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Impacts of Internal Financing on Investment Decisions by Managers with Cognition Biases

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Abstract

This paper extends the standard model of real option by allowing managerial biases originating from the cognition about the economic states in the future to examine the impacts of internal liquid funds on the firm's investment decisions. The model indicates that the interactions between the cognition biases and firm's volatility determine the extent to which the manager overestimates the growth rate, hence subjectively overvalues the firm's value and distorts the investment decisions. The managerial cognition biases enhance the sensitivity of corporate investment to cash flow and increase the convexity of the investment decisions against the firm's volatility. Relying on the amount of internal liquidity, the manager with cognition biases can both delay and expedite investing.

Keywords

Cognition Biases, Internal Financing, Real Option, Investment, Behavioral Finance, Subjective Evaluation

1. Introduction

Many financial economists have exerted great efforts to studying the interactions between corporate investment and financing strategies by assuming that the firm's managers are fully rational, largely ignoring common personality traits of managers in modeling the complex decision-making process of corporate executives. However, at the present time, a lot of studies about human behavior find that managers are inclined to display irrational characteristics, such as time-inconsistent preferences, optimism, overconfidence and over-extrapo-

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lative biases, and so on[1]. In fact, all of these characteristics can be generalized as the managerial cognition biases about the economic states. Besides, to study how the firm's internal capital impacts its investment decisions is another important topic. Thus, in this paper, based on the standard theories of real options, we develop a continuous-time model with managerial biases originating from the manager's cognition about the economic states in the future to investigate the impacts of internal liquid funds on the firm's investment decisions.

The cognition biases capturing the manager's characteristics also can be interpreted as a bias of ability to derive information from the real world. We consequently look at what happens inside the firm when the manager is rational all respects, except for his cognition associated with the economic states. Interestingly, our model shows that it is the interaction between the biased cognition of the manager and the firm's volatility determines the extend to which the manager who has biased cognition about the economic states perceives the firm's growth rate. In contrast, in the literature studying optimism and overconfidence, such as [4]-[6], overconfidence associated with the firm's volatility has no impacts on the growth rate from manager's perspective. In fact, in the existing literature, an overconfident agent is defined to undervalue the riskiness of the firm in the future, for instance [7]-[9], and the optimistic one overestimate the growth of the firm, for example, [4]-[6].

By assuming the manager has biased cognition on the economic states, our paper finds that the corresponding biases enhance the sensitivity of corporate investment to cash flow. Meanwhile, managerial cognition biases result in distortion of the firm's investment. Specifically, the manager with biased cognition tends to delay investment if the firm just has few internal funds, but is inclined to accelerate the investment timing if the firm has sufficient internal liquidity. In addition, differing from the studies about overconfidence, the interaction between the managerial biased cognition and the firm's volatility imposes effects on the growth rate of the firm in the future from the managerial point of view and hence gives rise to that the managerial cognition biases increase the convexity of the investment decision against the firm's volatility.

This paper relates to several strands of literature. Firstly, by incorporating the managerial cognition biases and indicating the corresponding effects on firms' investment decisions, it extends the study on real options developed by [10]-[12]. Additionally, based on the real option framework, [4] [5] investigate investment and financing strategies undertaken by optimistic and overconfident managers and find that these managerial characteristics induce the manager to invest early, mitigating the agency cost of debt caused by debt-overhang. By employing the definitions of optimism and overconfidence in [4]-[6] looks at the impacts of internal financing on investment decisions undertaken by an optimistic and overconfident manager.

Secondly, our paper continues the line of research about the effects of internal liquidity on firm's investment decisions. [13] considers a model in which the firm possibly encounters financing constraints arising from capital market frictions and examines the influence of the amount of internal liquidity on firm's investment strategies. [14] extends the model in [13] by incorporating costly external financing.

Finally, our paper is related to the studies about subjective evolution, such as [15]-[18]. [15] provides conditions for the existence of efficient self-enforcing contracts when the principal and agent have the same beliefs regarding a subjective evaluation. [16] extends the standard principal-agent model to allow for subjective evaluation and consequently looks for the optimal payout policies. [17] develops a model by allowing for imperfect subjective evaluation on the part of principal and derives an optimal contract. [18] presents a dynamic contracting model in which the principal and agent disagree about the resolution of uncertainty. In our paper, we focus on the impacts of the subjective evaluation on firm's investment by ruling out the agency problem for simplicity.

The rest of the paper is organized as follows. Section 2 describes the setup of the model with managerial cognition biases. Section 3 provides solutions for the model. Implication analysis and numerical results are given in Section 4. Finally, Section 5 concludes the whole paper. All Proofs are presented in the **Appendix** section.

