

The Giants Arcs as Modeled by the Superbubbles

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How to cite this paper: Zaninetti, L. (2019) The Giants Arcs as Modeled by the Superbubbles. *Journal of High Energy Physics, Gravitation and Cosmology*, 5, 442-463.

<https://doi.org/10.4236/jhepgc.2019.52026>

Received: February 23, 2019

Accepted: April 5, 2019

Published: April 8, 2019

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Abstract

The giant arcs in the clusters of galaxies are modeled in the framework of the superbubbles. The density of the intracluster medium is assumed to follow a hyperbolic behavior. The analytical law of motion is function of the elapsed time and the polar angle. As a consequence the flux of kinetic energy in the expanding thin layer decreases with increasing polar angle making the giant arc invisible to the astronomical observations. In order to calibrate the arc-sec-parsec conversion three cosmologies are analyzed.

Keywords

Galaxy Groups, Clusters and Superclusters, Large Scale Structure of the Universe Cosmology

1. Introduction

The giant arcs in the cluster of galaxies start to be observed as narrow-like shape by [1] [2] [3] and the first theoretical explanation was the gravitational lensing, see [4] [5] [6] [7]. The determination of the statistical parameters of the giant arcs has been analyzed in order to derive the cosmological parameters, see [8], in connection with the Λ CDM cosmology, see [9], in the framework of the triaxiality and substructure of CDM halo, see [10], in connection with the Wilkinson Microwave Anisotropy Probe (WMAP) data, see [11], including the effects of baryon cooling in dark matter N-body simulations, see [12] and in order to derive the photometric properties of 105 giant arcs in the Second Red-Sequence Cluster Survey (RCS-2), see [13]. The gravitational lensing is the most common theoretical explanation; we select some approaches among others: [14] evaluated the mass distribution inside distant clusters; [15] studied the statistics of giant arcs in flat cosmologies with and without a cosmological constant; [16] analyzed how

the gravitational lensing influences the surface brightness of giant luminous arcs and [17] used the warm dark matter (WDM) cosmologies to explain the lensing in galaxy clusters. Another theoretical line of research explains the giant arcs as shells originated by the galaxies in the cluster: [18] [19] analyzed the limb-brightened shell model, the gravitational lens model and the echo model; [20] suggested that the Gamma-ray burst (GRB) explosions are the sources of the shells with sizes of many kpc, and [21] suggested a connection between the Einstein ring associated to SDP.81 and the evolution of a superbubble (SB) in the intracluster medium.

This paper analyzes in Section 2 three cosmologies in order to calibrate the transversal distance which allows to convert the arcsec in pc . Section 3 is devoted to the evolution of a SB in the intracluster medium. Section 4 reports the observations of the giant arcs and the first phase of a SB. Section 5 reports the various steps which allow to reproduce the shape of the giant arc A2267 and the multiple arcs visible in the cluster of galaxies. Section 6 is dedicated to theory of the image: analytical formulae explain the hole in the central part of the SBs and numerical results reproduce the details of a giant arc.

2. Adopted Cosmologies

In the following we review three cosmological theories.

2.1. Λ CDM Cosmology

The basic parameters of Λ CDM cosmology are: the Hubble constant, H_0 , expressed in $\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, the velocity of light, c , expressed in kms^{-1} , and the three numbers Ω_M , Ω_K , and Ω_Λ , see [22] for more details. In the case of the Union 2.1 compilation, see [23], the parameters are $H_0 = 69.81 \text{ km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, $\Omega_M = 0.239$ and $\Omega_\Lambda = 0.651$. To have the luminosity distance, $D_L(z; H_0, c, \Omega_M, \Omega_\Lambda)$, as a function of the redshift only, we apply the minimax rational approximation, which is characterized by two parameters, p and q . The luminosity distance, $D_{L,3,2}$, when $p = 3$ and $q = 2$

$$D_{L,3,2} = \frac{-7.761 - 1788.53z - 3203.06z^2 - 65.8463z^3}{-0.438025 - 0.334872z + 0.0203996z^2} \text{Mpc} \quad \text{for } 0.001 < z < 4. \quad (1)$$

The transversal distance in Λ CDM cosmology, $D_{T,3,2}$, which corresponds to the angle δ expressed in arcsec is

$$D_{T,3,2} = \frac{4.84813\delta(2.328 + 502.067z + 113.03z^2)}{0.124085 + 0.149501z + 0.0932928z^2} \text{pc} \quad (2)$$

2.2. Flat Cosmology

The two parameters of the flat cosmology are H_0 , the Hubble constant expressed in $\text{km}\cdot\text{s}^{-1}\cdot\text{Mpc}^{-1}$, and Ω_M which is

$$\Omega_M = \frac{8\pi G\rho_0}{3H_0^2}, \quad (3)$$

where G is the Newtonian gravitational constant and ρ_0 is the mass density at the present time. In the case of $m = 2$ and $n = 2$ the minimax rational expression for the luminosity distance, $d_{L,m,2,2}$, when $H_0 = 70 \text{ km} \cdot \text{s}^{-1} \cdot \text{Mpc}^{-1}$ and $\Omega_M = 0.277$, is

$$d_{L,m,2,2} = \frac{0.0889 + 748.555z + 5.58311z^2}{0.175804 + 0.206041z + 0.068685z^2} \text{Mpc} \quad (4)$$

The transversal distance in flat cosmology, $D_{Tf,3,2}$, which corresponds to the angle δ expressed in arcsec is

$$D_{Tf,3,2} = \frac{4.84813\delta(0.0889 + 748.555z + 5.58311z^2)}{0.175804 + 0.206041z + 0.068685z^2} \text{pc} \quad (5)$$

2.3. Modified Tired Light

In an Euclidean static framework the modified tired light (MTL) has been introduced in Section 2.2 in [24]. The distance in MTL is

$$d = \frac{c}{H_0} \ln(1+z). \quad (6)$$

The distance modulus in the modified tired light (MTL) is

$$m - M = \frac{5}{2} \frac{\beta \ln(z+1)}{\ln(10)} + 5 \frac{1}{\ln(10)} \ln \left(\frac{\ln(z+1)c}{H_0} \right) + 25 \quad (7)$$

Here β is a parameter comprised between 1 and 3 which allows to match theory with observations. The number of free parameters in MTL is two: H_0 and β . The fit of the distance modulus with the data of Union 2.1 compilation gives $\beta = 2.37$, $H_0 = 69.32 \pm 0.34$, see [22], which means the following distance

$$d = 4324.761 \ln(1+z)(1+z)^{1.185} \text{Mpc}. \quad (8)$$

The transversal distance in MTL, d_T , which corresponds to the angle δ expressed in arcsec is

$$d_T = 20967\delta \ln(1+z)(1+z)^{1.185} \text{pc} \quad (9)$$

We report the angular distance for a fixed δ as function of redshift for the three cosmologies, see **Figure 1**. The angular distance in flat and Λ CDM cosmology does not increase with z , see [25], in contrast with the modified tired light.

3. The Motion of a SB

We now summarize the adopted profile of density and the equation of motion for a SB.

