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Duopolistic Competition and Capacity Choice with Jump-Diffusion Process

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Abstract

This paper studies the effects of sudden events on the optimal timing and capacity choice in a duopoly market. According to the characteristics of economic environment, we assume that the product demand follows geometric Brownian motion with a Poisson jump process. Under the settings, the firms face the risk of a sudden drop in demand which is caused by sudden events. We develop the real option game model to derive the investment equilibrium strategies. Moreover, the effects of sudden events on investment decisions are obtained by numerical analysis.

Keywords

1. Introduction

We develop the real option game model to discuss the effects of sudden events on the optimal timing and capacity choice in a duopoly market. When sudden events occur, such as the financial crisis, economic policy from government, and the emergence of new products, discontinuous change in product demand appears. We use jump-diffusion process to capture the discontinuous changes of product demand.

Most real option game models suppose that the uncertainty variables such as asset price or product demand follow the geometric Brown motion (GBM) to describe the characteristics of continuous changes (e.g. Smets [1]; Dixit and Pindyck [2]; Grenadier [3]; Weeds [4]; Mason and Weeds [5]).

However, the GBM cannot explain some important empirical features of asset price or product demand dynamics. Jorion [6] and Bates [7] discovered the presence of jumps in asset price through empirical research. Recent studies have pointed out the importance of allowing for jumps, or discontinuities of asset price or product demand due to the effects of random sudden events in the economic environment. Merton [8] assumed that the

stock price follows a jump-diffusion process with Poisson jump to model sudden events. Kou [9] proposed a double exponential jump-diffusion model. Mason and Wilmot [10] investigated the potential presence of jumps in natural gas price. Ko *et al.* [11] established real option game model with jump process to investigate the effects of sudden events on investment timing. Pereira and Armada [12] assumed that the entrance of the hidden rivals follows a Poisson process. The project value of the positioned firm has a sudden drop as the hidden rivals enter the market. They presented a model suitable for investment decisions under a hidden competition environment. Pereira and Rodrigues [13] assumed that firms face the risk of demonopolization from government that can occur as a random or a certain event. They studied the optimal timing in finite-lived monopolies.

The large majority of real option game models focus on the investment timing without considering production capacity choice. However, in reality, production capacity decision is a key factor when one firm invests products. Few studies have considered the interaction between the investment timing and the production capacity in a real option framework. Besanko *et al.* [14] considered the investment decisions of the heterogeneous products under discrete time framework. Jou and Lee [15] assumed that all firms use the same investment strategy, obtaining the investment timing and the optimal capacity under imperfect competition. Huisman and Kort [16] provided a dynamic analysis of entry deterrence strategies, they discovered the leader overinvest in capacity in order to delay entry of the follower. The paper has close connection with these studies, which are extended by introducing the effects of sudden events and pre-emptive competition on the investment decisions. In this model, two firms are allowed to produce the homogeneous products; the product demand is assumed to obey the geometric Brown motion with a Poisson jump process. We discuss strategic investment decisions under duopolistic competition.

The remainder of the paper is organized as follows. Section 2 introduces the basic assumption of the real option game model. In Section 3, we derive the equilibrium strategies in a duopoly market. Section 4 exercises numerical analysis. Section 5 concludes the paper.

2. Basic Assumption of Real Option Game Model

In the section, we assume two firms have the chance to produce the homogeneous products in a duopoly market. Time is continuous and horizon is infinite. So every firm can defer the investment timing until the optimal moment to enter the market. The firm that enters first is known as the leader and the other as the follower. The product price at time t in market is given as follows:

$$P_t = X_t (a - Q) \tag{1}$$

where X_t is the exogenous demand shock, Q is the total market output, unit production cost is c, so the total costs of production are cQ. Similar to Huisman and Kort [16], every firm cannot adjust production capacity after entering the market and the two firms must make full use of production capacity. The exogenous demand shock is affected by sudden events of external market environment. Suppose that X_t obey the geometric Brown motion with the Poisson jump process:

$$\mathrm{d}X_t = \mu X_t \mathrm{d}t + \sigma X_t \mathrm{d}Z_t + \theta X_t \mathrm{d}q_t$$

