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A Predictive Functional Regression Model for Asset Return

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ABSTRACT

Since many of predictive financial variables are highly persistent and non-stationary, it is challenging econometrically to explore the predictability of asset returns. Predictability issues are generally addressed in parametric regressions [1,2] in which rates of asset returns are regressed against the lagged values of stochastic explanatory variables, but three limitations stand ahead [3-5]. This paper studies a predictive functional regression model for asset returns, which takes account of endogeneity and integrated or nearly integrated explanatory variables. The regression function is expressed in terms of distribution of the vector of the observable variables. Estimators are nonlinear functionals of a kernel estimator for the distribution of the observable variables [6]. We find that the estimators for the distribution of the unobservable random terms and the nonparametric function are consistent and asymptotically normal. This paper obtains the similar results in many literatures, for example [1-5], but in different method.

Keywords: Asset Return; Functional Regression; Consistentcy

1. Introduction

People routinely examine the predictability problem, for example, the mutual fund performance, the conditional capital asset pricing, and the optimal asset allocations. For the predictability of stock returns, various lagged financial variables are used, for example, the log dividend-price ratio, the log earning-price ratio, the log book-to-market ratio, the dividend yield, the term spread, default premium, and the interest rates [3]. Since many of the predictive financial variables are highly persistent and even non-stationary, it is challenging econometrically to explore the predictability of asset returns.

Predictability issues are generally addressed in parametric regressions in which rates of returns are regressed against the lagged values of stochastic explanatory variables. In predictive linear structure model [1,2], excess stock return is the predictable variable at time *t*, innovations $\{(\varepsilon_t, \mu_t)\}$ are independently and identically distributed bivariate normal and the log dividend-price ratio is a financial variable at time $t-1$, which is modelled by an AR(1) model.

There are three limitations. At first, two innovations are unfortunately correlated in real applications [3,4]. The second difficulty arises from the unknown parameter for financial variable regression, for stationary case, see [4,5,7,8], for unit root or integrated, see [9-11], and for local-to-unity or nearly integrated, see [3,12-16]. The third difficulty comes from the instability of the predictive regression model. It concluded from many evidences on the dividend and earnings yield and the sample from the second half of the 1990s that the coefficients should change over time, see, for example [4,5,7,17-19].

In finite samples, the ordinary least squares estimate of the slope coefficient and its standard errors are substantially biased if explanatory variable is highly persistent, not really exogenous, and even non-stationary, see [20]. To avoid over-rejecting the null of non-predictability, some improvements arise, such as the first order bias-correction estimator [2], the two-stage least squares estimator [8], and the conservative bias-adjusted estimator [21], but the instability difficulty was kept silent. To deal with this issue, some predictive regression models were analyzed, for example, excess return predictive regression model on international equity indices [4], equity return predictive regression model [5] with random coefficients generated from a unit root process, asset regression model with varying coefficients [22]. A predictive functional regression model has not touched, though not

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only interesting in its applications to finance and economics, but also enriching the econometric theory.

The rest of this paper runs as follows. Section 2 proposes basic functional regression model. Section 3 is for nonparametric estimation. Section 4 derives the consistency for the proposed estimator. Section 5 concludes the paper.

2. Basic Model

We propose a functional regression model to capture the stability of asset returns. It is well known that a nonlinear function would better to characterize dynamic relationship between the stock return and the related financial variables, the two innovations may have a time dependent nonlinear relationship, and the log dividend-price ratio x_t, is a integrated or nearly integrated process [3, 22]. Our model runs as follows.

$$y_t = f\left(x_{t-1}, \epsilon_t\right), \tag{1}$$

$$x_t = \rho x_{t-1} + \mu_t,\ \ \rho = 1 + \frac{c}{n},\ \ c \le 0,\ \ 1 \le t \le n, \tag{2}$$

where innovation ϵ_t is exogenous.

