fails to provide us with a quantitative

measure of corruption. A quantitative measure is not only important for researchers but necessary for planners and policy makers in determining the size of resources lost to corrupt practices of government agents.

The purpose of this paper is to provide a method of measuring the size of corruption. This is done by providing a formula for pricing a corrupt activity. This is a departure from the literature where the focus of research is on the aggregate determinants of corruption. Corruption is an activity that is done in secrecy and hard to detect. However, in most developing countries, there are “secret” markets for most of the government’s goods and services that are difficult to obtain through the normal channels. These markets are run by government officials or their proxies. Although the official prices of these goods are known and set by the government, the prices of the goods and services in these markets are determined by the government official based on the maximum rent that could be extracted. The price in the secret market could be higher or lower than the official set price, depending on the demand for the goods or services.

The question asked in the paper is: in a corrupt environment, what is the price a corrupt government official must pay for the right to sell a government goods or services at a future date? This question is very important because in a country where it is very difficult for a government to curb corruption, it could impose a price or a special “tax” on its corrupt officials. The special taxes collected by the government could then be used to provide public goods for the citizens, or under some circumstance returned to those who purchased the goods or services. In the spirit of Coase theorem, the significant contribution of this paper is an attempt to find a market solution to the problem of corruption. The theoretical basis of the paper is to provide a pricing formula to value a corrupt activity. It draws on the option pricing literature by considering a corrupt activity as a European call option.

In this paper, corruption is defined as “the use of public office for private gains, where an official (the agent) entrusted with carrying out a task by the public (the principal) engages in some sort of malfeasance for private enrichment which is difficult to monitor for the principal” ([4], p. 1321). Despite this definition, corruption could mean different things in different contexts. For example, there can be political corruption where the ill-gotten gains are in terms of political power. Corruption could also occur in the private sector. Instead of allowing the market forces to operate, a seller could ration the supply of scarce goods and demand bribes from those who are in need of the goods (for example, paying a higher price to a “scalper” for a sold-out basketball game featuring Lebron James). It must be noted that this paper does not seek to answer the distortionary effects of bribery on resource allocation neither does it answer the question of the impact of corruption on economic development.

This paper also assumes that the punishment for detecting corruption is negligible since there are a lot of corrupt officials. [5] has a model that demonstrates the expected punishment for corruption when detected declines as more officials become corrupt. This is because it is preferable to be discovered by a corrupt superior than a non-corrupt one. Furthermore, our model implies that the potential bribers’ demand for corrupt

services decreases as the bribe size increases.

This paper is organised as follows. Section 2 presents a brief review of issues related to corruption. Section 3 lays out a model for pricing a corruption activity. Concluding remarks are made in Section 4.

2. A Brief Review of the Issues¹

Corruption, or the perception of corruption, has been observed to have a pervasive impact on a nation's economic development. [4] reports that corruption is not a new phenomenon. Bardhan points out that in the fourth century B.C, then prime minister of an Indian King, Kautiliya, writes in his book *Arthasastra*:

Just as it is impossible not to taste the honey (or the poison) that finds itself at the tip of the tongue, so it is impossible for a government servant not to eat up, at least, a bit of the king's revenue. Just as fish moving under water cannot possibly be found out either as drinking or not drinking water, so government servants employed in the government work cannot be found out (while) taking money (for themselves) ([4], p 1320).

Dante and Shakespeare also give prominence to the negative effects of corruption in their works and in the Bible, Jesus Christ condemns tax collectors for their corrupt practices (I Timothy 4:5).

In the literature, corruption is observed to grow out government regulations and controls, as government officials embark on rent-seeking activities which in turn slows down economic growth ([4] [6]-[13] among others). In most developing countries, government authorization is required for the opening of factories, for exportation and importation of goods and services and others. The procurement of the authorization notes or permits may involve different government agencies and several government officials. As pointed out by [11], the presence of regulation and authorizations gives monopoly power to government officials. These officials tend to use this power to extract bribes from those who need government authorization or permits.

Trade restrictions could also foster corruption. Some countries impose import restriction on certain goods so as to protect home industry. However, the quantity restriction on the flow of foreign goods makes import licences for the restricted goods very valuable and therefore attracts bribes from importers. On the other hand home producers of the good could also corrupt influential lawmakers to maintain such restriction even if it is through the imposition of tariffs.

Another source of corruption in some countries is the tax system. [11] indicates that corruption tend to be higher in countries where, the tax code is complicated, or the settlement of taxes require frequent meetings between the taxpayer and tax official, or the wages of tax officials are low, or there is a lack of transparency, or lack or mild penalties for corrupt official who are discovered.

Price controls, differential exchange rates, exchange rate controls and lower paid civil servants are part of the contributing factors of corruption.² Keeping the price of gov-

¹Our review of the literature is heavily influenced by [4] [14] [15] [16].