3.1. The Profile

The density is assumed to have the following hyperbolic dependence on Z which is the third Cartesian coordinate,

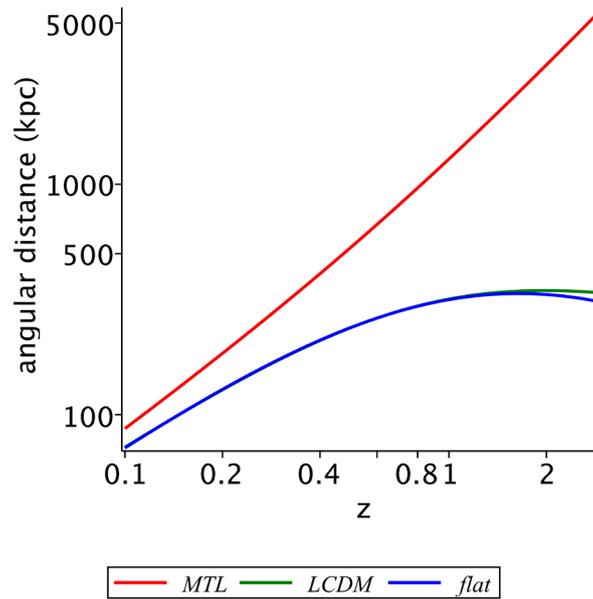


Figure 1. Angular distances in kpc for the three cosmologies here considered when $\delta = 38.913456$.

$$\rho(Z; Z_0, \rho_0) = \begin{cases} \rho_0 & \text{if } z \leq Z_0 \\ \rho_0 \frac{Z_0}{z} & \text{if } z > Z_0 \end{cases} \quad (10)$$

where the parameter Z_0 fixes the scale and ρ_0 is the density at $Z = Z_0$. In spherical coordinates the dependence on the polar angle is

$$\rho(r; \theta, Z_0, \rho_0) = \begin{cases} \rho_0 & \text{if } \cos(\theta) \leq Z_0 \\ \rho_0 \frac{Z_0}{r \cos(\theta)} & \text{if } r \cos(\theta) > Z_0 \end{cases} \quad (11)$$

Given a solid angle $\Delta\Omega$ the mass M_0 swept in the interval $[0, r_0]$ is

$$M_0 = \frac{1}{3} \rho_0 r_0^3 \Delta\Omega. \quad (12)$$

The total mass swept, $M(r; r_0, Z_0, \alpha, \theta, \rho_0)$, in the interval $[0, r]$ is

$$M(r; r_0, Z_0, \alpha, \theta, \rho_0) = \left(\frac{1}{3} \rho_0 r_0^3 + \frac{1}{2} \frac{\rho_0 Z_0 (r^2 - r_0^2)}{\cos(\theta)} \right) \Delta\Omega. \quad (13)$$

and its approximate value at high values of r is

$$M(r; Z_0, \alpha, \theta, \rho_0) \approx \frac{1}{2} \frac{r^2 \rho_0 Z_0}{\cos(\theta)} \Delta\Omega. \quad (14)$$

The density ρ_0 can be obtained by introducing the number density, n_0 , expressed in particles cm^{-3} , the mass of hydrogen, m_H , and a multiplicative factor f , which is chosen to be 1.4, see [26],

$$\rho_0 = f m_H n_0. \quad (15)$$

The astrophysical version of the total approximate swept mass as given by Equa-

tion (14), expressed in solar mass units, M_{\odot} , is

$$M(r_{pc}; Z_{0,pc}, n_0, \theta) \approx \frac{0.0172 n_0 z_{0,pc} r_{pc}^2}{\cos(\theta)} M_{\odot} \Delta\Omega, \tag{16}$$

where $Z_{0,pc}$, and $r_{0,pc}$ are Z_0 , and r expressed in pc .

3.2. The Equation of Motion

The conservation of the classical momentum in spherical coordinates along the solid angle $\Delta\Omega$ in the framework of the thin layer approximation states that

$$M_0(r_0)v_0 = M(r)v, \tag{17}$$

where $M_0(r_0)$ and $M(r)$ are the swept masses at r_0 and r , and v_0 and v are the velocities of the thin layer at r_0 and r . This conservation law can be expressed as a differential equation of the first order by inserting $v = \frac{dr}{dt}$:

$$M(r) \frac{dr}{dt} - M_0 v_0 = 0. \tag{18}$$

The velocity as a function of the radius r is

$$v(r; r_0, Z_0, v_0, \theta) = 2 \frac{r_0^3 v_0 \cos(\theta)}{2r_0^3 \cos(\theta) - 3r_0^2 Z_0 + 3r^2 Z_0}. \tag{19}$$

The differential equation which models the momentum conservation in the case of a hyperbolic profile is

$$\left(\frac{1}{3} r_0^3 + \frac{1}{2} \frac{Z_0 (-r_0^2 + (r(t))^2)}{\cos(\theta)} \right) \frac{d}{dt} r(t) - \frac{1}{3} r_0^3 v_0 = 0, \tag{20}$$

where the initial conditions are $r = r_0$ and $v = v_0$ when $t = t_0$.

The variables can be separated and the radius as a function of the time is

$$r(t; t_0, r_0, Z_0, v_0, \theta) = \frac{HN}{HD}, \tag{21}$$

where

$$HN = -\sqrt[3]{3} \left(2 \cos(\theta) \sqrt[3]{3} r_0 - 3 \sqrt[3]{3} Z_0 - (-9Z_0^{3/2} + ((9t - 9t_0)v_0 + 9r_0) \cos(\theta) \sqrt{Z_0} + \sqrt{3} \sqrt{27} \sqrt{AHN})^{2/3} \right) r_0, \tag{22}$$

with

$$AHN = \left(\frac{8(\cos(\theta))^2 r_0^3}{27} + Z_0 \left((t - t_0)^2 v_0^2 + 2r_0(t - t_0)v_0 - \frac{1}{3} r_0^2 \right) \cos(\theta) - 2v_0 Z_0^2 (t - t_0) \right) \cos(\theta) \tag{23}$$

and

$$HD = 3\sqrt{Z_0} \times \sqrt[3]{-9Z_0^{3/2} + ((9t - 9t_0)v_0 + 9r_0) \cos(\theta) \sqrt{Z_0} + 9\sqrt{BHD}}, \tag{24}$$

with

$$BHD = \left(\frac{8(\cos(\theta))^2 r_0^3}{27} + Z_0 \left((t-t_0)^2 v_0^2 + 2r_0(t-t_0)v_0 - \frac{1}{3}r_0^2 \right) \cos(\theta) - 2v_0 Z_0^2 (t-t_0) \right) \cos(\theta). \quad (25)$$

As a consequence the velocity as function of the time is

$$v(t; t_0, r_0, Z_0, v_0, \theta) = \frac{dr(t; t_0, r_0, Z_0, v_0, \theta)}{dt}. \quad (26)$$

More details as well the exploration of other profiles of density can be found in [27]. We now continue evaluating the flux of kinetic energy, F_{ek} , in the thin emitting layer which is supposed to have density ρ_l

$$F_{ek}(t; t_0, r_0, Z_0, v_0, \theta) = \frac{1}{2} \rho_l 4\pi r(t)^2 v(t)^3. \quad (27)$$