Among the above factors, μ represents the drift rate, σ represents the volatility, $\mathrm{d}Z_t$ is the increment of a standard Brownian motion. We assume $\rho > \mu$ to ensure that the option is exercised within a finite period of time. We assume random sudden events follow the Poisson jump process of intensity λ. This means sudden events occur with probability $\lambda \mathrm{d}t$ during the time interval $\mathrm{d}t$. Sudden drop in product demand as the events occur. θ represents the deterministic amplitude of the downward jumps satisfying $-1 \leq \theta \leq 0$. Assume $\mathrm{d}q_t$ and $\mathrm{d}Z_t$ are independent, so $E(\mathrm{d}q_t \mathrm{d}Z_t) = 0$:

$$\mathrm{d}q_t = \begin{cases} 1, & \text{with probability } \lambda \mathrm{d}t \\ 0, & \text{with probability } 1 - \lambda \mathrm{d}t \end{cases}$$

3. Investment Decisions in a Duopoly Market

In the section, we develop the model to determine the values and the investment decisions of two firms in a duopoly market, facing the risk of random sudden drop in product demand.

3.1. The Follower's Value Function

We need to consider the game backwards. When the leader has invested the project, the follower can make his decisions optimally in response to capacity of the leader. Suppose that the leader has invested the project with capacity q_l, the investment threshold and capacity of the follower are chosen as X_f and q_f, so the investment costs are cq_f. The value function of the follower $V_f(X, q_f)$ is recorded as $V_f(X)$ for short. By using the standard real option method, the Bellman equation can be expressed as:

$$\rho V_f(X)\mathrm{d}t = E\big(\mathrm{d}V_f(X)\big).$$

According to Itô's Lemma, the value of the follower $V_f(X)$ satisfies the following differential equation:

$$\rho V_f(X) = \mu X \frac{\partial V_f(X)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_f(X)}{\partial X^2} + \lambda V_f\big(X(1+\theta)\big) - \lambda V_f(X).\tag{2}$$

The general solution of (1) is of the form:

$$V_f(X) = A_1 X^{\beta_1} + A_2 X^{\beta_2}.\tag{3}$$

Among them, A_1, A_2 are the undetermined coefficients, β_1, β_2 are the roots of the equation $\frac{1}{2}\sigma^2\beta^2 + \left(\mu - \frac{1}{2}\sigma^2\right)\beta + \lambda(1+\theta)^{\beta_1} - \lambda - \rho = 0$:

$$\beta_1 = \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) + \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\left(\lambda + \rho - \lambda(1+\theta)^{\beta_1}\right)}}{\sigma^2} > 1,$$

$$\beta_2 = \frac{-\left(\mu - \frac{1}{2}\sigma^2\right) - \sqrt{\left(\mu - \frac{1}{2}\sigma^2\right)^2 + 2\sigma^2\left(\lambda + \rho - \lambda(1+\theta)^{\beta_2}\right)}}{\sigma^2} < 0.$$

Moreover, the value of the follower $V_f(X)$ must satisfy three boundary conditions:

$$V_f(0) = 0,\tag{4}$$

$$V_f(X_f) = \frac{(a - q_l - q_f)q_f X_f}{\rho - \mu - \lambda\theta} - cq_f,\tag{5}$$

$$\left.\frac{\partial V_f(X)}{\partial X}\right|_{X = X_f} = \frac{(a - q_l - q_f)q_f}{\rho - \mu - \lambda\theta}.\tag{6}$$

Condition (4) says that the value will be 0 if $X = 0$. Condition (5) and (6) are the value-matching and the smooth-pasting conditions. The two conditions ensure that $V_f(X)$ can be maximized when the firm invests at the threshold X_f.