²For an elaborate discussion of factors affecting corruption see [11].

ernment goods and services below their market value allows government officials to engage in rent-seeking activities. Corruption grows in countries with differential exchange rates—one for importers, one for exporters, one for multinationals, one for tourists, one for investors, etc.—because individuals or firms would pay a bribe to corrupt official to get the “best” rate. In countries where public servants are paid significantly lower than their counterparts in the private sector and sanctions are non-existent for corrupt officials, public servants tend to take bribes in performing their duties.

Corruption has been observed to impact negatively on economic growth ([4] [6] [8] [9] [17] [18] [19] and others). This is because corruption distorts incentives and market signals leading to productive resources, such as human talent, to be diverted into rent-seeking activities. Financial rewards gained from corruption act as an incentive to lure the more talented and better educated to pursue rent-seeking activities instead of productive engagements, which in turn impact negatively on economic growth. Corruption impacts adversely on investment and growth because the payment of bribes by investors to secure investment licenses increases the cost of doing business and consequently reduces the incentive to invest. [4] points out that the tax code in some countries allow for losses to be deducted from taxable investment income, while losses from the payment of bribes can not be offset, making bribery harmful for risk-taking in the context of innovation. [20] finds corruption to be more distortionary than taxes. Wei argues that because of the secrecy and arbitrariness of corruption, the implicit contract between the briber and bribe cannot be enforced in a proper court of law. [8] shows theoretically that countries with a more disorganised pattern of corruption are not attractive for investors. Holding the level of corruption constant, [20] empirically finds countries with disorganized corruption structure receive far less direct foreign investment.³

Corruption has negative effects on productivity enhancing growth. For example, computers meant for the work place that are diverted for personal use by government officials, in some countries, do affect output growth negatively. Corruption also cut into profit margins of productive investments relative to rent seeking investments. As pointed out by [21], there are increasing returns to rent-seeking, as a rise in rent-seeking activity reduces the cost of further rent-seeking relative to that of productive investment. It is generally known that a fall in the returns to productive investment have a negative impact on growth. However, corruption could also induce an “accelerator effect” on growth. The fall in the return to productive investment could in turn raise the returns to rent-seeking activities, which in turn would slow down growth. [22] has indicated that corruption acts as a tax on profits and could hinder the entry of new goods or technology to the market. This is because innovators or new producers need permits and licenses which are obtained by paying hefty bribes.

Extending the corruption literature to the open economy, [17] find that the correlation between corruption and growth in a closed economy to be positive but negative in an open economy. The authors’ analysis is founded on the observation that: richer

³[20] measure of corruption structure is based on the dispersion of corruption ratings by survey respondents.

countries are less corrupt, while corrupt economies are poor. Based on this observation ([17], p. 2) pose the following puzzle: “If poorer countries do indeed experience higher levels of corruption, and if indeed as suggested by a number of empirical studies ([19] and others), corruption hampers growth, then how did rich countries, who were poor once, become rich?” [17] provides an answer to the puzzle by conjecturing that the western economies of the 19th century were relatively closed economies and that the gains of corruption remained in the country as part of the productive capital. However, because of the inter-connectedness of current economies, the rewards of corruption are dispatched to foreign bank accounts, robbing the country of productive capital. The implication of this analysis is that poor developing countries should be slow at becoming global partners.

Corruption also affects economic efficiency. In the literature there are proponents who argue that corruption is the oil needed to grease the economic engine of an over-regulated economy. [23] argues that corruption could improve economic efficiency by correcting for the ills of bad government regulation. [24] buttresses this view by pointing out that, in a rigid and over-centralized environment, it is better to have a dishonest bureaucracy than an honest one. Using Coase’s analysis, [4] argues that allocation efficiency is achieved in a competitive bidding by private firms for a government procurement contract and the corrupt government official awards the contract to the highest bidder in bribes. In this example, allocation efficiency is unaffected even though the producer surplus goes to the corrupt official.⁴

Another efficiency argument in favour of corruption in some developing countries is to consider corruption as “speed money.” A payment of bribe to a government official could hasten the movement of files in a bureaucracy. Corrupt officials take advantage of slow grinding bureaucracy by varying the size of bribes in accordance to the time preferences of the clients. The argument here is that in a country with bad and heavy regulations, the use of “speed money” to circumvent bad government control is like deregulation and therefore a good thing. Note that in some cases government regulations could require frequent contacts between government officials and citizens. Citizens may have to spend a lot of time in acquiring government licences. To cut down the processing time, citizens could pay bribes to officials and thus economic efficiency is achieved. Theoretically, [25] has used queuing models to show that “speed money” reduces the inefficiency in public administration. [28] finds corruption to be efficiency improving in a country with bad regulation and lack of punishment for corrupt officials. [11] points out the growth of international trade and business may have encouraged the payment of bribes by multinational companies in exchange for profitable contracts over competitors.