The volume of the thin emitting layer, V_l , is approximated by

$$V_l = 4\Delta\pi r^2, \quad (28)$$

where Δ is thickness of the layer; as an example [28] quotes $\Delta = \frac{r}{12}$. The two approximations for mass, Equation (14), and volume, Equation (28), allows to derive an approximate value for the density in the thin layer

$$\rho_l = \frac{1}{8} \frac{\rho_0 Z_0 f}{\cos(\theta) r \pi}. \quad (29)$$

Inserted in Equation (27) the radius, velocity and density as given by Equation (21), Equation (26) and Equation (29), we obtain

$$F_{ek}(t; t_0, r_0, Z_0, v_0, \theta) = \frac{FN}{FD}, \quad (30)$$

where

$$\begin{aligned} FN &= -\sqrt{27} \left(-3\sqrt{3}Z_0^{3/2} + \sqrt{27}\sqrt{F_1 \cos(\theta)} + 3F_5 \right)^3 f \\ &\times \left(2\sqrt[3]{3}F_3 + \left(-9Z_0^{3/2} + F_2 + \sqrt{27}\sqrt{F_1 \cos(\theta)}\sqrt{3} \right)^{2/3} \right)^3 r_0^4 \\ &\times \cos(\theta) v_0^3 \sqrt{Z_0} \left(2\sqrt[3]{3}F_3 - \left(-9Z_0^{3/2} + F_2 + \sqrt{27}\sqrt{F_1 \cos(\theta)}\sqrt{3} \right)^{2/3} \right) \sqrt[3]{3}\rho_0 \end{aligned} \quad (31)$$

and

$$FD = 108\sqrt{F_1 \cos(\theta)} \times \left(-9Z_0^{3/2} + F_2 + \sqrt{27}\sqrt{F_1 \cos(\theta)}\sqrt{3} \right)^{13/3} F_4 \quad (32)$$

being

$$\begin{aligned} F_1 &= \frac{8(\cos(\theta))^2 r_0^3}{27} + \left((t-t_0)^2 v_0^2 + 2r_0(t-t_0)v_0 - \frac{1}{3}r_0^2 \right) Z_0 \cos(\theta) \\ &- 2v_0 Z_0^2 (t-t_0), \end{aligned} \quad (33)$$

$$F_2 = ((9t - 9t_0)v_0 + 9r_0)\cos(\theta)\sqrt{Z_0}, \quad (34)$$

$$F_3 = \cos(\theta)r_0 - 3/2Z_0 \quad (35)$$

$$F_4 = 8(\cos(\theta))^2 r_0^3 + 27\left((t-t_0)^2 v_0^2 + 2r_0(t-t_0)v_0 - \frac{1}{3}r_0^2\right)Z_0 \cos(\theta) - 54v_0Z_0^2(t-t_0), \quad (36)$$

$$F_5 = (v_0(t-t_0) + r_0)\cos(\theta)\sqrt{3}\sqrt{Z_0}. \quad (37)$$

We now assume that the amount of luminosity, L_{theo} , reversed in the shocked emission is proportional to the flux of kinetic energy as given by Equation (30)

$$L_{theo}(t; t_0, r_0, Z_0, v_0, \theta) \propto F_{ek}(t; t_0, r_0, Z_0, v_0, \theta). \quad (38)$$

The theoretical luminosity is not equal along all the SB but is function of the polar angle θ . In this framework is useful to introduce the ratio, κ , between theoretical luminosity at θ and that one at $\theta = 0$,

$$\kappa = \frac{L_{theo}(t; t_0, r_0, Z_0, v_0, \theta)}{L_{theo}(t; t_0, r_0, Z_0, v_0, \theta = 0)}. \quad (39)$$

The above model for the theoretical luminosity is independent from the image theory, see Section 6, and does not explain the hole of luminosity visible in the shells.

4. Astrophysical Environment

We now analyze the catalogue for the giant arcs, the two giant arcs SDP.81 and A2267 and the initial astrophysical conditions for the SBs.

4.1. The Catalogue

Some parameters of the giant arcs as detected as images by cluster lensing and the supernova survey with Hubble (CLASH) which is available as a catalogue at <http://vizier.u-strasbg.fr/viz-bin/VizieR>, see [29]. We are interested in the arc length which is given in arcsec, the arc length to width ratio, the photometric redshift, and in the radial distance from the arc center to the cluster center in arcsec. **Table 1** reports the statistical parameters of the radial distance from the arc center to the cluster center in kpc and **Figures 2-4** the histogram of the frequencies in the framework of flat, Λ CDM and MTL cosmology respectively.

4.2. Single Giant Arcs

The ring associated with the galaxy SDP.81, see [30], is characterized by a foreground galaxy at $z = 0.2999$ and a background galaxy at $z = 0.3042$. This ring has been studied with the Atacama Large Millimeter/sub-millimeter Array (ALMA) by [31]-[36] and has the observed parameters as in **Table 2**.

Another giant arc is that in A2667 which is made by three pieces: A, B and C, see **Figure 1** in [37]. The radius can be found from the equation of the circle

Table 1. Statistical parameters of the radial distance from the arc center to the cluster center in kpc.

Cosmology	minimum (kpc)	average (kpc)	maximum (Mpc)
Flat	89	270	313
Λ CDM	89	288	323
MTL	7	3810	1540

Table 2. Observed parameters of the giants arcs.

Name	redshift	radius arcsec
SDP.81	3.04	1.54
A2667	1.033	42

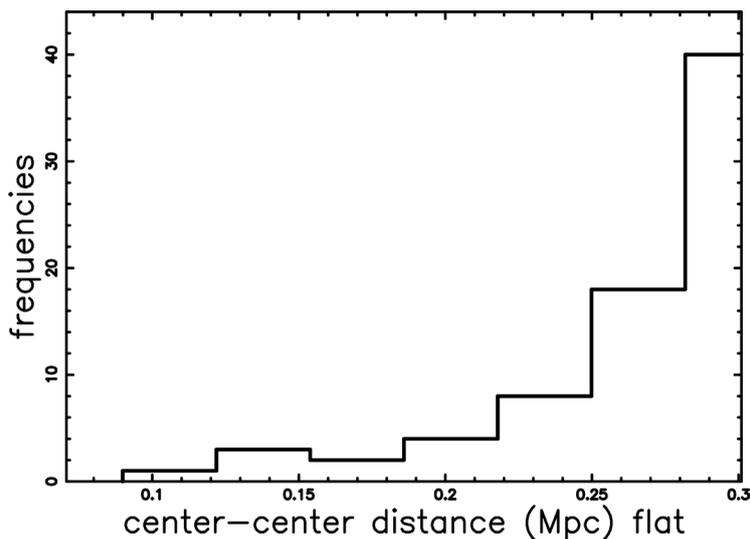


Figure 2. Histogram of the radial distance from the arc center to the cluster center in Mpc in flat cosmology with conversion formula (5).