2. The Model

In this section, we develop a continuous-time model with managerial cognition biases associated with the cash flows of the firm in the future. Firstly, we assume that the firm has an investment project and some internal funds at the initial time 0. The investment perpetually generates cash flows X_t at any time t after it is undertaken with irrecoverable investment costs I. The cash flows X_t follow a geometric Brownian motion:

[1]There is a large body of literature studying time-inconsistent preferences, such as [1]-[3].

$$\frac{\mathrm{d}X_t}{X_t} = \mu \mathrm{d}t + \sigma \mathrm{d}Z_t^{\mathbb{R}}, \tag{1}$$

where μ is the instantaneous growth rate of the cash flows, constant σ is the instantaneous standard deviation, $Z_t^{\mathbb{R}}$ is a standard Brownian motion on a complete probability space $(\Omega, \mathcal{B}, \mathbb{R})$. In order to distinguish it from the other probability measure used to capture the managerial cognition biases about the firm's cash flows in the future, we call this probability measure $\mathbb{R}$ as the physical probability measure. Namely, the complete probability space $(\Omega, \mathcal{B}, \mathbb{R})$ represents the real world, in which the firm's cash flows X_t in the future evolves according to Equation (1). Next, we introduce another probability space to characterize the managerial subjective world, where the manager has biased cognition on the firm's cash flows X_t.

In addition to the real probability space $(\Omega, \mathcal{B}, \mathbb{R})$, we assume that from the managerial point of view, the probability space is $(\Omega, \mathcal{B}, \mathbb{A})$. Thus, the future cash flow X_t has another dynamics and evolves according to

$$\frac{\mathrm{d}X_t}{X_t} = \mu \mathrm{d}t + \sigma \mathrm{d}Z_t^{\mathbb{A}}. \tag{2}$$

Comparing Equation (1) with (2) indicates that the manager can accurately predict the firm's growth rate μ and volatility σ, but he can not completely forecast what will happen in the future because $Z^{\mathbb{A}}$ does not need to equal $Z^{\mathbb{R}}$. In other words, the manager has ability to precisely estimate the growth rate and volatility via various instruments, but has cognition biases on the economic states in the future. In the real world, $Z^{\mathbb{R}}$ may be upward, but the manager perceives that it has the opposite direction or they has different magnitude of change. In fact, $\mathrm{d}Z^{\mathbb{R}}$ in Equation (1) and $\mathrm{d}Z^{\mathbb{A}}$ in Equation (2) are able to characterize these differences.

Conceptionally speaking, probability measures $\mathbb{R}$ and $\mathbb{A}$ are all mutually absolutely continuous and agree on zero-probability events. In order to measure the cognition biases, we let $\xi_t = (\mathrm{d}\mathbb{R}/\mathrm{d}\mathbb{A})$ denote the Radon-Nikodym derivative of the probability measure $\mathbb{A}$ with respect to $\mathbb{R}$. Meanwhile, $Z_t^{\mathbb{A}}$ is the Brownian motion under $\mathbb{A}$. Thus, we have

$$\mathrm{d}Z_t^{\mathbb{A}} = \delta_t \mathrm{d}t + \mathrm{d}Z_t^{\mathbb{R}}, \tag{3}$$

$$\xi_t = \exp\left[-\frac{1}{2}\int_0^t \delta_s^2 \mathrm{d}s - \int_0^t \delta_s \mathrm{d}Z_t^{R}\right], \tag{4}$$

where δ_t captures the managerial cognition biases associated with the economic states in the future. In fact, δ_t also can be interpreted as managerial knowledge biases about the real world. Hereafter, we call δ_t as cognition biases or knowledge biases. Additionally, $\delta_t \in \mathcal{L}_2$ is a technical requirement[2]. Moreover, it is easy to check that ξ_t is a martingale. As the studies associated with dynamic learning process, such as [18], it does not require any particular type of learning process. But for simplificity, we assume a particular process in our application so as to shed light on the effects arising from the managerial cognition biases by restricting δ to a constant. In fact, according to the studies about psychology, individuals, especial top-managers, are inclined to underestimate the bad states, which implies δ in Equation (3) is positive.