The efficiency-enhancing view of corruption is not shared by all in the literature. As [4] points out, corruption does not always remove distortions created in an economic environment. [4]’s argument is that distortions are not exogenous to the economic en-

⁴In a repeated game with incomplete information, [26] [27] have also shown a similar outcome where bribery produces efficiency in competitive bidding procedures.

vironment and are part of built-in corrupt practices. Inefficient allocation outcomes may be attained in the bidding exercise if the bribee is influenced by other factors than the size of the bribe or the briber could get away with supplying low-quality good at high-quality price. With respect to corruption as the oil for a “squeaky” economic engine, anecdotal evidence points to countries where corrupt officials slows down the wheels of bureaucracy in order to attract more bribes. Theoretically, there are no efficiency gains in an environment of asymmetric information as the briber and bribee may not have an agreement on the size of the bribe. Moreover, because of its illegality, corruption agreements are not protected by the courts of law, making it easier for the bribee to renege on the contract. Corruption also fails to achieve economic efficiency when the procurement of the government good or service requires the approval of several government officials. [29] notes that corrupt government officials could, instead of speeding up, reduce the speed of processing files thereby attracting more bribes. [8] also points out that corruption aggravates distortions in the economy because of its secrecy and resources spent by government officials to prevent detection and punishment. [30] contends that existing practices in more corrupt societies tend to encourage rent-seeking activities. Thus, the beneficiaries of corrupt practices are the most successful at rent-seeking, and not necessarily the most economically efficient.

Empirically, the literature has also investigated the effects of corruption on economic growth. Using data from some developed and developing countries, [19] finds that corruption has a negative and significant impact on economic growth, with decreased investment in physical capital responsible for most of the negative effect. Based on a cross-sectional dataset for a number of countries, [31] finds that corruption decreases private investment and education, and increases political instability, all combining to push down economic growth. [11] and [32] find that higher corruption is associated with lower quality of public infrastructure and decreasing private investment. Using a measure of corruption-induced uncertainty, [33] finds that the uncertainty associated with corruption reduces foreign direct investment. [34] finds that corruption increases income inequality and poverty, distorts the progressivity of the tax system and stifle the formation of human capital and above all reduces growth. [9] examines the impact of corruption in a number of African countries and concludes that corruption decreases economic growth and that increased corruption leads to increased income inequality. [35] find corrupt governments to receive more foreign aid under some circumstances.

Examining the effects of corruption on income and the Gini coefficient of income distribution, [36] find a non linear relationship between corruption and the Gini coefficient; positive relationship between the Gini coefficient and corruption in countries with intermediate level of corruption, and negative relationship for countries with high or low levels of corruption. [37] finds that reductions in the level of corruption results in significant social welfare as proxied by child and infant mortality rates, percentage of low-birth weight babies and primary school dropout rates. [38] also finds corruption to be significantly associated with higher military spending in about 120 countries.

In the empirical studies cited above, the corruption variable is based on a qualitative measure rather than a quantitative one. The qualitative measure or corruption index is constructed from either published reports on corruption, such as newspapers or the internet, or questionnaire-based surveys. A popular index used by researcher and the business community is the Transparency International index.⁵ This index measures the perception of corruption on a scale of 0 to 10, with 0 indicating that most transactions or relations in the country are tainted by corruption and 10 indicates the country is free of corruption.

Despite the usefulness of the corruption index it has some drawbacks. First, it measures perceptions of corruption and not the size of corruption. This imposes a potential problem as the index may be fraught with perception bias, as perceptions of corruption change from country to country and from culture to culture. Second, the index is not helpful to governments or economic planner who would like to know the resource cost of corruption as the index fails to provide one. In the next section we would attempt to provide a method of measuring the size of corruption. This is done by providing a formula for pricing a corrupt activity.

3. Valuation of Corruption

Corruption is a secret activity that is very difficult to detect in some countries. Even when discovered, the monetary value of the transaction tends to be elusive. In some countries around the world, “secret” or unofficial markets exist for most of the government produced goods and services that are difficult to obtain through the normal process. These markets, which are run by government officials or their proxies, exist because government officials restrict the quantity of the goods or services sold in the “normal” markets. Furthermore, although the official price of the good is set by the government, the price in the unofficial markets fluctuates, depending on the maximum rent that could be extracted by the government official. With this observation as the back drop, we attempt to provide a model for valuing corruption. The focus of this section is to answer the question: in a corrupt environment, what is the maximum rent a corrupt government official could extract from the sale of the good or service at a future date?

In this paper, we consider a homogenous government produced goods. Following [8] the paper assumes that the government’s agent has the opportunity to restrict the quantity of the good sold.⁶ We also assume that corruption can not be detected or punished because of the corrupt environment. This is a reasonable assumption given that in many corrupt nations, corrupt officials incur no penalties when caught because their bosses share in the “spoils” and the law is blind. Furthermore, the total cost of producing the good is assumed to be borne by only the government and is of no concern to the corrupt official.

A corrupt official is faced with two options when he or she decides to sell the gov-

⁵The website for the Transparency Index is: www.transparency.com.