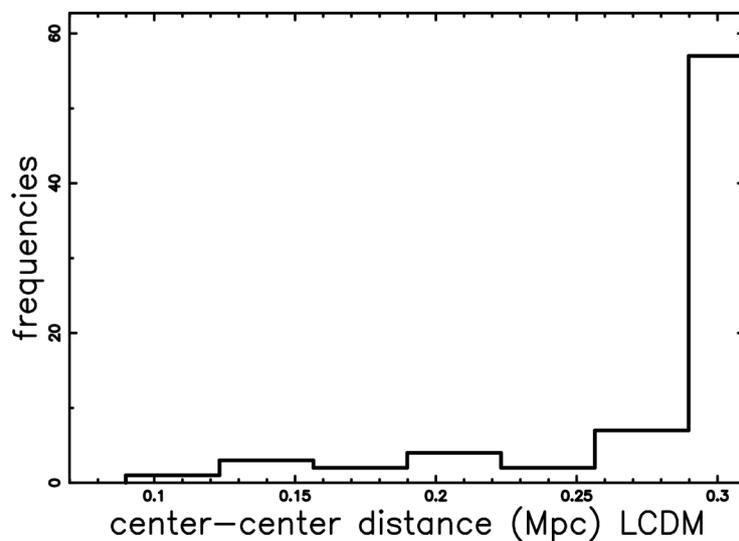


Figure 3. Histogram of the radial distance from the arc center to the cluster center in Mpc in Λ CDM cosmology with conversion formula (2).

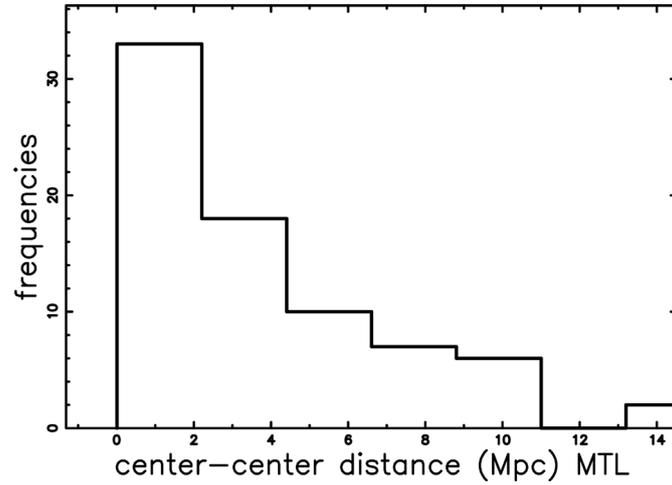


Figure 4. Histogram of the radial distance from the arc center to the cluster center in Mpc in MTL cosmology with conversion formula (9).

given the three points A, B and C, see **Table 3**. The three pieces can be digitalized for a further comparison with a simulation, see empty red stars in **Figure 8**.

4.3. The Initial Conditions

We review the starting equations for the evolution of the SB [38] [39] [40] which can be derived from the momentum conservation applied to a pyramidal section. The parameters of the thermal model are N^* , the number of SN explosions in 5.0×10^7 yr, Z_{OB} , the distance of the OB associations from the galactic plane, E_{51} , the energy in 10^{51} erg usually chosen equal to one, v_0 , the initial velocity which is fixed by the bursting phase, t_0 , the initial time in yr which is equal to the bursting time, and t the proper time of the SB. With the above definitions the radius of the SB is

$$R = 111.56 \left(\frac{E_{51} t_7^3 N^*}{n_0} \right)^{\frac{1}{5}} \text{ pc}, \quad (40)$$

and its velocity

$$V = 6.567 \frac{1}{t_7^{2/5}} \sqrt[5]{\frac{E_{51} N^*}{n_0}} \text{ km} \cdot \text{s}^{-1}. \quad (41)$$

In the following, we will assume that the bursting phase ends at $t = t_{7,0}$ (the bursting time is expressed in units of 10^7 yr) when N_{SN} SN are exploded

$$N_{SN} = N^* \frac{t_{7,0} \times 10^7}{5 \times 10^7}. \quad (42)$$

The two following inverted formula allows to derive the parameters of the initial conditions for the SB with ours r_0 expressed in pc and v_0 expressed in $\text{km} \cdot \text{s}^{-1}$ are

$$t_{7,0} = 0.05878095238 \frac{r_0}{v_0}, \quad (43)$$

Table 3. Radius of the giant arcs in kpc.

Cosmology	SDP.81	A2667
Flat	12.09	345.39
Λ CDM	13.33	347.42
MTL	235.82	1456.32

and

$$N^* = 2.8289 \times 10^{-7} \frac{r_0^2 n_0 v_0^3}{E_{51}}. \quad (44)$$

5. Astrophysical Simulation

We simulate a single giant arc, A2267, and then we simulate the statistics of many giant arcs.

5.1. Simulation of A2667

The final stage of the SB connected with A2267 is simulated with the parameters reported in **Table 4**; in particular **Figure 5** displays the 3D shape and **Figure 6** reports the 2D section.

Figure 7 reports the 2D section of the SB as well the three pieces of the giant arc connected with A2267. The similarity between the observed radius of curvature of the giant arc as well the theoretical one is reported in a zoom, see **Figure 8**.

We can understand the reason for which the giant arc A2267 has a limited angular extension of $\approx 31^\circ$ by plotting the ratio κ , equation (37), between the theoretical luminosity as function of θ and the theoretical luminosity at $\theta = 0$ with parameters as in **Table 4**, see **Figure 9**. As a practical example at $\approx 31^\circ/2$, where the factor two arises from the symmetry of the framework, the theoretical luminosity is decreased of a factor $\kappa = 0.987$ in respect to the value at $\theta = 0$. We now introduce the threshold luminosity, L_{tr} , which is an observational parameter. The theoretical luminosity will scale as function of the polar angle as $L_{theo}(\theta) \propto L_0 * r$ where L_0 is the theoretical luminosity at $\theta = 0$ and κ has been defined in Equation (39). When the inequality $L_{theo} < L_{tr}$ is verified the giant arc is impossible to detect and only the zone characterized by low values of the polar angle will be detected.