From Equations (1) and (3) , we have the following process

$$\frac{\mathrm{d}X_t}{X_t} = (\mu + \delta\sigma)\mathrm{d}t + \sigma \mathrm{d}Z_t^{\mathbb{R}}, \tag{5}$$

which implies that it is the interaction between the biased cognition of the manager and the firms volatility determines the extend to which the manager overestimates the growth rate of firm in the future. However, this effects are completely different from those arising from overconfidence and/or overoptimism because the volatility has no impacts on the growth rate such as [4] and [6], where overconfident managers overestimate the precision of the uncertainty and overoptimistic ones overvalue the growth rate. Obviously, evolution (5) demonstrates that the volatility imposes effects on the growth grate in our model.

Finally, we assume that the manager run the firm to maximize the value of the existing shareholders subject to his knowledge biases on the economic environment. This also implies that there is not agency conflict between shareholders and the manager. In addition, the internal funds $C_0 (\leq I)$ are generated by assets in place at the

[2]The space $\mathcal{L}_2$ is defined so that $\mathcal{L}_2 = \left\{X : \int_0^{\tau} X_t^2 \mathrm{d}t < \infty \text{ a.s.}\right\}$.

initial time $t=0$. In order to focus on the effect of the managerial biases on investment decisions, we assume that the assets are fully depreciated at time 0 and that the internal funds are continuously reinvested in a risk-free asset over time until the manager undertakes the investment. Therefore, the amount of internal funds at any time t equals $C_t = C_0 e^{rt}$, which suggests that when $t > r^{-1}\ln\left(I/C_0\right)$, the amount of internal funds exceeds the investment costs I.

Valuation of the Investment Project

Once the investment is undertaken, the profit is perpetually generated, following the process (1). Thus, the real valuation of the investment conditional on the current cash flow X_t, denoted by $V_R\left(X\right)$, is

$$V_R\left(X_t\right) = \mathbb{E}_t\left[\int_t^{\infty} e^{-r(s-t)} X_s \mathrm{d}s\right] = \frac{X_t}{r-\mu}. \tag{6}$$

On the other hand, based on his knowledge about the project, the manager thinks that the generated cash flow by the investment evolves according to Equation (5) and hence his subjective value of the investment conditional on X_t denoted by $V_A\left(X\right)$ has the following form[3]:

$$V_A\left(X_t\right) = \mathbb{E}_t\left[\int_t^{\infty} e^{-r(s-t)} X_s \mathrm{d}s\right] = \frac{X_t}{r-\left(\mu+\delta\sigma\right)}. \tag{7}$$

This implies that the managerial subjective evaluation of the investment depends on his subjective biases, which result in an overestimated value of the firm after investment.

3. Solutions for Managerial Decisions

Suppose that the manager undertakes the project at time τ. The investment costs I are financed with internal capital $C_\tau\left(\le I\right)$ and external equity issuance $I-C_\tau$ for outside investors. Thus, based on the managerial cognition biases, the net present value for the existing shareholders at time τ is

$$\begin{aligned} V &= \frac{V_R\left(X_\tau\right)-\left(I-C_\tau\right)}{V_R\left(X_\tau\right)} V_A\left(X_\tau\right) - C_\tau \\ &= \frac{X_\tau}{r-\left(\mu+\delta\sigma\right)} - \frac{r-\mu}{r-\left(\mu+\delta\sigma\right)}\left(I-C_\tau\right) - C_\tau. \end{aligned} \tag{8}$$

In fact, the first term in the second line is the firm value based on the managerial cognition biases, and the second and third terms capture the costs of external equity funding in the manager's point of view. If the manager has not any knowledge biases, that is $\delta = 0$, then V becomes the classical form in the standard models of real options. However, with cognition biases, the manager has to make a trade-off between the benefits from undertaking the project and the costs arising from issuing external equity. Specifically, the knowledge biases induce the manager to undertake the project earlier because the project is valuable from his perspective and hence it is valuable to accelerate the investment timing. On the other hand, due to his cognition biases, the manager thinks that it is costly to issue external equity to fund the investment if the internal capital is not enough to cover the costs. Accordingly, he tends to delay investing. Not surprisingly, the internal funds also impose effects on the managerial investment decisions.

Note that before investment, internal capital is reinvested in a risk-free asset and hence generates interest rC_t for $0 \le t < \tau$. If the internal funds is not enough for investment costs I, the high costs to issue external equity forces the manager to finance the project with internal funds as much as possible, alleviating the costs of external equity issuance from his perspective. In addition, we assume that once the internal funds exceed investment costs I before the investment is undertaken, the manager distributes the surplus funds rI to the existing shareholders as dividends.