⁶The restriction could take the form of the government official refusing to issue a passport, or deny access to a road or place a lot of impediments or requirements in the way of a purchaser of the goods.

ernment good. The official could decide to sell the good by a multiple of the government price and then pocket proceeds without accounting for it. This first option is what [8] refer to as *corruption with theft*. The official has a second option, where he/she sells the good at a price (government price plus bribe) prevailing in the secret market and truthfully accounts to the government the quantity of goods or services sold at the government price. Under the second option, the government official keeps the difference between the secret market price and government set price and returns the rest to the government. [8] refers to the second option as *corruption without theft*.

Let K be the government set price of the good, and P be the price (government price plus bribe) of the good in the secret market. The options facing the government official can then be summarized as:

$$\text{Max}[\alpha K, P - K] \quad (1)$$

where α is a positive number. The first part of Equation (1) summarises the first option while the second part capture the second option. What Equation (1) suggests is that a rational corrupt official will choose which ever options give the maximum return. Equation (1) could then be rewritten as:

$$\alpha K + \text{Max}[0, P - (1 + \alpha)K] \quad (2)$$

If we assume that corruption activity will take place T period from now, then we can interpret the present value of the corrupt official's gains from the sale of a unit of government produced good or service as being equivalent to a bond with face value αK and a call option on the good, in the secret market, at an exercise price of $(1 + \alpha)K$. With this interpretation, we can proceed to use the option pricing literature to provide a formula for quantifying the value of a corruption activity.

In addition to the above assumption we shall make the usual assumptions for modeling continuous time asset pricing models: 1) in the secret market, the trading of the government good is traded continuously; and 2) the price of the good follows a continuous time diffusion process of the form:

$$\frac{dP}{P} = \alpha_p dt + \sigma_p dz_p \quad (3)$$

where α_p is the instantaneous average return of holding one unit of the good, σ_p is the instantaneous standard deviation of rate of change of the price and dz_p has a standard normal distribution with mean of zero and variance dt . Note that α_p and σ_p may be functions of P and t . However, for the purpose of this exercise we shall assume they are constants. Given that the government price is fixed, we must also mention that it is the size of the bribe that is responsible for the diffusion process.

3.1. Pricing a Corrupt Activity

Given the secret market price of the government produced good, P , let the value of the corrupt activity at any time be $C(P, t)$. Then applying Ito's lemma the drift and the diffusion of the value of the activity is given as:

$$dC = C_p dP + \frac{1}{2} C_{pp} (dP)^2 + C_t dt \tag{4}$$

which upon simplifying, yields:

$$\frac{dC}{C} = \alpha_c dt + \sigma_c dz_p \tag{5}$$

where

$$\alpha_c = \frac{\frac{1}{2} \sigma_p^2 P^2 C_{pp} + \alpha_p P C_p + C_t}{C} \tag{6}$$

$$\sigma_c = \frac{\sigma_p P C_p}{C} \tag{7}$$

Proposition 1: The partial differential equation governing the corruption activity is:

$$\frac{1}{2} \sigma_p^2 P^2 C_{pp} + r P C_p + C_t - r C = 0 \tag{8}$$

Proof: See Appendix.

Remarks: Intuitively, the differential equation governing the valuation of the corruption activity is explained as follows. The first term on the left hand side of Equation (8) captures the Jensen’s inequality effect coming from the variance of the price of the commodity. The second term represents the risk-adjusted expected drift of the price of the good. The third term reflects the shrinking time to maturity. The last term represent the net flows to the corrupt official. It is important to note that the drift term of the price of the good plays no role in the valuation of the corruption activity. However, the variance of the price of the goods plays an important role.

Proposition 2: Under the set of assumptions given earlier, the present value of the gains from corruption activity is:

$$C(P, \tau) = \alpha K e^{-r\tau} + E(P, K, \tau) \tag{9}$$

where $E(P, K, \tau)$ is an European call option to purchase the government produced good in the secret market at an exercise price of $(1 + \alpha)K$. The value of the call option is

$$E(P, K, \tau) = P N(d_1) - (1 + \alpha) K e^{-r\tau} N(d_2) \tag{10}$$

where

$$d_1 = \frac{\log\left(\frac{P}{(1 + \alpha)K}\right) + \left(r + \frac{1}{2} \sigma_p^2\right) \tau}{\sigma_p \sqrt{\tau}}$$

$$d_2 = d_1 - \sigma_p \sqrt{\tau}$$

and $N(.)$ is the cumulative normal distribution function.

Proof:

Based on Equation (2), it is shown that the present value of the corrupt official’s gains from the sale of a unit of government produced good or service is a equivalent to a bond with face value αK and a call option on the good, in the secret market, at an ex-

ercise price of $(1 + \alpha)K$. The first term of Equation (10) corresponds to the present value of a bond and the second term is the value of the call option. The call value option follows from [39]-[44]. Note that the differentiation of Equation (10) yields the stochastic partial differential (Equation (8)).

Remarks: Propositions (2) is very important on a number of reasons. First, it provides us with a method of placing a monetary value on a corruption activity. Second, for budgetary purposes, a government facing a pervasive level of corruption could use this formula to gauge the level resources lost to corruption activity. Third, the formula could be used to tax government officials for their corrupt activities. Fourth, the recognition of corruption activity as a European call option provides an analytical framework for improved understanding of the corruption literature. In the next section, we will examine how the underlying parameters of the model influence corrupt activities.