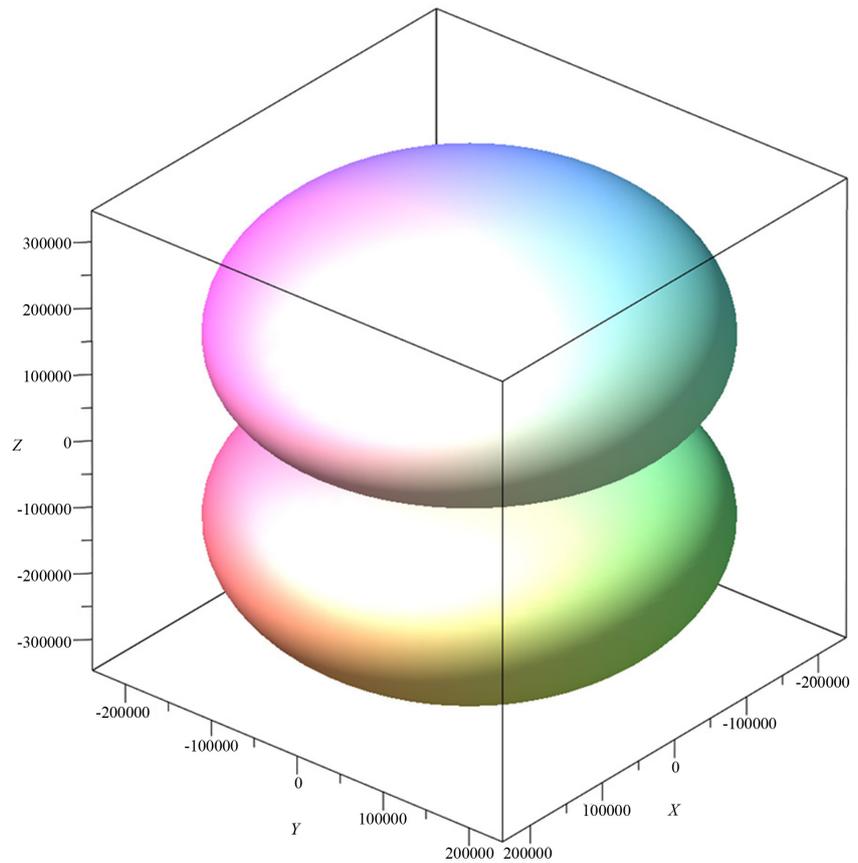
In our model the velocity with parameters as in **Table 4** is function of the polar angle, see **Figure 10**, and has range $37 \text{ km/s} < v(\theta) < 142 \text{ km/s}$. As a comparison a velocity $50 \text{ km/s} < v < 75 \text{ km/s}$ is measured in A2267, see **Figure 5** in [37].

5.2. Simulation of Many Giants Arcs

The presence of multiple giants arcs in the CLASH cluster, see as an example **Figure 11** in [29], can be simulated adopting the following steps:

Table 4. Theoretical parameters of the SB connected with A2267.

theory	parameter	value
initial thermal model	E_{s1}	1
initial thermal model	n_0	1
initial thermal model	$t_{7,0}$	0.0078
initial thermal model	N^*	1.22×10^{14}
initial thermal model	N_{SN}	1.91×10^{11}
SB	r_0	4000 pc
SB	Z_0	74.07 pc
SB	v_0	30,000 km/s
SB	t	8×10^8 yr

**Figure 5.** 3D surface of the SB connected with A2267, parameters as in **Table 4** and axes in *pc*.

- A given number of SBs, as an example 15, are generated with variable lifetime, t , see **Figure 11**.
- For each SB we select a section around polar angle equal to zero characterized by a fixed angle of $\approx 31^\circ$ and we randomly rotate it around the origin, see **Figure 12**.

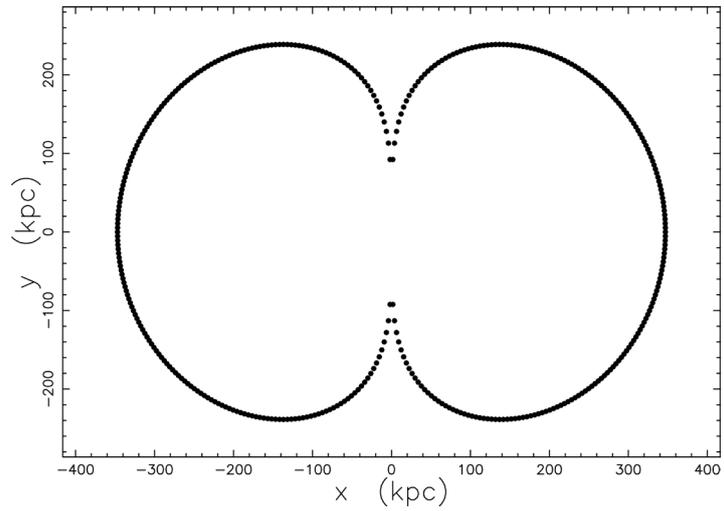


Figure 6. 2D section in the $z = 0$ plane of the SB connected with A2267, parameters as in Table 4 and axes in kpc.

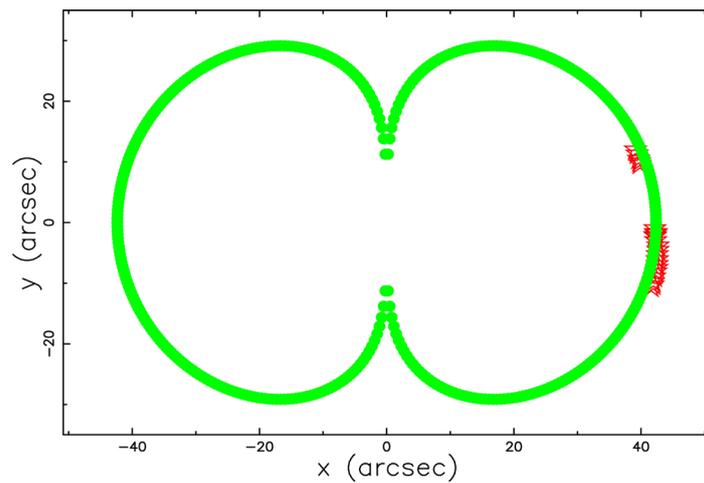


Figure 7. 2D section in the $z = 0$ plane of the SB connected with A2267, parameters as in Table 4 (full green points) and the three pieces of the giant arc in A2667 (empty red stars); axes in arcsec.

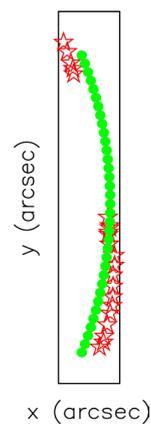


Figure 8. Enlarged view of the three pieces of the giant arc in A2667 (empty red stars) and the theoretical radius (full green points); axes in arcsec.

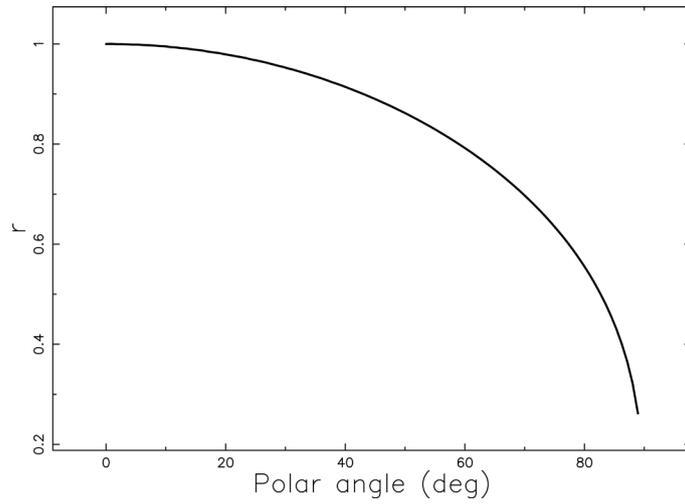


Figure 9. Normalized luminosity as function of the polar angle in deg, parameters as in Table 4.