To remove the endogeneity, we project ϵ_t onto μ_t by $\epsilon_t = g\left(\mu_t, \nu_t\right)$, which is strictly increasing in ν_t and ν_t is uncorrelated with μ_t and $\nu_t \sim \mathbb{N}(0,1)$. See, for example, [23] for endogenous variable. Thus the model becomes

$$y_t = f\left(x_{t-1}, g\left(\mu_t, \nu_t\right)\right) \equiv h\left(x_{t-1}, \mu_t, \nu_t\right), \tag{3}$$

$$x_t = \rho x_{t-1} + \mu_t,\ \ \rho = 1 + \frac{c}{n},\ \ c \le 0,\ \ 1 \le t \le n. \tag{4}$$

The function *f* can be estimated once function *h* is estimated due to the strict increasing of $\epsilon_t = g\left(\mu_t, \nu_t\right)$ with respect to ν_t and the Equation (4). Indeed if the functions *f* and *g* are linear, the model reduces to [22].

3. Nonparametric Estimation

Once parametric structures are not specified for the functions *h* in the economic model, the function *h* is nonadditive in ν. If the function is additive in unobservable random term ν, one can interpret this added unobservable random term as being a function of the observable and other unobservable variables, which is hard to estimate this function of the observable and unobservable variables. Here we estimate a nonparametric function *h*, not necessarily additive.

To estimate the regression function *h* in the basic model (3), we will derive its expression in terms of the distribution of the vector of the observable variables. Once the unknown regression function is expressed in terms of the distribution of $\left(y_t, x_{t-1}, \mu_t\right)$, we will derive its nonparametric estimator for the unknown regression function by substituting the distribution of the observable variables. Though any type of nonparametric estimator for this distribution can be used, we present here the details and asymptotic properties for the case in which the conditional cumulative distributed functions are estimated by the method of kernels. To express the unknown function in terms of the distribution of the observable variables, we need the following assumptions [24].

Assumption 1 *ν_t is independent of μ_t and x_{t-1}, and $\nu_t \sim \mathbb{N}(0,1)$.*

Assumption 2 *For all values of x_{t-1} and μ_t, the function h is strictly increasing in ν_t.*

Assumption 1 guarantees that the distribution of ν_t is the same for all values of x_{t-1} and μ_t. Assumption 2 guarantees that the distribution of ν_t can be obtained from the conditional distribution of y_t given x_{t-1} and μ_t.

Theorem 3 *Under Assumptions* 1 *and* 2, *the mapping between the unknown regression function h and $F_{\nu_t|x_{t-1},\mu_t}$, the distribution of the observable variables $F_{y_t|x_{t-1},\mu_t}$ is given by*

$$F_{\nu_t|x_{t-1},\mu_t}(\nu) = F_{y_t|x_{t-1},\mu_t}\left(h\left(x_{t-1}, \mu_t, \nu\right)\right), \tag{5}$$

for all $\nu \in \mathbb{E}$ with $f\left(x_{t-1}, \mu_t\right) > 0$.

Proof.

$$F_{\nu_t|x_{t-1},\mu_t}(\nu) = \mathbb{P}r\left(\nu_t \le \nu \middle| x_{t-1}, \mu_t\right) \tag{6}$$

$$= \mathbb{P}r\left(h\left(x_{t-1}, \mu_t, \nu_t\right) \le h\left(x_{t-1}, \mu_t, \nu\right) \middle| x_{t-1}, \mu_t\right) \tag{7}$$

$$= \mathbb{P}r\left(y_t \le h\left(x_{t-1}, \mu_t, \nu\right) \middle| x_{t-1}, \mu_t\right) \tag{8}$$

$$= F_{y_t|x_{t-1},\mu_t}\left(h\left(x_{t-1}, \mu_t, \nu\right)\right). \tag{9}$$