3.2. Properties of the Valuation Model

In this section, we attempt to learn more about the factors that influence corruption activity, and the implied policy implications.

Proposition 3: The value of corruption activities rises with the increase in the “secret” market price of the government produced goods.

Proof:

Differentiating Equation (10) with respect to P and using Equation (A16) in the appendix:

$$\frac{\partial C}{\partial P} = \frac{\partial E}{\partial P} = N(d_1) \geq 0 \quad (11)$$

Remarks: The results suggest that a strong demand for the good in the secret market would enhance the value of corrupt activities. The result corroborates the view that government officials could deliberately restrict the supply of goods and services so as to profit substantially from corruption activities.

On the other hand, a government interested in curtailing corruption could embark on policies that could contribute to the fall in the price of the good in the secret market. Such policies could include more government officials supplying the government produced goods. For example, the government could decentralize the issuing of passports by having more passport offices. Because of the difficulty for several government officials to collude, such a measure would reduce the amount of bribe and therefore the price the good in the “secret” market which in turn leads to a fall in monetary value of corruption.

Proposition 4: Corruption activities increases with the rise in the volatility of “secret” market price of the goods.

Proof:

Differentiating Equation (10) with respect to the variance of P , σ_p , and using Equation (A22) in the appendix:

$$\frac{\partial C}{\partial \sigma_p} = \frac{\partial E}{\partial \sigma_p} = \sqrt{\tau} PN'(d_1) \geq 0 \quad (12)$$

Remarks: The result demonstrates that corrupt government officials profit a great deal from increased volatility of the price of the good in the secret market. The increased volatility of the price increases the value of the call option implied in the value of the corrupt activity. An explanation for this is that a call option has no downside risk as the value of the call is zero irrespective of how far it finishes out of the money. Hence, an increase in the volatility of the price of the good goes to increase the chances that the call option will expire in the money. The rise in the volatility of the price of the good also increases the general uncertainty in the secret market which in turn enhances the gains from corruption.

To curb corruption, the government could embark on measures that will reduce the volatility of the price of the good in the secret market. One measure would be for the government allowing competition in the supply of the good. Such an action would not only reduce the level of bribe but also the volatility of the price in the secret market, and eventually decrease the incentive to engage in corrupt activities.

Proposition 5: A rise in the government set price, K , may or may not reduce the gains from corruption activity.

Proof:

Differentiating Equation (10) with respect to K and using Equation (A28) of the appendix:

$$\frac{\partial C}{\partial K} = \alpha e^{-rt} - (1 + \alpha) e^{-rt} N(d_1) \quad (13)$$

which is clearly indeterminate.

Remarks: As mentioned earlier a corrupt official has two options: corruption with theft and corruption without theft. Corruption with theft gives the implied “bond” feature to the gains from corruption and corruption without theft gives the embedded call value in the gains. Proposition (5) is explained by the fact that an increase in government set price leads to an increase in the value of the “bond” implied in the gains from corruption and a decrease in the value of the embedded call option. The value of the call falls because the rise in the government set price increases the probability that the call finishes out of the money. The net impact of the rise in the government set price on corruption activity depends on which effects dominate. In other words, if official’s accountability of the goods and services is weak then we may see corruption with theft rising. In which case the first term in Equation (13) will dominate the second term and therefore corruption activity will rise with the increase in K .

Corollary: A rise in the government set price, K , would reduce the monetary value of corruption if it is backed by tough accountability for all government-produced goods sold by officials.

Remarks: If the government mechanism of ensuring that all goods supplied to government officials are properly accounted for then officials would find it very difficult to carry out corruption with theft activities. Thus in Equation (13) α would be close to 0, and the impact of the rise in K on corruption activity will be negative. The policy implication of this result is that, in an environment of unstoppable levels of corruption, a

government could reduce corruption by raising its price of goods while maintaining strong accounting standards.

Proposition 6: The rise in the factor (α) by which the government official, engaged in corruption with theft, sells the good or service for leads to an increase in the gains from corruption.

Proof:

Differentiating Equation (10) with respect to α and using Equation (A34):

$$\frac{\partial B}{\partial \alpha} = Ke^{-rt}(1 - N(d_2)) \geq 0 \quad (14)$$

since $0 \leq N(d_1) \leq 1$.

Remarks: Following the explanation given for the remarks made for Proposition 5, an intuitive explanation for this result is that a rise in the parameter α , enhances the value of the implied bond and decreases the embedded call value. However, in this case, the net impact is positive because the bond dominates the call. Although corruption without theft falls, the implication of the results is that the rise in the parameter α raises the rewards to corruption because of the increase in corruption with theft.

Proposition 7: Interest rates have indeterminate effects on the gains from corrupt activities.