As stated above, the manager undertakes the investment to maximize the firm value for the existing shareholders based on his biased knowledge. It is equivalent to choose the timing of the investment. Ac-

[3]The convergence condition requires $r > \mu + \delta\sigma$.

cordingly, the investment decisions determined by the manager can be represented as the following optimal problem before the investment

$$V\left(X_t,C_t\right)=\max_{\tau}\mathbb{E}_t^{\mathbb{R}}\left[\mathrm{e}^{-r(\tau-t)}\left(\frac{X_t}{r-(\mu+\delta\sigma)}-\frac{r-\mu}{r-(\mu+\delta\sigma)}\left(I-C_t\right)-C_t\right)\right] \tag{9}$$

subject to

$$\frac{\mathrm{d}X_t}{X_t}=\left(\mu+\delta\sigma\right)dt+\sigma\mathrm{d}Z_t^{\mathbb{R}}, \tag{10}$$

$$\mathrm{d}C_t=rC_t\mathrm{d}t. \tag{11}$$

Intuitively, Equation (9) shows that the manager exercises the option to realize the value of the project by costing C_t in the interest of the existing equityholders. As the standard theories of real options, the investment threshold X^* is chosen by the manager to maximized the value of real option for the existing shareholders according to his own cognition.

Note that once the accruing internal funds exceed the investment costs I before undertaking the investment, the manager distributes the interests rI to the existing equityholders as dividend. Therefore, according to the procedure as in [12], the value function $V\left(X,C\right)$ satisfies the following partial differential equation:

$$rV=\begin{cases} rCV_C+\left(\mu+\delta\sigma\right)XV_X+\dfrac{1}{2}\sigma^2X^2V_{XX} & \text{if } t<r^{-1}\ln\left(I/C_0\right) \\ rK+\left(\mu+\delta\sigma\right)XV_X+\dfrac{1}{2}\sigma^2X^2V_{XX} & \text{if } t\geq r^{-1}\ln\left(I/C_0\right) \end{cases} \tag{12}$$

with boundary conditions:

$$V\left(0,C_t\right)=C_t, \tag{13}$$

$$V\left(X^*,C_t\right)=\frac{X^*}{r-\left(\mu+\delta_t\sigma\right)}-\frac{r-\mu}{r-\left(\mu+\delta_t\sigma\right)}\left(I-C_t\right) \tag{14}$$

and

$$\left.\frac{\partial I\left(X_t,C_t\right)}{\partial X_t}\right|_{X_t=X^*}=\frac{1}{r-\left(\mu+\delta_t\sigma\right)}. \tag{15}$$

These are very intuitive. Condition (13) represents that the firm value valuated by the manager equals the value of internal capital C_t if the cash flow X_t of the project is absorbed to zero before the investment. Conditions (14) and (15) characterize the value-matching and smoothing-pasting conditions respectively, which ensure the optimality of investment threshold X^*. Under these conditions, we can solve the equation of $V\left(X,C\right)$ and have the following solutions:

$$V\left(X_t,C_t\right)=\begin{cases} \left(\dfrac{X^*}{r-\left(\mu+\delta\sigma\right)}-C_t-\dfrac{r-\mu}{r-\left(\mu+\delta\sigma\right)}\left(I-C_t\right)\right)\left(\dfrac{X_t}{X^*}\right)^{\beta}+C_t & \text{if } X_t<X^* \\ \dfrac{X_t}{r-\left(\mu+\delta\sigma\right)}-\dfrac{r-\mu}{r-\left(\mu+\delta\sigma\right)}\left(I-C_t\right) & \text{if } X_t\geq X^*, \end{cases} \tag{16}$$

where

$$X^*\left(C_t\right)=\frac{\beta}{\beta-1}\left[\left(r-\mu\right)I-\delta\sigma C_t\right], \tag{17}$$

$$\beta=\frac{1}{2}-\frac{\mu+\delta\sigma}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\mu+\delta\sigma}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}}>1 \tag{18}$$

and

$$C_t = \begin{cases} C_0 e^{rt} & \text{if } t < r^{-1}\ln(I/C_0) \\ I & \text{if } t \ge r^{-1}\ln(I/C_0). \end{cases} \tag{19}$$

Equation (16) implies that it is optimal for the manager to put off investing if the current profit X_t remains below the investment boundary $X^*(C_t)$. Otherwise, it is best for the manager to invest at once. The investment threshold in Equation (17) intuitively depends on the amount of internal funds C_t, the parameter of managerial cognition biases δ and the firm's volatility σ. In the standard model of real options in [12], the firm's volatility influences the investment threshold only through the term of β. However, the managerial cognition biases provide an additional channel for the volatility to influence the investment threshold, *i.e.* the term of $\delta\sigma$, which arises in β and another term $\delta\sigma C_t$. As the existing literature associated with overconfidence and/or overoptimism, such as [6], there is no the interaction between the internal funds *C* and the volatility σ and hence no the corresponding effects on the investment decisions. Thus, differing from the behavior arising from overconfidence and overoptimism, the managerial cognition biases distort the investment strategies in an exclusive way. In the next section, we will analyse the effects of managerial cognition biases on the investment threshold specifically.