Under these conditions, the value $V_f(X)$, the investment threshold X_f and capacity q_f of the follower are calculated as follows:

$$V_f(X) = \begin{cases} A_1 X^{\beta_1} & X < X_f \\ \dfrac{(a - q_l - q_f)q_f X}{\rho - \mu - \lambda\theta} - cq_f & X \geq X_f, \end{cases}\tag{7}$$

where

$$X_f = \frac{\beta_1}{\beta_1 - 1}\frac{c(\rho - \mu - \lambda\theta)}{(a - q_l - q_f)},\tag{8}$$

$$A_1 = \frac{X_f^{1-\beta_1}}{\beta_1} \frac{(a-q_l-q_f)q_f}{\rho-\mu-\lambda\theta}. \tag{9}$$

We assume that the initial demand level is sufficiently low, the follower will not start production immediately. According to (7), the value of the follower $V_f(X)$:

$$V_f(X) = A_1 X^{\beta_1} \tag{10}$$

See from (10), $V_f(X)$ is function of q_f. We apply (8), (9) to (10), the follower considers the capacity to maximize the value, $V_f(X)$ satisfies the first order condition:

$$\left. \frac{\partial V_f(X)}{\partial q_f} \right|_{q_f = q_f^*} = 0. \tag{11}$$

Combining (8) and (11), we obtain the optimal threshold and the capacity of the follower:

$$q_f^* = \frac{a-q_l}{\beta_1+1}, \quad X_f^* = \frac{\beta_1+1}{\beta_1-1} \frac{c(\rho-\mu-\lambda\theta)}{a-q_l} \tag{12}$$

Substitute (12) into (9) and (10), the value of the follower $V_f(X, q_f^*)$ is as follows:

$$V_f(X, q_f^*) = \frac{c(a-q_l)X^{\beta_1}}{(\beta_1-1)(\beta_1+1)} \left(\frac{\beta_1-1}{\beta_1+1}\right)^{\beta_1} \left[\frac{a-q_l}{c(\rho-\mu-\lambda\theta)}\right]^{\beta_1} \tag{13}$$

Let $q_l = 0$ in (12), the optimal threshold X_m^* and the capacity q_m^* of the monopolist are as follows:

$$q_m^* = \frac{a}{\beta_1+1}, \quad X_m^* = \frac{\beta_1+1}{\beta_1-1} \frac{c(\rho-\mu-\lambda\theta)}{a} \tag{14}$$

3.2. The Leader's Value Function

When the follower is out of the market, the leader earns profits $(a-q_l)q_l X_t$ at time t. When the follower enters the market, the leader's profits decreases to $(a-q_l-q_f^*)q_l X_t$ at time t. Suppose that the value function of the leader is $V_l(X, q_l)$. $V_l(X, q_l)$ is also written as $V_l(X)$ for simplicity. Let $V_l(X) = v_l(X) - cq_l$, so $v_l(X)$ represent that the value subtract the investment costs when the leader has invested. $v_l(X)$ satisfies the following differential equation:

$$\rho v_l(X) = \mu X \frac{\partial v_l(X)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 v_l(X)}{\partial X^2} + \lambda v_l(X(1+\theta)) - \lambda v_l(X) + (a-q_l)q_l X. \tag{15}$$

The general solution of (15) is of the form:

$$v_l(X) = B_1 X^{\beta_1} + B_2 X^{\beta_2} + \frac{(a-q_l)q_l X}{\rho-\mu-\lambda\theta} \tag{16}$$

The value of the leader $V_l(X)$ must satisfy two boundary conditions:

$$v_l(0) = 0, \tag{17}$$

$$v_l(X_f^*) = \frac{(a-q_l-q_f^*)q_l X_f^*}{\rho-\mu-\lambda\theta}. \tag{18}$$

If we apply (17), (18) to (16), $v_l(X)$ is given by:

$$v_l(X) = B_1 X^{\beta_1} + \frac{(a-q_l)q_l X}{\rho-\mu-\lambda\theta}, \tag{19}$$

where

$$B_1 = -\frac{q_l q_f^*}{\rho - \mu - \lambda\theta} X_f^{*1-\beta_1}. \tag{20}$$