Proof:

Differentiating Equation (10) with respect to the interest rate, r , and making use of Equation (A40) yields:

$$\frac{\partial B}{\partial r} = \tau Ke^{-rt}((1 + \alpha)N(d_2) - \alpha) \quad (15)$$

Remarks: The result shows that rising interest rates have an ambiguous impact on corruption. Again, following the remarks for Proposition 5, the rise in interest rates reduces the value of the implied bond in the gains of corruption activity, discouraging corruption with theft. At the same time the rewards from corruption without theft rises since the exercise price on the embedded option reduces. The impact on corruption activity depends on which effect dominates. Intuitively, interest rate increases tends to slowdown the economy and consequently raise the level of unemployment. Using the efficiency wage argument, increased unemployment pool would serve as a deterrent to officials who would want to keep their jobs from engaging in corruption with theft for fear of being fired. If that is the case then α would be close to zero and therefore the impact of the rise in interest rate on corruption activity would be positive.

Corollary: In a country with strict accounting standards for all government-produced goods sold by officials, a rise in the interest rate would result in higher corruption.

Remarks: A country with strict accounting standards suggests that α would be close to zero and therefore the sign for Equation (15) would be clearly positive. The monetary value of corruption activity rises because the embedded option rises in value as the rise in interest rate reduces the exercise price of the option. The implication of this result is that a government committed to a higher accounting standard could reduce cor-

ruption by cutting interest rates, which in turn spurs economic growth. The rise in economic growth would in turn curtail acts of corruption, since the acts would be less lucrative.

4. Conclusions

Corruption is a costly activity to a nation as it leads to the diversion of public resources for private use. It has the potential of shifting resources away from the productive sector of an economy. This paper has attempted to address the issue by providing a market based solution by using a financial model to quantify the value of corrupt activities. The approach taken in the paper is very innovative as it provides a solution for a country where it is very difficult for a government to curb corruption. The paper proposes that the government could impose a price or a special “tax” on its corrupt officials. The special taxes collected by the government could then be used to provide public goods for the citizens, or under some circumstance returned to those who purchased the goods or services.

Using option pricing techniques, the paper shows that the monetary value of a corrupt activity is equivalent to a regular bond and an embedded European call option. Unlike the current measures in the literature, where corruption is constructed as an index, this paper provides a formula for quantifying corruption. This formula is very important because it could be used by governments, private sectors and interested parties to gauge the level resources lost to corrupt activities. The formula could also be used to determine the level of tax that corrupt-government officials could be levied. Furthermore, the recognition of corruption as a European call option provides an analytical framework for improved understanding of the corruption literature.

Results in the paper show that a government committed to reducing corruption should institute measures that will reduce the level and the volatility of the price of the goods or services it provides in the parallel markets. The measures could include the government allowing for competition in the supply of the goods or services. Such an action would not only reduce the level of bribe but also the volatility of the price in the secret market and eventually a decrease in the incentive to engage in corrupt activities. The paper also finds that a government committed to a higher accounting standard could reduce corruption by cutting interest rates, which would spur economic growth and render corruption as an unprofitable exercise.

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Appendix

A.1. Proof of Proposition 1

To prove *Proposition 1*, we use standard arbitrage arguments common in the options pricing literature. Construct a hypothetical portfolio by investing ω in the government produced goods and $(1 - \omega)$ in corruption activity. If L is the value of the portfolio then the instantaneous return on portfolio, dL , is:

$$dL = \omega(\alpha_p dt + \sigma_p dz_p) + (1 - \omega)[\alpha_c dt + \sigma_c dz_c] \tag{A1}$$

or:

$$dL = (\omega\alpha_p + (1 - \omega)\alpha_c)dt + (\omega\sigma_p + (1 - \omega)\sigma_c)dz_p \tag{A2}$$

The portfolio, L , could be made riskless if ω is chosen to satisfy:

$$\omega\sigma_p + (1 - \omega)\sigma_c = 0 \tag{A3}$$

implying:

$$\omega = \frac{\sigma_c}{\sigma_c - \sigma_p} \tag{A4}$$

Holding $\frac{\sigma_c}{(\sigma_c - \sigma_p)}$ units of the government produced goods and in the corruption activity ensures that portfolio L is riskless. In the absence of arbitrage profits, the instantaneous return on the portfolio must be equal to that of a risk-free asset r . Or:

$$dL = r \left[\frac{\sigma_c}{\sigma_c - \sigma_p} - \frac{\sigma_p}{\sigma_c - \sigma_p} \right] dt \tag{A5}$$

Equating Equation (A5) to the drift term of Equation (A2) with the substitution of ω we have:

$$\left[\left(\frac{\sigma_c}{\sigma_c - \sigma_p} \right) \alpha_p - \left(\frac{\sigma_p}{\sigma_c - \sigma_p} \right) \alpha_c \right] = r \left[\frac{\sigma_c}{\sigma_c - \sigma_p} - \frac{\sigma_p}{\sigma_c - \sigma_p} \right] \tag{A6}$$

Substituting Equations (6) and (7) into Equation (A6) and re-arranging yields the differential equation governing the loan contract (Equation (8)).