Particularly, the solutions in Equations (16)-(18) are consistent with those to maximize the physical value of the firm if no managerial biases, *i.e.* $\delta = 0$. For convenience, we call the solutions to the special case with $\delta = 0$ as the physical solutions, and denote them as V_e, X_e^* and β_e respectively. In fact, these solutions are consistent with those derived from the standard model of real options in [12]. With simple computation, we have $1 < \beta < \beta_e$, which is very useful in our proofs.

4. Analysis of Managerial Investment Decisions

In this section, we focus on the effects of the managerial cognition biases on investment decisions by comparing the solutions in our model with those for the standard model. In the numerical analysis, according to [6], we use the following parameter values in our model: $r = 0.08$, $\mu = 0.02$, $\sigma = 0.2$, $I = 100$.

Similarly, we define the first-order derivative $\frac{\mathrm{d}X^*(C_t)}{\mathrm{d}C_0}$ as an indicator for investment-cash flow sensitivity in the perspective of the timing of undertaking investment because a reduction in $C^*(C_t)$ on average increases the amount of investments for a given period. Thus, we can investigate that how the managerial biases influence the sensitivity. And the following proposition formalizes the relevant effects on the investment-cash flow sensitivity.

Proposition 1. *The increase in the magnitude of cognition biases* δ *enhances investment-cash flow sensitivity due to*

$$\frac{\partial X^*(C_t)}{\partial C_0} < 0 \text{ and } \frac{\partial^2 X^*(C_t)}{\partial C_0 \partial \delta} < 0. \tag{20}$$

Proposition (1) shows that increments in internal funds induce the manager to expedite investing sooner when his cognition biases are greater, providing useful empirical implications about the relationship between the managerial cognition biases and the sensitivity of corporate investment to cash flow.

Proposition 2. *If* $C_t = I$, *then* $X^*(I) < X_e$; *if* $C_t = 0$, *then* $X^*(0) > X_e$. *In addition, there is an optimal level of internal funds* $C^* \in (0, I)$ *that induces the manager with biased cognition to undertake the investment at* $X^*(C^*) = X_e$, *where the physical value of the firm* V_e *is optimized. The optimal level of internal capital* C^* *has the following expression*:

$$C^* = \frac{\beta_e - \beta}{\beta(\beta_e - 1)} \frac{r - \mu}{\delta\sigma} I. \tag{21}$$

The first part of Proportion (2) indicates that if the internal capital of the firm is enough to cover the whole investment costs *I*, the manager with biased cognition prefers to expedite investing relative to the manager without biased cognition. This is due to that the manager overestimates the economic states of the firm, that is he thinks that the firm has a higher growth rate so it is optimal to invest earlier. This result is consistent with the empirical studies about behavioral corporate finance, such as [5] [19], among others. In contrast, if whole costs *I* are funded by issuing external equity, the manager with cognition biases will delay undertaking the investment. Because without enough capital to cover the costs, the manager overvalues the costs of issuing external equity, which erodes the value of existing equityholders. Thus, he is inclined to wait the internal capital to accrue until it reaches a certain level. In addition, the second part shows that an appropriate level of internal funds can induce the manager with biased cognition to take the investment strategies to optimize the physical value of the firm V_e.

Figure 1 plots the investment threshold X^* against the internal funds C_t under different managerial cognition biases, *i.e.* $\delta = 0$, $\delta = 0.03$ and $\delta = 0.06$ respectively. The case with $\delta = 0$ represents the threshold X_e^* for the standard model, which is a constant and consistent with the traditional one. Firstly, **Figure 1** indicates the results in proposition (2) graphically. On one hand, the manager's investment decisions are dependent on the firm's internal funds. Specifically, if the firm just has a low level of internal funds, manager's cognition biases induce him to delay investment. The reason is that the manager with cognition biases overestimates the costs of external equity issuance and hence tends to finance the investment by using internal funds, putting off investing. And as the internal capital increases, reducing the costs for issuing new equity, the manager with biased cognition is inclined to expedite investing. In special, if the internal funds exceed a suitable level that is C^* defined in proposition (2), the manager tends to invest earlier than the manager without cognition biases. On the other hand, the graph illustrate the existence of optimal internal capital C^*, at which the manager with cognition biases can maximize the physical value of the firm by experiencing the investment option. Moreover, as the magnitude of cognition biases increases, the optimal level C^* of internal funds increases monotonically.