Before the leader invests the project, the value of the leader $V_l(X)$ satisfies the following differential equation:

$$\rho V_l(X) = \mu X \frac{\partial V_l(X)}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 V_l(X)}{\partial X^2} + \lambda V_l(X(1+\theta)) - \lambda V_l(X). \tag{21}$$

The general solution of (21) is of the form:

$$V_l(X) = C_1 X^{\beta_1} + C_2 X^{\beta_2} \tag{22}$$

In addition, the value of the leader $V_l(X)$ must satisfy three boundary conditions:

$$V_l(0) = 0, \tag{23}$$

$$V_l(X_l) = \frac{(a-q_l)q_l X_l}{\rho - \mu - \lambda\theta} - \left(\frac{X_l}{X_f^*}\right)^{\beta_1} \frac{q_l q_f^* X_f^*}{\rho - \mu - \lambda\theta} - cq_l, \tag{24}$$

$$\left.\frac{\partial V_l(X)}{\partial X}\right|_{X=X_l} = \frac{(a-q_l)q_l}{\rho - \mu - \lambda\theta} - \frac{\beta_1}{X_f^*}\left(\frac{X_l}{X_f^*}\right)^{\beta_1-1}\frac{q_l q_f^* X_f^*}{\rho - \mu - \lambda\theta}. \tag{25}$$

Condition (24) says that:

$$V_l(X_l) = v_l(X_l) - cq_l. \tag{26}$$

If we apply (23), (24), (25) to (22), we obtain:

$$V_l(X) = \begin{cases} C_1 X^{\beta_1} & X < X_l \\ \dfrac{(a-q_l)q_l X}{\rho - \mu - \lambda\theta} - \left(\dfrac{X}{X_f^*}\right)^{\beta_1} \dfrac{q_l q_f^* X_f^*}{\rho - \mu - \lambda\theta} - cq_l & X_l \le X < X_f^* \\ \dfrac{(a-q_l-q_f^*)q_l X}{\rho - \mu - \lambda\theta} - cq_l & X \ge X_f^*, \end{cases} \tag{27}$$

where

$$X_l = \frac{\beta_1}{\beta_1 - 1} \frac{c(\rho - \mu - \lambda\theta)}{a - q_l}, \tag{28}$$

$$C_1 = \frac{X_l^{1-\beta_1}}{\beta_1}\left[\frac{(a-q_l)q_l}{\rho - \mu - \lambda\theta} - \frac{\beta_1}{X_f^*}\left(\frac{X_l}{X_f^*}\right)^{\beta_1-1}\frac{q_l q_f^* X_f^*}{\rho - \mu - \lambda\theta}\right]. \tag{29}$$

We assume that the initial demand level is sufficiently low, the leader will not start production immediately. To maximize the value, we apply (28), (29) to (27), and substitute (12) into (27), the value of the leader is calculated as:

$$V_l(X) = \frac{cq_l X^{\beta_1}}{\beta_1 - 1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left[1 - \left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}\right]\left[\frac{a - q_l}{c(\rho - \mu - \lambda\theta)}\right]^{\beta_1} \tag{30}$$

3.3. Equilibrium

Here, in order to examine impacts of pre-emptive competition, we assume that the roles of the leader and the follower are designated exogenously. The follower enters the market only after the leader has entered. This

means that one firm is designated as the leader beforehand. So, the risk of pre-emption is eliminated, two firms can delay their investment to maximize their values. When the initial demand is sufficiently low, we suppose the leader (follower) select the optimal capacity q_{ln}^* $\left(q_{fn}^*\right)$, the investment threshold X_{ln}^* $\left(X_{fn}^*\right)$. Thus, the value of the leader (follower) is denoted as $V_l\left(X, q_{ln}^*\right)$ $\left(V_f\left(X, q_{fn}^*\right)\right)$. Similar to the above, $V_l(X)$ must satisfy:

$$\left.\frac{\partial V_l(X)}{\partial q_l}\right|_{q_l=q_{ln}^*}=0. \tag{31}$$

Combining (28) and (31), we have:

$$q_{ln}^*=q_m^*=\frac{a}{\beta_1+1}, \quad X_{ln}^*=X_m^*=\frac{\beta_1+1}{\beta_1-1}\frac{c(\rho-\mu-\lambda\theta)}{a}. \tag{32}$$