According to the theorem above, the following four cases hold. □

Lemma 4 (*Case* 1) *For all $\nu \in \mathbb{E}$ and some $\bar{x}_{t-1}, \bar{\mu}_t$ with $f\left(\bar{x}_{t-1}, \bar{\mu}_t\right) > 0$,*

$$h\left(\bar{x}_{t-1}, \bar{\mu}_t, \nu\right) = \nu, \tag{10}$$

and Assumptions 1 *and* 2 *hold. Then*

$$F_{\nu_t}(\nu) = F_{y_t|\bar{x}_{t-1},\bar{\mu}_t}(\nu), \tag{11}$$

$$h\left(x_{t-1}, \mu_t, \nu\right) = F^{-1}_{y_t|x_{t-1},\mu_t} F_{y_t|\bar{x}_{t-1},\bar{\mu}_t}(\nu). \tag{12}$$

Lemma 5 (*Case* 2) *For all $\nu \in \mathbb{E}$ and some $\bar{x}_{t-1}, \bar{\mu}_t$ with $f\left(\bar{x}_{t-1}, \bar{\mu}_t\right) > 0$, and $\lambda \in \mathbb{R}$ such that $\lambda\nu \in \mathbb{E}$ and $f\left(\lambda\bar{x}_{t-1}, \lambda\bar{\mu}_t\right) > 0$,*

$$h\left(\bar{x}_{t-1}, \bar{\mu}_t, \bar{\nu}\right) = \alpha, \tag{13}$$

$$h\left(\lambda\bar{x}_{t-1}, \lambda\bar{\mu}_t, \lambda\bar{\nu}\right) = \lambda\alpha, \tag{14}$$

and Assumptions 1 *and* 2 *hold. Then*

$$F_{\nu_t}(\nu) = F_{y_t\left|\frac{\nu}{\bar{\nu}}\bar{x}_{t-1},\frac{\nu}{\bar{\nu}}\bar{\mu}_t\right.}\left(\frac{\nu}{\bar{\nu}}\alpha\right), \tag{15}$$

$$h(x_{t-1},\mu_t,\nu) = F^{-1}_{y_t|x_{t-1},\mu_t}\left(F_{y_t\left|\frac{\nu}{\bar{\nu}}\bar{x}_{t-1},\frac{\nu}{\bar{\nu}}\bar{\mu}_t\right.}\left(\frac{\nu}{\bar{\nu}}\alpha\right)\right). \tag{16}$$

Lemma 6 (*Case* 3) *For some unknown function* $s(.)$, *all* $\nu\in\mathbb{E}$ *and some* $\alpha\in\mathbb{R}$, *some* $\bar{y}\in\mathbb{R}$, *and some* $\bar{x}_{t-1}$ *such that* $f(\bar{x}_{t-1},\mu_t)>0$, *and*

$$h(x_{t-1},\mu_t,\nu) = s(x_{t-1},\nu-\mu_t), \tag{17}$$

$$s(\bar{x}_{t-1},\alpha) = \bar{y}. \tag{18}$$

Assumptions 1 *and* 2 *hold, and for all* x_{t-1}, $s(x_{t-1},.)$ *is strictly increasing. Then, for* $f(\bar{x}_{t-1},\nu-\alpha)>0$,

$$F_{\nu_t}(\nu) = F_{y_t|\bar{x}_{t-1},\nu-\alpha}(\bar{y}), \tag{19}$$

$$s(x_{t-1},\nu-\mu_t) = F^{-1}_{y_t|x_{t-1},\mu_t}\left(F_{y_t|\bar{x}_{t-1},\nu-\alpha}(\bar{y})\right). \tag{20}$$