In addition, because the manager with biased cognition trades-off the subjective benefits from undertaking the investment and costs arising from external equity issuance, the effects of cognition biases on the investment threshold intuitively depend on the level of internal funds. When the internal funds are less, the more the cognition biases of the manager, the later he undertakes the investment. Otherwise, the manager expedites investing. **Figure 2** graphs these effects. Specifically, with a low level of internal funds, *i.e.* $C_t = 0$ and $C_t = 50$, as the magnitude of the cognition biases increases, the investment threshold increases, which implies that the manager delays investing. In contrast, if the firm has a high level of internal capital, *i.e.* $C_t = 95$, the cognition biases induce the manager to accelerate the investment timing. Meanwhile, given a level of cognition biases δ, the investment threshold $X^*(C_t)$ decreases in C_t, consistent whit **Figure 1** and implies that the manager with cognition biases undertakes the investment early.

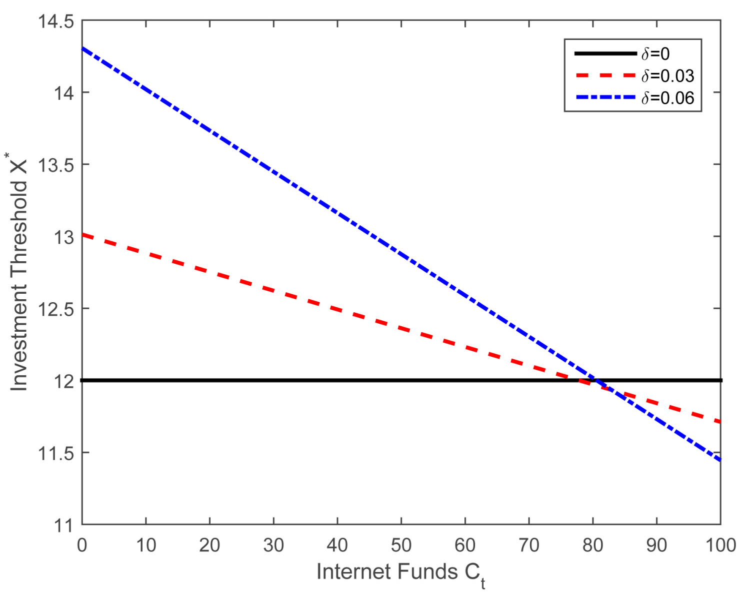

Figure 1. The effects of internal financing on the investment strategies for the manager with biased cognition. The baseline parameters are: $r = 0.08$, $\mu = 0.02$, $\sigma = 0.2$, $I = 100$.

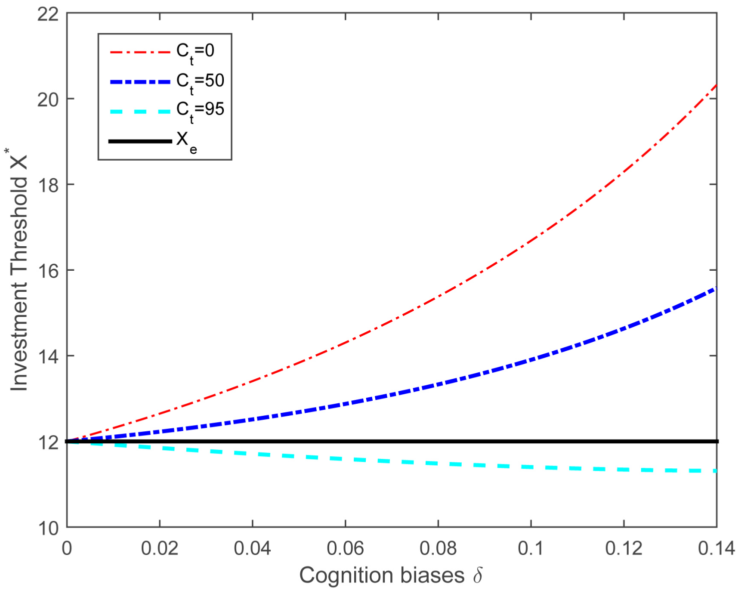

Figure 2. The effects of internal financing on the investment strategies for the manager with biased cognition. The baseline parameters are: $r = 0.08$, $\mu = 0.02$, $\sigma = 0.2$, $I = 100$. In order to ensure $r > \mu + \delta\sigma$, we take a rang of $\delta \in [0, 0.14]$ in the figure.