So, by substituting (32) into (12), the optimal threshold and the capacity of the follower are given by:

$$q_{fn}^*=\frac{a-q_{ln}^*}{\beta_1+1}=\frac{a\beta_1}{\left(\beta_1+1\right)^2}, \quad X_{fn}^*=\frac{\beta_1+1}{\beta_1-1}\frac{c(\rho-\mu-\lambda\theta)}{a-q_{ln}^*}=\frac{\left(\beta_1+1\right)^2}{\beta_1\left(\beta_1-1\right)}\frac{c(\rho-\mu-\lambda\theta)}{a} \tag{33}$$

Comparing (14) and (32), Proposition 1 is obtained.

Proposition 1. The investment threshold of the designated leader is the same as that of the monopolist.

The designated leader has valuable option to defer investment at the optimal threshold of the monopolist as he need not face the risk of being preempted.

However, two firms are allowed to invest first in reality. This means that firm roles are endogenous. So, the risk of pre-emption exists. We assume that the initial demand level is sufficiently low, two firms are induced to delay their investment. When firm roles are endogenous, according to Fudenberg and Tirole [17], if one firm intends to invest at the threshold X_m^* first, the other firm will pre-empt it as long as the value of the firm is greater than that of the other firm. So the value is equal for both firms in equilibrium. Suppose that the value of the leader (follower) $V_l\left(X, q_{le}^*\right)$ $\left(V_f\left(X, q_{fe}^*\right)\right)$, the optimal capacity q_{le}^* $\left(q_{fe}^*\right)$, the investment threshold X_{le}^* $\left(X_{fe}^*\right)$. So,

$$V_l\left(X, q_{le}^*\right)=V_f\left(X, q_{fe}^*\right). \tag{34}$$

Proposition 2 describes the sequential equilibrium. For the proof of Proposition 2, see the Appendix.

Proposition 2. (sequential equilibrium). If the initial demand level is lower than X_{le}^*, the equilibrium investment is sequential, the leader invests with capacity q_{le}^* until the demand X_t reaches X_{le}^*, the follower invests with capacity q_{fe}^* until the demand X_t reaches X_{fe}^*. Where, the optimal capacity of the leader q_{le}^* is a unique solution of the equation:

$$\left[1-\left(\frac{\beta_1}{\beta_1+1}\right)^{\beta_1+1}\right]q_{le}^*-\left(\frac{\beta_1}{\beta_1+1}\right)^{\beta_1}\frac{a}{\beta_1+1}=0, \tag{35}$$

the optimal capacity X_{le}^*:

$$X_{le}^*=\frac{\beta_1}{\beta_1-1}\frac{c(\rho-\mu-\lambda\theta)}{a-q_{le}^*}, \tag{36}$$

the optimal capacity q_{fe}^* and investment threshold q_{fe}^* of the follower:

$$q_{fe}^*=\frac{a-q_{le}^*}{\beta_1+1}, \quad X_{fe}^*=\frac{\beta_1+1}{\beta_1-1}\frac{c(\rho-\mu-\lambda\theta)}{a-q_{le}^*}. \tag{37}$$

Proposition 3, 4 describe the impacts of pre-emptive competition. The proofs are in Appendix.

Proposition 3. When the risk of pre-emption exists, the leader reduce its capacity to invest early, the follower increases its capacity to invest early. That is, $q_{le}^*<q_{ln}^*$, $X_{le}^*<X_{ln}^*$, $q_{fe}^*>q_{fn}^*$, $X_{fe}^*<X_{fn}^*$.