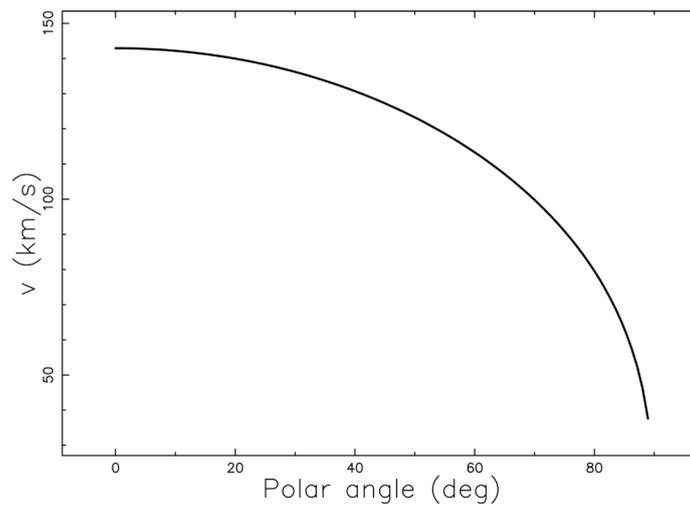


Figure 10. Velocity in km/s as function of the polar angle in deg.

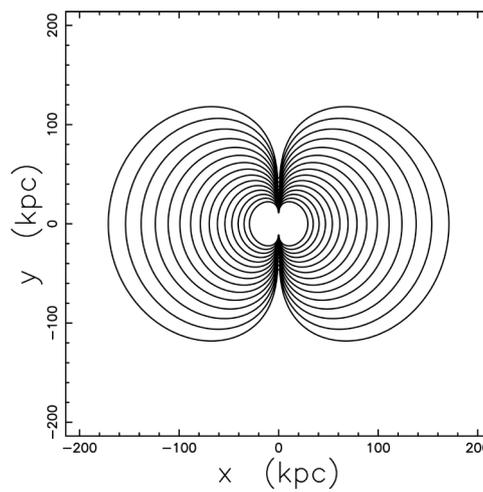


Figure 11. Multiple sections of the SB with time, t comprised in $[10^6 \text{ yr}, 10^8 \text{ yr}]$ and other parameters as in Table 4.

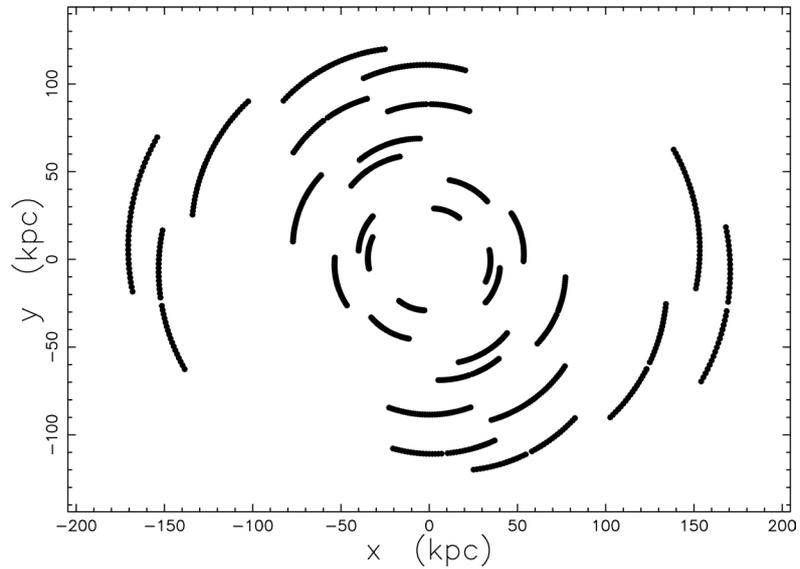


Figure 12. Multiple sections of the SB as in **Figure 11** with angular extension of the polar angle, θ , of $\approx 31^\circ$ and progressive rotation of the selected piece of section.

- The centers of the SBs are randomly placed in a squared box with side of 300 kpc, see **Figure 13**.

Table 5 reports the theoretical statistical parameters of the above simulation for the radial distance from the arc center to the cluster center in kpc. A comparison should be done with the astronomical parameters for the CLASH clusters of **Table 1**.

6. Theory of the Image

We now review the theory of the image for the case of optically thin medium both from an analytical and an analytical point of view.

6.1. The Elliptical Shell

A real ellipsoid represents a first approximation of the asymmetric giants arcs and has equation

$$\frac{z^2}{a^2} + \frac{x^2}{b^2} + \frac{y^2}{d^2} = 1, \quad (45)$$

in which the polar axis is the z -axis.

We are interested in the section of the ellipsoid $y = 0$ which is defined by the following external ellipse

$$\frac{z^2}{a^2} + \frac{x^2}{b^2} = 1. \quad (46)$$

We assume that the emission takes place in a thin layer comprised between the external ellipse and the internal ellipse defined by

$$\frac{z^2}{(a-c)^2} + \frac{x^2}{(b-c)^2} = 1, \quad (47)$$

Table 5. Statistical parameters of the radial distance from the theoretical arc center to the cluster center in kpc.

	50	165	349
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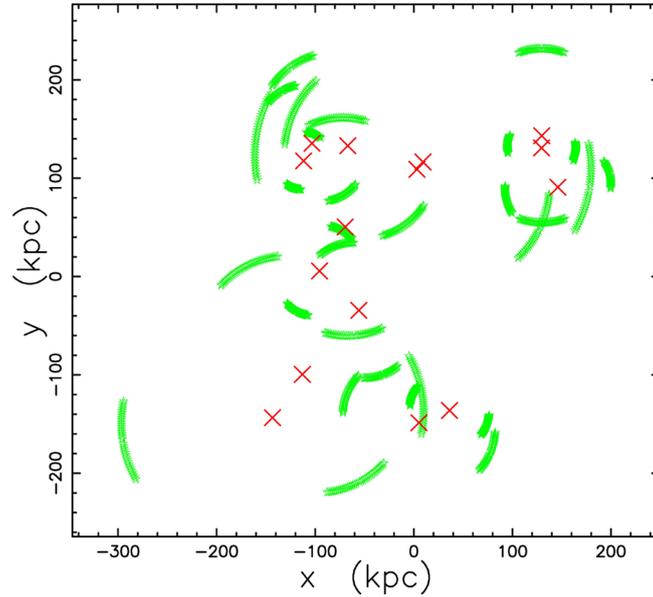


Figure 13. Multiple sections of SB as in **Figure 12** with random shift of the origin of the selected SB (green empty stars). The random shift denotes the galaxies (red crosses).