Lemma 7 (*Case* 4) *For some unknown function* $s(.)$, *all* $\nu\in\mathbb{E}$ *and some* $\alpha\in\mathbb{R}$, *some* $\bar{y}\in\mathbb{R}$, *and some* $\bar{\mu}_t$ *such that* $f(x_{t-1},\bar{\mu}_t)>0$, *and*

$$h(x_{t-1},\mu_t,\nu_t) = s(\mu_t,\nu-x_{t-1}), \tag{21}$$

$$s(\bar{\mu}_t,\alpha) = \bar{y}. \tag{22}$$

Assumptions 1 *and* 2 *hold, and for all* x_{t-1}, $s(\mu_t,.)$ *is strictly increasing. Then, for* $f(\bar{\mu}_t,\nu-\alpha)>0$,

$$F_{\nu_t}(\nu) = F_{y_t|\bar{\mu},\nu-\alpha}(\bar{y}), \tag{23}$$

$$s(\mu_t,\nu-x_{t-1}) = F^{-1}_{y_t|x_{t-1},\mu_t}\left(F_{y_t|\bar{\mu}_t,\nu-\alpha}(\bar{y})\right). \tag{24}$$

Let $(y_t,x_{t-1},\mu_t)_{t=1}^{\infty}$ denote the data, $f(y_t,x_{t-1},\mu_t)$ and $F(y_t,x_{t-1},\mu_t)$, respectively, the joint probability distribution function and cumulative distribution function of (y_t,x_{t-1},μ_t), $\hat{f}(y_t,x_{t-1},\mu_t)$ and $\hat{F}(y_t,x_{t-1},\mu_t)$, respectively, their kernel estimators, and $\hat{f}_{y_t|x_{t-1},\mu_t}(y)$ and $\hat{F}_{y_t|x_{t-1},\mu_t}(y)$ the kernel estimators of the conditional probability distribution function and cumulative distribution function of y given x_{t-1} and μ_t. Then, according to [6], for all $(y_t,x_{t-1},\mu_t)\in\mathbb{R}^3$,

$$\hat{f}_N(y,x,\mu) = \frac{1}{Nh_N^3}\sum_{t=1}^{N}K\left(\frac{y-y_t}{h_N},\frac{x-x_t}{h_N},\frac{\mu-\mu_t}{h_N}\right), \tag{25}$$

$$\hat{F}(y,x,\mu) = \int_{-\infty}^{y}\mathrm{d}s\int_{-\infty}^{x}\mathrm{d}t\int_{-\infty}^{\mu}\mathrm{d}z\hat{f}_N(s,t,z), \tag{26}$$

$$\hat{f}_{y|x,\mu}(y) = \frac{\hat{f}_N(y,x,\mu)}{\int_{-\infty}^{x}\mathrm{d}t\int_{-\infty}^{\mu}\mathrm{d}z\hat{f}_N(s,t,z)}, \tag{27}$$

$$\hat{F}_{y|x,\mu}(y) = \frac{\int_{-\infty}^{y}\mathrm{d}s\hat{f}_N(s,x,\mu)}{\int_{-\infty}^{x}\mathrm{d}t\int_{-\infty}^{\mu}\mathrm{d}z\hat{f}_N(s,t,z)}, \tag{28}$$

where $K:\mathbb{R}^3\to\mathbb{R}$ is a kernel function and h_N is the bandwidth. Hence, for case 1,

$$\hat{h}(x_{t-1},\mu_t,\nu) = \hat{F}^{-1}_{y_t|x_{t-1},\mu_t}\hat{F}_{y_t|\bar{x}_{t-1},\bar{\mu}_t}(\nu); \tag{29}$$

for case 2,

$$\hat{h}(x_{t-1},\mu_t,\nu) = \hat{F}^{-1}_{y_t|x_{t-1},\mu_t}\left(\hat{F}_{y_t\left|\frac{\nu}{\bar{\nu}}\bar{x}_{t-1},\frac{\nu}{\bar{\nu}}\bar{\mu}_t\right.}\left(\frac{\nu}{\bar{\nu}}\alpha\right)\right); \tag{30}$$

for case 3,

$$\hat{s}(x_{t-1},\nu-\mu_t) = \hat{F}^{-1}_{y_t|x_{t-1},\mu_t}\left(\hat{F}_{y_t|\bar{x}_{t-1},\nu-\alpha}(\bar{y})\right); \tag{31}$$

for case 4,

$$\hat{s}(\mu_t,\nu-x_{t-1}) = \hat{F}^{-1}_{y_t|x_{t-1},\mu_t}\left(\hat{F}_{y_t|\bar{\mu}_t,\nu-\alpha}(\bar{y})\right). \tag{32}$$