Proposition 4. If the initial demand level is lower than X_{le}^*, the value of the leader is less than that of the designated leader, the value of the follower is larger than that of the designated follower. That is,

$$V_l\left(X, q_{le}^*\right) < V_l\left(X, q_{ln}^*\right), \quad V_f\left(X, q_{fe}^*\right) > V_f\left(X, q_{fn}^*\right).$$

4. Numerical Results Analysis

4.1. The Impacts of Pre-Emptive Competition

The subsection describes the impacts of pre-emptive competition on the firms. The parameters are as follows: $\rho = 0.05$, $\mu = 0.03$, $c = 10$, $a = 1$, $\lambda = 0.1$, $\theta = -0.1$, $\sigma = 0.3$.

Based on the given parameters, we calculate to obtain the optimal capacities and thresholds respectively as the roles of the firms are endogenous or exogenous: $X_{ln}^* = 1.9427$, $X_{fn}^* = 3.3656$, $q_{ln}^* = 0.4228$, $q_{fn}^* = 0.2440$, $X_{le}^* = 1.5456$, $X_{fe}^* = 2.6776$, $q_{le}^* = 0.2745$, $q_{fe}^* = 0.3067$. We can see that $X_{le}^* < X_{ln}^*$, $q_{le}^* < q_{ln}^*$ hold from the numerical results. This means that the leader will reduce production to speed up the investment due to fear of being pre-empted. Comparison of the numerical results, we find that $X_{fe}^* < X_{fn}^*$, $q_{fe}^* > q_{fn}^*$ stand. This is because when the leader reduces production to invest ahead, the production in the market is not enough, the price is relatively high, the follower is willing to increase production to invest ahead. These results indicate pre-emptive competition makes the two firms to accelerate investment.

Figure 1 further shows the relationship between the values of the two firms as the roles are endogenous or exogenous. The numerical analysis is based on the assumptions that the initial demand is less than X_{le}^*, the firms will wait until the optimal timing to enter the market. See **Figure 1**, the value of the designated leader is the largest, that of the leader is the second, that of the designated follower is the smallest. That is to say, $V_f\left(X, q_{fn}^*\right) < V_f\left(X, q_{fe}^*\right) < V_l\left(X, q_{ln}^*\right)$. When the firm roles are designated, the designated follower is at a disadvantage in the game. The designated leader is dominant, he will invest at the optimal timing not considering the risk of pre-emption. So, the conclusion is intuitive.

So, **Figure 1** suggests the conclusions of Proposition 3, 4.

4.2. Sensitivity Analysis

In the subsection, we perform a comparative static analysis, focusing on the impacts of different parameter values such as the Poisson jump process of intensity λ, the deterministic amplitude of the jumps θ, the volatility σ.

Table 1 illustrates the impacts of different values of λ. The parameters are as follows: $\rho = 0.05$, $\mu = 0.03$, $c = 10$, $a = 1$, $X = 1$, $\theta = -0.1$, $\sigma = 0.3$. The intensity λ measures the arrival probability of sudden events such as financial crisis or financial tsunami. When sudden events occur, the economy will be immersed in depression, a sudden decline in market demand. When λ increases, the leader will reduce its production to invest later, the follower will reduce its production to invest earlier first, then later. Increasing λ brings more risks which will make the leader more conservative. The investment threshold of the follower is non monotonic with
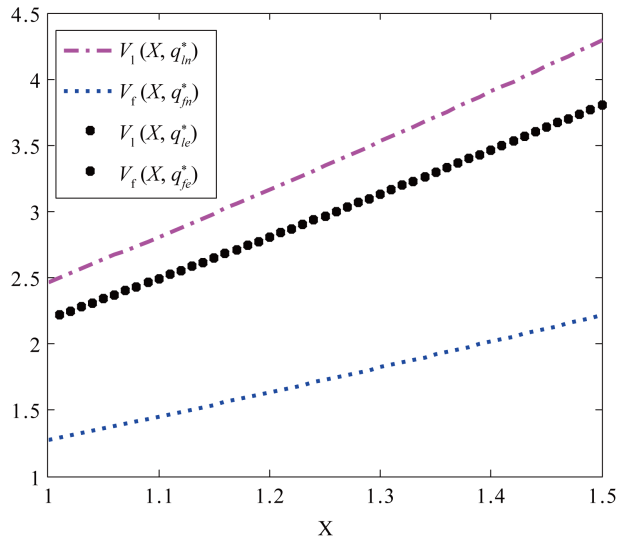
Figure 1. Values of the two firms.