Now, we focus on that how the firm's volatility influences the firm's investment decisions and that how the managerial cognition biases distort the investment decisions along with the firm's volatility. For convenience, we rewrite Equation (17) as follows:

$$X^*(\delta,\sigma,C) = \frac{\beta(\delta,\sigma)}{\beta(\delta,\sigma)-1}\left[(r-\mu)I - \delta\sigma C\right]. \tag{22}$$

Thus, the corresponding threshold for the standard model can be represented as

$$X^*(0,\sigma,C) = \frac{\beta(0,\sigma)}{\beta(0,\sigma)-1}\left[(r-\mu)I\right]. \tag{23}$$

Comparing Equation (22) with Equation (23) shows that how the volatility influences the investment threshold through δ when the manager has biased cognition. Specifically, including the direct effect as in Equation (23), the volatility σ indirectly impacts the investment threshold through an additional channel included in the term of $\delta\sigma$, which arises in the term of drift and an alone term $-\delta\sigma C$. This implies that with managerial cognition biases, changes in the firm's volatility not only influence the volatility itself, but also result in a biased growth rate and an additional term with negative effects. In addition, it is intuitive that these effects also depend on the amount of internal capital C_t.

Figure 3 plots the investment threshold against the firm's volatility for two cases with $\delta = 0$ and $\delta = 0.12$, under two different levels of internal funds, $C_t = 25$ and $C_t = 95$, respectively. Obviously, as the standard deviation σ increases, the investment threshold increases, which is consistent with the wisdom in the standard theories of real options that the investment boundary is a monotonically increasing function of volatility σ. Moreover, it is interesting that the managerial cognition biases increase the convexity of the investment threshold against volatility σ. This is completely different from the effect arising from the manager's overconfidence which just makes parallel translations of curve of the investment boundary against the volatility, such as [6]. In addition, the left panel shows that if the firm has a low level of internal funds, *i.e.* $C = 25$, the manager with biased cognition is inclined to wait and hence delay investing relative to the manager without biases. However, if the firm has a promising level of internal funds, *i.e.* $C = 95$, the managerial cognition biases distort the investment decisions more seriously. Specifically, relative to the manager without biases, the