4. Consistency

The consistency and asymptotic normality of the estimator of the marginal or conditional distribution of μ will follow from the consistency and asymptotic normality of the kernel estimator for the conditional distribution of y given x and μ. In particular, the asymptotic properties for each of the estimators for the distribution of ν given above can be derived from Theorem 13 after substituting the corresponding values of y, x, and μ. For this result, we need following assumptions.

Assumption 8 *The sequence* (y_t,x_t,μ_t) *is independently identically distributed.*

Assumption 9 $f(y_t,x_t,\mu_t)$ *has compact support* $\Theta\subset\mathbb{R}^3$ *and it is continuously differentiable on* $\mathbb{R}^3$ *up to the order* s' *for some* $s'>0$.

Assumption 10 *The kernel function* $K(.,.,.)$ *is differentiable of order* $\tilde{s}$, *the derivatives of K of order* $\tilde{s}$ *are Lipschitz,* $K(.,.,.)$ *vanishes outside a compact set, integrates to* 1, *and is of order* s'' *where* $\tilde{s}+s''\le s'$.

Assumption 11 *As* $N\to\infty$ *and* $h_N\to 0$, $\frac{\ln N}{Nh_N^3}\to 0$, $\sqrt{N}h_N^{1/2}\to\infty$, $\sqrt{N}h_N^{1/2+s''}\to 0$, *and* $\sqrt{N}h_N^{1/2}\left(\sqrt{\frac{\ln N}{Nh_N^3}}+h_N^{s''}\right)^2\to 0$.

Assumption 12 $0<f(x,\mu)<\infty$.

Assumptions 8, 9, 10, 11 and 12 for (y_t,x_t,μ_t) are similar to Assumptions $C.1-C.5$ in [24] for (y_t,x_t).

Theorem 13 *Let* $\hat{F}_{Y|x,\mu}(y)$ *denote the kernel estimator for the conditional distribution of Y conditional on x and* μ *evaluated at* $Y=y$. *Assumptions* 8, 9, 10, 11 *and* 12 *hold. Then, for* $\tilde{s}>0$ *and* $s''>2$,

$$\sup_{y\in\mathbb{R}}\left|\hat{F}_{Y|x,\mu}(y)-F_{Y|x,\mu}(y)\right|\to 0 \tag{33}$$

in probability, and

$$\sqrt{N}h_N^{1/2}\left(\hat{F}_{Y|x,\mu}(y)-F_{Y|x,\mu}(y)\right)\to\mathbb{N}(0,V_F) \tag{34}$$

in distribution, where

$$V_F=\frac{\left[\int\mathrm{d}s\left(\int\int\mathrm{d}t\mathrm{d}zK(s,t,z)\right)^2\right]\left[F_{Y|x,\mu}(y)\left(1-F_{Y|x,\mu}(y)\right)\right]}{f(x)}. \tag{35}$$

Proof. It is the case for $d=1$ in the Theorem 1 in [24] in their notations when X_0 is not an argument. □

Theorem 13 states that $\hat{F}_\epsilon$ converges to F_ϵ in the supremum norm, and $\hat{F}_\epsilon$ is asymptotically normal with mean F_ϵ and variance equal to

$$\frac{\left[\int\mathrm{d}s\left(\int\int\mathrm{d}t\mathrm{d}zK(s,t,z)\right)^2\right]\left[F_\epsilon(e)\left(1-F_\epsilon(e)\right)\right]}{Nf(x,\mu)\sigma^2}.$$

To study the asymptotic properties of the estimator for the unknown function *h*, notice that Equation (3), the estimator for the unknown regression function *h* can be obtained by substituting the true conditional distributions of *Y* by their kernel estimators, the consistency and asymptotic normality of the estimator of *h* will follow from the consistency and asymptotic normality of the functional, $\hat{F}_{y|x,\mu}^{-1}\left(\hat{F}_{y|\tilde{x},\tilde{\mu}}(\tilde{v})\right)$, of the kernel estimator for the distribution of (Y,X,μ). For this result, one more assumption is required as follows.