A.2. Properties of the Call Option

1) The value of the call option

In the main text, an expression is derived for valuing a European call option as:

$$E(P, K, \tau) = PN(d_1) - (1 + \alpha)Ke^{-r\tau}N(d_2) \tag{A7}$$

where

$$d_1 = \frac{\log\left(\frac{P}{(1 + \alpha)K}\right) + \left(r + \frac{1}{2}\sigma_p^2\right)\tau}{\sigma_p\sqrt{\tau}} \tag{A8}$$

$$d_2 = d_1 - \sigma_p\sqrt{\tau} \tag{A9}$$

and $N(\cdot)$ is the cumulative normal distribution function.

The paper now turns to the sensitivity of the value of the call to the parameters.

2) The effect of the change of the “secret” market price on the call value

Differentiate the call with respect to the secret market price, P :

$$\frac{\partial E}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{\tau}} N'(d_1) - \frac{(1 + \alpha) K e^{-r\tau}}{P \sigma_p \sqrt{\tau}} N'(d_2) \tag{A10}$$

but

$$N'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2} \tag{A11}$$

thus:

$$\frac{\partial E}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} e^{-\frac{1}{2}d_1^2} - \frac{(1 + \alpha) K e^{-r\tau}}{P \sigma_p \sqrt{2\pi\tau}} e^{-\frac{1}{2}d_2^2} \tag{A12}$$

$$\frac{\partial E}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[e^{-\frac{1}{2}d_1^2} - e^{-\log\left(\frac{P}{(1+\alpha)K}\right) - r\tau - \frac{1}{2}d_2^2} \right] \tag{A13}$$

Substitute Equations (A2) and (A3) in the last part of Equation (A7):

$$\frac{\partial E}{\partial P} = N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[e^{-\frac{1}{2}d_1^2} - e^{-d_1\sigma_p\sqrt{\tau} + \frac{1}{2}\sigma_p^2 - \frac{1}{2}(d_1 - \sigma_p\sqrt{\tau})^2} \right] \tag{A14}$$

which simplifies into:

$$\frac{\partial E}{\partial P} = -N(d_1) + \frac{1}{\sigma_p \sqrt{2\pi\tau}} \left[e^{-\frac{1}{2}d_1^2} - e^{-\frac{1}{2}d_1^2} \right] \tag{A15}$$

hence:

$$\frac{\partial E}{\partial P} = N(d_1) \geq 0 \tag{A16}$$

3) The change of the variance of the “secret” market price on the call value

Differentiate the call price with respect to σ_p :

$$\frac{\partial E}{\sigma_p} = P N'(d_1) \left[\sqrt{\tau} - \frac{d_1}{\sigma_p} \right] - (1 + \alpha) K e^{-r\tau} N'(d_2) \left[-\frac{d_1}{\sigma_p} \right] \tag{A17}$$

or

$$\frac{\partial E}{\sigma_p} = \sqrt{\tau} P N'(d_1) - P \frac{d_1}{\sigma_p} \left[N'(d_1) - \frac{(1 + \alpha) K e^{-r\tau}}{P} N'(d_2) \right] \tag{A18}$$

Using Equation (A11),

$$\frac{\partial E}{\sigma_p} = \sqrt{\tau} P N'(d_1) - P \frac{d_1}{\sigma_p} \left[e^{-\frac{1}{2}d_1^2} - e^{-\log\left(\frac{P}{(1+\alpha)K}\right) - r\tau - \frac{1}{2}d_2^2} \right] \tag{A19}$$

substituting Equations (A8) and A(9),

$$\frac{\partial E}{\sigma_p} = \sqrt{\tau}PN'(d_1) - P \frac{d_1}{\sigma_p} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-d_1\sigma_p\sqrt{\tau} + \left(\frac{1}{2}\right)\tau\sigma_p^2 - \left(\frac{1}{2}\right)(d_1 - \sigma_p\sqrt{\tau})^2} \right] \tag{A20}$$

which simplifies to:

$$\frac{\partial E}{\sigma_p} = \sqrt{\tau}PN'(d_1) - P \frac{d_1}{\sigma_p} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A21}$$

hence,

$$\frac{\partial E}{\sigma_p} = \sqrt{\tau}PN'(d_1) \geq 0 \tag{A22}$$

4) The change of the government set price, K , on the call value

Differentiate the call value with respect to K :

$$\frac{\partial E}{\partial K} = PN'(d_1) \left[-\frac{1}{K\sigma_p\sqrt{\tau}} \right] - (1 + \alpha)e^{-r\tau}N(d_2) + \frac{(1 + \alpha)e^{-r\tau}N'(d_2)}{K\sigma_p\sqrt{\tau}} \tag{A23}$$

which could be expressed as:

$$\frac{\partial E}{\partial K} = -(1 + \alpha)e^{-r\tau}N(d_2) + \frac{P}{K\sigma_p\sqrt{\tau}} \left[\frac{(1 + \alpha)e^{-r\tau}N'(d_2)}{P} - N'(d_1) \right] \tag{A24}$$

manipulating further:

$$\frac{\partial E}{\partial K} = -(1 + \alpha)e^{-r\tau}N(d_2) + \frac{P}{K\sigma_p\sqrt{\tau}} \left[e^{-\log\left(\frac{P}{(1+\alpha)K}\right) - r\tau - \left(\frac{1}{2}\right)d_2^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A25}$$

which reduces to:

$$\frac{\partial E}{\partial K} = -(1 + \alpha)e^{-r\tau}N(d_2) + \frac{P}{K\sigma_p\sqrt{\tau}} \left[e^{-d_1\sigma_p\sqrt{\tau} + \left(\frac{1}{2}\right)\tau\sigma_p^2 - \left(\frac{1}{2}\right)(d_1 - \sigma_p\sqrt{\tau})^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A26}$$

or

$$\frac{\partial E}{\partial K} = -(1 + \alpha)e^{-r\tau}N(d_2) + \frac{P}{K\sigma_p\sqrt{\tau}} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A27}$$

hence

$$\frac{\partial E}{\partial K} = -(1 + \alpha)e^{-r\tau}N(d_2) \leq 0 \tag{A28}$$

5) The change of the parameter α on the call option value

Differentiate with respect to α :

$$\frac{\partial E}{\partial \alpha} = PN'(d_1) \left[-\frac{1}{(1 + \alpha)\sigma_p\sqrt{\tau}} \right] - Ke^{-r\tau}N(d_2) + \frac{1}{\sigma_p\sqrt{\tau}}N'(d_2) \tag{A29}$$

simplifying,

$$\frac{\partial E}{\partial \alpha} = -KN(d_2) + \frac{P}{(1 + \alpha)\sigma_p\sqrt{\tau}} \left[\frac{(1 + \alpha)Ke^{-r\tau}}{P}N'(d_2) - N'(d_1) \right] \tag{A30}$$

or,

$$\frac{\partial E}{\partial \alpha} = -Ke^{-r\tau}N(d_2) + \frac{P}{(1+\alpha)\sigma_p\sqrt{\tau}} \left[e^{-\log\left(\frac{P}{(1+\alpha)K}\right) - r\tau - \left(\frac{1}{2}\right)d_2^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A31}$$

which reduces to:

$$\frac{\partial E}{\partial \alpha} = -Ke^{-r\tau}N(d_2) + \frac{P}{(1+\alpha)\sigma_p\sqrt{\tau}} \left[e^{-d_1\sigma_p\sqrt{\tau} + \left(\frac{1}{2}\right)r\sigma_p^2 - \left(\frac{1}{2}\right)(d_1 - \sigma_p\sqrt{\tau})^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A32}$$

or

$$\frac{\partial E}{\partial \alpha} = -Ke^{-r\tau}N(d_2) + \frac{P}{(1+\alpha)\sigma_p\sqrt{\tau}} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A33}$$

hence

$$\frac{\partial E}{\partial \alpha} = -Ke^{-r\tau}N(d_2) \leq 0 \tag{A34}$$

6) The change of interest rate on the call option value

Differentiate with respect to r:

$$\frac{\partial E}{\partial r} = PN'(d_1) \left[\frac{\sqrt{\tau}}{\sigma_p} \right] + (1+\alpha)Ke^{-r\tau}N(d_2) - (1+\alpha)Ke^{-r\tau}N'(d_2) \left[\frac{\sqrt{\tau}}{\sigma_p} \right] \tag{A35}$$

rearranging:

$$\frac{\partial E}{\partial r} = (1+\alpha)Ke^{-r\tau}N(d_2) - \frac{P\sqrt{\tau}}{\sigma_p} \left[N'(d_1) - \frac{(1+\alpha)Ke^{-r\tau}}{P} N'(d_2) \right] \tag{A36}$$

or:

$$\frac{\partial E}{\partial r} = (1+\alpha)Ke^{-r\tau}N(d_2) - \frac{P\sqrt{\tau}}{\sigma_p} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-\log\left(\frac{P}{(1+\alpha)K}\right) - r\tau - \left(\frac{1}{2}\right)d_2^2} \right] \tag{A37}$$

which reduces to:

$$\frac{\partial E}{\partial r} = (1+\alpha)Ke^{-r\tau}N(d_2) - \frac{P\sqrt{\tau}}{\sigma_p} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-d_1\sigma_p\sqrt{\tau} + \left(\frac{1}{2}\right)r\sigma_p^2 - \left(\frac{1}{2}\right)(d_1 - \sigma_p\sqrt{\tau})^2} \right] \tag{A38}$$

or:

$$\frac{\partial E}{\partial r} = (1+\alpha)Ke^{-r\tau}N(d_2) - \frac{P\sqrt{\tau}}{\sigma_p} \left[e^{-\left(\frac{1}{2}\right)d_1^2} - e^{-\left(\frac{1}{2}\right)d_1^2} \right] \tag{A39}$$

hence:

$$\frac{\partial E}{\partial r} = (1+\alpha)Ke^{-r\tau}N(d_2) \geq 0 \tag{A40}$$



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