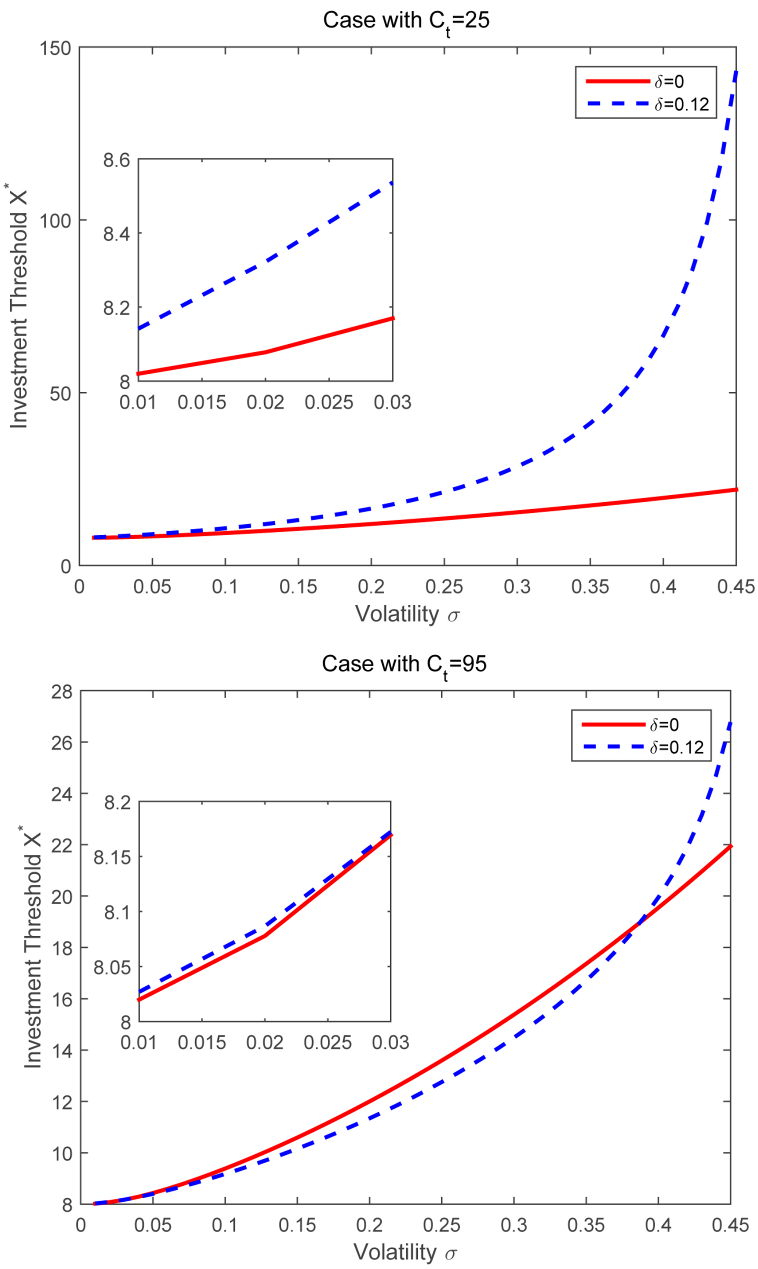

Figure 3. The effects of firm's volatility on the investment strategies in the model with and without cognition biases. The baseline parameters are: $r = 0.08$, $\mu = 0.02$, $\sigma = 0.2$, $I = 100$. And the amount of internal funds $C_t = 25$ in the left panel and $C_t = 95$ in the right panel. In order to make sure $r > \mu + \delta\sigma$, we take a rang of $\sigma \in (0, 0.45]$ in this figure.

extreme low or high level of the firm's volatility induces the manager who has biased cognition to delay undertaking the investment. In contrast, the moderate volatility induces the manager with cognition biases to accelerate the investment timing.

5. Conclusions

In this paper, based on the standard theories of real options, we develop a continuous-time model with managerial cognition biases associated with the firm's cash flow in the future to investigate the impacts of internal funds on the firm's investment decisions and provide closed-form solutions for the relationship among the managerial cognition biases, the internal funds and investment strategies. Our model shows that the interactions between the biased cognition of the manager and the firm's volatility determine the extend to which the manager with cognition biases overestimates the growth rate and accordingly the consequent valuation of the firm.

In addition, we find that the biased cognition of the manager enhances the sensitivity of corporate investment to cash flow. Moreover, relative to the manager without any cognition biases as in the standard models of real options, the limited internal funds induce the manager who has biased cognition to put off investing, but the sufficient internal capital in the firm induces the manager to accelerate the investment timing. This implies that the managerial cognition biases greatly distort the effects of the internal financing on the firm's investment decisions. Finally, our paper concludes that the managerial cognition biases increase the convexity of the investment decisions against the firm's volatility, which implies that the impacts resulting from managerial cognition biases are completely different from those arising from the manager's overconfidence, which only translates the curve of the investment threshold against the firm's volatility.

The paper just uses a parsimonious model to study that how managerial cognition biases influence the firm's investment strategies by using internal liquidity. There is some limitations in our research. But our model can serve as a benchmark for future work. For example, if we allow the firm to issue risky corporate debt, what are firm's investment decisions and how managerial cognition bases impact the firm's capital structure. In fact, to study the role played by human behavior in finance has very extensive applications and significant economic implications.

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Appendix

1. Proof of Proposition 1

Proof. Note that $\beta > 1$ and $\delta > 0$, therefore

$$\frac{\partial X^*(C_t)}{\partial C_0} = -\frac{\beta}{\beta - 1}\delta\sigma e^{rt} < 0.$$

Meanwhile, note that $\beta > 1$ is a solution to the following equation

$$\frac{\sigma^2}{2}\beta^2 + \left(\mu + \delta\sigma - \frac{\sigma^2}{2}\right)\beta - r = 0.$$

Differentiating it with respect to δ gives

$$\frac{\partial\beta}{\partial\delta} = -\frac{\sigma\beta}{\sigma^2\beta + \mu + \delta\sigma - \sigma^2/2} < 0.$$

For convenience, we denote $\frac{\beta}{\beta - 1}$ by $F(\beta)$, then

$$\begin{aligned}\frac{\partial^2 X^*(C_t)}{\partial C_0 \partial\delta} &= -\sigma e^{rt}\frac{\partial\left[F(\beta)\delta\right]}{\partial\delta} \\ &= -\sigma e^{rt}\left(-\frac{1}{(\beta - 1)^2}\frac{\partial\beta}{\partial\delta} + F(\beta)\right) < 0.\end{aligned}$$

2. Proof of Theorem 2

Proof. The proof for the first part, please refer to the proof of proposition 2 in [6] just note that in our model, $1 < \beta < \beta_e$ always holds, which satisfies the required conditions. And the second part is akin to the proof in proposition 3 in [6].