see **Figure 14**. We therefore assume that the number density C is constant and in particular rises from 0 at $(0, a)$ to a maximum value C_m , remains constant up to $(0, a-c)$ and then falls again to 0. The length of sight, when the observer is situated at the infinity of the x -axis, is the locus parallel to the x -axis which crosses the position z in a Cartesian x - z plane and terminates at the external ellipse. The locus length is

$$l_I = 2 \frac{\sqrt{a^2 - z^2} b}{a} \tag{48}$$

when $(a - c) \leq z < a$

$$l_{II} = 2 \frac{\sqrt{a^2 - z^2} b}{a} - 2 \frac{\sqrt{a^2 - 2ac + c^2 - z^2} (b - c)}{a - c} \tag{49}$$

when $0 \leq z < (a - c)$

In the case of optically thin medium, the intensity is split in two cases

$$I_I(z; a, b) = I_m \times 2 \frac{\sqrt{a^2 - z^2} b}{a} \tag{50}$$

when $(a - c) \leq z < a$

$$I_{II}(z; a, c) = I_m \times \left(2 \frac{\sqrt{a^2 - z^2} b}{a} - 2 \frac{\sqrt{a^2 - 2ac + c^2 - z^2} (b - c)}{a - c} \right) \tag{51}$$

when $0 \leq z < (a - c)$

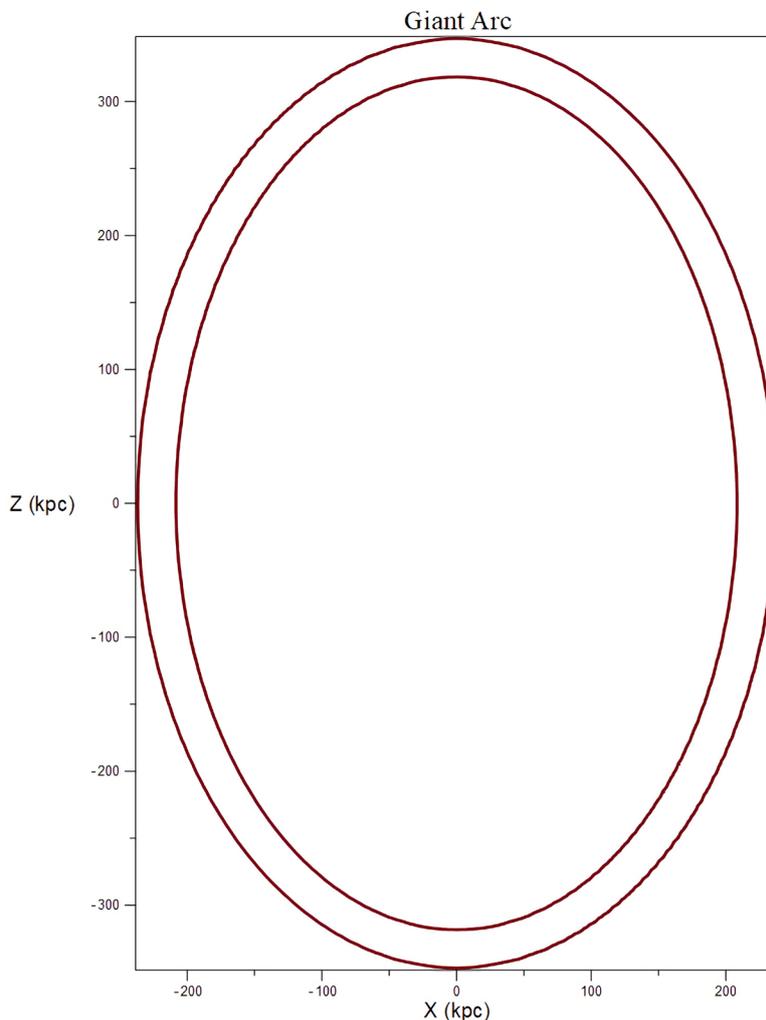


Figure 14. Internal and external ellipses when $a = 347$ kpc, $b = 237$ kpc and $c = \frac{a}{12}$ kpc .

where I_m is a constant which allows to compare the theoretical intensity with the observed one. A typical profile in intensity along the z -axis is reported in **Figure 15**. The ratio, κ , between the theoretical intensity at the maximum, ($z = a - c$), and at the minimum, ($z = 0$), is given by

$$\frac{I_I(z = a - c)}{I_{II}(z = 0)} = \kappa = \frac{\sqrt{2a - cb}}{\sqrt{ca}}. \tag{52}$$

As an example the values $a = 6$ kpc, $b = 4$ kpc, $c = \frac{a}{12}$ kpc gives $\kappa = 3.19$.

The knowledge of the above ratio from the observations allows to deduce c once a and b are given by the observed morphology

$$c = 2 \frac{ab^2}{a^2 r^2 + b^2}. \tag{53}$$

The above analytical model explains the hole in luminosity visible in the astrophysical shells such as supernovae and SBs. More details can be found in [41].

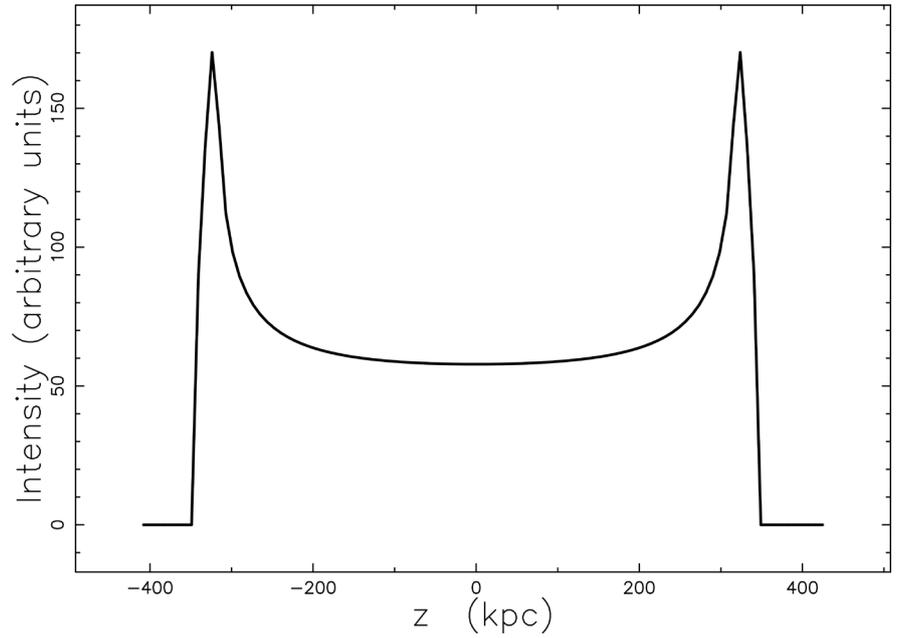


Figure 15. The intensity profile along the z -axis when $a = 347$ kpc, $b = 237$ kpc, $c = \frac{a}{12}$ kpc and $I_m = 1$.

6.2. The Numerical Shell

The source of luminosity is assumed here to be the flux of kinetic energy, L_m ,

$$L_m = \frac{1}{2} \rho A V^3, \quad (54)$$

where A is the considered area, V is the velocity and ρ is the density. In our case $A = r^2 \Delta\Omega$, where $\Delta\Omega$ is the considered solid angle and $r(\theta)$ the temporary radius along the chosen direction. The observed luminosity along a given direction can be expressed as

$$L = \epsilon L_m, \quad (55)$$

where ϵ is a constant of conversion from the mechanical luminosity to the observed luminosity.