Assumption 14 *The vectors* (X,μ) *and* $(\tilde{X},\tilde{\mu})$ *have at least one coordinate in common, and the values* (x,μ) *and* $(\tilde{x},\tilde{\mu})$ *are different at one such coordinate*; $0<f(x,\mu)$, $f(\tilde{x},\tilde{\mu})<\infty$ *; and there exist* $\delta,\xi>0$ *such that* $s\in N\left(h(x,\mu,v),\xi\right)$, $f(s,x,\mu)>\delta$.

Assumption 14 is the Assumption $C.5'$ if $W\equiv(X,\mu)$ in their notations.

Theorem 15 *Assumptions* 8, 9, 10, 11 *and* 14 *hold for* $s''>2$ *and* $s'>s''$. *Let* $\hat{h}(x,\mu,v)\equiv\hat{F}_{y|x,\mu}^{-1}\left(\hat{F}_{y|\tilde{x},\tilde{\mu}}(\tilde{v})\right)$, $h(x,\mu,v)\equiv F_{y|x,\mu}^{-1}\left(F_{y|\tilde{x},\tilde{\mu}}(\tilde{v})\right)$. *Then,*

$$\hat{h}(x,\mu)\to h(x,\mu) \tag{36}$$

in probability, and

$$\sqrt{N}h_N^{1/2}\left(\hat{h}(x,\mu)-h(x,\mu)\right)\to\mathbb{N}(0,V_n) \tag{37}$$

in distribution, where

$$V_n=\left[\int\mathrm{d}s\left(\int\int\mathrm{d}t\mathrm{d}zK(s,t,z)\right)^2\right]\cdot\frac{F_{Y|\tilde{x},\tilde{\mu}}(\tilde{v})\left(1-F_{Y|\tilde{x},\tilde{\mu}}(\tilde{v})\right)}{f_{Y|x,\mu}\left(h(x,\mu)\right)^2}\left[\frac{1}{f(\tilde{x},\tilde{\mu})}+\frac{1}{f(x,\mu)}\right]. \tag{38}$$

Proof. It is the case for $d_1=d_2=1$ of the Theorem 2 in [24] in their notations when X_0 is not an argument. □

Theorem 15 implies that $\hat{h}(x,\mu)$ is consistent and asymptotically normal with mean $h(x,\mu)$ and asymptotic variance equal to

$$\left[\int\mathrm{d}s\left(\int\int\mathrm{d}t\mathrm{d}zK(s,t,z)\right)^2\right]\cdot\frac{F_\epsilon(e)\left(1-F_\epsilon(e)\right)}{Nf_{Y|x,\mu}\left(\sigma h(x,\mu)\right)^2}\left[\frac{1}{f(\tilde{x},\tilde{\mu})}+\frac{1}{f(x,\mu)}\right]. \tag{39}$$

5. Conclusions

This paper studied a predictive regression model which includes the state variable of NI(1) or I(1) and allows endogeneity, where nonlinear regression function is not necessarily additive in unobservable random terms.

We develop a nonparametric method for estimating the functional regression and find that the estimators for the distribution of the unobservable random terms and the nonparametric function are consistent and asymptotically normal. The estimators are nonlinear functionals of a kernel estimator for the distribution of the observable variables. However, the model specification or stationary is not discussed here.

More investigations are worth for the predictive application of this functional regression model due to its importance in various applications in economics and finance. For example, we here keep silent of mixing of ν_t and μ_t in the context of nonparametric functional predication, though a time-varying coefficient model is valid in [22].

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