λ. As we mentioned above, higher λ makes the investors reduce output to invest later. On the other hand, higher price which due to less output encourages the follower to invest earlier. As λ increases, the values of both the leader and the follower will decline.

Table 2 illustrates the impacts of different values of θ. The parameters are as follows: $\rho = 0.05$, $\mu = 0.03$, $c = 10$, $a = 1$, $X = 1$, $\lambda = 0.1$, $\sigma = 0.3$. The deterministic amplitude of the jumps θ measures the level of the effect of the sudden events on investment environment. As θ decreases, both the leader and the follower will reduce production to invest later. Decreasing θ means worse investment environment. Thus, the values of both will decline.

Table 3 illustrates the impacts of different values of σ. The parameters are as follows: $\rho = 0.05$, $\mu = 0.03$, $c = 10$, $a = 1$, $X = 1$, $\lambda = 0.1$, $\theta = -0.1$. As σ increases, the uncertainties and risks linked with investment also increase. When the uncertainties and risks rise, both the leader and the follower will prefer to wait for better chance rather than now. So, they will increase production to invest later, the values of both will increase.

5. Conclusions

In this paper, we examine the impact of sudden events on the investment timing and production capacity decisions of a firm that faces competition. We obtain the investment equilibrium strategies.

Table 1. Impacts of different values of λ.

λ	q_{le}^*	q_{fe}^*	X_{le}^*	X_{fe}^*	$V_l(X, q_{le}^*)$	$V_f(X, q_{fe}^*)$
0.1	0.2745	0.3067	1.5456	2.6776	2.1887	2.1887
0.15	0.2657	0.3017	1.5747	2.6727	1.6971	1.6971
0.2	0.2572	0.2965	1.6054	2.6723	1.3385	1.3385
0.25	0.2491	0.2914	1.6377	2.6762	1.0688	1.0688
0.3	0.2414	0.2862	1.6715	2.6839	0.8612	0.8612

Table 2. Impacts of different values of θ.

θ	q_{le}^*	q_{fe}^*	X_{le}^*	X_{fe}^*	$V_l(X, q_{le}^*)$	$V_f(X, q_{fe}^*)$
−0.1	0.2745	0.3067	1.5456	2.6776	2.1887	2.1887
−0.15	0.2660	0.3019	1.5815	2.6861	1.6994	1.6994
−0.2	0.2581	0.2971	1.6239	2.7086	1.3447	1.3447
−0.25	0.2510	0.2925	1.6727	2.7447	1.0800	1.0800
−0.3	0.2445	0.2883	1.7279	2.7939	0.8783	0.8783

Table 3. Impacts of different values of σ.

σ	q_{le}^*	q_{fe}^*	X_{le}^*	X_{fe}^*	$V_l(X, q_{le}^*)$	$V_f(X, q_{fe}^*)$
0.2	0.2495	0.2916	1.0963	1.7928	2.0273	2.0273
0.25	0.2633	0.3003	1.3053	2.2034	2.1005	2.1005
0.3	0.2745	0.3067	1.5456	2.6776	2.1887	2.1887
0.35	0.2835	0.3116	1.8177	3.2168	2.2819	2.2819
0.4	0.2908	0.3154	2.1227	3.8225	2.3746	2.3746

We find that pre-emptive competition and sudden events have great influence on investment decisions; pre-emptive competition makes firms accelerate investment. Higher uncertainty for market demand increases the values of both the leader and the follower. When sudden events occur more frequently or product demand declines in greater magnitude, the values of both firms will decline.

This paper considers the case of two firms. Consequently, a natural idea is to consider the case of a number of firms. Future research can also be concerned with the application of a different random process, e.g., arithmetic Brownian motion.

Acknowledgements

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Appendix

The proof for Proposition 2:

When pre-emptive competition exists, the follower makes his decisions reacting to the capacity of the leader. According to (28), we can obtain (36). According to (12), we can obtain (37).