We review the algorithm that allows to build the image, see [42]:

- An empty memory grid $\mathcal{M}(i, j, k)$ which contains $NDIM^3$ pixels is considered.
- We first generate an internal 3D surface of revolution by rotating the ideal image of 360° around the polar direction and a second external surface of revolution at a fixed distance ΔR from the first surface. As an example, we fixed $\Delta R = R/12$, where R is the momentary radius of expansion. The points on the memory grid which lie between the internal and external surfaces are memorized on $\mathcal{M}(i, j, k)$ by a variable integer number according to formula (52) and density ρ proportional to the swept mass.
- Each point of $\mathcal{M}(i, j, k)$ has spatial coordinates x, y, z which can be represented by the following 1×3 matrix, A ,

$$A = \begin{bmatrix} x \\ y \\ z \end{bmatrix}. \quad (56)$$

The orientation of the object is characterized by the Euler angles (Φ, Θ, Ψ) and therefore by a total 3×3 rotation matrix, E . The matrix point is represented by the following 1×3 matrix, B ,

$$B = E \cdot A. \quad (57)$$

- The intensity map is obtained by summing the points of the rotated images along a particular direction.

The image of A2267 built with the above algorithm is shown in **Figure 16**.

The threshold intensity, I_r , is

$$I_{\max} \kappa = I_r, \quad (58)$$

where I_{\max} , is the maximum value of intensity characterizing the ring and κ is a parameter which allows matching theory with observations and was previously defined in Equation (39). A typical image with a hole is visible in **Figure 17**. The opening angle of the visible arc can be parametrized as function of the ratio κ , see **Figure 18**. An opening of $\approx 31^\circ$ is reached at $\kappa \approx 0.95$.

7. Conclusions

7.1. The Equation of Motion

The giants arcs are connected with the visible part of the SBs which advance in the intracluster medium surrounding the host galaxies. The chosen profile of density is hyperbolic, see Equation (10), and the momentum conservation along

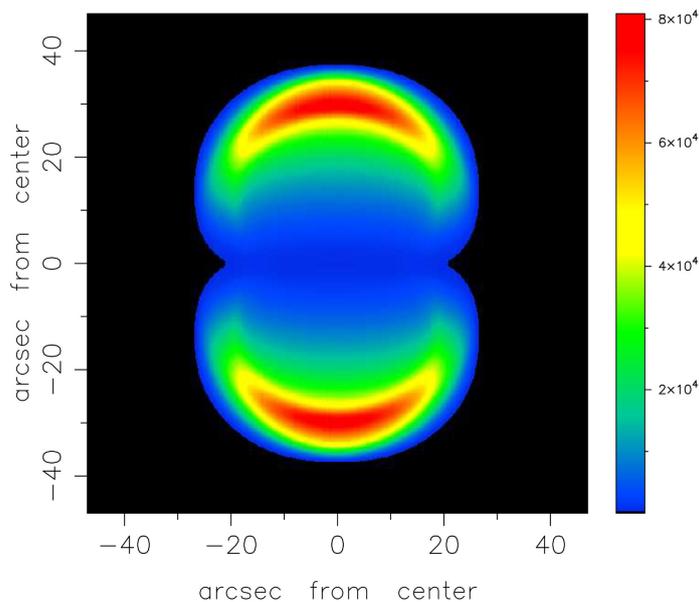


Figure 16. Contour map of I for A2267, the x and y axes are in arcsec. The three Euler angles characterizing the orientation are $\Phi = 0^\circ$, $\Theta = 90^\circ$ and $\Psi = 90^\circ$, and $NDIM = 400$.

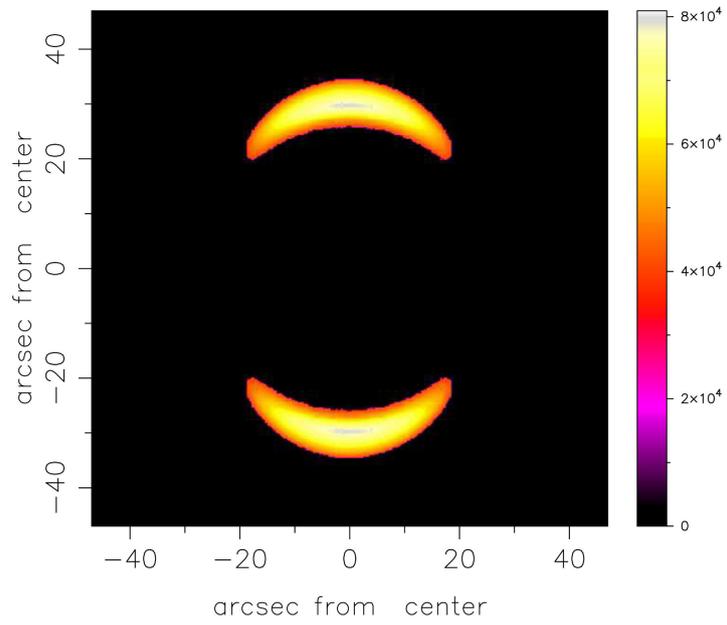


Figure 17. The same as **Figure 16** parameters as in **Figure 16** and $\kappa = 0.5$.

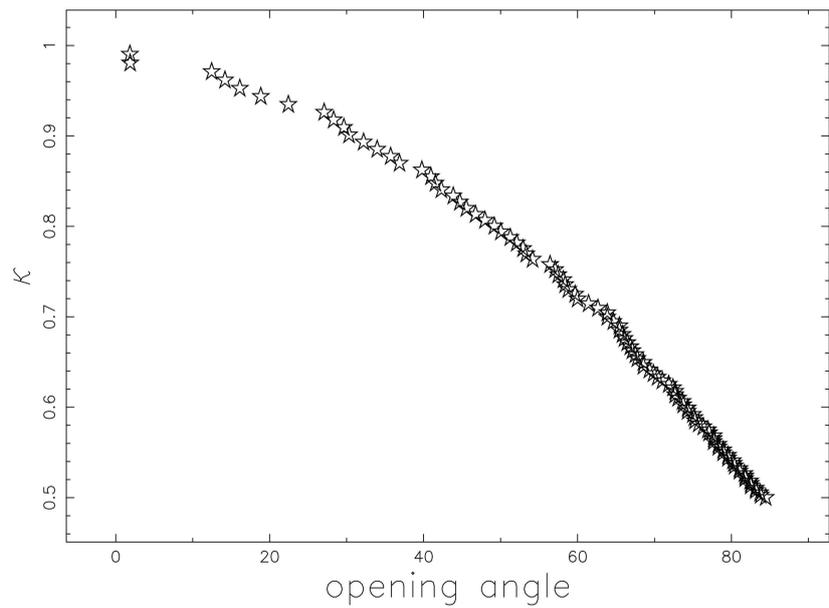


Figure 18. The opening angle of the visible simulated arc as function of κ ; parameters as in **Figure 16**.

a given direction allows to derive the equation of motion as function of the polar angle, see Equation (21).

7.2. The Image

According to the theory here presented the giants arcs are the visible part of an advancing SB. An analytical explanation for the limited angular extent of the giant arcs is represented by the theoretical luminosity as function of the polar angle, see Equation (39). An increase in the polar angle produces a decrease of

the theoretical luminosity and the arc becomes invisible. Selecting a given number of SBs with variable lifetime and randomly inserting them in a cubic box of side ≈ 600 kpc is possible to simulate the giants arcs visible in the clusters of galaxies, see **Figure 13** and relative statistical parameters in **Table 5**.

Acknowledgements

This research has made use of the VizieR catalogue access tool, CDS, Strasbourg, France.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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