The value is the same for both firms at the threshold $q_l = q_{le}^*$. Substitute $q_l = q_{le}^*$ into (13) and (30), we have:

$$\frac{c q_{le}^* X^{\beta_1}}{\beta_1 - 1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left[1 - \left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}\right]\left[\frac{a - q_{le}^*}{c(\rho - \mu - \lambda\theta)}\right]^{\beta_1} = \frac{c(a - q_{le}^*) X^{\beta_1}}{(\beta_1 - 1)(\beta_1 + 1)}\left(\frac{\beta_1 - 1}{\beta_1 + 1}\right)^{\beta_1}\left[\frac{a - q_{le}^*}{c(\rho - \mu - \lambda\theta)}\right]^{\beta_1}$$

Simplifying the above equation, we conclude that (35) stands and q_{le}^* is a unique root of (35).

The proof for Proposition 3:

We proof that when $\beta_1 > 1$, $\left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1} < \dfrac{1}{2}$ first.

$$\lim_{\beta_1 \to 1}\left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1} = \frac{1}{2}, \quad \lim_{\beta_1 \to \infty}\left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1} = \frac{1}{e}, \tag{38}$$

$$\frac{\partial\left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}}{\partial\beta_1} = \frac{1}{\beta_1 + 1}\left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}\left(1 + (\beta_1 + 1)\ln\left(\frac{\beta_1}{\beta_1 + 1}\right)\right).$$

Let $f(\beta_1) = \ln\left(\dfrac{\beta_1 + 1}{\beta_1}\right) - \dfrac{1}{\beta_1 + 1}$, we have $\lim_{\beta_1 \to 1} f(\beta_1) = \ln 2 - \dfrac{1}{2} > 0$, $\lim_{\beta_1 \to \infty} f(\beta_1) = 0$, and if $\beta_1 > 1$, then

$f'(\beta_1) = \dfrac{1}{(\beta_1 + 1)^2} - \dfrac{1}{\beta_1(\beta_1 + 1)} < 0$. So, $f(\beta_1) > 0$. Thus, we conclude $1 + (\beta_1 + 1)\ln\left(\dfrac{\beta_1}{\beta_1 + 1}\right) < 0$ and

$\dfrac{\partial\left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}}{\partial\beta_1} < 0$. According to (38), we obtain that $\beta_1 > 1$, $\left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1} < \dfrac{1}{2}$.

Substitute q_{ln}^* into (35), we have $\left[1 - \left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1 + 1}\right]\dfrac{a}{\beta_1 + 1} - \left(\dfrac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}\dfrac{a}{\beta_1 + 1} > 0$. So, $q_{le}^* < q_{ln}^*$. Combining

(12) and (28), Proposition 3 holds.

The proof for Proposition 4:

If $q_l \le q_{ln}^*$, taking the first derivatives on (13) and (30) respectively:

$$\frac{\partial V_l(X)}{\partial q_l} = \frac{c(a - q_l)^{\beta_1 - 1}\left[a - (\beta_1 + 1)q_l\right] X^{\beta_1}}{\beta_1 - 1}\left(\frac{\beta_1 - 1}{\beta_1}\right)^{\beta_1}\left[1 - \left(\frac{\beta_1}{\beta_1 + 1}\right)^{\beta_1}\right]\left[\frac{1}{c(\rho - \mu - \lambda\theta)}\right]^{\beta_1} \ge 0,$$

$\dfrac{\partial V_f(X, q_f^*)}{\partial q_l} = -\dfrac{cX^{\beta_1}}{(\beta_1 - 1)}\left(\dfrac{\beta_1 - 1}{\beta_1 + 1}\right)^{\beta_1}\left[\dfrac{a - q_l}{c(\rho - \mu - \lambda\theta)}\right]^{\beta_1} < 0$. So, if $q_l \le q_{ln}^*$, $V_l(X)$ is a monotonically increasing function of q_l, $V_f(X, q_f^*)$ is a monotonically decreasing function of q_l. Because $q_{le}^* < q_{ln}^*$, we have Proposition 4 holds.